

# Efficient determination of soft spots in amorphous solids only using local structural information

Ekin Dogus Cubuk

Samuel Schoenholz, Jennifer Rieser,  
Brad Malone, Joerg Rottler, Doug  
Durian, Andrea J Liu, Efthimios  
Kaxiras



HARVARD  
UNIVERSITY



Penn  
UNIVERSITY of PENNSYLVANIA

# Introduction

- Soft spots are structurally different.
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\*L. Manning & A. Liu, PRL **107**, 108301 (2011)

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- We propose a method that can efficiently predict regions vulnerable to rearrangement purely from local geometric quantities.
- We look at where rearrangements occur, and ask what was structurally different before the rearrangement.

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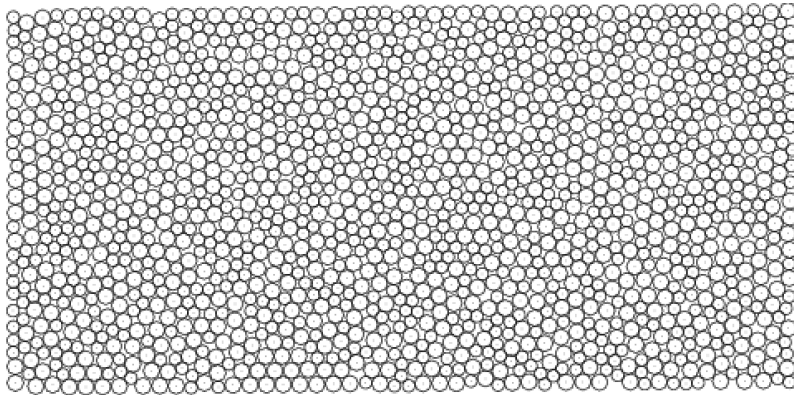
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- calculate many descriptors for every particle's local environment

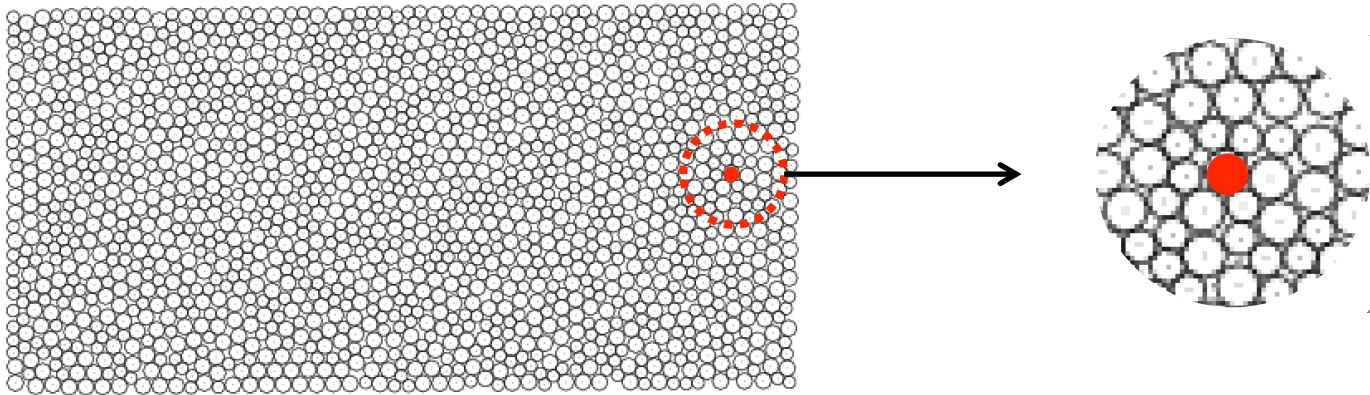
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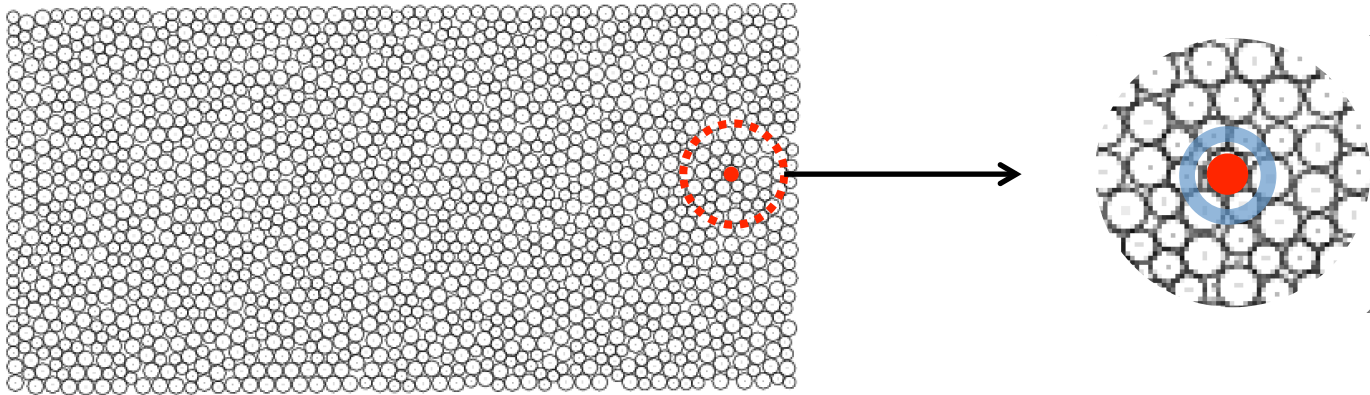
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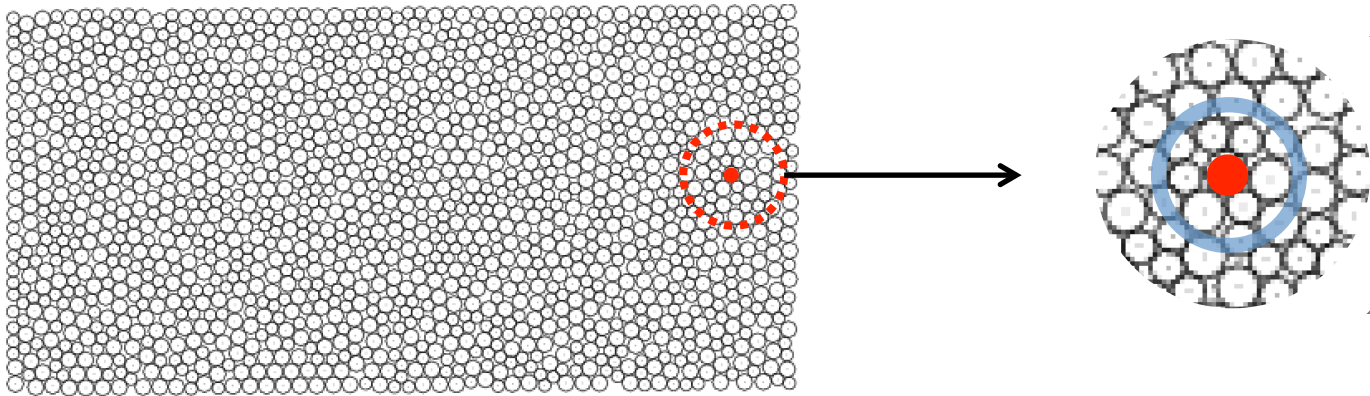
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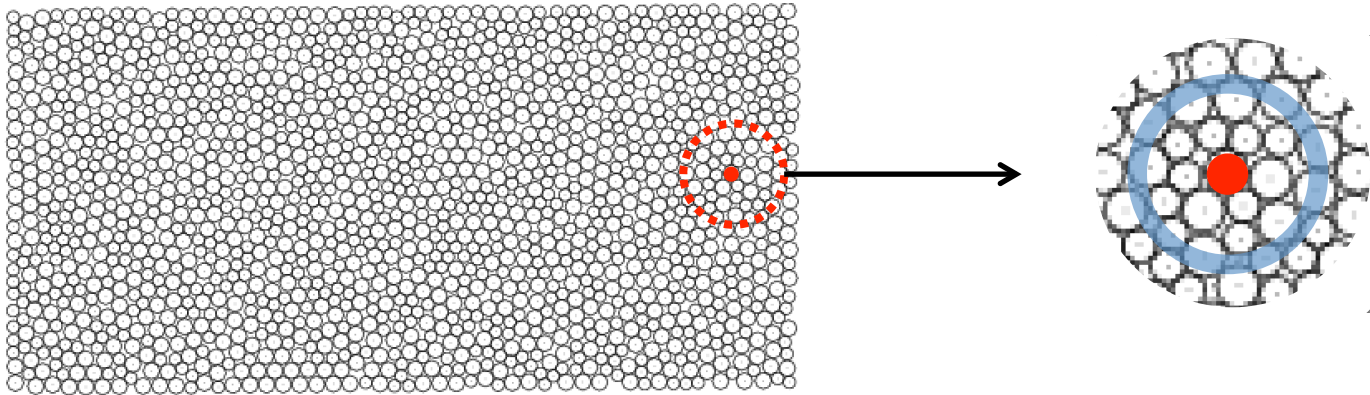
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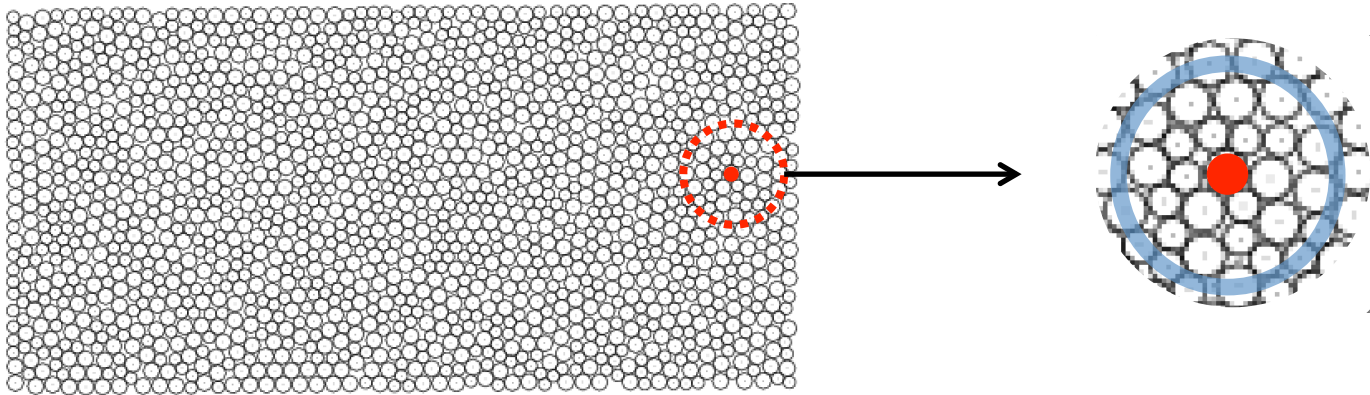
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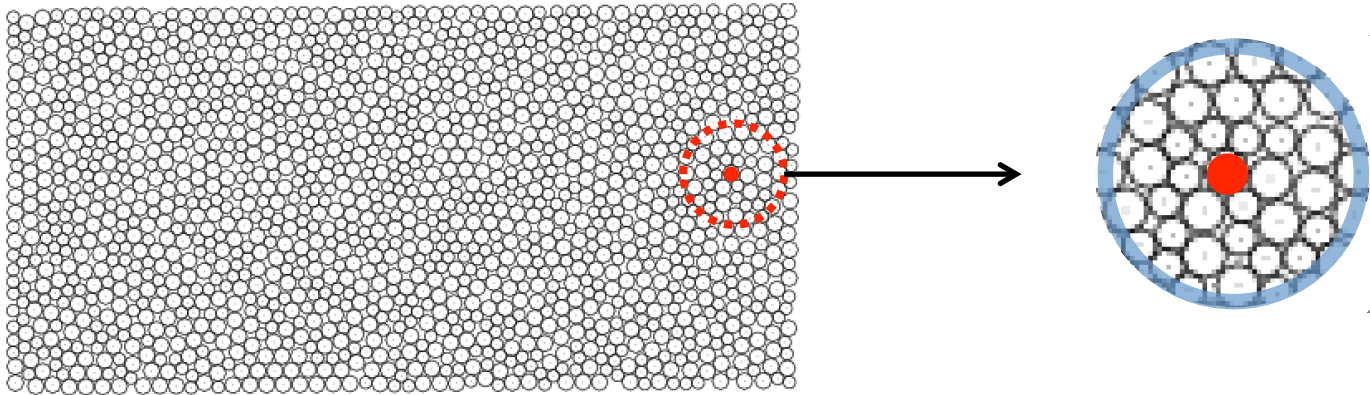
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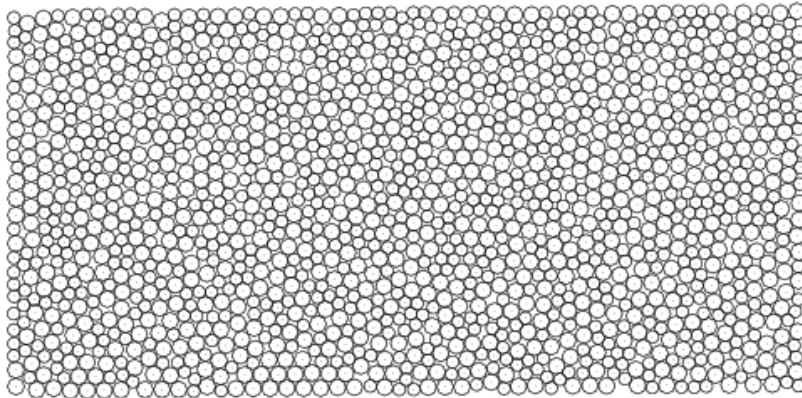
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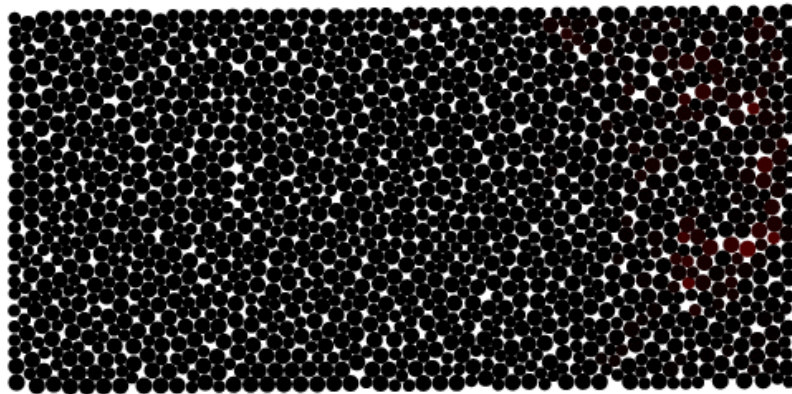
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- observe the system and record plastic behavior
- learn (from examples) the structural quantities that make up a soft spot

	Descriptors ( $t_0$ )	$D^2_{\text{Min}}(t_0, \Delta T)$	Rearranged?
Particle 1	3.2,4.5,1.5 .....	1.8	Yes
Particle 2	1.3,5.6,7.2 .....	0.6	No
Particle 3	1.2,4.2,1.4 .....	0.2	No
⋮	⋮	⋮	⋮



We propose to identify soft spots by supervised learning:

- calculate many descriptors for every particle's local environment
- observe the system and record plastic behavior
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Potential advantages of this method:

- Extremely fast,  $O(N)$  scaling
- Does not need vibrational modes
- The system does not need to be quenched
- Does not require the Hamiltonian

# The local descriptors

- Calculate descriptors for each particle  $i$
- They are sums over neighbors  $j$  and  $k$  of  $i$
- Constants  $\mu$ ,  $\lambda$ ,  $\zeta$  and  $\xi$  are varied

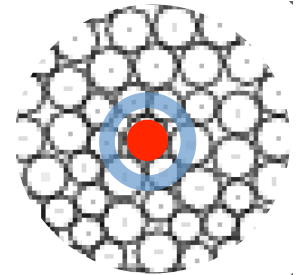
$$G^1(i; \mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \sum_j e^{-(R_{ij}-\mu)^2/2\sigma^2}$$

$$G^2(i; \xi, \lambda, \zeta) = \sum_{j,k} e^{-\xi(R_{ij}^2+R_{ik}^2+R_{jk}^2)} (1 + \lambda \cos \theta_{ijk})^\zeta$$

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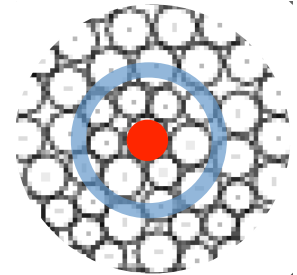


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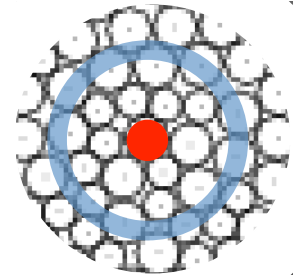


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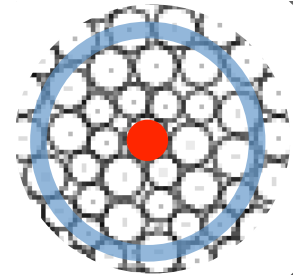


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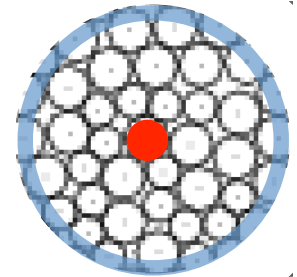


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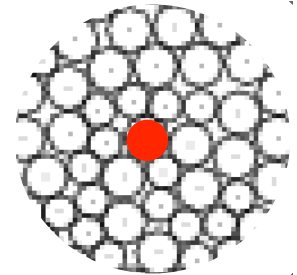


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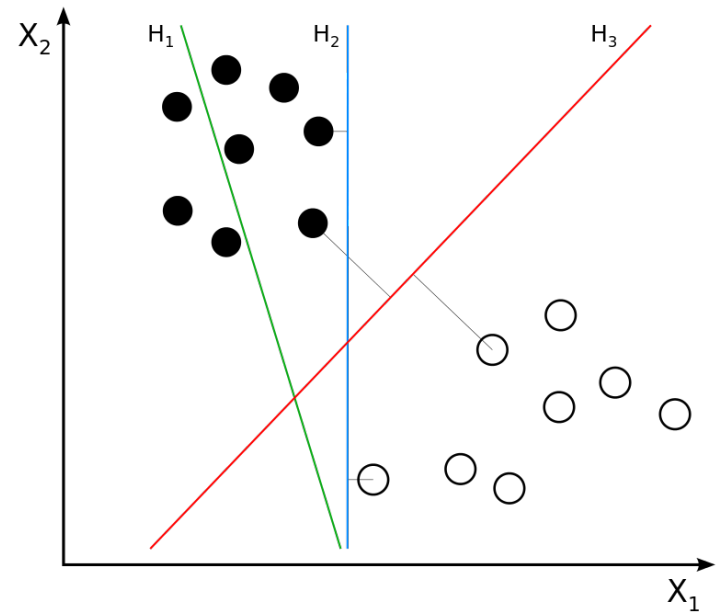


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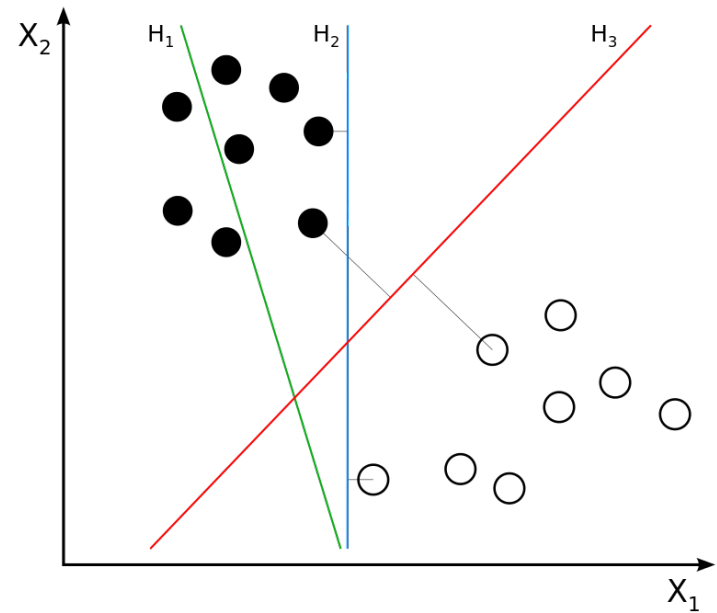
# Support Vector Machines

- A simple supervised learning model.



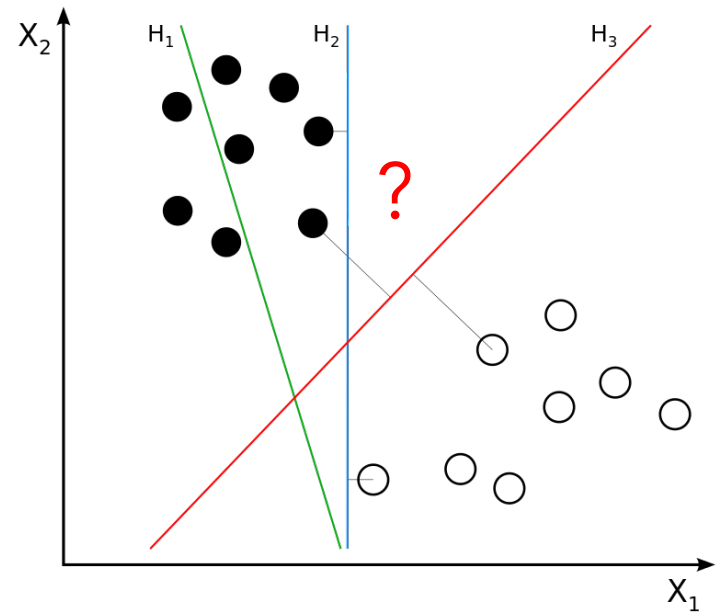
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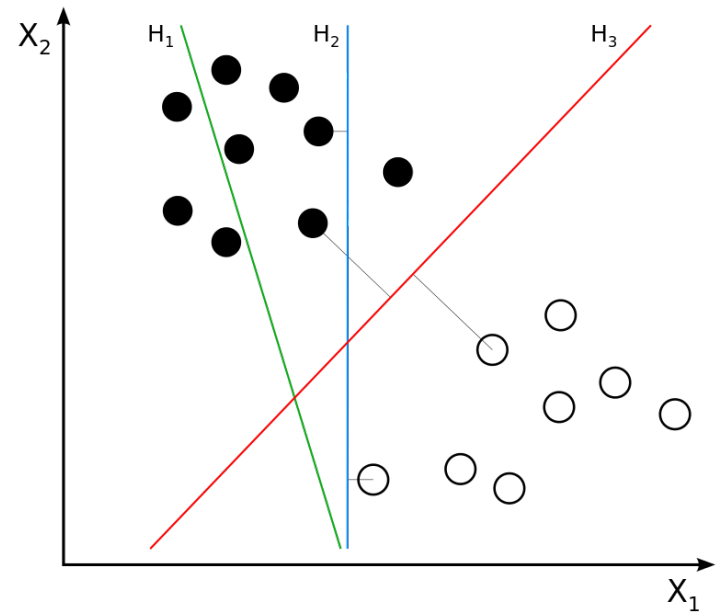
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Simulation of a 2D glass being sheared (LJ temperatures 0.1 to 0.4).

MD simulations done by Joerg Rottler.

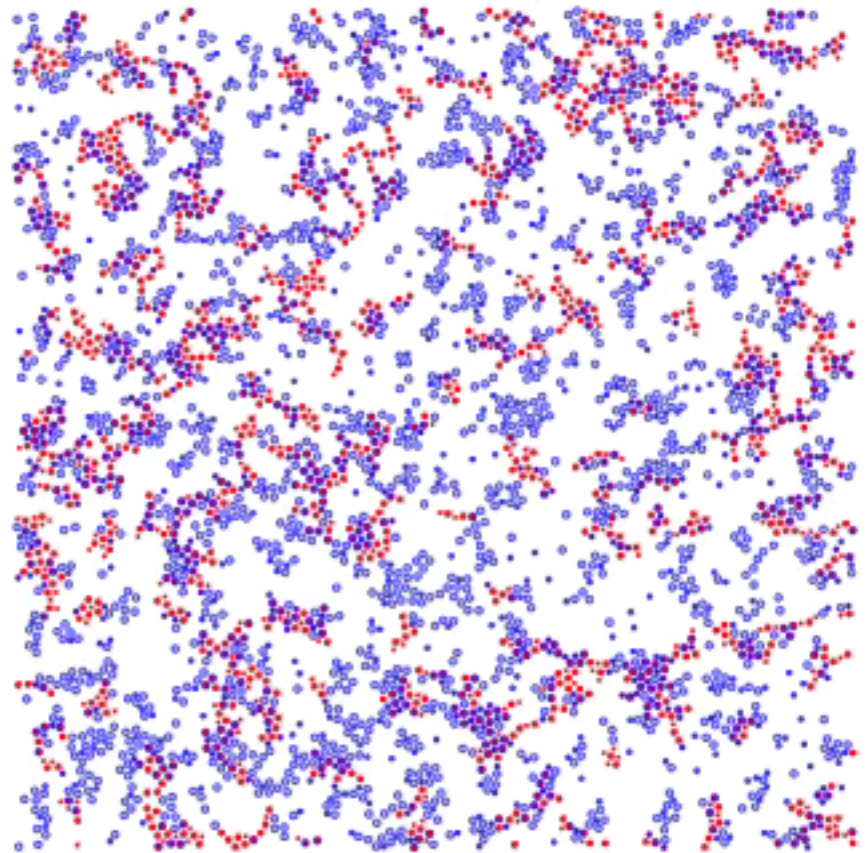
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Blue → machine learning soft spots  
Red → phonon method soft spots

Our soft spots are very robust until rearrangement.



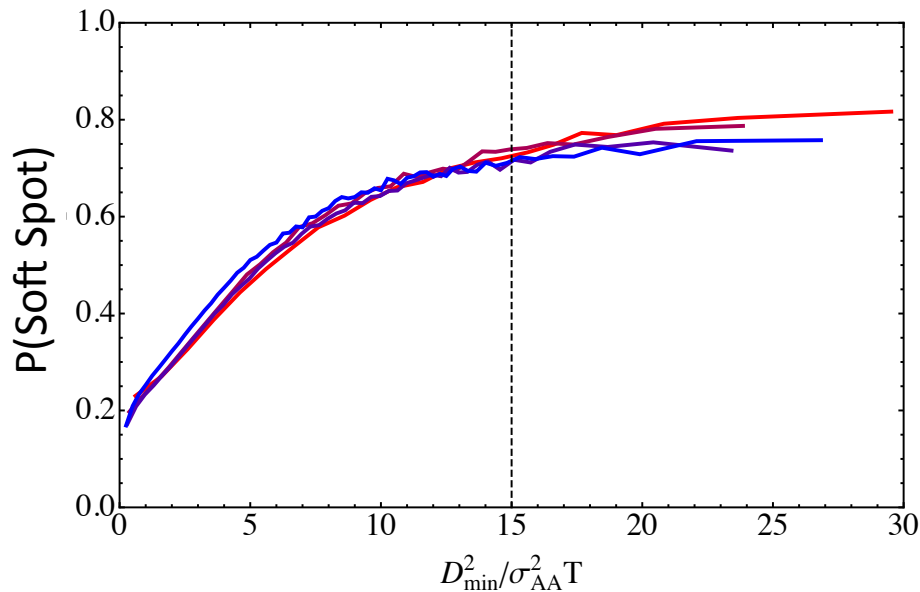
# Results (sheared system)

- For the simulation of a sheared glass,  
(for  $T = 0.1, 0.2, 0.3$  and  $0.4$ ) we correctly predict
  - 78% of the rearrangements
  - with a 24% soft spot coverage

20K training samples  
20M test samples  
 $T_g = 0.33$

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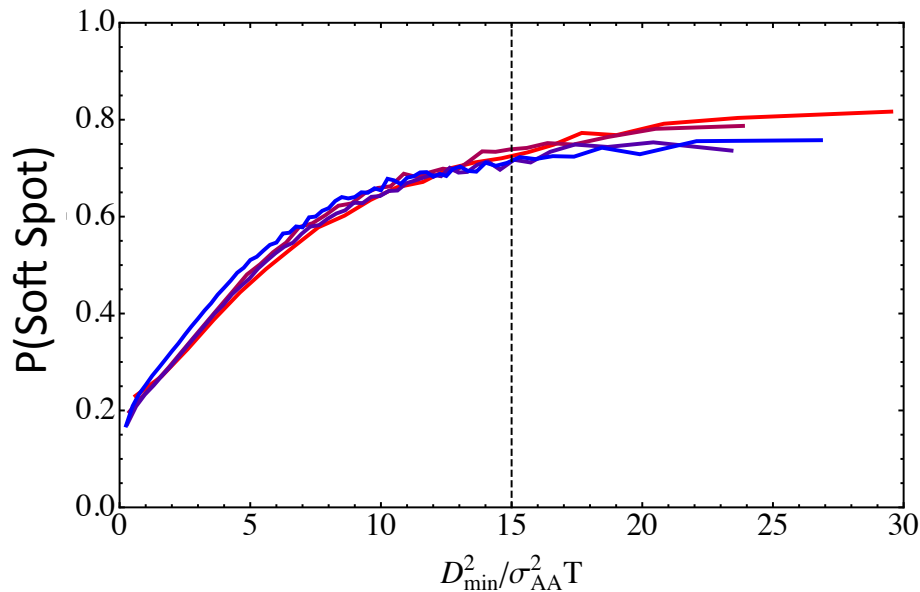


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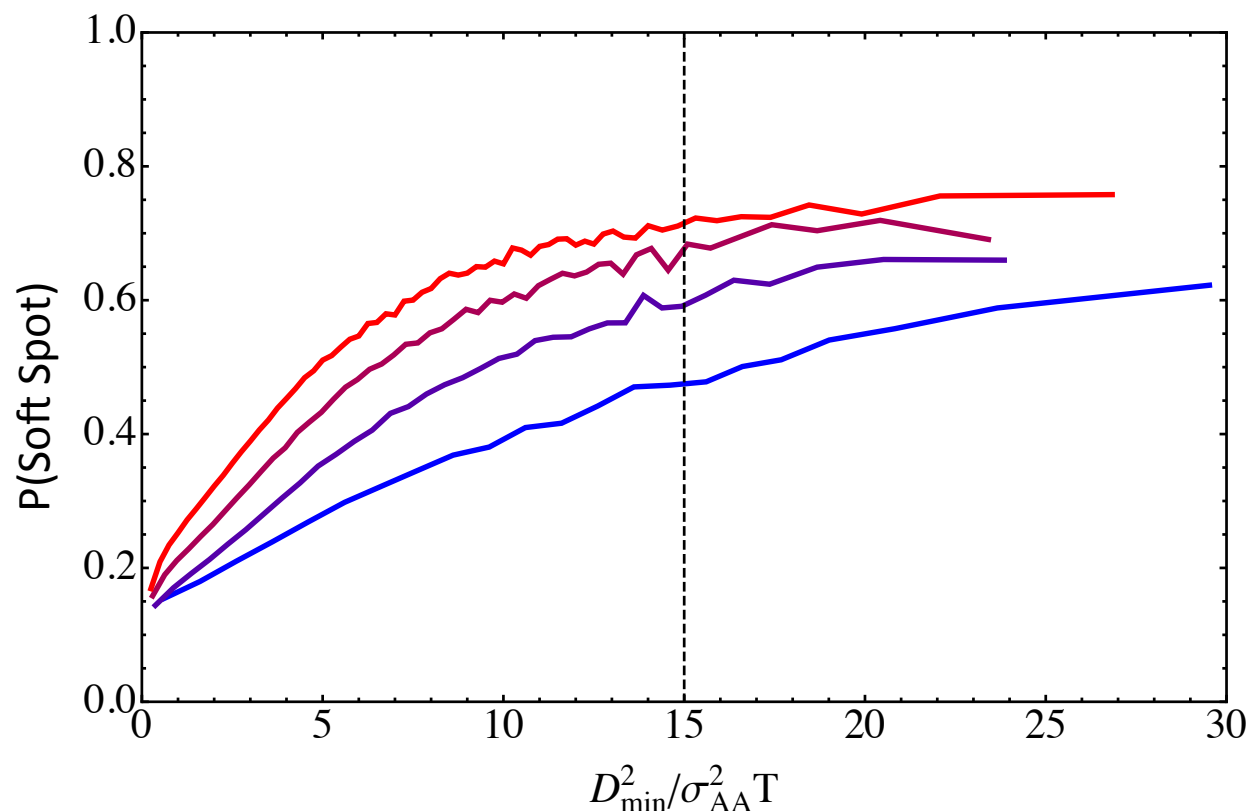


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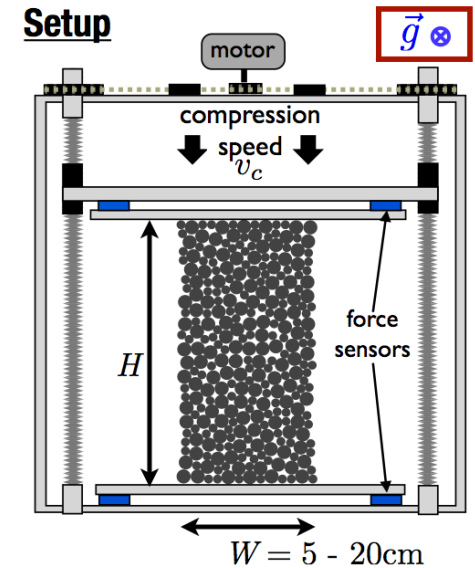
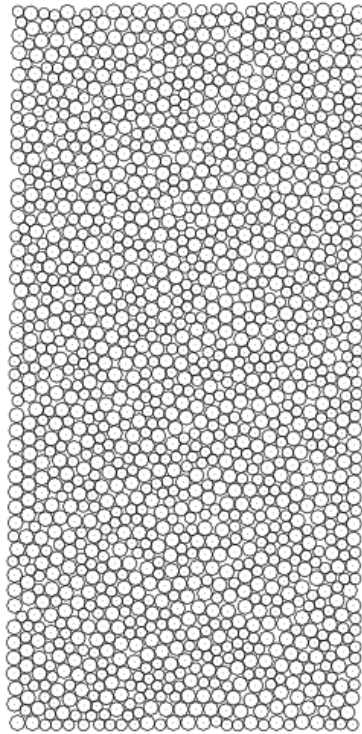
**Soft spots predict rearrangements just as well for  $T > T_g$  as for  $T < T_g$  !!!!**

# Results (sheared system)

- Results are transferable:  
train on  $T = 0.4$ , test on  $T = 0.1, 0.2, 0.3$  and  $0.4$



# Results (granular pillar)

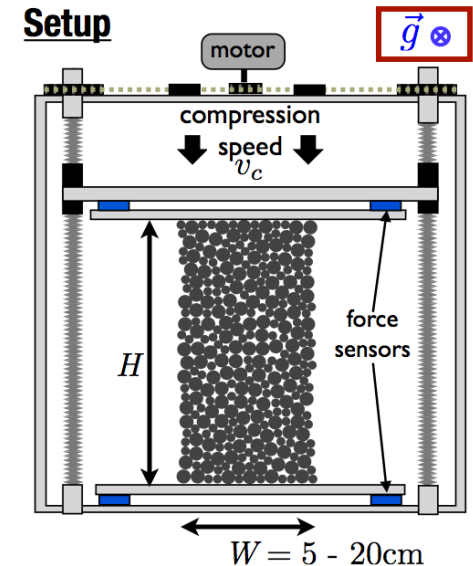


Cannot be analyzed with the phonon method!

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Doug Durian

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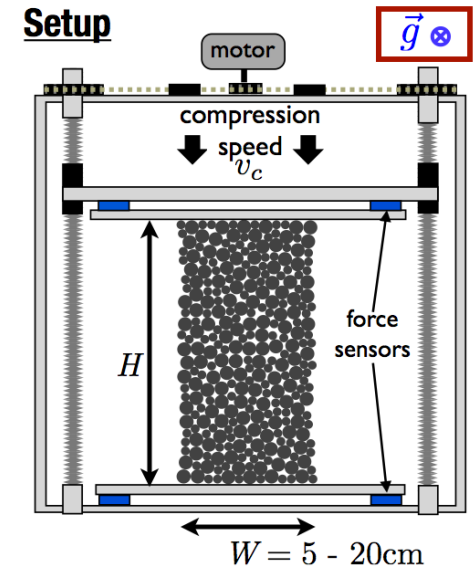
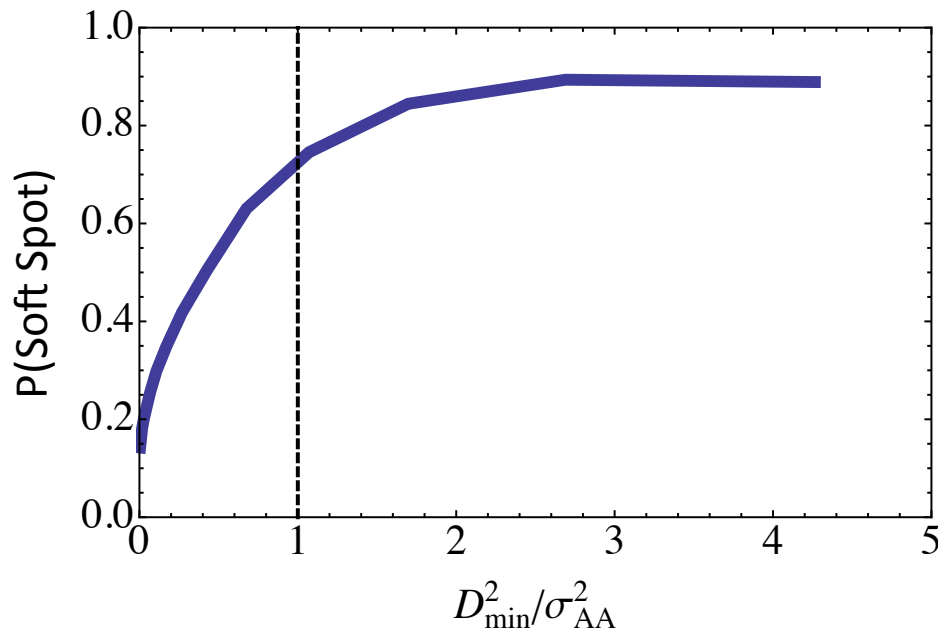
- Train on 10 pillars.
- For 10 test pillars, we correctly predict
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- Very efficient method for identifying soft spots.
- Only requires observation of representative plastic behavior (training set).
- Can be applied to a large variety of systems (experimental/computational, quenched/thermal, 2D/3D).

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- What in the structure determines if a spot is soft?
- Use the hyperplane to explore new physics.

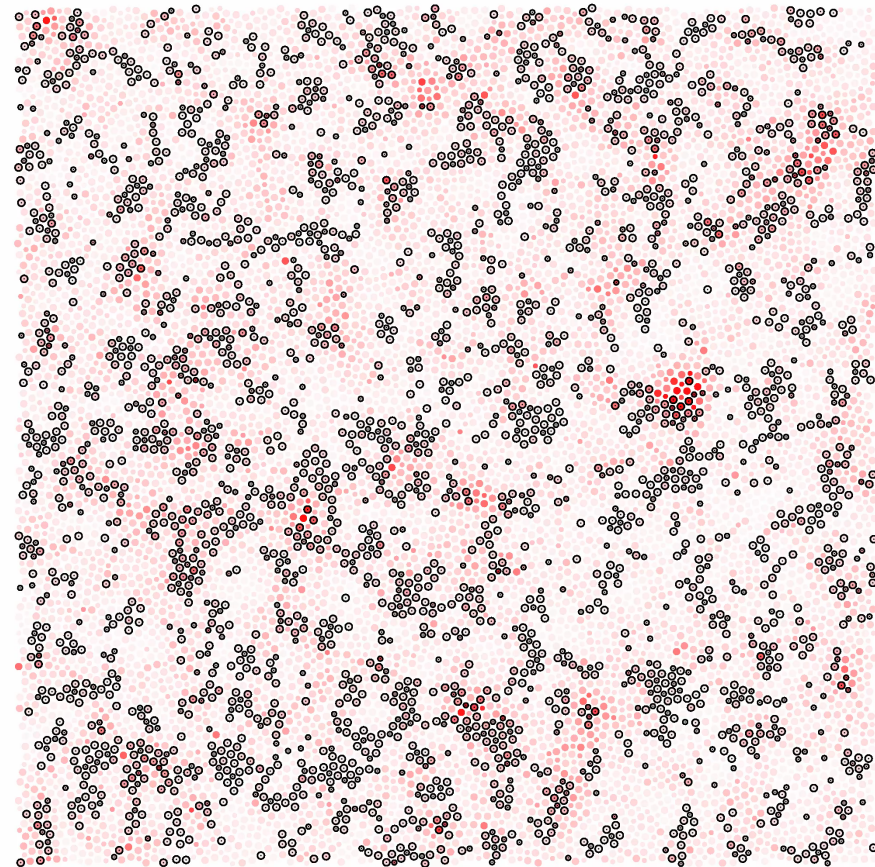
email: [cubuk@fas.harvard.edu](mailto:cubuk@fas.harvard.edu)

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