Alex was asked to solve $2x^2 + 5x + 1 = 0$, and Morgan was asked to solve $a^2 + bx + c = 0$

Alex's "complete the square" way

First I divided both sides of the equation by the coefficient of x^2 . Then I subtracted the constant term from both sides.

I completed the square by adding

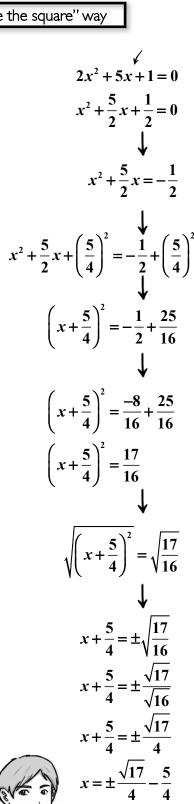
$$\left(\frac{b}{2a}\right)^2$$
 to both sides of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side.

I simplified and solved for x..



Morgan's "complete the square and derive the quadratic formula" way

$$2x^{2} + 5x + 1 = 0$$

$$x^{2} + \frac{5}{2}x + \frac{1}{2} = 0$$

$$x^{2} + \frac{5}{2}x + \left(\frac{5}{4}\right)^{2} = -\frac{1}{2} + \left(\frac{5}{4}\right)^{2}$$

$$x^{2} + \frac{5}{2}x + \left(\frac{5}{4}\right)^{2} = -\frac{1}{2} + \left(\frac{5}{4}\right)^{2}$$

$$\left(x + \frac{5}{4}\right)^{2} = -\frac{1}{2} + \left(\frac{5}{4}\right)^{2}$$

$$\left(x + \frac{5}{4}\right)^{2} = -\frac{1}{2} + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^{2} = \frac{-8}{16} + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^{2} = \frac{17}{16}$$

$$\left(x + \frac{5}{4}\right)^{2} = \sqrt{\frac{17}{16}}$$

$$\left(x + \frac{5}{4}\right)^{2} = \sqrt{\frac{17}{16}}$$

$$\left(x + \frac{5}{4}\right)^{2} = \sqrt{\frac{17}{16}}$$

$$\left(x + \frac{5}{2a}\right)^{2} = -\frac{4ac + b^{2}}{4a^{2}}$$

$$\left(x + \frac{b}{2a}\right)^{2} = -\frac{4ac + b^{2}}{4a^{2}}$$

$$\left(x + \frac{b}{2a}\right)^{2} = \sqrt{\frac{-4ac + b^{2}}{4a^{2}}}$$

$$\left(x + \frac{b}{2a}\right)$$

First I divided both sides of the equation by the coefficient of x^2 . Then I subtracted

the constant term from both sides.

I completed the square by adding

$$\left(\frac{b}{2a}\right)^2$$
 to both sides

of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side.

I simplified and solved for x.

^{*} How did Alex solve the equation? How did Morgan solve the equation?

^{*} What does Morgan's answer look like to you?

^{*} What are some similarities and differences between Alex's and Morgan's problems?

^{*} What are some similarities and differences between Alex's and Morgan's ways?

st What happens when you substitute the values for a, b, and c from Alex's equation into Morgan's answer?

^{*} Where does the quadratic formula come from?

Why does it work? Alex was asked to solve $2x^2 + 5x + 1 = 0$, and Morgan was asked to solve $a^2 + bx + c = 0$ Morgan's "complete the square and derive the quadratic Alex's "complete the square" way formula" way First I divided $2x^2 + 5x + 1 = 0$ First I divided $ax^2 + bx + c = 0$ both sides of the both sides of the $x^2 + \frac{5}{2}x + \frac{1}{2} = 0$ $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ equation by the equation by the coefficient of x^2 . coefficient of x^2 . Then I subtracted Then I subtracted the constant term the constant term from both sides. from both sides. I completed the I completed the square by adding square by adding to both to both sides of the equation he equation. Hey Alex, what did comparing these two examples help us to bot see? I simplifie d further on the righ aht-hand side. I These examples help us see where the quadratic right her. them tog formula comes from. It is I took the uare derived from solving a root oz side. quadratic equation for x so ved to by completing the square. * How did Alex solve the equ * What does Morgan's answe broblems? * What are some similarities of * What are some similarities a differences bet ex's and Morgan * What happens when you substitute the values and c from Alex's equation into Morgan's answer?

8.1.1

* Where does the quadratic formula come from?

Student Worksheet 8.1.1

1a	How did Alex solve the equation? 1b How did Morgan solve the equation?
2	What does Morgan's answer look like to you?
3	What are some similarities and differences between Alex's and Morgan's problems?
4	What happens when you substitute the values for <i>a</i> , <i>b</i> , and <i>c</i> from Alex's equation into Morgan's answer?
_	which improves which you constitute the talest sold with the state of the state with the state w
-	
5	Where does the quadratic formula come from?

Alex and Morgan were asked to find the x-intercepts of the graph given by the equation $y=x^2-2x-3$

Alex's "use the quadratic formula" way

Since the xintercepts occur when y is equal to zero, I substituted 0 for y in the equation.

Then I wrote down the **quadratic** formula.

I plugged in the values for a, b, and c from the original equation, and solved for x.

$$y = x^{2} - 2x - 3$$

$$0 = x^{2} - 2x - 3$$

$$\downarrow$$

$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$\downarrow$$

$$(-2) \pm \sqrt{(-2)^{2} - 4(1)(-3)}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$$x = \frac{6}{2} \text{ and } \frac{-2}{2}$$

$$x = 3 \text{ and } x = -1$$



Morgan's "factor" way

 $0=x^2-2x-3$

0 = (x-3)(x+1)

x - 3 = 0

x = 3

x+1=0

x = -1

x = 3 and x = -1

Since the xintercepts occur when y is equal to zero, I substituted O for y in the equation.

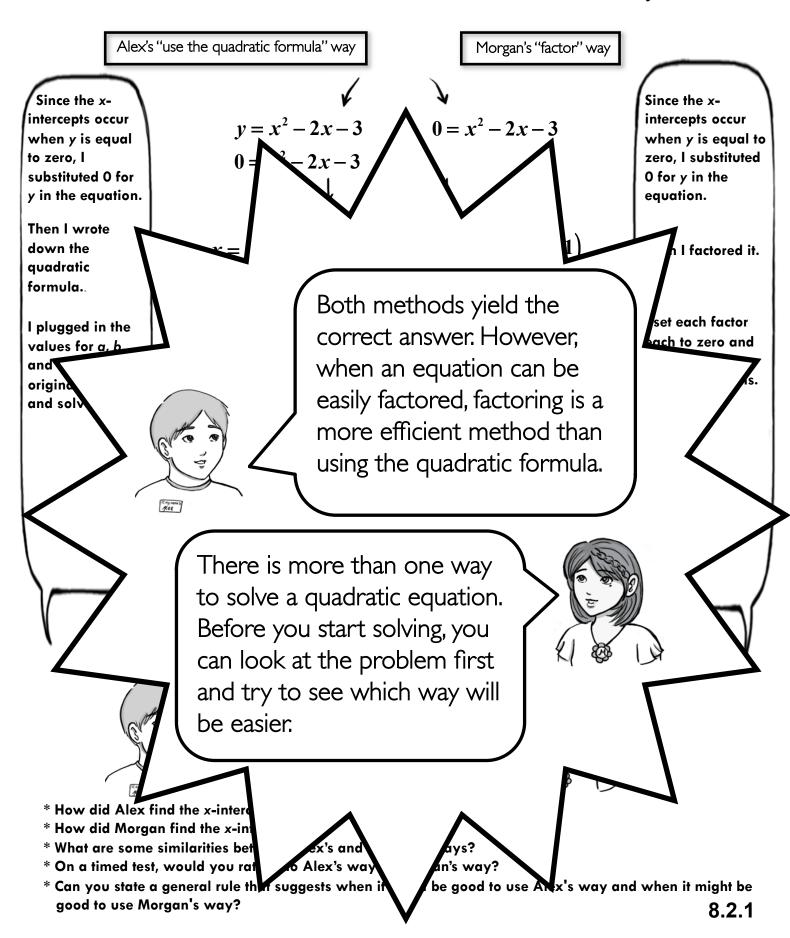
Then I factored it.

I set each factor each to zero and solved. And here are my solutions.



- * How did Alex find the x-intercepts?
- * How did Morgan find the x-intercepts?
- * What are some similarities between Alex's and Morgan's ways?
- * On a timed test, would you rather do Alex's way or Morgan's way?
- * Can you state a general rule that suggests when it might be good to use Alex's way and when it might be good to use Morgan's way? 8.2.1

Alex and Morgan were asked to find the x-intercepts of the graph given by the equation $y=x^2-2x-3$



Student Worksheet 8.2.1

1a	How did Alex find the <i>x</i> -intercepts?	b How did Morgan find the <i>x</i> -intercepts?
	· -	
2	What are some similarities between Alex's and Morg	an's ways?
3	On a timed test, would you rather do Alex's way or I	Morgan's way?
4	Can you state a general rule that suggests when it mi	ght be good to use Alex's way and when it might be
	good to use Morgan's way?	Site so good to dee Them of why dista which it implies so
	good to doe intergante way.	

Alex and Morgan were asked to solve $8x^2 - 24x = 0$

Alex's "factor first" way

Morgan's "divide by x" way

l

1

$$8x^2 - 24x = 0$$

 $8x^2 - 24x = 0$

1

1

First I factored out an 8x term from the left side.

$$8x(x-3)=0$$

 $8x^2 - 24x = 0$ + 24x + 24x

$$8x^2 = 24x$$

1

Then I set 8x equal to zero and (x-3) equal to zero. I solved both of the equations for x.

$$8x = 0$$
 or $(x-3) = 0$

$$x = 0$$
 or $x = 3$

 $\frac{8x^2}{8x} = \frac{24x}{8x}$

$$x = 3$$

First added 24x on both sides.

Then I divided by 8x on both sides. I got my answer, x = 3.





^{*} How did Morgan solve the equation?

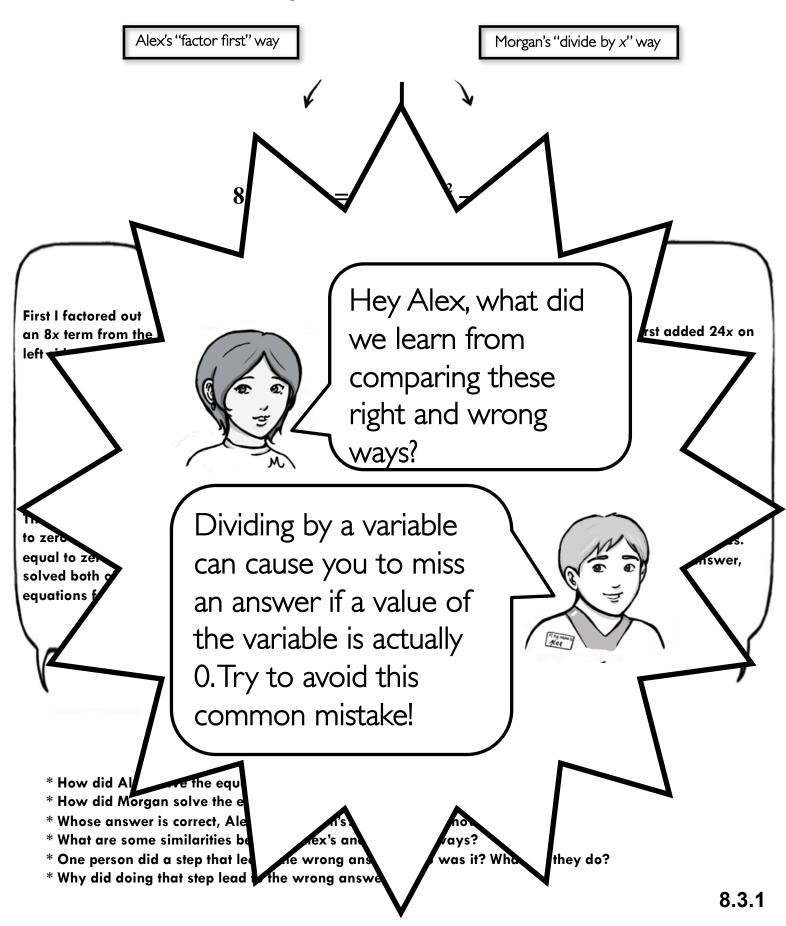
^{*} Whose answer is correct, Alex's or Morgan's? How do you know?

^{*} What are some similarities between Alex's and Morgan's ways?

^{*} One person did a step that led to the wrong answer. Who was it? What did they do?

^{*} Why did doing that step lead to the wrong answer?

Alex and Morgan were asked to solve $8x^2 - 24x = 0$



Student Worksheet 8.3.1

1a	How did Alex solve the equation? 1b How did Morgan solve the equation?
2	Whose answer is correct, Alex's or Morgan's? How do you know?
	,
2	Wileston and a circulation between Alexand Manager and
3	What are some similarities between Alex's and Morgan's ways?
4	One person did a step that led to the wrong answer. Who was it? What did they do?
4	One person did a step that led to the wrong answer. Who was it: what did they do:
5	Why did doing that step lead to the wrong answer?

Alex and Morgan were asked to solve $-x^2 + 5x + 3 = 0$

Alex's "confuse the sign of a" way

Morgan's "substitute into the quadratic formula correctly" way

First I wrote the quadratic formula.

Then I substituted the values for a, b, and c from the original equation into the formula.

I simplified.

I subtracted the values inside the radical.

 $-x^2 + 5x + 3 = 0$

$$2(-1)$$

$$\downarrow$$

$$x = \frac{-5 \pm \sqrt{25 - 12}}{-2}$$

$$x = \frac{-5 \pm \sqrt{13}}{2}$$

 $-x^2 + 5x + 3 = 0$ $x = -b \pm \sqrt{b^2 - 4ac} \quad x = -b \pm \sqrt{b^2 - 4ac}$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(3)}}{2(-1)}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(-1)(3)}}{2(-1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 12}}{-2}$$

$$x = -5 \pm \sqrt{37}$$

First I wrote the quadratic formula.

Then I substituted the values for a, b, and c from the original equation into the formula.

I simplified.

I subtracted the values inside the radical.





* Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?

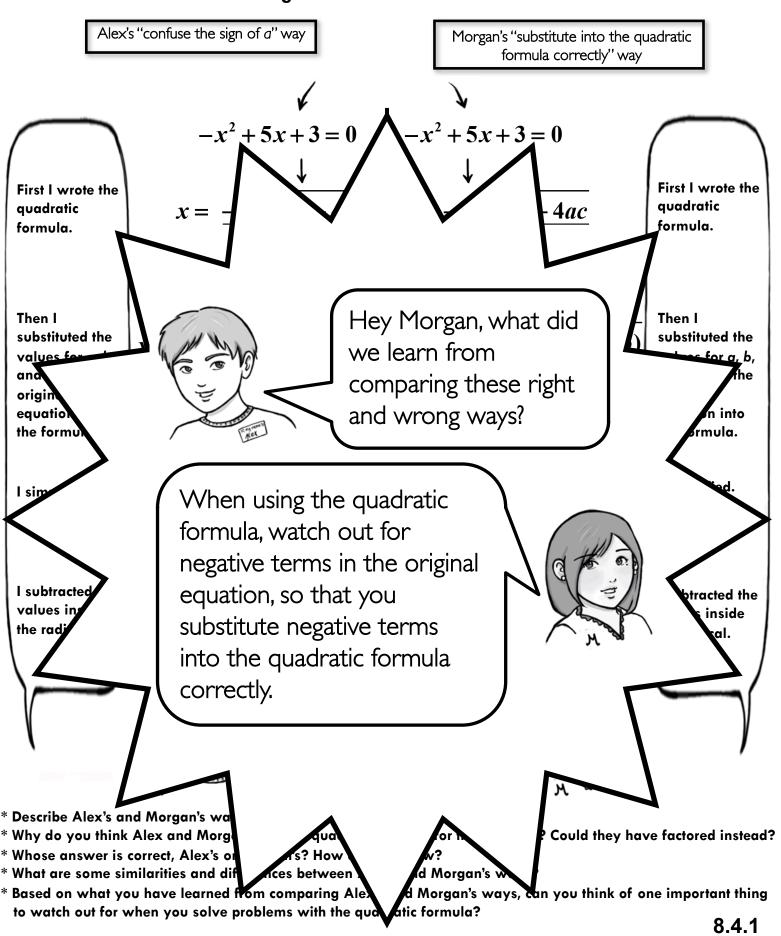
* Whose answer is correct, Alex's or Morgan's? How do you know?

* What are some similarities and differences between Alex's and Morgan's ways?

* Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

8.4.1

Alex and Morgan were asked to solve $-x^2 + 5x + 3 = 0$



Student Worksheet 8.4.1

1	Describe Alex's and Morgan's ways to a new student in your class.
2	Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have
	factored instead?
3	Whose answer is correct, Alex's or Morgan's? How do you know?
4	What are some similarities and differences between Alex's and Morgan's ways?
4	what are some similarness and differences between thex's and morgan's ways:
5	Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one
	important thing to watch out for when you solve problems with the quadratic formula?

Alex and Morgan were asked to solve $4x^2 - 6x + 1 = 0$

Alex's "substitute into the quadratic formula correctly" way

Morgan's "confuse the sign of b" way

First I wrote the avadratic formula.

Then I substituted the values for a, b, and c from the original equation into the formula.

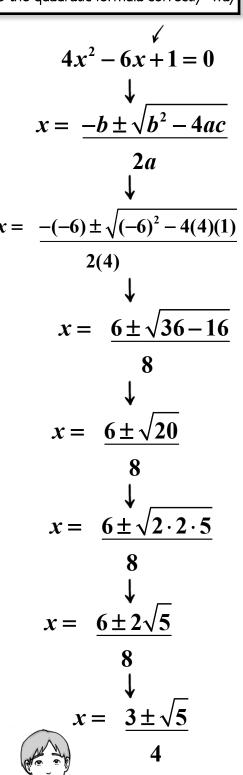
I simplified.

I subtracted the values inside the radical.

I rewrote 20 as the product of prime factors.

I simplified the expression inside the radical.

Then I simplified the fraction. Here is my answer.



Morgan's "confuse the sign
$$4x^{2} - 6x + 1 = 0$$

$$x = -b \pm \sqrt{b^{2} - 4ac}$$

$$2a$$

$$x = -6 \pm \sqrt{(-6)^{2} - 4(4)(1)}$$

$$x = -6 \pm \sqrt{36 - 16}$$

$$8$$

$$x = -6 \pm \sqrt{20}$$

$$x = -6 \pm \sqrt{2 \cdot 2 \cdot 5}$$

$$x = -6 \pm 2\sqrt{5}$$

$$x = -6 \pm 2\sqrt{5}$$

 $-3\pm\sqrt{5}$

I subtracted the values inside the radical. I rewrote 20 as the product of prime factors. I simplified the expression inside the radical. Then I simplified the fraction. Here is my answer.

First I wrote the

Then I substituted

the values for a, b,

original equation

into the formula.

I simplified.

and c from the

auadratic

formula.

* Describe Alex's and Morgan's ways to a new student in your class.

* Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?

x =

- * Whose answer is correct, Alex's or Morgan's? How do you know?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

Alex and Morgan were asked to solve $4x^2 - 6x + 1 = 0$

Alex's "substitute into the quadratic formula correctly" way

Morgan's "confuse the sign of b" way

First I wrote the avadratic formula.

Then I substituted the values for a, b, and c from the original equation into the formula.

x =

I simplified.

l subtrad values ins radical.

I simplified expressi the rad

Then I simplified the fraction. Here is my answer.

 $4x^2 - 6x + 1 = 0$

 $4x^2 - 6x + 1 = 0$ $x = -b \pm \sqrt{b^2 - 4ac}$ $x = -b \pm \sqrt{b^2 - 4ac}$

4(4)(1)

First I wrote the avadratic formula.

Then I substituted the values for a, b, and c from the iginal equation to the formula.

I simplified.

Hey Alex, what did we learn from comparing these right and wrong ways?

and Morgan's wa

When using the quadratic formula, watch out for negative terms in the original equation, so that you substitute negative terms into the quadratic formula correctly.



ed the inside

side the

Then I simplified the fraction. Here is my answer.

- * Describe Alex's and Morgan's w
- * Why do you think Alex and More
- * Whose answer is correct, Alex's o
- * What are some similarities and differences between
- * Based on what you have learned from comparing Ale and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

n? Could they have factored instead?

Student Worksheet 8.4.2

1	Describe Alex's and Morgan's ways to a new student in your class.
2	Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have
L	factored instead?
	ractored mistead:
3	Whose answer is correct, Alex's or Morgan's? How do you know?
5	whose answer is correct, ruck's or morgan's: from do you know:
4	What are some similarities and differences between Alex's and Morgan's ways?
	6
J	
5	Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one
	important thing to watch out for when you solve problems with the quadratic formula?

Alex and Morgan were asked to solve $x^2 + 5x - 3 = 0$

Alex's "substitute into the quadratic formula correctly" way

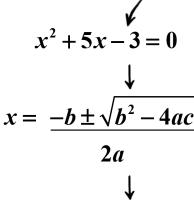
Morgan's "confuse the sign of c" way

First I wrote the quadratic formula.

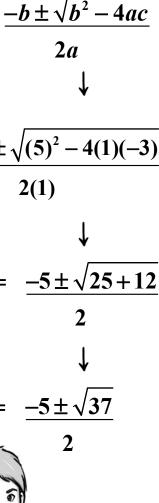
Then I substituted the values for a, b, and c from the original equation into the formula.

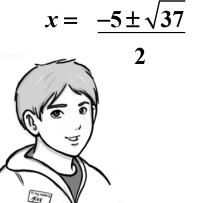
I simplified.

I added the values inside the radical.



$$\begin{array}{c}
2(1) \\
\downarrow \\
x = \frac{-5 \pm \sqrt{25 + 12}}{2} \\
\downarrow
\end{array}$$





$$x^{2} + 5x - 3 = 0$$

$$\downarrow$$

$$x = -b \pm \sqrt{b^{2} - 4ac}$$

$$\downarrow$$

$$2a$$

$$\downarrow$$

$$\downarrow$$

$$2a$$

$$\downarrow$$

$$\downarrow$$

$$\downarrow$$

$$2a$$

$$\downarrow$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-3)}}{2(1)} \quad x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 12}}{2}$$

$$x = -5 \pm \sqrt{13}$$

First I wrote the **quadratic** formula.

Then I substituted the values for a, b, and c from the original equation into the formula.

I simplified.

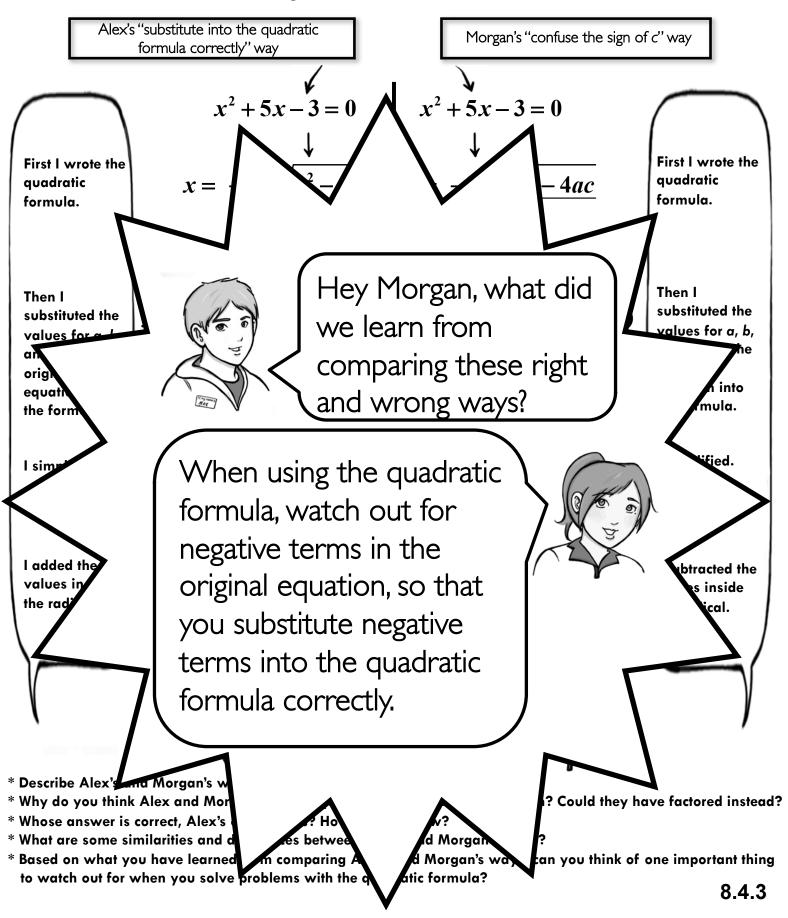
I subtracted the values inside the radical.



- * Describe Alex's and Morgan's ways to a new student in your class.
- * Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have factored instead?
- * Whose answer is correct, Alex's or Morgan's? How do you know?
- * What are some similarities and differences between Alex's and Morgan's ways?
- * Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one important thing to watch out for when you solve problems with the quadratic formula?

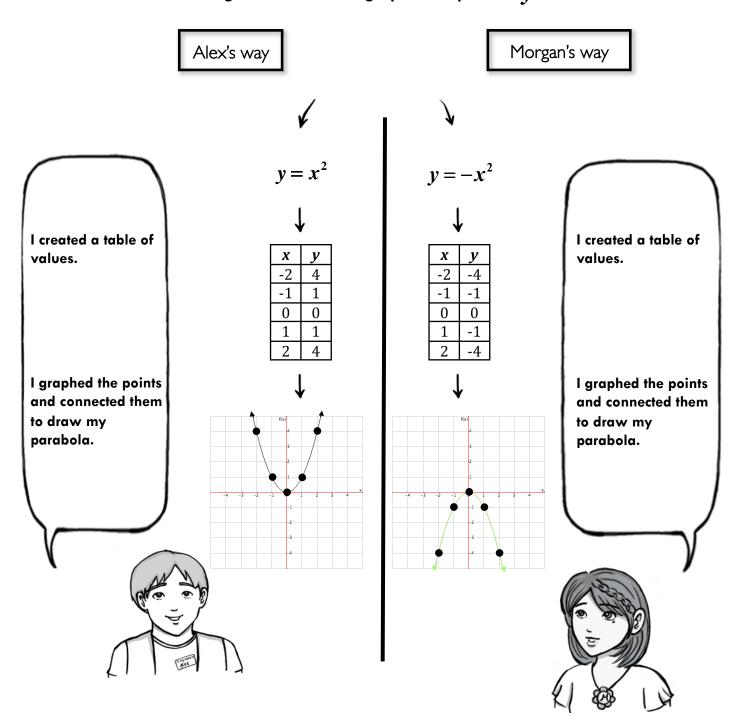
8.4.3

Alex and Morgan were asked to solve $x^2 + 5x - 3 = 0$



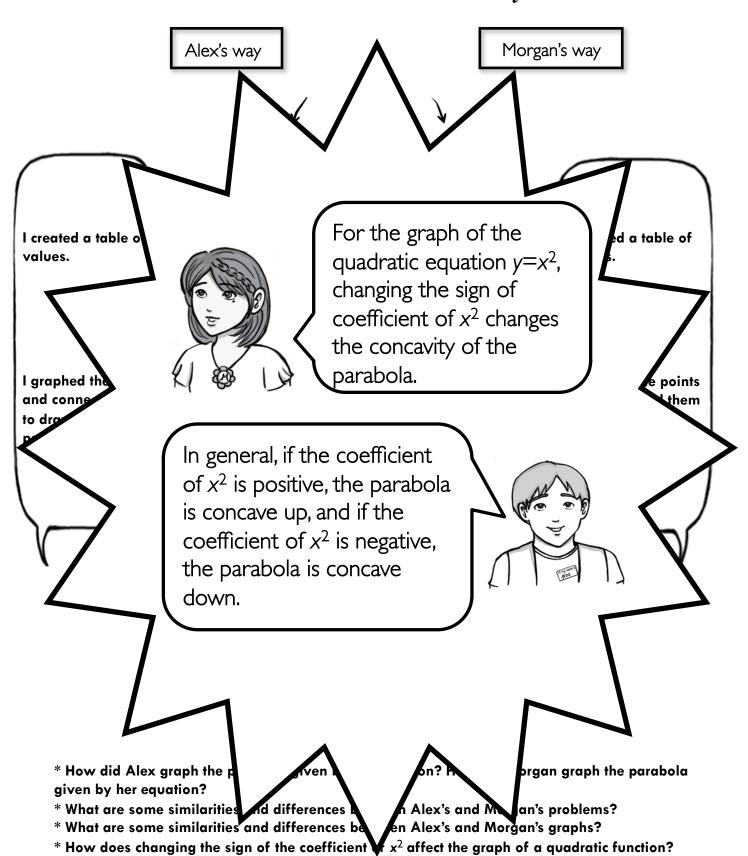
1	Describe Alex's and Morgan's ways to a new student in your class.
	, , , , , , , , , , , , , , , , , , ,
2	Why do you think Alex and Morgan used the quadratic formula for this problem? Could they have
	factored instead?
	ractored motead:
3	Whose answer is correct, Alex's or Morgan's? How do you know?
7	whose answer is correct, thex s or morgan s. from do you know.
4	What are some similarities and differences between Alex's and Morgan's ways?
.	D
5	Based on what you have learned from comparing Alex's and Morgan's ways, can you think of one
	important thing to watch out for when you solve problems with the quadratic formula?

Alex was asked to graph the equation $y = x^2$, and Morgan was asked to graph the equation $y = -x^2$.



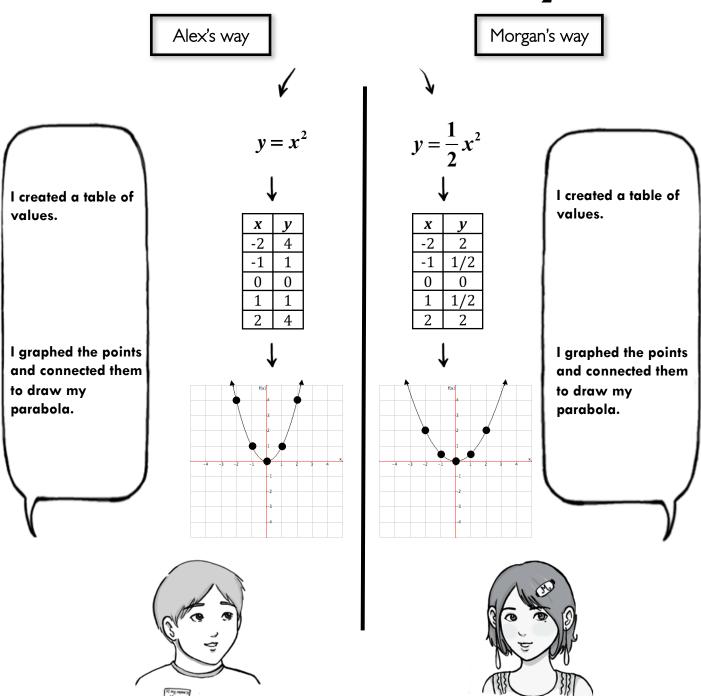
- * How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * What are some similarities and differences between Alex's and Morgan's graphs?
- * How does changing the sign of the coefficient of x^2 affect the graph of a quadratic function?

Alex was asked to graph the equation $y = x^2$, and Morgan was asked to graph the equation $y = -x^2$.



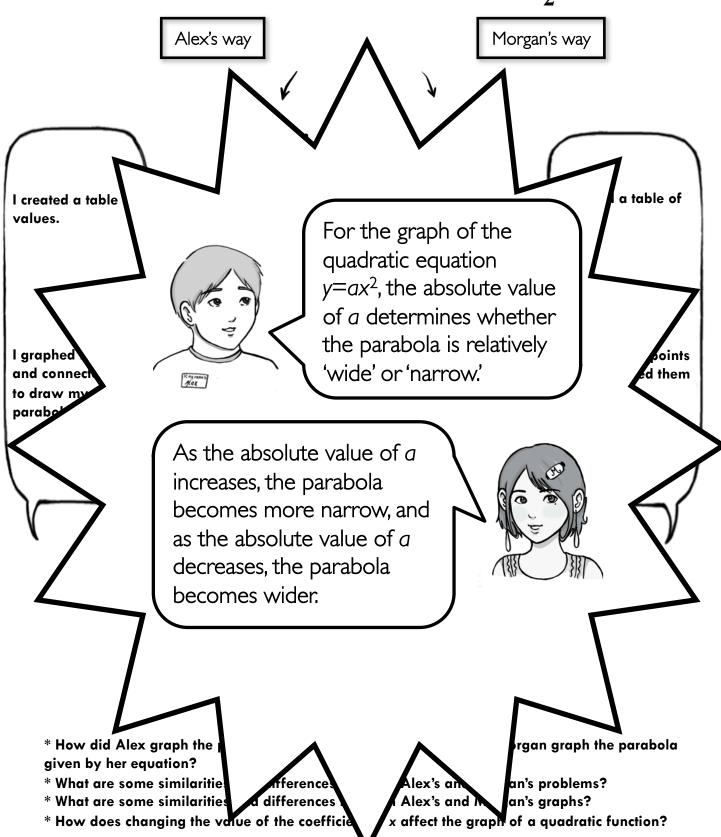
1a	How did Alex graph the parabola given by his equation?	1b	How did Morgan graph the parabola given by her equation?
	equation:		equation:
2	What are some similarities and differences between	zeen Ale	ex's and Morgan's problems?
l			
2	Wil. 4	A 1-	
3	What are some similarities and differences between	een Ale	ex's and Morgan's graphs?
	How does changing the sign of the coefficient of	of v ² off	eat the graph of a quadratic function?
4	Trow does changing the sign of the coefficient of	oi x aii	eet the graph of a quadratic function:

Alex was asked to graph the equation $y=x^2$, and Morgan was asked to graph the equation $y=\frac{1}{2}x^2$.



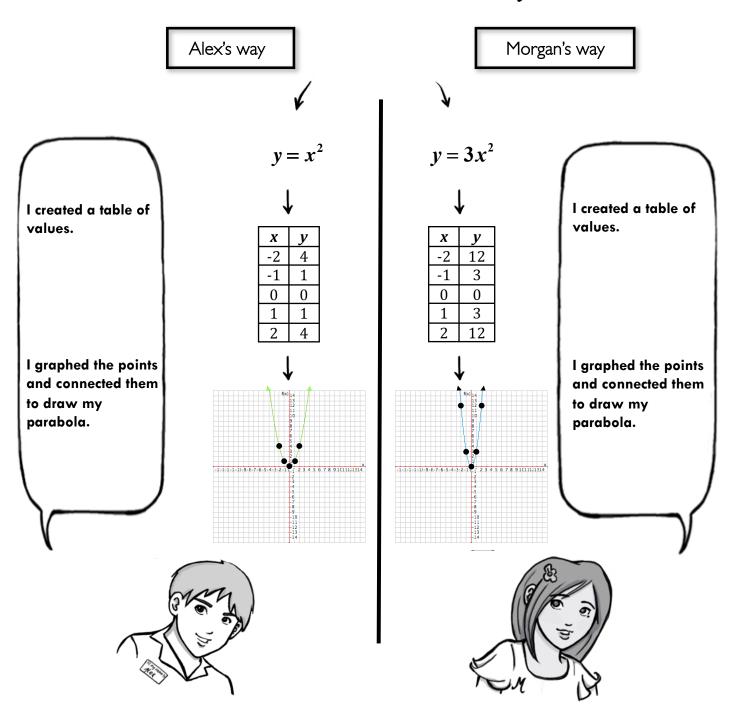
- * How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * What are some similarities and differences between Alex's and Morgan's graphs?
- * How does changing the value of the coefficient of x^2 affect the graph of a quadratic function?

Alex was asked to graph the equation $y=x^2$, and Morgan was asked to graph the equation $y=\frac{1}{2}x^2$.



1a	How did Alex graph the parabola given by his equation?	1b	How did Morgan graph the parabola given by her equation?
	equation:		equation:
2	What are some similarities and differences between	een Ale	ex's and Morgan's problems?
3	What are some similarities and differences between	zeen Ale	ex's and Morgan's graphs?
ļ			
4	How does changing the value of the coefficient	$\cot x^2$ af	ffect the graph of a quadratic function?

and Morgan was asked to graph the equation $y = 3x^2$.



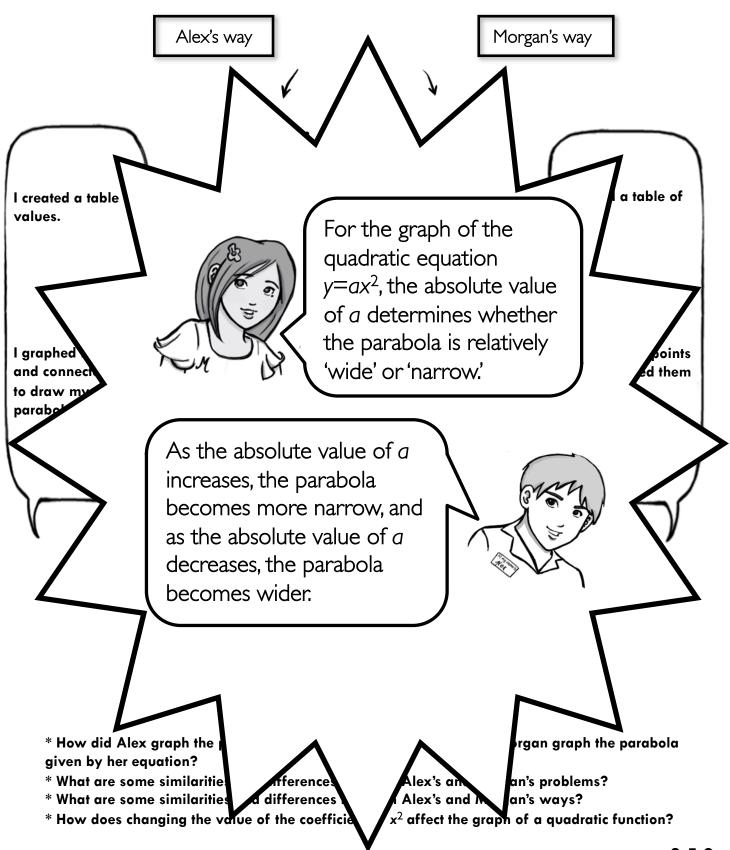
^{*} How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?

^{*} What are some similarities and differences between Alex's and Morgan's problems?

^{*} What are some similarities and differences between Alex's and Morgan's graphs?

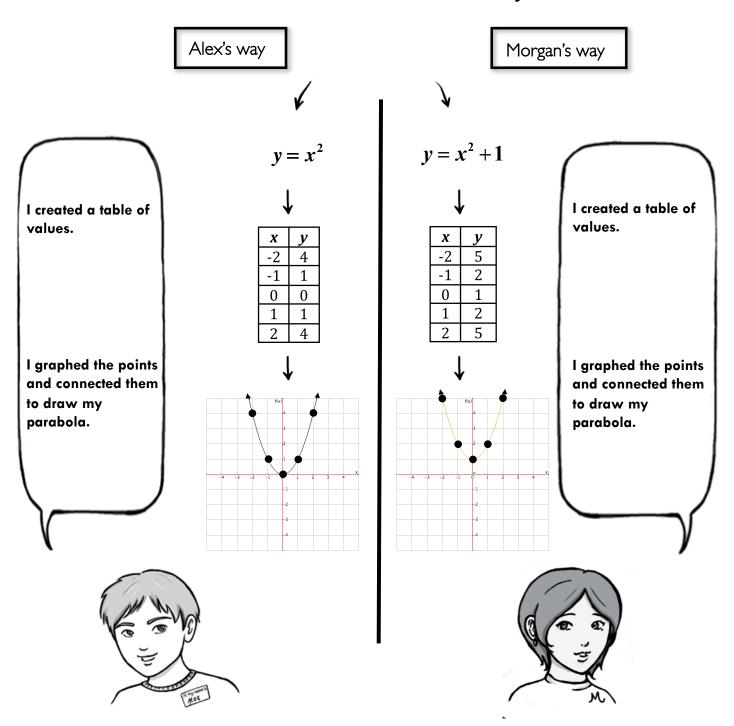
^{*} How does changing the value of the coefficient of x^2 affect the graph of a quadratic function?

and Morgan was asked to graph the equation $y = 3x^2$.



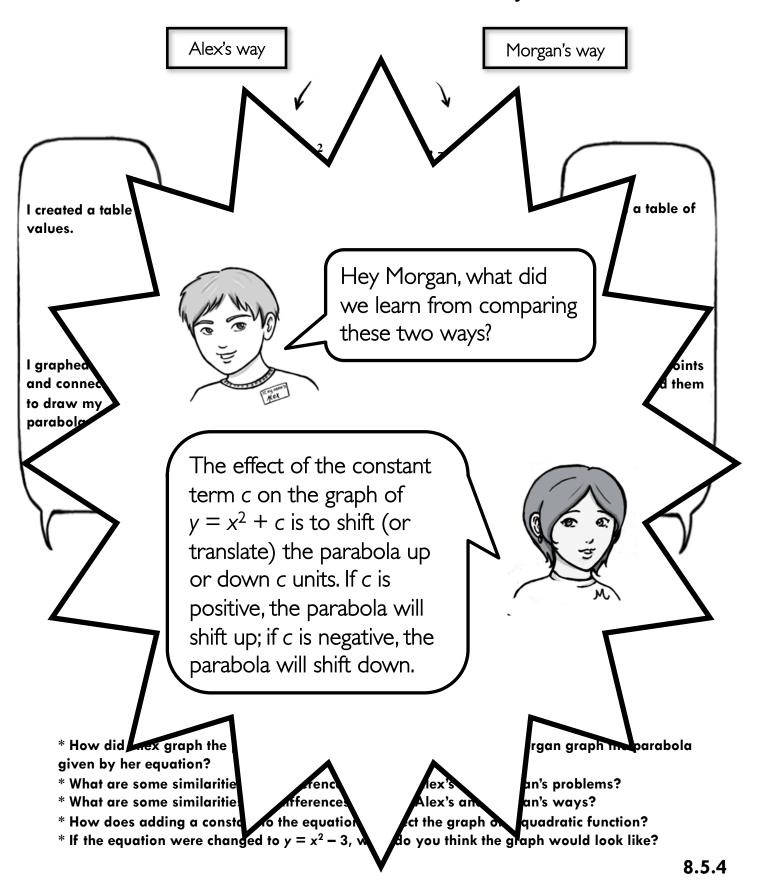
1a	How did Alex graph the parabola given by his	1b	How did Morgan graph the parabola given by her
	equation?		equation?
2	What are some similarities and differences between	veen Ale	ex's and Morgan's problems?
!			
3	What are some similarities and differences between	veen Ale	ex's and Morgan's graphs?
ļ			
4	How does changing the value of the coefficient	$t ext{ of } x^2 ext{ at}$	ffect the graph of a quadratic function?

and Morgan was asked to graph the equation $y = x^2 + 1$.



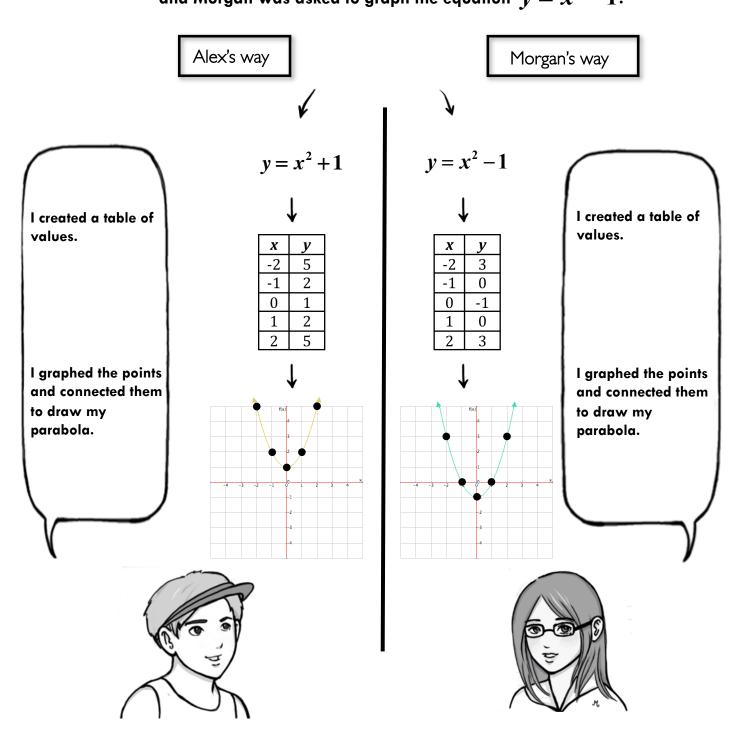
- * How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * What are some similarities and differences between Alex's and Morgan's graphs?
- * How does adding a constant to the equation affect the graph of a quadratic function?
- * If the equation were changed to $y = x^2 3$, what do you think the graph would look like?

and Morgan was asked to graph the equation $y = x^2 + 1$.



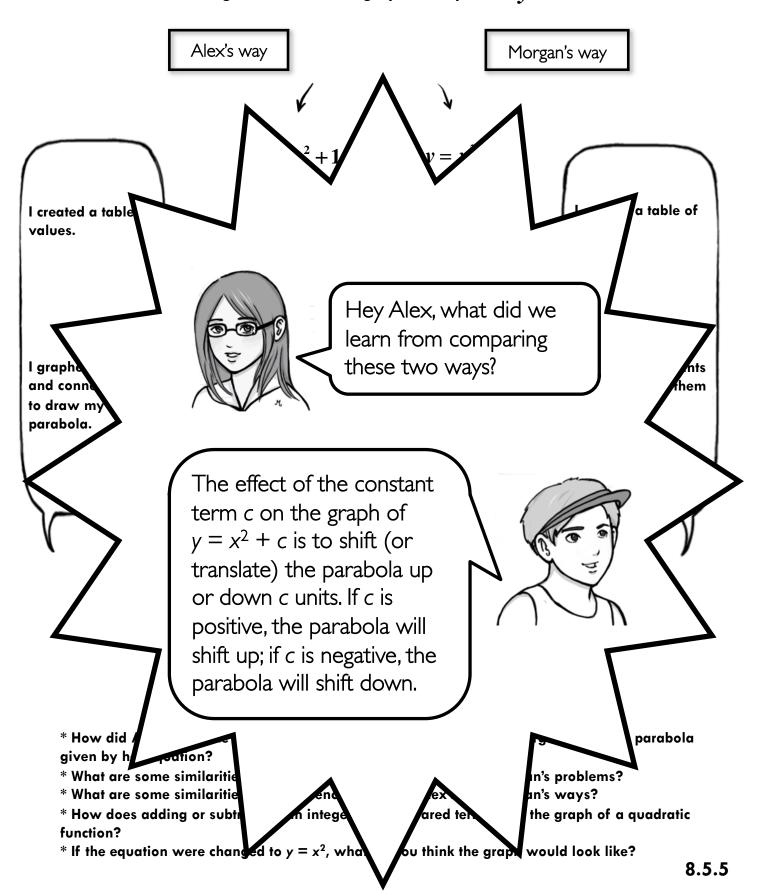
1a	How did Alex graph the parabola given by his 1b How did Morgan graph the parabola given by her
	equation? equation?
•	W/I
2	What are some similarities and differences between Alex's and Morgan's problems?
3	What are some similarities and differences between Alex's and Morgan's graphs?
4	How does adding a constant to the equation affect the graph of a quadratic function?
•	210 W does daming to the equation arrest the Simple of a quantitate function.
ı	
5	If the equation were changed to $y = x^2 - 3$, what do you think the graph would look like?

Alex was asked to graph the equation $y=x^2+1$, and Morgan was asked to graph the equation $y=x^2-1$.



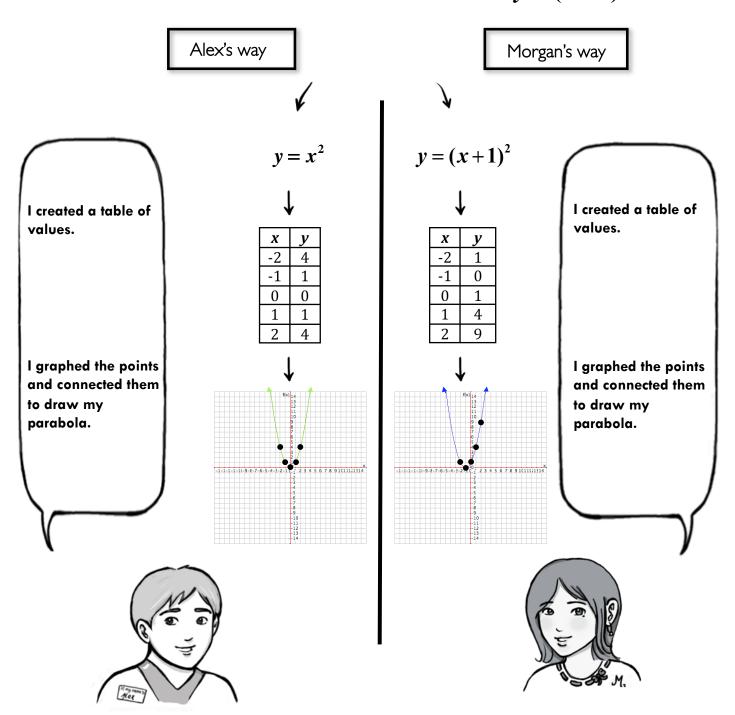
- * How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * What are some similarities and differences between Alex's and Morgan's graphs?
- * How does adding or subtracting a constant to the equation affect the graph of a quadratic function?
- * If the equation were changed to $y = x^2$, what do you think the graph would look like?

Alex was asked to graph the equation $y=x^2+1$, and Morgan was asked to graph the equation $y=x^2-1$.



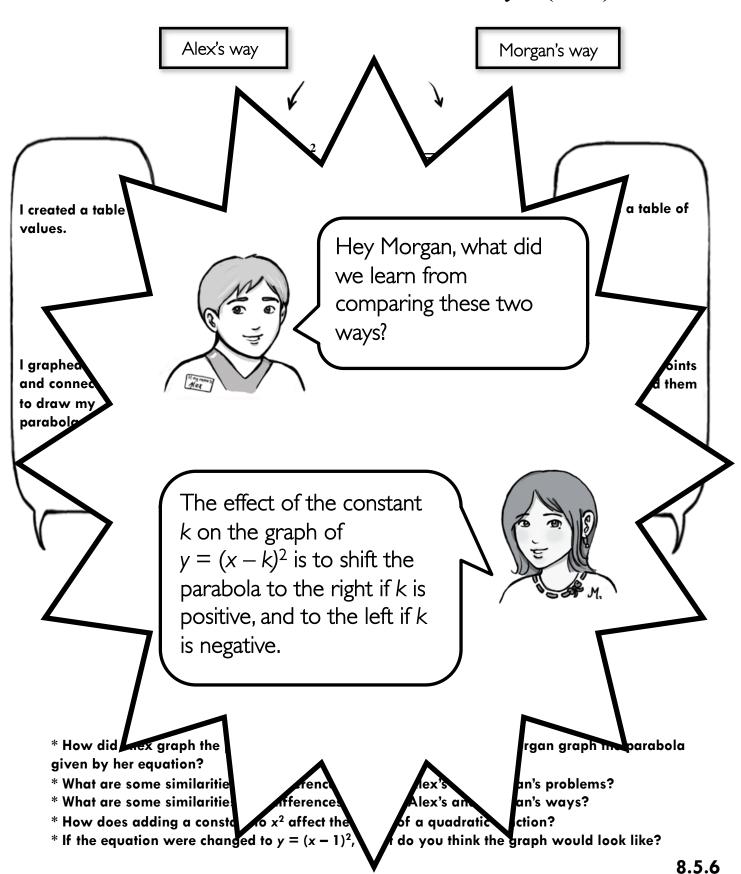
1a	How did Alex graph the parabola given by his equation?	How did Morgan graph the parabola given by her equation?
	equation:	equation:
2	What are some similarities and differences between	Alex's and Morgan's problems?
	1	3 1
3	What are some similarities and differences between	Alex's and Morgan's graphs?
4	How does adding or subtracting a constant to the ed	quation affect the graph of a quadratic function?
5	If the equation were changed to $y = x^2$, what do you	think the graph would look like?

and Morgan was asked to graph the equation $y = (x+1)^2$.



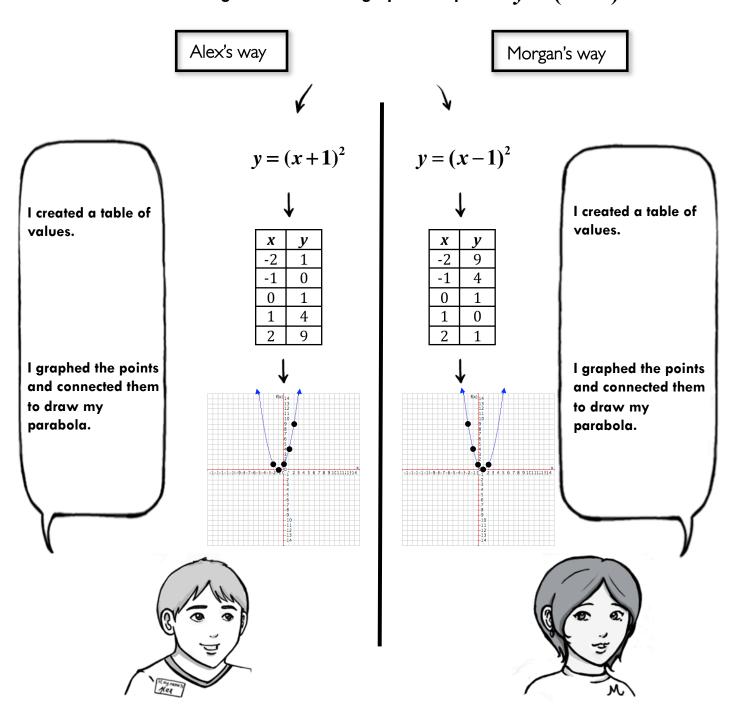
- * How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * What are some similarities and differences between Alex's and Morgan's graphs?
- * How does adding a constant to x^2 affect the graph of a quadratic function?
- * If the equation were changed to $y = (x 1)^2$, what do you think the graph would look like?

and Morgan was asked to graph the equation $y = (x+1)^2$.



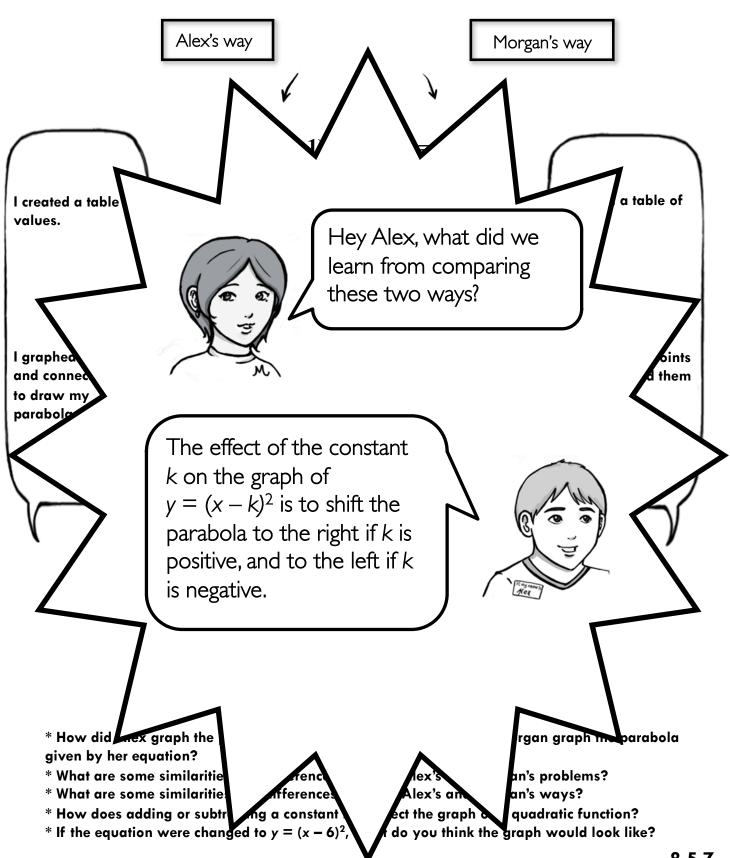
1a	How did Alex graph the parabola given by his 1b How did Morgan graph the parabola given by her
	equation? equation?
2	W/L - t
2	What are some similarities and differences between Alex's and Morgan's problems?
•	W/I
3	What are some similarities and differences between Alex's and Morgan's graphs?
l	
4	How does adding a constant to x^2 affect the graph of a quadratic function?
	TC1 1 1
5	If the equation were changed to $y = (x - 1)^2$, what do you think the graph would look like?

Alex was asked to graph the equation $y=(x+1)^2$, and Morgan was asked to graph the equation $y=(x-1)^2$.



- * How did Alex graph the parabola given by his equation? How did Morgan graph the parabola given by her equation?
- * What are some similarities and differences between Alex's and Morgan's problems?
- * What are some similarities and differences between Alex's and Morgan's graphs?
- * How does adding or subtracting a constant to x^2 affect the graph of a quadratic function?
- * If the equation were changed to $y = (x 6)^2$, what do you think the graph would look like?

Alex was asked to graph the equation $y=(x+1)^2$, and Morgan was asked to graph the equation $y=(x-1)^2$.



1a	How did Alex graph the parabola given by his 1b How did Morgan graph the parabola given by her
	equation? equation?
2	What are some similarities and differences between Alex's and Morgan's problems?
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<u> </u>	what are some similarities and differences between Alex's and Morgan's graphs?
4	How does adding or subtracting a constant to x^2 affect the graph of a quadratic function?
	Trow does adding or substances a constant to w arrest the Stapes of a domaining random
5	If the equation were changed to $y = (x - 6)^2$, what do you think the graph would look like?
5	If the equation were changed to $y = (x = 0)$, what do you think the graph would look like: