Why does it work?
Alex was asked to solve $2 x^{2}+5 x+1=0$, and Morgan was asked to solve $a^{2}+b x+c=0$

Alex's "complete the square" way

First I divided both sides of the equation by the coefficient of $\mathbf{x}^{2}$. Then I subtracted the constant term from both sides.

I completed the square by adding $\left(\frac{b}{2 a}\right)^{2}$ to both sides of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side.

I simplified and


$$
2 x^{2}+5 x+1=0
$$

$$
x^{2}+\frac{5}{2} x+\frac{1}{2}=0
$$

$$
\begin{gathered}
\downarrow \\
x^{2}+\frac{5}{2} x=-\frac{1}{2}
\end{gathered}
$$

$$
x^{2}+\frac{5}{2} x+\left(\frac{5}{4}\right)^{2}=\stackrel{\stackrel{1}{2}+\left(\frac{5}{4}\right)^{2}}{\downarrow}
$$

$$
\left(x+\frac{5}{4}\right)^{2}=-\frac{1}{2}+\frac{25}{16}
$$

$$
\downarrow
$$

$$
\left(x+\frac{5}{4}\right)^{2}=\frac{-8}{16}+\frac{25}{16}
$$

$$
\left(x+\frac{5}{4}\right)^{2}=\frac{17}{16}
$$

$$
\sqrt{\left(x+\frac{5}{4}\right)^{2}}=\sqrt{\frac{17}{16}}
$$

$$
\begin{gathered}
\downarrow \\
x+\frac{5}{4}= \pm \sqrt{\frac{17}{16}}
\end{gathered}
$$

$$
x+\frac{5}{4}= \pm \frac{\sqrt{17}}{\sqrt{16}}
$$

$$
x+\frac{5}{4}= \pm \frac{\sqrt{17}}{4}
$$

$$
x= \pm \frac{\sqrt{17}}{4}-\frac{5}{4}
$$

$$
x=\frac{-5 \pm \sqrt{17}}{4}
$$

Morgan's "complete the square and derive the quadratic formula" way

> First I divided both sides of the equation by the coefficient of $x^{2}$. Then I subtracted the constant term from both sides.
> I completed the square by adding $\left(\frac{b}{2 a}\right)^{2}$ to both sides of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side.

I simplified and solved for $x$.

[^0]Why does it work?
Alex was asked to solve $2 x^{2}+5 x+1=0$, and Morgan was asked to solve $a^{2}+b x+c=0$ Alex's "complete the square" way

Morgan's "complete the square and derive the quadratic formula" way

First I divided both sides of the equation by the coefficient of $\mathbf{x}^{2}$. Then I subtracted the constant term from both sides.


I simplifia on the righ
side. If

These examples help us see where the quadratic formula comes from. It is derived from solving a quadratic equation for $x$ by completing the square.

* How did Alex solve the equ
* What does Morgan's answe
* What are some similarities od
* What are some similarities a differences beth
* What happens when you substitute the values fo
* Where does the quadratic formula come from?

ex's and Morgan ways?
, and c from Alex's equation into Morgan's answer?
8.1.1

1a How did Alex solve the equation?
1b How did Morgan solve the equation?

What does Morgan's answer look like to you?

3 What are some similarities and differences between Alex's and Morgan's problems?

4 What happens when you substitute the values for $a, b$, and $c$ from Alex's equation into Morgan's answer?

5 Where does the quadratic formula come from?

## Which is better?

Alex and Morgan were asked to find the $x$-intercepts of the graph given by the equation $y=x^{2}-2 x-3$

Alex's "use the quadratic formula" way

Since the $x$ intercepts occur when $y$ is equal to zero, I
substituted 0 for $y$ in the equation.

Then I wrote down the quadratic formula.

I plugged in the values for $a, b$, and $c$ from the original equation, and solved for $x$.

$$
x=\frac{-(-2) \pm \sqrt{(-2)^{2}-4(1)(-3)}}{2(1)}
$$

> ailasoriven a

$$
1
$$

$$
x=3 \text { and } x=-1
$$


Morgan's "factor" way


$$
\begin{aligned}
& x=\frac{2 \pm \sqrt{4+12}}{2} \\
& x=\frac{2 \pm \sqrt{16}}{2}
\end{aligned}
$$

$$
x=\frac{2 \pm 4}{2}
$$

$$
x=\frac{6}{2} \text { and } \frac{-2}{2}
$$

[^1]
## Which is better?

Alex and Morgan were asked to find the $x$-intercepts of the graph given by the equation $y=x^{2}-2 x-3$


| 1 a | How did Alex find the $x$-intercepts? | How did Morgan find the $x$-intercepts? |
| :--- | :--- | :--- | :--- |

2 What are some similarities between Alex's and Morgan's ways?

3 On a timed test, would you rather do Alex's way or Morgan's way?

4 Can you state a general rule that suggests when it might be good to use Alex's way and when it might be good to use Morgan's way?

## Alex and Morgan were asked to solve $8 x^{2}-24 x=0$

Alex's "factor first" way

Morgan's "divide by $x$ " way


* How did Alex solve the equation?
* How did Morgan solve the equation?
* Whose answer is correct, Alex's or Morgan's? How do you know?
* What are some similarities between Alex's and Morgan's ways?
* One person did a step that led to the wrong answer. Who was it? What did they do?
* Why did doing that step lead to the wrong answer?

Alex and Morgan were asked to solve $8 x^{2}-24 x=0$


Student Worksheet 8.3.1

1a How did Alex solve the equation?
1b How did Morgan solve the equation?


2 Whose answer is correct, Alex's or Morgan's? How do you know?

3 What are some similarities between Alex's and Morgan's ways?

4 One person did a step that led to the wrong answer. Who was it? What did they do?

5 Why did doing that step lead to the wrong answer?

Alex and Morgan were asked to solve $-x^{2}+5 x+3=0$


* Describe Alex's and Morgan's ways to a new student in your class.
* Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?
* Whose answer is correct, Alex's or Morgan's? How do you know?
* What are some similarities and differences between Alex's and Morgan's ways?
* Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

Alex and Morgan were asked to solve $-x^{2}+5 x+3=0$


1 Describe Alex's and Morgan's ways to a new student in your class.

2 Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?

3 Whose answer is correct, Alex's or Morgan's? How do you know?

4 What are some similarities and differences between Alex's and Morgan's ways?

5 Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

## Alex and Morgan were asked to solve $4 x^{2}-6 x+1=0$



Morgan's "confuse the sign of b" way


* Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?
* Whose answer is correct, Alex's or Morgan's? How do you know?
* What are some similarities and differences between Alex's and Morgan's ways?
* Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

Alex and Morgan were asked to solve $4 x^{2}-6 x+1=0$


1 Describe Alex's and Morgan's ways to a new student in your class.

2 Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?

3 Whose answer is correct, Alex's or Morgan's? How do you know?

4 What are some similarities and differences between Alex's and Morgan's ways?

5 Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

## Which is correct?

Alex and Morgan were asked to solve $x^{2}+5 x-3=0$


* Describe Alex's and Morgan's ways to a new student in your class.
* Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?
* Whose answer is correct, Alex's or Morgan's? How do you know?
* What are some similarities and differences between Alex's and Morgan's ways?
* Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

Alex and Morgan were asked to solve $x^{2}+5 x-3=0$


1 Describe Alex's and Morgan's ways to a new student in your class.

2 Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?

3 Whose answer is correct, Alex's or Morgan's? How do you know?

4 What are some similarities and differences between Alex's and Morgan's ways?

5 Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

How do they differ?
Alex was asked to graph the equation $y=x^{2}$, and Morgan was asked to graph the equation $\boldsymbol{y}=-\boldsymbol{x}^{2}$.
Alex's way

Morgan's way


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does changing the sign of the coefficient of $x^{2}$ affect the graph of a quadratic function?

How do they differ?
Alex was asked to graph the equation $y=\boldsymbol{x}^{2}$, and Morgan was asked to graph the equation $\boldsymbol{y}=-\boldsymbol{x}^{2}$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does changing the sign of the coefficient of $x^{2}$ affect the graph of a quadratic function?

How do they differ?
Alex was asked to graph the equation $y=x^{2}$,
and Morgan was asked to graph the equation $y=\frac{1}{2} x^{2}$.


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does changing the value of the coefficient of $x^{2}$ affect the graph of a quadratic function?

How do they differ?
Alex was asked to graph the equation $y=x^{2}$,
and Morgan was asked to graph the equation $y=\frac{1}{2} x^{2}$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does changing the value of the coefficient of $x^{2}$ affect the graph of a quadratic function?

How do they differ?
Alex was asked to graph the equation $\boldsymbol{y}=\boldsymbol{x}^{2}$,
and Morgan was asked to graph the equation $\boldsymbol{y}=3 \boldsymbol{x}^{2}$.


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does changing the value of the coefficient of $x^{2}$ affect the graph of a quadratic function?

Alex was asked to graph the equation $y=x^{2}$,
and Morgan was asked to graph the equation $y=3 x^{2}$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does changing the value of the coefficient of $x^{2}$ affect the graph of a quadratic function?

Alex was asked to graph the equation $y=x^{2}$,
and Morgan was asked to graph the equation $y=x^{2}+1$.


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does adding a constant to the equation affect the graph of a quadratic function?
* If the equation were changed to $y=x^{2}-3$, what do you think the graph would look like?

Alex was asked to graph the equation $y=x^{2}$,
and Morgan was asked to graph the equation $y=x^{2}+1$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does adding a constant to the equation affect the graph of a quadratic function?

5 If the equation were changed to $y=x^{2}-3$, what do you think the graph would look like?

Alex was asked to graph the equation $y=x^{2}+1$,
and Morgan was asked to graph the equation $y=x^{2}-1$.


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does adding or subtracting a constant to the equation affect the graph of a quadratic function?
* If the equation were changed to $y=x^{2}$, what do you think the graph would look like?


## How do they differ?

Alex was asked to graph the equation $y=x^{2}+1$,
and Morgan was asked to graph the equation $y=x^{2}-1$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does adding or subtracting a constant to the equation affect the graph of a quadratic function?

5 If the equation were changed to $y=x^{2}$, what do you think the graph would look like?

Alex was asked to graph the equation $\boldsymbol{y}=\boldsymbol{x}^{2}$,
and Morgan was asked to graph the equation $y=(x+1)^{2}$.


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does adding a constant to $x^{2}$ affect the graph of a quadratic function?
* If the equation were changed to $y=(x-1)^{2}$, what do you think the graph would look like?

Alex was asked to graph the equation $y=x^{2}$,
and Morgan was asked to graph the equation $y=(x+1)^{2}$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does adding a constant to $x^{2}$ affect the graph of a quadratic function?

5 If the equation were changed to $y=(x-1)^{2}$, what do you think the graph would look like?

Alex was asked to graph the equation $y=(x+1)^{2}$,
and Morgan was asked to graph the equation $y=(x-1)^{2}$.


* How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
* What are some similarities and differences between Alex's and Morgan's problems?
* What are some similarities and differences between Alex's and Morgan's graphs?
* How does adding or subtracting a constant to $x^{2}$ affect the graph of a quadratic function?
* If the equation were changed to $y=(x-6)^{2}$, what do you think the graph would look like?

Alex was asked to graph the equation $y=(x+1)^{2}$,
and Morgan was asked to graph the equation $y=(x-1)^{2}$.


1a How did Alex graph the parabola given by his equation?

1b How did Morgan graph the parabola given by her equation?

2 What are some similarities and differences between Alex's and Morgan's problems?

3 What are some similarities and differences between Alex's and Morgan's graphs?

4 How does adding or subtracting a constant to $x^{2}$ affect the graph of a quadratic function?

5 If the equation were changed to $y=(x-6)^{2}$, what do you think the graph would look like?


[^0]:    * How did Alex solve the equation? How did Morgan solve the equation?
    * What does Morgan's answer look like to you?
    * What are some similarities and differences between Alex's and Morgan's problems?
    * What are some similarities and differences between Alex's and Morgan's ways?
    * What happens when you substitute the values for $a, b$, and $c$ from Alex's equation into Morgan's answer?
    * Where does the quadratic formula come from?

[^1]:    * How did Alex find the x-intercepts?
    * How did Morgan find the x-intercepts?
    * What are some similarities between Alex's and Morgan's ways?
    * On a timed test, would you rather do Alex's way or Morgan's way?
    * Can you state a general rule that suggests when it might be good to use Alex's way and when it might be good to use Morgan's way?

