LECTURE 1: THE ONE IN WHICH WE SEE SOME MIRACLES AT p

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We stated the following result last time:

Theorem 0.0.1. *Let κ be a perfect field of characteristic $p > 0$ and X a smooth projective k-scheme which lifts to $\mathrm{W}_2\kappa$ and such that $\dim(\mathrm{X}) < p$, then the Hodge-to-de Rham spectral sequence for $\Omega^\bullet_{\mathrm{X}/\kappa}$ degenerates at the E_1-page.*

Our goal now is to work towards the proof of Theorem 0.0.1. Let us recall some terminology from derived categories (which we always regard as derived ∞-categories). Fix a commutative ring A. Then $\mathbf{D}(\mathrm{A})$ admits the canonical t-structure where the non-negative part is given by complexes which are **connective**: those whose homology groups are concentrated in non-negative degrees; cohomologically this means those in non-positive degrees. We will now work with cohomological indexing but we will be very clear about what we mean. So we use the "upper truncation" notation, so we have endofunctors $\tau^{\geqslant i} : \mathbf{D}(\mathrm{A}) \to \mathbf{D}(\mathrm{A})$ and we have the "interval-wise" truncation $\tau^{[i,j]}$, where $i \leqslant j$.

For each consecutive interval we have a cofiber sequence

$$\mathrm{H}^{i-1}\mathrm{K}[-i+1] \to \tau^{[i-1,i]}\mathrm{K} \to \mathrm{H}^i\mathrm{K}[-i] \xrightarrow{\delta} \mathrm{H}^{i-1}\mathrm{K}[-i+2].$$

The map δ defines a class

$$\delta \in \mathrm{Ext}^2(\mathrm{H}^i\mathrm{K}, \mathrm{H}^{i-1}\mathrm{K}) = [\mathrm{H}^i\mathrm{K}, \mathrm{H}^{i-1}\mathrm{K}[2]]_{\mathbf{D}(\mathrm{A})}.$$

If the class δ disappears then the triangle above splits so that we have a decomposition

$$\tau^{[i-1,i]}\mathrm{K} \simeq \mathrm{H}^{i-1}\mathrm{K}[-i+1] \oplus \mathrm{H}^i\mathrm{K}[-i].$$

In general we can look at "wider" truncations of the complex K, $\tau^{[a,b]}\mathrm{K}$ and there are higher obstruction classes living in groups like $\mathrm{Ext}^3(\mathrm{H}^i\mathrm{K}, \mathrm{H}^{i-2}\mathrm{K})$ and so on. Note that we have a filtered object

$$0 \to \mathrm{H}^b\mathrm{K}[-b] \to \cdots \tau^{[a+2,b]}\mathrm{K} \to \tau^{[a+1,b]}\mathrm{K} \to \tau^{[a,b]}\mathrm{K},$$

which defines a spectral sequence converging to the cohomology of $\tau^{[a,b]}\mathrm{K}$. The higher obstructions can be interpreted as differentials in the spectral sequence and if K is decomposable, then this spectral sequence degenerates.

We then say that K is **decomposable** if there is an equivalence in $\mathbf{D}(\mathrm{A})$:

$$\mathrm{K} \cong \bigoplus \mathrm{H}^i(\mathrm{K})[-i],$$

which induces the identity on cohomology. Now the choice of splitting is the same datum as a map $\mathrm{H}^i\mathrm{K}[-i] \to \mathrm{H}^{i-1}\mathrm{K} - [i+1]$; therefore the set of splittings is a torsor under the group $\mathrm{Ext}^1(\mathrm{H}^i\mathrm{K}, \mathrm{H}^{i-1}\mathrm{K}) = [\mathrm{H}^i\mathrm{K}, \mathrm{H}^{i-1}\mathrm{K}[1]]_{\mathbf{D}(\mathrm{A})}$. Thus the collection of splittings are also objects which are reasonably parametrized; we will take this into account in what follows.

Theorem 0.0.2. [DI87, Corollaire 3.7] *Let S be scheme over a perfect field κ of characteristic $p > 0$, let X be a smooth S-scheme and let $\mathrm{F}_{\mathrm{X}/\mathrm{S}} : \mathrm{X} \to \mathrm{X}^{(1)}$ be the relative Frobenius. Fix a lift $\widetilde{\mathrm{S}}$ to $\mathrm{W}_2(\kappa)$.*

1. *assume that there is a smooth lift $\widetilde{\mathrm{X}}^{(1)}$ to $\widetilde{\mathrm{S}}$, then $\tau^{[0,p]}\mathrm{F}_{\mathrm{X}/\mathrm{S}*}\Omega^\bullet_{\mathrm{X}/\mathrm{S}}$ is decomposable;*
2. *the collection of such lifts are in bijection with all possible liftings of X.*

Remark 0.0.3 (Properness)**.** The role of properness is to ensure that the numbers h^{ji} are actually finite so that we can run the numerical argument to degenerate the spectral sequence.

From the Deligne-Illusie theorem is an immediate consequence. Before we proceed, let us discuss what the de Rham complex is all about.

0.1. **The de Rham complex.** We fix a commutative ring A. We would like to understand the true nature of the functor of A-algebras given by

$$\mathrm{B} \mapsto \Omega^{\bullet}_{\mathrm{B/A}}.$$

We first ask ourselves: where does this functor land? It is good to keep in mind the following caution:

Remark 0.1.1. The object $\Omega^j_{\mathrm{B/A}}$ is naturally a B-module. However, the exterior derivative is not B-linear but only A-linear. Hence there is no way that the above assignment lands in a category "varying in B."

So we first ask ourselves: what is the nature of the functor $\mathrm{B} \mapsto \Omega^1_{\mathrm{B/A}}$?

Definition 0.1.2. Let B be an A-algebra. Then a **A-derivation** of B is the datum of a B-module M and a map

$$\mathrm{D} : \mathrm{B} \to \mathrm{M},$$

such that D is A-linear (where M is an A-module via the forgetful functor) and the **Leibniz rule** holds:

$$\mathrm{D}(fg) = f\mathrm{D}g + g\mathrm{D}f.$$

In this case, we say that D is an A-derivation of B valued in M.

One way to define $\Omega^1_{\mathrm{B/A}}$ is then as the **universal derivation**, it is a B-module which comes equipped with a derivation $d : \mathrm{B} \to \Omega^1_{\mathrm{B/A}}$, inducing an isomorphism

$$\mathrm{Hom}_{\mathrm{B}}(\Omega^1_{\mathrm{B/A}}, \mathrm{M}) \cong \mathrm{Der}_{\mathrm{A}}(\mathrm{B}, \mathrm{M}),$$

where the target is the set of A-derivations of B with valued in M.

Remark 0.1.3. Here is another way to think about derivations. Let M be an B-module. We can then form the **square-zero** extension of B by M, denoted by $\mathrm{B} \oplus \mathrm{M}$ which is B-algebra whose multiplication is given by

$$(b, m) \cdot (b', m') = (bb', bm' + b'm).$$

It comes equipped with a projection map $\mathrm{B} \oplus \mathrm{M} \to \mathrm{B}$. A derivation is then the same thing as an A-algebra section $s : \mathrm{B} \to \mathrm{B} \oplus \mathrm{M}$ of the projection map.

Now, the de Rham complex is a priori given as the cochain complex whose j-th term is given by the B-linear exterior power

$$\wedge^j_{\mathrm{B}} \Omega^1_{\mathrm{B/A}} =: \Omega^j_{\mathrm{B/A}}$$

It has two more pieces of structure:

(1) the graded module

$$\bigoplus_{j \geqslant 0} \Omega^j_{\mathrm{B/A}} =: \Omega^{\bullet}_{\mathrm{B/A}}$$

has the structure of a **strict commutative differential graded algebra** or just **strict dga**. The strict adjective says that $x^2 = 0$.

(2) there are A-linear maps called the **exterior derivative** or the **de Rham differential**

$$d : \Omega^j_{\mathrm{B/A}} \to \Omega^{j+1}_{\mathrm{B/A}}$$

determined by

$$d(b_0 db_1 \wedge \cdots db_j) = db_0 \wedge db_1 \wedge \cdots db_j.$$

In fact, these two pieces of structure pins down the de Rham complex; we can think of it as a "dga version" of the universal property of Ω^1:

Theorem 0.1.4. *Let* $\mathrm{A} \to \mathrm{B}$ *be a map of rings. Then* $\Omega^{\bullet}_{\mathrm{B/A}}$ *is the initial strict cdga equipped with a map from* B *to its degree zero component.*

Proof. Recall that the exterior algebra $\wedge^{\bullet}_{\mathrm{B}}(\mathrm{M})$ is the quotient of the tensor algebra generated by M modulo the two sided ideal generated by the degree two elements $\{m \otimes m\}$; it inherits a natural grading where M itself sits in degree one. In particular, it is a strict graded commutative A-algebra (ignoring differentials!). Now, we see that to define a A-linear map $\wedge^{\bullet}_{\mathrm{B}}(\mathrm{M}) \to \mathrm{C}^{\bullet}$ where $\mathrm{C}^{\bullet}$ is a strict graded commutative A-algebra, we need to define a map of degree zero part $\mathrm{B} \to \mathrm{C}^0$, a map of the degree one part $\mathrm{M} \to \mathrm{C}^1$; the maps on the higher degrees are determined by these datum. Here is where the differentials are helpful: if we impose further that the maps must commute with differentials, then all we need is to define a map $\mathrm{B} \to \mathrm{C}^0$ because the composite $\mathrm{B} \to \mathrm{C}^0 \to \mathrm{C}^1$ is an A-derivation and thus the universal property of $\Omega^1_{\mathrm{B/A}}$ furnishes the map on degree 1.

Now, we claim that the de Rham differentials as above define the unique structure of a strict *differential* graded commutative A-algebra such that on $\bullet = 0$, the map is given by $\mathrm{B} \xrightarrow{\mathrm{id}} \mathrm{B}$. In particular, this means that d must satisfy the higher Liebniz rules:

$$d^{i+j}(ab) = d^i(a)b + (-1)^i a d^j b \qquad |a| = i, |b| = j.$$

This last claim follows from an explicit presentation of $\Omega^j_{\mathrm{B/A}}$ as an A-*module*, generated by $b_0 dx_1 \wedge \cdots \wedge dx_j$ and checking directly that the higher Leibniz rules must be satisfied. This latter presentation then also gives us that the map constructed in the previous paragraph must define a map of strict commutative graded A-algebras. □

Remark 0.1.5. It is not hard to give a globalization of the de Rham complex using the language of a ringed topos; I encourage the reader to prove and formulate it.

0.2. **The Cartier isomorphism.** Throughout this class, we will refer to the following diagram. Let S be a scheme of characteristic $p > 0$ and let $\mathrm{Frob} : \mathrm{S} \to \mathrm{S}$ be the **absolute Frobenius**. This is a map which, on the level of rings, is given by

$$\mathrm{A} \xrightarrow{\mathrm{Frob}} \mathrm{A} \qquad x \mapsto x^p.$$

For any S-scheme X with structure map $\pi : \mathrm{X} \to \mathrm{S}$, we then form:

$$\begin{array}{ccccc}
\mathrm{X} & & \xrightarrow{\quad\mathrm{Frob}\quad} & & \\
& \searrow^{\mathrm{F_{X/S}}} & & & \\
& & \mathrm{X}^{(1)} & \xrightarrow{\mathrm{W}} & \mathrm{X} \\
& & \downarrow{\scriptstyle \pi^{(1)}} & & \downarrow{\scriptstyle \pi} \\
& & \mathrm{S} & \xrightarrow{\mathrm{Frob}} & \mathrm{S}.
\end{array}$$

Here $\mathrm{F_{X/S}}$ is called the **relative Frobenius**, in contrast to the absolute one.

Remark 0.2.1 (The object $\mathrm{F}_*\Omega^{\bullet}_{\mathrm{X/S}}$)**.** *A priori* the object $\mathrm{F}_*\Omega^{\bullet}_{\mathrm{X/S}}$ does not quite make sense since $\Omega^{\bullet}_{\mathrm{X/S}}$ is *not* $\mathcal{O}_{\mathrm{X}}$-linear. However, observe that the differential is actually $\mathcal{O}_{\mathrm{X}^{(1)}}$-linear: for any $f \in \mathcal{O}_{\mathrm{X}}$ we have

$$d(f^p g) = f^p dg + 0.$$

This is what the de Rham complex $\mathrm{F}_{\mathrm{X/S}*}\Omega^{\bullet}_{\mathrm{X/S}}$ is all about. This alone buys us a little miracle: the cohomology sheaves $\mathcal{H}^i(\mathrm{F}_{\mathrm{X/S}*}\Omega^{\bullet}_{\mathrm{X/S}})$ are, in fact, linear over $\mathcal{O}_{\mathrm{X}^{(1)}}$.

Here is a remarkable construction in characteristic $p > 0$:

Lemma 0.2.2. *Let* S *be an* $\mathbb{F}_p$*-algebra. Then there exists a* $\mathcal{O}_{\mathrm{X}^{(1)}}$*-linear map called the* ***inverse Cartier map***

$$\mathrm{C}^{-1} : \Omega^j_{\mathrm{X}^{(1)}/\mathrm{S}} \to \mathcal{H}^j(\mathrm{F}_{\mathrm{X/S}*}\Omega^j_{\mathrm{X/S}}).$$

which are determined uniquely by the following properties:

(1) $C^{-1}(1) = 1$*;*
(2) $C^{-1}(\omega \wedge \tau) = C^{-1}(\omega) \wedge C^{-1}(\tau)$*;*
(3) $C^{-1}(df) = f^{p-1}df$*.*

Let us postpone the construction of Construction 0.2.2 for a moment. The reason why C^{-1} has a -1 on it is because the inverse map was first defined, and is an isomorphism within the context of smooth schemes:

Theorem 0.2.3. *Let* $X \to S$ *be a smooth morphism of characteristic* $p > 0$ *schemes. Then the map*

$$C^{-1} : \Omega^j_{X^{(1)}/S} \to \mathcal{H}^j(F_{X/S*}\Omega^j_{X^{(1)}/S}),$$

is an isomorphism for all $j \geqslant 0$*.*

Proof. The easiest example of a smooth morphism is the projection map $\mathbb{A}^n_S \to S$, so let us try to prove the result in this generality first. Let us observe the following: the cdga $F_*\Omega^\bullet_{(\mathbb{A}^n_S)^{(1)}/S}$ is the $\mathcal{O}_{(\mathbb{A}^n_S)^{(1)}}$-linear complex generated by

$$x_1^{w_1} \cdots x_n^{w_n} dx_{\alpha_1} \cdots d_{x_{\alpha_j}}$$

where

$$w_i \in [0, p-1] \qquad 1 \leqslant \alpha_1 < \alpha_2 < \cdots \alpha_j \leqslant n,$$

and the differential is given by the usual exterior derivative. More precisely, let us write

$$K(n)^\bullet$$

as the $\mathbb{F}_p$-linear cdga generated by the above. Then we have an isomorphism of $\mathcal{O}_{(\mathbb{A}^n)^{(1)}}$-linear complexes

$$K(n)^\bullet \otimes_{\mathbb{F}_p} \mathcal{O}_{(\mathbb{A}^n_S)^{(1)}} \cong F_*\Omega^\bullet_{(\mathbb{A}^n_S)^{(1)}/S}.$$

With this presentation, the cohomology of this complex is given by

$$H^i(F_*\Omega^\bullet_{(\mathbb{A}^n_S)^{(1)}/S}) \cong H^i(\mathrm{Kos}(n)^\bullet) \otimes_{\mathbb{F}_p} \mathcal{O}_{(\mathbb{A}^n_S)^{(1)}}$$

Now, we have an equivalences:

$$K(n)^\bullet \simeq K(1)^\bullet \otimes^L \cdots \otimes^L K(1)^\bullet \simeq K(1)^\bullet \otimes \cdots \otimes K(1)^\bullet,$$

where there are n-tensor factors. Therefore, by the Künneth formula it suffices to prove the result for $n = 1$ in which we are reduced to the following claims:

(1) $H^0(K(1)^\bullet) = \mathbb{F}_p$;
(2) $H^1(K(1)^\bullet) = x^{p-1}dx$;
(3) $H^j(K(1)^\bullet) = 0$ for $j \geqslant 2$.

This is easy to verify by hand: the point is that if $n < p-1$, then $x^n dx$ has a primitive given by $\frac{1}{n+1}x^{n+1}$ since $n+1$ is a unit in $\mathbb{F}_p$.

We now reduce to the general case. We note that both source and target of C^{-1} are actually "local" on X in the sense that they form Zariski sheaves on X:

$$\Omega^j_{(-)^{(1)}/S}, \mathcal{H}^j(F_{X/S*}\Omega^j_{(-)^{(1)}/S}) : X^{op}_{Zar} \to \mathrm{Ab},$$

and that they map C^{-1} is a morphism of sheaves. Therefore it suffices to prove the result for local rings of X. However any smooth morphism $X \to S$ is, Zariski-locally on X, an étale morphism over an affine space, i.e., X is of the form

$$X \xrightarrow{f} \mathbb{A}^n_S \to S,$$

where f is an étale morphism and the first map is the projection map. We now conclude the result from the previous computation and the fact that if $f : X \to Y$ is an étale morphism of S-schemes, then we have isomorphisms:

$$\Omega^j_{X^{(1)}/S} \cong (f^{(1)})^*\Omega^j_{Y^{(1)}/S} \qquad \mathcal{H}^j(F_{X/S*}\Omega^j_{X^{(1)}/S}) \cong (f^{(1)})^*\mathcal{H}^j(F_{Y/S*}\Omega^j_{Y^{(1)}/S}).$$

□

Remark 0.2.4. We can enhance the above theorem slightly. Recall that a morphism of schemes $f : \mathrm{X} \to \mathrm{Y}$ is said to be **regular** if it is flat, every fiber X_y is locally noetherian and X_y is geometrically regular over $\kappa(y)$ in the sense that any for any finite, purely inseparable field extension $\kappa'/\kappa(y)$, $\mathrm{X}_{\kappa'}$ is a regular scheme. The following is landmark result in commutative algebra:

Theorem 0.2.5 (Popescu)**.** *Any regular morphism of rings* $\mathrm{A} \to \mathrm{B}$ *can written as a cofiltered limit of smooth ring maps* $\mathrm{A} \to \mathrm{A}_\alpha$.

Therefore, the Cartier isomorphism holds for $\mathrm{X} \to \mathrm{S}$ which is regular. A useful situation is this: suppose that X is a regular $\mathbb{F}_p$-scheme; this means that the structure map $\mathrm{X} \to \operatorname{Spec} \mathbb{F}_p$ is a regular. Then the inverse Cartier map is an isomorphism:

$$\Omega^j_{\mathrm{X}/\mathbb{F}_p} \xrightarrow{\mathrm{C}^{-1}, \cong} \mathcal{H}^j(\Omega^j_{\mathrm{X}/\mathbb{F}_p}).$$

Here we have used the fact that the frobenius on $\mathbb{F}_p$ is just the identity.

Remark 0.2.6. It is useful to isolate the following property from the proof above, first recognized by Achinger and Suh:

Definition 0.2.7. Let $(\mathrm{X}, \mathcal{O})$ be a ringed topos. A coconnective commutative differential $\mathcal{O}$-algebra $\mathrm{K}^\bullet$ is said to be an **abstract Koszul complex** if

(1) the map $\mathcal{O} \to \mathcal{H}^0(\mathrm{K}^\bullet)$ is an isomorphism;
(2) for every $q \geqslant 1$, the induced multiplication $\mathcal{H}^1(\mathrm{K}^\bullet)^{\otimes q} \to \mathcal{H}^q(\mathrm{K}^\bullet)$ factors through an isomorphism
$$\wedge^q_{\mathcal{O}} \mathcal{H}^1(\mathrm{K}^\bullet) \to \mathcal{H}^q(\mathrm{K}^\bullet).$$

The above result, and its generalization as in Remark 0.2.4 states that the de Rham complex is abstract Koszul in the above sense. This notion also captures seemingly unrelated phenomenon: let X be the topological torus of complex dimension g, i.e., it is homotopy equivalent to $(\mathrm{S}^1)^{2g}$. Then the complex $\mathrm{C}^*(\mathrm{X}; \mathbb{Q})$ is also abstract Koszul in the sense that H^0 is just $\mathbb{Q}$ and we have an isomorphism

$$\wedge^q_{\mathbb{Q}} \mathrm{H}^1(\mathrm{X}; \mathbb{Q}) \cong \mathrm{H}^q(\mathrm{X}; \mathbb{Q}) \qquad j \geqslant 1,$$

via the multiplication.

We now discuss the proof of Lemma 0.2.2.

Proof of Lemma 0.2.2. We work with X, S affine and globalize as usual. First let me say what the Cartier operator is all about. Let p be a prime and pretend that $p \neq 0$; so for example we are working in something like $\mathbb{Z}/p^2$. Then the Frobenius pullback of differential form gives us

$$\mathrm{F}^*(dx) = dx^p = px^{p-1}dx.$$

The observation here is that F^* is p-divisible on Ω^1. More generally, on a j-form, F^* is divisible by p^j on something like $\mathbb{Z}/p^j$. Morally speaking, the Cartier operator is a "divided" or "weighted" Frobenius — a very common theme among all Frobenius action in motives. Hence to compute C^{-1} what one does is to lift a differential form ω in degree j to an algebra over $\mathbb{Z}/p^j$, divided by p^j and reduce back. This is why the definition involves choices and only makes sense after taking cohomology. More concretely, the point is that we want to define a map $\mathrm{C}^{-1}(dx) = x^{p-1}dx$. But to ensure that this is a homomorphism we need to say that

$$(x + y)^{p-1}d(x + y) = x^{p-1}dx + y^{p-1}dy.$$

This is not true, but off by a factor of d of a sum involving factorials. For details on the construction, we refer the reader to [Kat70].

Now, on a ring A, it is not necessarily the case that A lifts to $\mathbb{Z}/p^2$ or even to $\mathbb{Z}_p$. But there is a gadget that accomplishes this called **crystalline cohomology**. For now let us construct the Cartier operator in a more restricted setting by assuming that $\mathrm{S} = \operatorname{Spec} \mathrm{A}$ admits a lift to p-torsion free algebra $\widetilde{\mathrm{A}}$ (this is equivalent to asking that $\widetilde{\mathrm{A}}$ is a flat lift). Then the relative crystalline cohomology is a complex of $\widetilde{\mathrm{A}}$-modules $\mathrm{R}\Gamma_{\mathrm{crys}}(\mathrm{X}/\widetilde{\mathrm{A}})$ with the property that there is a natural quasi-isomorphisms:

$$\mathrm{R}\Gamma_{\mathrm{crys}}(\mathrm{X}/\widetilde{\mathrm{A}}) \otimes^{\mathrm{L}}_{\widetilde{\mathrm{A}}} \mathrm{A} \simeq \Omega^{\bullet}_{\mathrm{X}/\mathrm{A}}.$$

Now, the upshot is that we have the Bockstein homomorphisms coming from the cofiber sequence of $\widetilde{\mathrm{A}}$-modules

$$0 \to \mathrm{A} \to \widetilde{\mathrm{A}}/p^2 \to \mathrm{A} \to 0,$$

whence the Bockstein map

$$\beta : \mathrm{R}\Gamma_{\mathrm{crys}}(\mathrm{X}/\widetilde{\mathrm{A}}) \otimes^{\mathrm{L}}_{\widetilde{\mathrm{A}}} \mathrm{A} \to \mathrm{R}\Gamma_{\mathrm{crys}}(\mathrm{X}/\widetilde{\mathrm{A}}) \otimes^{\mathrm{L}}_{\widetilde{\mathrm{A}}} \mathrm{A}/p[1].$$

We thus can form the chain complex (because the Bockstein is a derivation)

$$(\mathrm{H}^i(\mathrm{F}_*\Omega^{\bullet}_{\mathrm{X}/\mathrm{S}}), \beta),$$

the universal property of the de Rham complex (note that, by $\mathcal{O}_{\mathrm{X}^{(1)}}$-linearity, we have a map $\mathcal{O}_{\mathrm{X}^{(1)}} \to \mathcal{H}^0(\mathrm{F}_*\Omega^{\bullet}_{\mathrm{X}/\mathrm{S}}))$ then furnishes a map

$$\mathrm{C}^{-1} : (\Omega^i_{\mathrm{X}^{(1)}/\mathrm{S}}, d) \to (\mathcal{H}^i(\mathrm{F}_*\Omega^{\bullet}_{\mathrm{X}/\mathrm{S}}), \beta),$$

which one can check satisfies the inverse Cartier map axioms. □

References

[DI87] P. Deligne and L. Illusie, *Relèvements modulo p^2 et décomposition du complexe de de Rham*, Invent. Math. **89** (1987), no. 2, pp. 247–270, https://doi.org/10.1007/BF01389078

[Kat70] N. M. Katz, *Nilpotent connections and the monodromy theorem: Applications of a result of Turrittin*, Inst. Hautes Études Sci. Publ. Math. (1970), no. 39, pp. 175–232, http://www.numdam.org/item?id=PMIHES_1970__39__175_0

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