

A DISTANCE MODEL FOR SOCIOMETRIC STRUCTURE†

Christopher Winship

Harvard University

The conceptual and mathematical framework of a general model for distance within sociometric structure is described. The model characterizes "balance" in terms of the triangle inequality, in which the distance between two people (A and C) should be less than or equal to the sum of the distances to a third person (B); i.e., $d(A,C) \leq d(A,B) + d(B,C)$. The notion of addition of distances is developed. Different ways of adding distances result in different models of sociometric structure. Two families of models for symmetric graphs are discussed. The general model is extended to asymmetric graphs by generalizing the notion of transitivity. The model's potential for resolving a problem of the transitivity model is then discussed. The general model provides a means of examining the relationship between stratification and clustering in the structure of groups.

During the past twenty years, a number of models have been developed to describe the sociometric structure of small groups. Some of the more successful models are those following the work of Heider (1958), especially the structural balance model of Cartwright and Harary (1956), the clustering model of Davis (1967), and the transitivity model of Holland and Leinhardt (1971). All of these models and most models in sociology are restricted to the analysis of binary relations. Typically, this has meant that they have only considered whether people are friends or not, or whether they like each other or not. These models do not consider differences between being good friends or just friends, or between a lot of liking or a little. This lack of refinement has been characterized as the problem of incorporating the notion of strength of relationship into a model. A relationship is considered stronger than another if it represents a greater

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degree of intensity of relationship, e.g., if it represents a greater degree of friendship or liking, or a greater degree of enmity or disliking.

A number of models incorporating the notion of strength have been proposed. (See Feather, 1967, and Taylor, 1970.) One of the most sophisticated is a model proposed by Cartwright and Harary (1970). All of these models are generalizations of Cartwright and Harary's model of structural balance. None incorporate either the clustering model of Davis (1967) or the transitivity model of Holland and Leinhardt (1971).

Davis (1967, 1970) has argued that the clustering model may be a more appropriate model for sociometric structure than the Cartwright and Harary model of structural balance. An important difference between the two models is that whereas the structural balance model postulates that groups tend to fragment into two cliques, the clustering model postulates only that groups tend to fragment into cliques, the number of which is not specified. Formally, this difference amounts to the prohibition or permissibility of the 003 triad within the sociometric data (see Holland and Leinhardt, 1970, for explanation of this notation of triads). The 003 triad is the triad where no one likes or chooses the others. The Cartwright and Harary model prohibits this triad type, whereas the Davis model permits it. In his analysis of 742 sociograms, Davis (1970) found no evidence indicating that the 003 triad type tended to be infrequent. When he coded the asymmetric relations as positive, he found the 003 triad to be infrequent in 41% of 649 matrices; when he coded the asymmetric ties as negative, he found the 003 triad to be infrequent in only 39% of 722 matrices. This evidence suggests that there is no empirical basis for believing that the Cartwright and Harary model is descriptive of the sociometric structure of small groups. Empirical evidence does support the Davis clustering model and its generalization, the Holland and Leinhardt transitivity model (Holland and Leinhardt, 1972). The model developed in this paper is an extension of these models.

A DISTANCE MODEL

In this paper, a general model for the sociometric structure of affect in small closed groups is developed. These groups and their relationships will be represented as complete, directed graphs with positive weights attached to each directed edge. The nodes represent group members; lines between pairs of nodes represent relationships between group members; and the weights represent the strength of the relations. For most of the discussion, we will assume that the graphs are either undirected or symmetric. Sociologically, this is equivalent to the assumption that person A and person B have the same type of relationship with or feeling toward one another.

The model assumes that relationships between members can be specified and linearly ordered in terms of strength. For example, it should be possible to specify whether people are good friends, friends, acquaintances, or enemies. Alternatively, it should be possible to specify whether people like each other very much, like each other somewhat, feel neutral

toward each other, dislike each other somewhat, or dislike each other very much. The main assumption of this paper is that this linear order of relationships (whether the relationships are degrees of friendship or degrees of liking) should be conceptualized in terms of distance. Two people will be considered closer together, or the distance between them smaller, if there is a greater degree of friendship or liking between them than between two other people. Conversely, two people will be considered farther apart, or at a greater distance from each other, if they dislike each other more or do not like each other as much as two other people. Strength or distance is assumed to be a positive quantity.

Mathematically, the concept of distance can be defined as follows:

Definition: (X, Y, d, \leq)

X is an unordered set of points that represents the people in the group: $X = \{A, B, C, \dots\}$.

Y is an ordered set of all the distances between the points in X: $Y = \{a, b, c, \dots\}$.

d is a function that assigns a distance to all ordered pairs (A, B) in $X \times X$ for $A \neq B$: $d: X \times X \rightarrow Y$.

\leq is a relation that defines the linear ordering of Y: for all a, b, c , elements of Y ($a, b, c \in Y$) either $a \leq b$ or $b \leq a$, and if $a \leq b$ and $b \leq c$ then $a \leq c$.

When we use this mathematical notation, the expression $d(A, B) = m$ means that the distance from A to B, or, equivalently, the degree to which A likes B, is m. Since we are assuming here that B has the same type of relationship with A that A has with B, i.e., $d(B, A) = d(A, B)$, we can say that the distance or the degree of liking between A and B is m.

The notation $a \leq b$ means that the distance a expresses a greater or equal degree of closeness than the distance b. I assume that the order of the alphabet represents the order of Y under \leq ; thus, $a \leq b$, $e \leq f$, and so on; that each letter represents a different distance, unless otherwise indicated; and that $>$, \geq and $<$ have their usual meaning.

Common to the class of models associated with balance theory is the notion that there are certain constraints on the way in which social relationships interconnect. Graphs of groups that are consistent with these constraints are termed balanced (Cartwright & Harary, 1956), clustered (Davis, 1967), or transitive (Holland & Leinhardt, 1971). Groups that are not constrained in the hypothesized fashion are termed, respectively, unbalanced, unclustered, or intransitive. Unconstrained groups are assumed to have both an unstable social structure and to be characterized by tension between group members.

The term "d-balance" will be used to refer to groups that are constrained by the distance model. (The use of the term "balance" is not meant to imply any direct connection with Cartwright and Harary's notion of "structural balance.") This notion is developed by introducing a concept of addition \oplus which is not identical with, but is closely related to the usual concept of addition.

A common axiom for any mathematical structure that incorporates the idea of distance is the triangle inequality. This inequality states that the distance between any two points cannot be longer than the sum of the distances from each of two points to any intermediary point. In mathematical notation, the idea is expressed as follows:

For any three points A,B,C and a distance function d ,
 $d(A,C) \leq d(A,B) + d(B,C)$.

The triangle inequality characterizes the notion of balance. If a structure satisfies the triangle inequality, it will be said to be balanced; if it does not, it will be said to be imbalanced.

This notion of d-balance can be best understood if we look at a physical analogy. For the moment, let us assume that the physical distance that separates two people is a measure of their degree of liking. Thus, if they stand very close to each other, this indicates that they like each other very much; if they stand very far apart, this suggests that they dislike each other very much. Let us now think about triads. On the one hand we could imagine a triad in which everyone could stand the distance they wanted to from each other. This would be a balanced triad. On the other hand, we could imagine a triad in which it is impossible for everyone to stand the distance he would like to stand from the others. This would be an imbalanced triad. Figures 1 and 2 illustrate. In Figure 1, A and B stand 3 feet apart, B and C stand 2 feet apart, and A and C stand 6 feet apart. Since

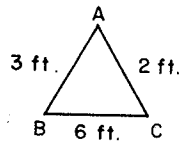


Figure 1. Imbalanced triad.

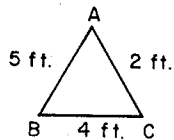


Figure 2. Balanced triad.

the distance between B and C is greater than the sum of the distances between the other two points ($6 > 3 + 2$), someone must stand closer or farther apart than he wants to. Presumably, this situation would create "tension," the principle idea behind the work of Heider (1958). In Figure 2, everyone can stand the distance he wants to from the others since the triangle inequality is satisfied ($5 < 4 + 2$, $4 < 5 + 2$, $2 < 4 + 5$).

The definition of d-balance is:

Definition: A structure is d-balanced if and only if for all A,B,C ∈ X†

$$d(A,C) \leq d(A,B) \oplus d(B,C)$$

What does the notion of d-balance mean in terms of social structure? For the moment, assume that the distance "a" between two people indicates that they are good friends, the distance "b" that they are friends, and the distance "c" that they are enemies. One might assume here that there is some social-psychological principle such that all good friends of good friends should be good friends. Mathematically, this is equivalent to saying that $a \oplus a = a$. If A and B are good friends [$d(A,B) = a$], B and C are good friends [$d(B,C) = a$], and A and C are also good friends [$d(A,C) = a$], then this triad is d-balanced. Formally, this is because $d(A,B) < d(A,C) \oplus d(B,C)$, $d(A,C) < d(A,B) \oplus d(B,C)$, and $d(B,C) < d(A,B) \oplus d(A,C)$. These inequalities all follow from the fact that $a \geq a \oplus a$. If, however, B and C are only friends [$d(B,C) = b$], the A,B,C triad fails to satisfy the principle that all good friends of good friends should be good friends. Its failure is formally equivalent to the fact that it is d-imbalanced. The triad is d-imbalanced, as $d(B,C) > d(A,B) \oplus d(A,C)$, or, equivalently, $b > a \oplus a$, which follows from $b > a = a \oplus a$.

When different degrees of a relationship can occur, a number of interrelationships must be specified and there are a number of different ways that they could be defined. Equivalently, within the graph of a group where there are a number of different distances, there are a number of sums that need to be specified and a number of things that these sums could equal.

TABLE 1
Operator Table for Propositions 1-9

\oplus	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

The easiest way to see how distances should be added is to use an operator table. Table 1 is an example. It specifies that $a \oplus a = a$, $a \oplus b = b$, and $b \oplus b = b$, etc. Assuming d-balance, the nine entries in the table are equivalent to nine propositions about social structure.

†Alternatively, d-balance may be defined for incomplete graphs in terms of paths. A graph may be said to be d-balanced if $d(A,C) \leq d(A,X_1) \oplus d(X_1,X_2) \oplus \dots \oplus d(X_n,C)$ for all A,C and X_i . It can be shown that the two definitions are mathematically equivalent for complete graphs.

- (1) $(a \oplus a = a)$ is equivalent to a good friend of a good friend should be a good friend, but not a friend or an enemy.
- (2) $(a \oplus b = b)$ is equivalent to a good friend of a friend should be either a good friend or a friend, but not an enemy.
- (3) $(a \oplus c = c)$ is equivalent to a good friend of an enemy may be either a good friend, a friend or an enemy.
- (4) $(b \oplus a = b)$ is equivalent to a friend of a good friend should be either a good friend or a friend, but not an enemy.
- (5) $(b \oplus b = b)$ is equivalent to a friend of a friend should be either a good friend or a friend, but not an enemy.
- (6) $(b \oplus c = c)$ is equivalent to a friend of an enemy may be either a good friend, a friend, or an enemy.
- (7) $(c \oplus a = c)$ is equivalent to an enemy of a good friend may be either a good friend, a friend, or an enemy.
- (8) $(c \oplus b = c)$ is equivalent to an enemy of a friend may be either a good friend, a friend, or an enemy.
- (9) $(c \oplus c = c)$ is equivalent to an enemy of an enemy may be either a good friend, a friend, or an enemy.

It is clear from the above nine propositions that an operator table specifies which triad types are allowed within the graph of a d-balanced group and which are not. Figure 3 shows the dichotomization for operator Table 1.

The relationship between the nine propositions and the triads contains a certain subtlety. Some propositions specify certain triad types as permissible, whereas others rule them out. For example, proposition (7) specifies that an enemy of a good friend may be a good friend. This is equivalent to stating that triad (10) is permissible. This triad type is not allowed, however, according to proposition (1), which specifies that a good friend of a good friend cannot be an enemy. The subtlety is that a proposition specifies only what is possible with respect to one inequality, but this does not rule out the possibility that another inequality may prohibit the specified triad. For instance, proposition (7) is equivalent to the statement that $a \leq c \oplus a$, $b \leq c \oplus a$, and $c \leq c \oplus a$, all of which are true for triad (10). But proposition (1) prohibits triad (10), as $c \geq a \oplus a$. Thus, with each triad A,B,C, there are three different inequalities that must be tested: $d(A,B) \leq d(A,C) \oplus d(B,C)$, $d(A,C) \leq d(A,B) \oplus d(B,C)$, and $d(B,C) \leq d(A,B) \oplus d(A,C)$. If the triad satisfies all three inequalities, then it is permitted, or d-balanced.

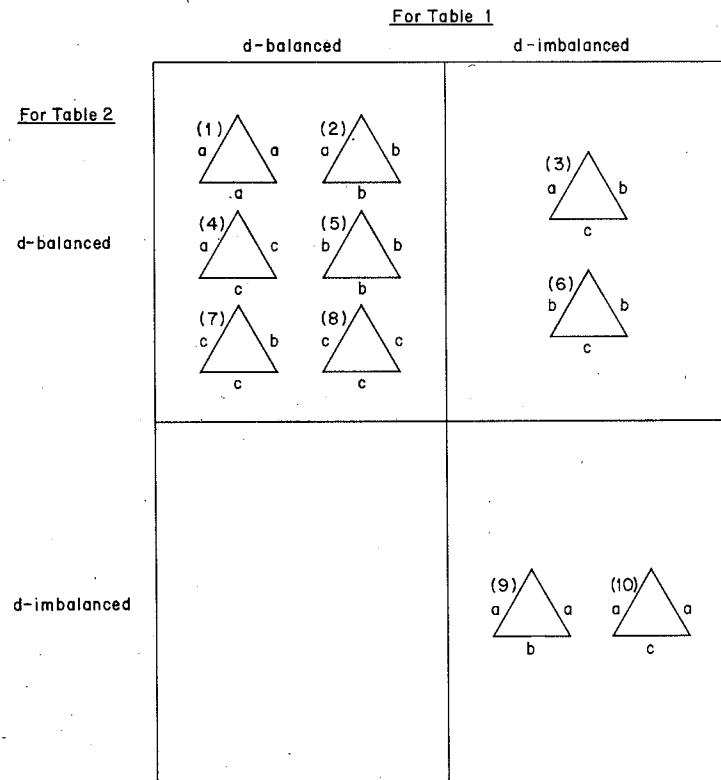


Figure 3

Different operator tables add distances in different ways. Operator tables, thus, are *specific* models for sociometric structure under the *general* model. Table 2 differs from Table 1. For example, Table 2 asserts that $b \oplus b = c$, where Table 1 asserts that $b \oplus b = b$. Sociologically, Table 2 asserts that a friend of a friend may be either a good friend, a friend, or an enemy, whereas Table 1 asserts that a friend of a friend should be either a good friend or a friend, but not an enemy. Tables 1 and 2 make different assertions about sociometric structure and different assertions about which triads are d-imbalanced. Figure 3 indicates which triads are allowed relative to Table 2.

TABLE 2

⊕	a	b	c
a	a	c	c
b	c	c	c
c	c	c	c

Figure 3, shows that Table 2 allows all the triads that Table 1 does, but that Table 1 prohibits triads permitted by Table 2. In other words, the triads that Table 2 prohibits are a subset of the triads that Table 1 prohibits. When this is the case, then one table is more restrictive than the other. For example, Table 1 is more restrictive than Table 2.

Mathematically, one table is more restrictive than another table if each entry in that table is less than or equal to the respective entry in the other table. Table 1 is more restrictive than Table 2 since in each entry where they are not equal, the entry in Table 1 is less than the entry in Table 2.

TABLE 3

⊕	a	b	c
a	b	b	c
b	b	b	c
c	c	c	c

It is not always the case that given two operator tables, one is more restrictive than the other. For example, operator Table 2 prohibits some of the triad types that Table 3 allows and Table 2 allows some of the triad types that Table 3 prohibits. (The reader is invited to work out the details.) The notion of restrictiveness produces a partial ordering of operator tables. This means that given three Tables I, II, and III, if I is more restrictive than II, and II is more restrictive than III, then it follows that I is more restrictive than III. This is because restrictiveness is simply a specific form of the notion of subset.

Figure 4 illustrates the partial ordering of all operator tables that contain only three distances.

Finding a specific model for social structure or for a particular type of group involves choosing the most restrictive model possible. If social structure was hypothesized to conform to the model at the bottom of the hierarchy in Figure 4, the table with all c's, then social relations could interrelate in any which way. On the other hand, saying that social structure conforms to the model at the top of the structure is equivalent to saying that social structure has as many constraints as the general model will allow. Clearly, the more restrictive the model, the better the nature of sociometric structure has been specified.

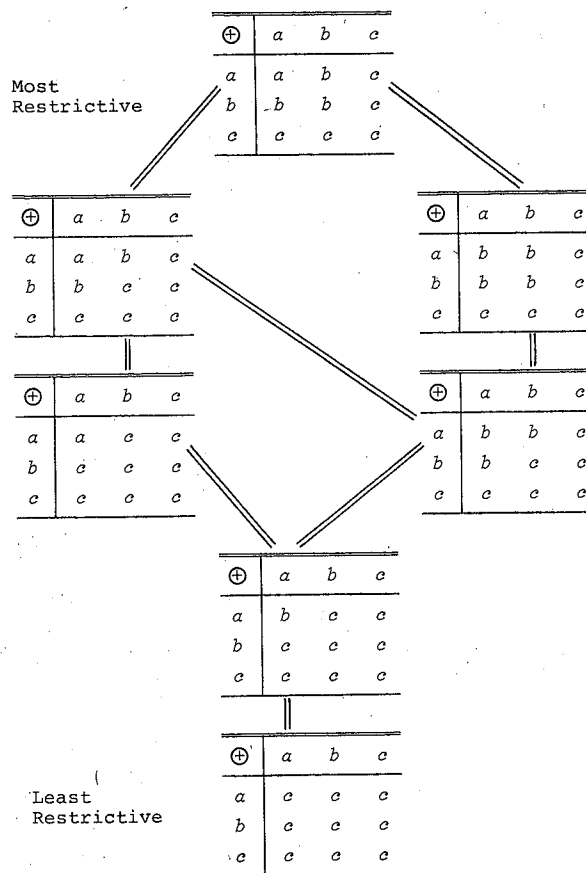


Figure 4. Partial ordering of all three distance models.

GENERAL NATURE OF OPERATOR TABLES

In the above treatment of operator tables and \oplus , it may be obvious that only a limited set of operator tables and propositions are dealt with. An axiomatic treatment of operator tables is given in Appendix A. Most of the axioms are not of sociological significance. They are incorporated into the model to insure that distances are added together in a consistent manner. One axiom, however, is of sociological importance. It is examined next.

Axiom: For $a, b \in Y$, the $\max(a, b) \leq a \oplus b$

The axiom states that for any distances a, b , the greater of the two elements must be less than or equal to the sum of the two. It rules out the possibility of having "negative" distances - that is, a distance c , which when added to another distance, d , sums to a distance smaller than d . This axiom has important sociological implications. Abstractly, it says that people's relationships are constrained by the interrelationship of their relations with other close people and by the relations of those people with close others. They are not constrained by the interrelationship of their relations with those who are distant. This is implicit in Davis' work and in his discussion of balance theory (Davis, 1967 and 1970). It can be better understood by examining the following two propositions.

- (1) ($a \oplus a = a$) a good friend of a good friend should be a good friend.
- (2) ($c \oplus c = a$) an enemy of an enemy should be a good friend

Proposition (1) is an example of people's relations being constrained by their relations with close others and relations these others have with the people that they are close to. Proposition (2), which is prohibited by the axiom [$\max(c, c) \leq c \oplus c$], is an example of people constrained by relations with distant people and their relations with other distant people.

This issue is the central difference between Davis' clustering model and the Cartwright and Harary model. For Davis, liking of people (i.e., close people) constrains one's relationships. For Cartwright and Harary, one's relations can constrain because of liking or disliking. This is a point of substantive significance. The model in this paper adopts Davis' point of view.

CLIQUE MODELS

The preceding has examined the microstructure of the distance model, i.e., the different types of triads that are permitted by different operator tables. Now two different types of specific models within the general model are examined: clique models and proximity models.

The notion of a clique is as old as sociometry. Within the class of models that has grown out of the work of Heider (1958), the word *clique* has had a very specific meaning. Cartwright and Harary (1956) define a clique as a set of people such that everyone in the set likes everyone else and no one likes anyone outside of the set. Clique is defined here analogously.

Definition: A clique is a set of points (Q) in X such that the distance between any points [$d(A, B)$] in Q is less than or equal to some element (g) of Y , and the distance between a point in Q and a point not in Q is always greater than g .

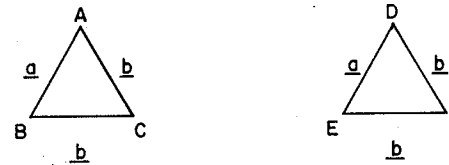
Using the operator table presented in Table 4 and assigning the category "like" to distance "a" and the category "dislike" to distance "b," the two definitions of cliques become equivalent with respect to Table 4. All of the people within a clique will like one another (will be of distance "a" from each other), and between cliques, they will dislike one another (will be of distance "b" from each other). Because the model allows for more than two distances, thereby allowing for cliques to form at more than one distance, the notion of clique needs qualification.

TABLE 4

\oplus	a	b
a	a	b
b	b	b

Definition: A g -clique, A , is a clique such that all members of A are no more than a distance g from each other, and there does not exist a $f < g$ such that all members of A are no more than a distance f from each other.

This definition implies that in the distance model there can be cliques within cliques. Figure 5 illustrates. In Figure 5, the set A, B, C is a b -clique, the set A, D is an a -clique, and the whole space X is the trivial c -clique.



All pairs whose distance from each other is not specified (e.g. A and E) are a distance c from one and other

Figure 5

Fundamental to the models that have been developed from the work of Heider is the notion that cliques exist within a group because of the type of constraints that have been put on the relationships within the group. A similar development can be provided for the distance model by introducing the notion of an idempotent element. An element is idempotent if its sum with itself is itself (e.g., $b \oplus b = b$). Sociologically, an idempotent distance (e.g., b), when taken together with the assumption of d -balance, requires that for any triad,

two sides of which are that distance (e.g., $d(A,B) = b$ and $d(B,C) = b$), the third side be no greater than the distance [e.g., $d(A,C) \leq b$]. The following theorem states this in terms of cliques

Theorem: Given a d -balanced graph (X, Y, d, \leq, \oplus) , for each idempotent element h there exists (an) h -clique(s).

Proof of the theorem is omitted. Descriptively, the theorem says that given a d -balanced graph and an idempotent element h , the graph can be subdivided into subsets such that within each subset all the people are distance h or less from each other (i.e., they like each other at least h) and between subsets people are more than a distance h from each other (i.e., they do not even like each other h).

A model is a clique model if it contains an idempotent element in addition to the largest element of Y . The axioms for operator tables guarantee that the largest element of Y is always idempotent. The Davis (1967) clustering model is an example of a clique model. Table 4 is the appropriate operator table. Davis defines a clustering of a graph as a partition of a graph into subsets such that people within each subset like each other and people between each subset dislike each other (Davis, 1967). If distance a in Table 4 is taken as like and distance b as dislike, then the above theorem guarantees that a graph d -balanced with respect to Table 4 is clustered in Davis' sense. The equivalence between the two models can also be seen by looking at triads. Davis' theorem 2 states that a complete graph is clustered if it contains no triads with exactly one negative relation (Davis, 1967). This is precisely the triad that operator Table 4 prohibits ($b > a \oplus a$).

Table 5 contains an operator table that has three additional idempotent elements. It represents a structure which is an example of a hierarchical tree, an idea common to the taxonomic literature. Figure 6 represents the hierarchical tree consistent with Table 5. Table 6 presents the matrix of relations of the seven people in Figure 6. A hierarchical tree is the extreme version of a clique model.

TABLE 5
Example of a Hierarchical Tree

\oplus	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	d
d	d	d	d	d

A basic problem underlies the clique type of model. Such a model suggests that groups tend to fragment into opposing and unconnected cliques. Although each clique may have a great deal of internal consistency, a group so fragmented is unlikely

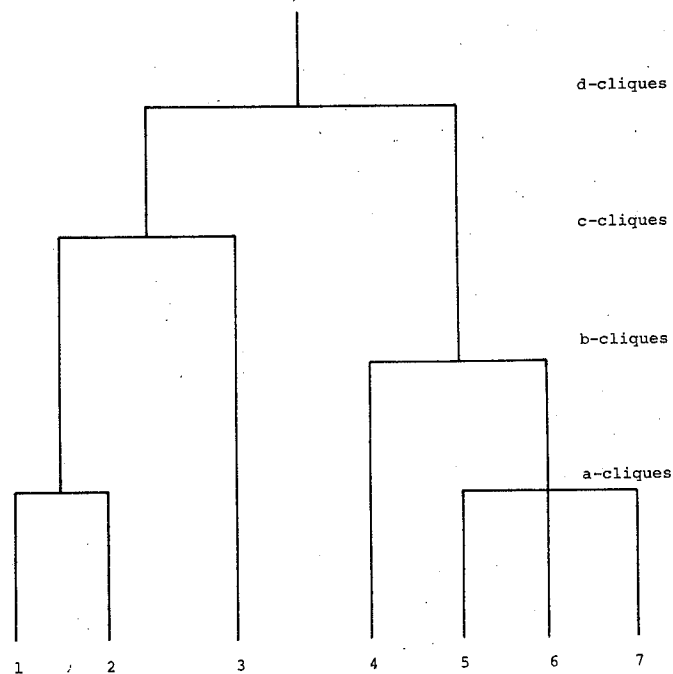


Figure 6

to be either typical, tension free, or desirable (see Davis, 1967; Granoveter, 1973; Killworth, 1974). Another type of model may provide a solution.

TABLE 6
Sociometric Data for Figure 6

	1	2	3	4	5	6	7
1	X	a	c	d	d	d	d
2	a	X	c	d	d	d	d
3	c	c	X	d	d	d	d
4	d	d	d	X	b	b	b
5	d	d	d	b	X	a	a
6	d	d	d	b	a	X	a
7	d	d	d	b	a	a	X

PROXIMITY MODELS

A basic idea in sociometry is that the amount of interaction between two people is directly related to the degree of liking between them. Homans made this point in *The Human Group* (1950), and it was later restated by Davis and Leinhardt (1972) and by Granoveter (1973). Granoveter (1973) points out that this relationship between interaction and liking provides an important way of thinking about positive sentiment. Consider the percentage of time that two people spend with each other as a direct measure of their degree of liking for each other. Assume that A and B spend 60% of their time together and B and C spend 40% of their time together. A and C should spend 24% of their time together. If A and C spend less time together, then they would be avoiding each other, an indication of tension. If they spent more than 24% of their time together, there would be no problem. (Granoveter, 1973, develops a model similar to this, p. 1362.)

An important distinction between the interaction model and the clique model is that in the former a triad is considered balanced even if the third side of a triad is less friendly than either of the other two sides. Alternatively, we can say that a good friend of a good friend should be at least a friend. With respect to the distance model, this idea implies that we have a model which has no idempotent elements except the largest one. When this is true for all distances except the largest, the model is a proximity model.

Table 7 is an example of such a model. Comparing Table 7 to Table 5, shows that the former is less restrictive than the latter. Proximity models are always less restrictive than clique models. The proximity model is important because it places restrictions on the way in which relationships interrelate, but it does not suggest that groups tend to fragment into cliques.

The proximity model provides an alternative to clique models. It does not postulate that groups fragment into cliques; however, it does not prohibit this. It postulates that cliques, when they exist, exist because of random variations in the group and not because of any socio-psychological constraints on the relationships in the group.

TABLE 7

Example of a Proximity Model

	a	b	c	d
a	b	c	d	d
b	c	c	d	d
c	d	d	d	d
d	d	d	d	d

ASYMMETRIC STRUCTURES

Until now, only symmetric structures, i.e., structures in which $d(A,B) = d(B,A)$, have been discussed. Symmetry may not, however, always be a reasonable assumption. Thus, A may like B very much, but B may not like A at all. Sociometric structures with asymmetric ties have been considered in the literature. Perhaps one of the most coherent and certainly the most relevant to the notions developed here is Holland and Leinhardt's revival of the concept of transitivity as an organizing principle for sociometric structure (Holland and Leinhardt, 1971).

Holland and Leinhardt define a graph to be transitive if (1) every person chooses themselves; and if (2) person A chooses person B and person B chooses person C, then A also chooses person C. Holland and Leinhardt show that the notion of a transitive graph leads to a partial ordering of cliques, such that within each clique everyone chooses everyone else and everyone within a clique chooses everyone in the cliques above it in the hierarchy. The clique at the top of the hierarchy contains the people who have been chosen the most and who are considered to have the highest status. People in lower level cliques have status intermediate between those in the cliques above them and those in the cliques below them. Holland and Leinhardt note that the model permits isolated cliques that neither choose anyone else nor are chosen by anyone else. These cliques have undetermined status.

In order to develop the model presented here in terms of asymmetric structures, some of the concepts that have been previously introduced need to be reexamined. What is meant by $d(A,B)$ and $d(B,A)$ must be specified. $d(A,B)$ will represent A's degree of liking for B and $d(B,A)$ will represent B's degree of liking for A. If $d(A,B) < d(B,A)$, this would presumably indicate that B was of higher status than A, since A liked B more than B liked A. Next, the definition of d-balance must be examined. A graph was defined as d-balanced if and only if for all $A, B, C \in X$, $d(A,C) \leq d(A,B) \oplus d(B,C)$. What the definition of d-balance states is that each triplet as defined by Davis, Holland and Leinhardt (1971) must satisfy the triangle inequality. A triplet is the two directed relations from a single point in a triad, plus one of the additional directed relations going from the second point in the triad to the third. Each triad has six triplets.

Table 4 shows that the assumption of asymmetric d-balance is equivalent to Holland and Leinhardt's notion of transitivity. Assume that $d(A,B) = a$ means that A likes or chooses B and that $d(A,B) = b$ means that A does not like or choose B. Then a graph is d-balanced according to the table if when A chooses B [$d(A,B) = a$] and B chooses C [$d(B,C) = a$], then A chooses C [$d(A,C) = a$], the definition of transitivity [$d(A,C) \leq d(A,B) \oplus d(B,C)$].

What is the sociological importance of the proximity type of operator table that we considered in the symmetric case? One of the objections that can be raised about the transitivity-

TABLE 8
Data for Figure 7

	1	2	3	4	5	6	7
1	X	a	a	b	c	c	c
2	a	X	a	b	c	c	c
3	c	c	X	a	b	b	b
4	d	d	d	X	a	a	a
5	d	d	d	b	X	a	a
6	d	d	d	b	a	X	a
7	d	d	d	b	a	a	X

ity model (White, 1973) is that it requires those at the bottom of a hierarchy to choose everyone. The proximity models offer a solution to this dilemma. Consider Table 7. Assume that a partial ordering of people in terms of status exists. Assume also that for every person A, the distance to the individuals in the next higher clique, the B's, is equal to *a*. This might be interpreted as A likes every B very much. If the model is to be *d*-balanced with respect to Table 7, how much must A like the C's, the people in the higher clique? A must be distance *b* from the C's, and this might be interpreted as A only likes the C's. The person next up in the hierarchy only have to be a distance *c* from A, which might be called indifference. Proximity models, then are a much more appropriate way to look at groups in that they maintain constraints on people's choices, but they do not force those on the bottom to choose or like everyone of higher status.

There is at least one more substantive issue that the distance model is capable of elucidating. This is the relationship between hierarchical structure and cliquing structures. In their paper on transitivity, Holland and Leinhardt (1971) argue that they have found a single mechanism to explain stratification and clustering in sociometric structures. They contend that it is the transitivity of the mutuals that forms cliques and the transitivity of the asymmetrics that forms hierarchical aspects of the structure. They argue that previously these two properties of sociometric structure have been treated differently (see Homans, 1950; Brown, 1965). But, Holland and Leinhardt's claim seems tautological. Since transitivity is a necessary condition for the existence of both cliques and hierarchies, there is no logical way to separate the two notions in their model. This is not the case in the distance model. Here there is no reason at all why the symmetrical structure of groups might be constrained in a way different from the hierarchical structure. The different ways that the interrelationship between these two types of structure might be considered are endless. One example of how the two types of structures might differ is considered below.

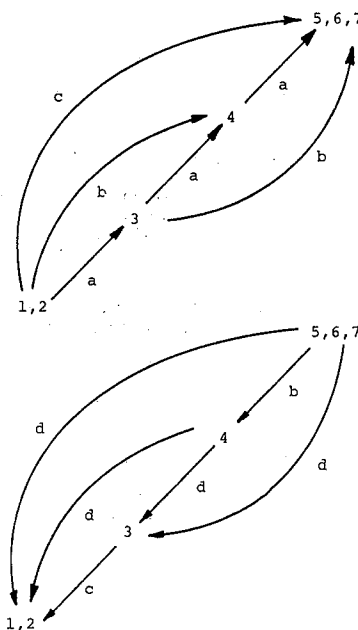


Figure 7

Consider the case in which there are seven people. Table 8 contains the raw distances between each of the individuals. In this case, the symmetric distance between two people is defined as the maximum of the two distances between them, i.e., symmetric distance = $\max [d(A,B), d(B,A)]$. This is a tenable position: If we have two people, one of whom would like to be friends and one who would not, then it is plausible to assume that they will not be friends. Table 6 contains the symmetric distances. When the operator tables for both the raw and the symmetric matrix are specified, the symmetric structure is clustered (Table 5) and the hierarchical structure is a proximity model (Table 7). Figures 6 and 7 illustrate the symmetric and hierarchical structures of the group.

I am not maintaining that stratification and clustering are two different processes in the sociometric structure of groups. What I want to suggest is that the distance model provides a very strong way of deciding whether this is true. In empirical data, if one operator table is found to be sufficient for describing both symmetric and asymmetric structure, then strong evidence exists that stratification and clustering are the same type of process. If two different operator tables are required, then this is evidence that they are in some sense two different types of processes.

CONCLUSION

My aim has been to outline the basic components of a model for sociometric structure that includes the notion of strength of relationship, or distance. The model provides a natural way to generalize the Davis (1967) clustering model and Holland and Leinhardt's (1971) transitivity model and a theorem is provided indicating the relationship between distance and cliquing. Finally, it is shown that specific models that are qualitatively different from clustering and transitivity models, can be formulated. It is argued that these models provide potential solutions to some issues.

APPENDIX A: AXIOMS

The general distance model contains twelve axioms. Only axiom 10 is of sociological relevance.

1. The set Y of distances is finite.
2. The set X of people is finite.

For any three elements, a, b, c , of the distance set Y:

3. Associativity: $a \oplus (b \oplus c) = (a \oplus b) \oplus c$
4. Closure: if $a, b \in Y$ then $a \oplus b \in Y$ Semigroup
Axioms
5. Commutativity: $a \oplus b = b \oplus a$
(necessary only for symmetric structures)
6. Completeness: either $a \leq b$ or $b \leq a$ Linear
Order
Axioms
7. Reflexivity: $a \leq a$
8. Strictness: if $a \leq b$ and $b \leq a$ then $b = a$
9. Transitivity: if $a \leq b$ and $b \leq c$ then $a \leq c$
10. Positiveness: sum of two numbers if greater than or equal to either number: $\max(a, b) \leq a \oplus b$
11. \oplus preserves order under \leq
if $a \leq b$ then $a \oplus c \leq b \oplus c$ and $c \oplus a \leq c \oplus b$
12. Existence: for all $a \in Y$ $\{(X, Y) / d(X, Y) = a\}$ is not empty.

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