Appendix 1: Examples

The three examples in this section illustrate the main channels through which the "divine coincidence" fails in the multi-sector model. The vertical chain isolates the effect of input-output linkages, while the horizontal economy highlights the role of heterogeneous adjustment frequencies and idiosyncratic shocks. The oil economy combines the two. This last example rationalizes the common wisdom that oil shocks generate a tradeoff between stabilizing output and consumer prices (an endogenous "cost-push" shock). The Example highlights the crucial role of wage rigidities and heterogeneous adjustment probabilities in generating this outcome.

Example 1. Vertical chain

Phillips curve

Consider an economy made of two sectors, which we label U (for "upstream") and D (for "downstream"), as in Figure 1. Both sectors use labor, and D also uses U as an intermediate input. Only D sells to final consumers.

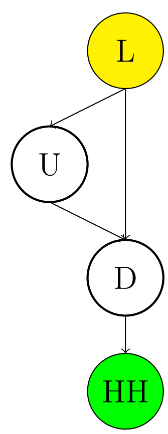

Figure 1: Vertical chain

Let's verify that in general consumer prices are not stabilized under zero output gap (the "divine coincidence" fails). In this example consumer prices coincide with the price of the downstream sector D, because this is the only consumption good. Consider first a negative productivity shock to D, $d \log A_D < 0$. The corresponding price responses are given by $\mathcal{V}_{UD} d \log A_D$ and $\mathcal{V}_{DD} d \log A_D$, where $\mathcal{V}_{UD}$ and $\mathcal{V}_{DD}$ can be derived from Proposition 2:

$$\mathcal{V}_{UD} d\log A_D = \overbrace{\underbrace{\delta_U}_{\text{pass-through}} \underbrace{\frac{1-\delta_D}{1-\bar{\delta}_w}}_{\text{multiplier}}}^{\text{nominal wages}} d\log A_D < 0 \tag{31}$$

$$\mathcal{V}_{DD} d\log A_D = \left[\overbrace{\underbrace{\bar{\delta}_w}_{\text{pass-through}} \underbrace{\frac{1-\delta_D}{1-\bar{\delta}_w}}_{\text{multiplier}}}^{\text{nominal wages}} - \overbrace{\delta_D}^{\text{productivity}} \right] d\log A_D > 0 \tag{32}$$

where

$$\bar{\delta}_w = \delta_D \left(\overbrace{\alpha_D}^{\text{direct pass-through}} + \overbrace{(1-\alpha_D)\,\delta_U}^{\text{through } U} \right)$$

From Equation (31) we see that inflation is negative in the upstream sector under zero output gap. This is because real wages fall to compensate the change in D's productivity, thereby reducing U's marginal cost. The downstream sector D instead experiences positive inflation (see Equation (32)), so that consumer inflation is also positive. In D the productivity shock has both a direct effect (lower productivity increases marginal costs), and an indirect effect through lower wages and input prices. As long as there is some price stickiness in U, input prices do not fully reflect the change in wages. The overall wage pass-through into D's marginal cost is given by

$$\alpha_D + (1-\alpha_D)\,\delta_U$$

which is less than 1 whenever $\delta_U < 1$. In this case the direct effect dominates.

In this example the "divine coincidence" fails because of input-output linkages. This is not merely a result of the asymmetric nature of the shock (it hits only one sector), as it is immediate to show that inflation is not stabilized after an aggregate Hicks-neutral shock either. The issue is that consumer inflation focuses only on the last stage of the chain. Indeed, given that inflation has opposite sign in the two sectors it is possible to construct a weighted average which is stabilized. Proposition 3 below shows that this is a general result, and the correct sectoral weights do not depend on the underlying productivity shock.

Optimal policy

Consider a negative downstream shock in a two-stage vertical chain, as in Example 1. Can monetary policy do better than implementing a zero output gap? Which sector shall it seek to stabilize?

Recall from Example 1 that, under zero output gap, marginal costs and prices increase downstream and fall upstream. A positive output gap raises wages, so that marginal costs and inflation increase everywhere, with a stronger effect on the upstream sector. As a result prices are stabilized in this sector, while inflation increases even more downstream. I argue that this is optimal, for two reasons. First, distortions are more costly in the upstream sector. Second, the positive effect on stabilization upstream from a given increase in the output gap is larger than the negative effect downstream (and viceversa).

Upstream distortions are more costly because they trigger both within- and cross-sector misallocation, while downstream distortions only trigger within-sector misallocation. This is because there is only one consumption good, and U only uses labor as an input. Therefore cross-sector misallocation can happen only across D's inputs (labor and U's product). Whenever U's price does not fully adjust to reflect changes in wages, D inefficiently substitutes between U and labor. Monetary policy can offset this effect by stabilizing wages, and thereby reducing U's desired price adjustment. Since wages fall after a negative productivity shock, wage stabilization requires to implement a positive output gap. This is reflected in the cross-sector component of the optimal output gap:

$$-\frac{(\gamma+\varphi)\,\theta_L^D}{\left(1-\delta_D\left(1-\left(1-\alpha_D\right)\delta_U\right)\right)^2}\left[\underbrace{\left(1-\alpha_D\right)\alpha_D}_{\text{input shares}}\quad\underbrace{\left(1-\delta_U\right)^2\left(1-\delta_D\right)}_{\propto\frac{1-\delta_U}{\delta_U}\pi_U}\right]d\log A_D>0$$

The relative marginal effect of monetary policy on inflation downstream relative to upstream is given by $1-(1-\alpha_D)\,\delta_U<1$. This reflects the fact that marginal costs are more sensitive to the output gap in the upstream sector, so that this sector is easier to stabilize. The within-sector component of the optimal output gap is

$$-\epsilon\frac{(\gamma+\varphi)(1-\alpha_D)(1-\delta_U)(1-\delta_D)}{\left(1-\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)\right)^2}\left[\underbrace{\delta_U}_{\text{benefit for }U}-\underbrace{\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)}_{\text{cost for }D}\right]d\log A_D$$

were $\frac{\delta_D}{\delta_U}$ is the relative cost of within-sector price dispersion in D and U. Implementing a positive output gap reduces overall within-sector misallocation if and only if the benefit for U (δ_U) is greater than the loss for D ($\delta_D\left(1-(1-\alpha_D)\,\delta_U\right)$), which is always the case when adjustment frequencies are the same in the two sectors.

Example 2. Horizontal economy

Phillips curve

Consider the horizontal economy in Figure (2): there are N sectors, $\{1,...,N\}$, with consumption shares $\beta_1,...,\beta_N$ and adjustment probabilities $\delta_1,...,\delta_N$. There are no input-output linkages, but sectors face idiosyncratic shocks and heterogeneous pricing frictions.

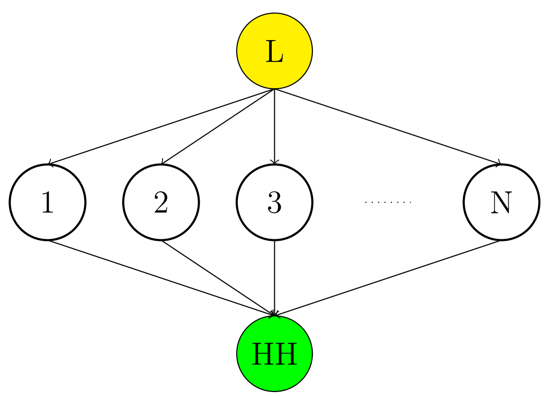

Figure 2: Horizontal economy

Under zero output gap wages adjust to reflect the "average" change in productivity $\mathbb{E}_\beta\left(d\log A\right)$. Sectors are equally exposed to wage changes, but they face different productivity shocks. Therefore marginal costs and prices cannot be stabilized everywhere. From Proposition 2, inflation in each sector i satisfies

$$\pi_i = \delta_i \left(\underbrace{\overbrace{\frac{1-\bar{\delta}_A}{1-\bar{\delta}_w}}^{\text{multiplier}} \overbrace{\mathbb{E}_\beta \left(d \log A \right)}^{\text{real wage}}}_{\text{wage}} - \underbrace{d \log A_i}_{\text{productivity}} \right) \tag{33}$$

where

$$\bar{\delta}_w = \mathbb{E}_\beta \left(\delta \right)$$
$$\bar{\delta}_A = \frac{\mathbb{E}_\beta \left(\delta d \log A \right)}{\mathbb{E}_\beta \left(d \log A \right)}$$

We see from (33) that inflation increases in sectors which received a worse shock than the average ($d \log A_i < \frac{1-\bar{\delta}_A}{1-\bar{\delta}_w} \mathbb{E}_\beta \left(d \log A \right)$), and vice versa. Consumer inflation is not stabilized either, because it overrepresents flexible sectors. It is negative if these sectors received a better shock, and vice versa:

$$\pi^C = -\frac{Cov_\beta \left(\delta, d \log A \right)}{1 - \mathbb{E}_\beta \left(\delta \right)} \tag{34}$$

As in the vertical chain, it would be possible to weight sectoral inflation rates in such a way that the average is stabilized. In the horizontal economy this can be achieved by discounting flexible sectors. Proposition 3 shows that this is a general results, and the correct sectoral weights do not depend on the underlying productivity shock.

Optimal policy

Consider the horizontal economy in Example 2. Here the tradeoff between within- and cross-sector misallocation is particularly stark. Intuitively, stabilizing within-sector misallocation would require all firms to charge the same price. Since some firms cannot adjust, this is achieved only if all prices remain constant. At the sector level however incomplete price adjustment results in a price response that is too small relative to the change in productivity. Therefore reducing cross-sector misallocation requires a larger price adjustment than that observed under zero output gap.

Formally, the marginal gain of increasing the output gap for within-sector misallocation is given by

$$\mathcal{B}^T \mathcal{D}_1 \pi = \epsilon \left(\gamma + \varphi \right) \mathbb{E}_{\beta(1-\delta)} \pi \tag{35}$$

while for cross-sector misallocation it is

$$\mathcal{B}^T \mathcal{D}_2 \pi = -\sigma \left(\gamma + \varphi\right) \mathbb{E}_{\beta(1-\delta)} \pi \tag{36}$$

I denote by $\mathbb{E}_{\beta(1-\delta)}$ the expectation computed with probability weights

$$\left\{ \frac{\beta_i \left(1 - \delta_i\right)}{\sum_j \beta_j \left(1 - \delta_j\right)} \right\}_{i=1,\dots,N}$$

The within- and cross-sector components are proportional, with opposite signs. Reducing within-sector misallocation is optimal if and only if the corresponding substitution elasticity is larger than the cross-sector one.

To determine the sign of the optimal output gap one needs to solve for the response of inflation in (35) and (36) as a function of sectoral productivity shocks. As in Example 2 the optimal output gap depends on the covariance between productivity shocks and adjustment frequencies (although with different weights). This covariance captures the competing effect of wage and productivity on marginal costs. To see this we can express inflation in (35) and (36) as a function of productivity shocks:

$$\mathbb{E}_{\beta(1-\delta)} \pi = Cov_{\beta(1-\delta)} \left(\delta, d \log A\right) = \frac{\sum_i \beta_i \left(1 - \delta_i\right) \delta_i \left(-d \log mc_i\right)}{\sum_j \beta_j \left(1 - \delta_j\right)} \tag{37}$$

Even though within- and cross-sector misallocation depend on the same covariance, this happens for different reasons. Within sectors, the marginal welfare loss is proportional to

$$\beta_i \left(1 - \delta_i\right) \delta_i \left(-d \log mc_i\right) \tag{38}$$

Misallocation is highest when the fraction of adjusting firms is closer to $\frac{1}{2}$, because price dispersion is maximal. To fix this, monetary policy should bring marginal cost changes closer to zero. Since marginal costs have unit elasticity with respect to the output gap, its marginal effect on within-sector misallocation is proportional to (38). This component of the optimal output gap is positive if and only if marginal costs decrease more in sectors with large consumption share and intermediate adjustment probabilities.

For cross-sector misallocation instead only the fraction $(1 - \delta_i)$ of non-adjusting firms matters, because these are the firms whose relative price is distorted with respect to producers in other sectors. The marginal contribution of sector i to the welfare loss is given by

$$\beta_i (1 - \delta_i) (-d \log mc_i)$$

Monetary policy should amplify the price adjustment by flexible firms, so that sectoral prices better reflect the change in productivity. In this case the relevant elasticity is not the marginal cost one, but the price one, which is given by δ_i. The marginal effect on cross-sector misallocation is then also proportional to (38), but with opposite sign.

Example 3. Oil economy

Phillips curve

This example presents a stylized "oil economy", showing that negative oil shocks lead to positive consumer inflation under zero output gap. Section 6.4.1 evaluates the quantitative importance of the channels highlighted here for the US economy. It finds that a 10% negative shock raises consumer prices by 0.22% under zero output gap.

Consider the production network in Figure 3. We can interpret our economy as a vertical chain where the upstream sector is labor, with sticky wages.[1] Then comes oil, and finally the last stage is broken down into multiple sectors, like a horizontal economy. Final sectors have heterogeneous consumption shares (β_i), oil input shares ($\omega_{i,oil}$) and adjustment frequencies (δ_i).

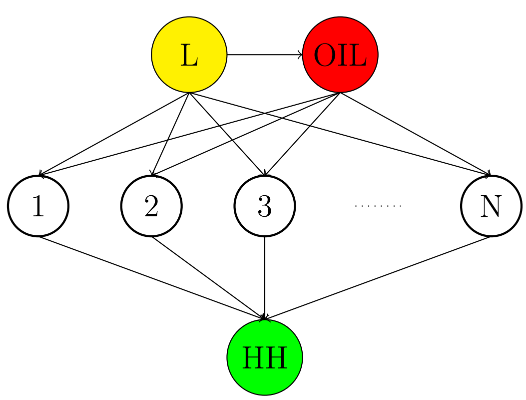

Figure 3: Oil economy

[1]See Remark 3 in Section 3.2.1 for a discussion of how I model wage rigidities.

Two channels determine the response of consumer inflation to oil shocks. First, since oil prices are very flexible, oil shocks are (almost) fully passed-through to the final goods sector. These shocks therefore act like downstream shocks, in spite of the role of oil as an intermediate input. We know from Example 1 that consumer prices increase in response to negative downstream shocks. Second, oil input shares and adjustment frequencies are positively correlated in the data. Consistent with the intuition from the horizontal economy in Example 2, this further increases the pressure on consumer prices.

Formally, if $\delta_{oil} = 1$ and wages adjust with probability δ_L, under zero output gap consumer inflation is given by

$$\pi^C = -\frac{\overbrace{Cov_\beta(\delta, \omega_{oil})}^{\text{horizontal}} + \overbrace{(1 - \delta_L)\, \mathbb{E}_\beta(\delta)}^{\text{vertical}}\, \mathbb{E}_\beta(\omega_{oil})}{1 - \delta_L \mathbb{E}_\beta(\delta)} d\log A_{oil} \tag{39}$$

For $d\log A_{oil} < 0$, consumer inflation π_C increases with wage stickiness $1 - \delta_L$ and with the covariance between oil shares and adjustment frequencies.

Table II in Section 6.4.1 reports the calibrated response of inflation to an oil shock in the US network, under different assumptions about δ_L and $Cov_\beta(\delta, \omega_{oil})$. The comparative statics are consistent with our discussion, even if the full network is much more complex than the stylized economy in this example.

Optimal policy

In this economy all the cross-sector misallocation comes from inefficient substitution by consumers across final goods. There is no misallocation across inputs in production: oil uses only labor as an input, and the oil price is fully flexible, so that it is never distorted relative to wages. Therefore the cross-sector component of the optimal output gap is the same as in the horizontal economy in Example 2, given by

$$\mathcal{B}^T \mathcal{D}_2 \pi = -\sigma(\gamma + \varphi)\, \mathbb{E}_{\beta(1-\delta)} \pi \tag{40}$$

Within-sector misallocation instead is present both at the final goods stage and at the labor stage. There is no misallocation across oil producers, because we assumed that they have flexible prices. The final good stage is the same as in the horizontal economy, and it is given by

$$\left(\mathcal{B}^T\mathcal{D}_1\pi\right)_{hor} = \epsilon\left(\gamma+\varphi\right)\mathbb{E}_{\beta(1-\delta)}\pi \tag{41}$$

From the vertical chain example we know that the labor stage (i.e. the most upstream stage) is stabilized with a positive output gap. With wage rigidities ($\delta_L < 1$) labor demand is inefficiently low for the workers who cannot lower their wage. A positive output gap reduces the desired wage cut, thereby shrinking the gap between adjusting and non-adjusting workers. The corresponding component of the optimal output gap is:

$$\left(\mathcal{B}^T\mathcal{D}_1\pi\right)_{vert} = \epsilon\left(\gamma+\varphi\right)\frac{\delta_L\mathbb{E}_\beta\left(1-\delta\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\left(1-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right) > 0$$

While the vertical component has a clear sign, to build intuition for the horizontal component we need to solve for the response of sectoral inflation rates in (41). This is different from the horizontal economy in Example 2. Substituting for inflation in Equation (41) as a function of the productivity shock, we obtain:

$$\begin{aligned}\mathcal{B}^T\mathcal{D}_2\pi = \sigma\left(\gamma+\varphi\right)\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}&\left[\mathbb{E}_{\beta(1-\delta)}\left(\delta\omega_{oil}\right)+\right.\\ &\left.-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\mathbb{E}_{\beta(1-\delta)}\left(\delta\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)\right]d\log A_{oil}\end{aligned} \tag{42}$$

Equation (42) differs from (37) because we introduced sticky wages. First, the expression in (42) is proportional to the wage flexibility δ_L. When wages are fully rigid ($\delta_L = 0$) monetary policy has no effect on marginal costs and markups, and the optimal output gap is zero. Second, the covariance $Cov_{\beta(1-\delta)}\left(\delta, d\log A\right)$ from Example 2 is replaced with the expression in square brackets in (42). Here sectoral productivity shocks are replaced by oil input shares, which reflect the pass-through of the oil shock into marginal costs. The first term in the expression reflects the effect of productivity on marginal costs, while the second is the effect of the equilibrium change in wages on marginal costs. In the presence of wage rigidities ($\delta_L < 1$) the latter is muted. As a result we have:

$$\underbrace{\mathbb{E}_{\beta(1-\delta)}\left(\delta\omega_{oil}\right)}_{\text{productivity channel}} - \underbrace{\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}}_{<1}\underbrace{\mathbb{E}_{\beta(1-\delta)}\left(\delta\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)}_{\text{wage channel}} > Cov_{\beta(1-\delta)}\left(\delta,\omega_{oil}\right) \tag{43}$$

Even though the mapping between productivity and inflation is different, the intuition behind the optimal policy remains the same as in the horizontal economy. The cross-sector component of the optimal output gap is negative if and only if marginal costs decreased relatively more in sectors with large consumption shares and/or intermediate adjustment probabilities.

The within-sector component instead combines insights from both the horizontal economy and the vertical chain. The optimal response to within-sector misallocation in the final goods stage is proportional to 43, as in the horizontal economy. However we have an additional term coming from misallocation in the labor sector, which plays the same role as the upstream sector in the vertical chain. Overall, the within-sector component of the optimal output gap is given by

$$\mathcal{B}^T\mathcal{D}_1\pi = \epsilon\left(\gamma+\varphi\right)\frac{\delta_L\mathbb{E}_\beta\left(1-\delta\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\left[\underbrace{\left(1-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)}_{\text{vertical chain}}\right.$$
$$\left.-\left[\underbrace{\mathbb{E}_{\beta(1-\delta)}\left(\delta\omega_{oil}\right)-\frac{\delta_L\left(1-\mathbb{E}_\beta\left(\delta\right)\right)}{1-\delta_L\mathbb{E}_\beta\left(\delta\right)}\mathbb{E}_{\beta(1-\delta)}\left(\delta\right)\mathbb{E}_{\beta(1-\delta)}\left(\omega_{oil}\right)}_{\text{horizontal economy}}\right]\right]d\log A_{oil} \tag{44}$$

In principle, the sign of this component depends on the "adjusted" covariance in (43). Quantitatively, the calibration in Section 6.4.1 shows that this covariance is positive. Given that the within-sector elasticity is larger than the cross-sector elasticity in our calibration, and the vertical chain term is also positive, the optimal output gap is positive.

Appendix 2: Cost-push shocks and Dynamics

2.1: Exogenous cost-push shocks

In this section I extend the model presented in the main text to allow for exogenous sector-level cost-push shocks, which I model as a change in producers' desired markups. I denote these changes by the $N\times 1$ vector $d\log\mu^D$. Lemma (1) derives sectoral inflation rates and the Phillips curve.

Lemma 1. *The elasticity of sectoral prices with respect to cost-push shocks is given by*

$$\left(\frac{\mathcal{B}\lambda^T}{\gamma+\varphi}-\mathcal{V}\right) \tag{45}$$

The "divine coincidence" Phillips curve is

$$DC = (\gamma+\varphi)\,\tilde{y} + \lambda^T d\log\mu^D \tag{46}$$

while the consumer-price Phillips curve is

$$\pi^C = \kappa\tilde{y} + u + v \tag{47}$$

where

$$u = \frac{\bar{\delta}_w - \bar{\delta}_A}{1-\bar{\delta}_w}\lambda^T d\log A$$

$$v = \frac{\bar{\delta}_\mu}{1-\bar{\delta}_w}\lambda^T d\log\mu^D$$

$$\delta_\mu = \frac{\beta^T\Delta\,(I-\Omega\Delta)^{-1}\,d\log\mu^D}{\lambda^T d\log\mu^D}$$

The two Phillips curves expressed in terms of deviations from steady-state output are

$$DC = (\gamma+\varphi)\,y + \lambda^T\left(d\log\mu^D - d\log A\right) \tag{48}$$

$$\pi^C = \kappa y + \frac{\bar{\delta}_\mu\lambda^T d\log\mu^D - \bar{\delta}_A\lambda^T d\log A}{1-\bar{\delta}_w} \tag{49}$$

Similar to the baseline model, the central bank faces a worse trade-off after a cost-push shock than after a negative productivity shock of the same size (i.e. $d\log A = -d\log\mu^D$). We can see this from the inequality

$$\frac{u}{\lambda^T d\log A} < \frac{v}{\lambda^T d\log\mu^D}$$

This is because the change in firms' desired price is the same for the two shocks, but after the cost-push shock natural output hasn't changed. In other words, inflation is the same after the two shocks for a given deviation of output from steady-state, but the output gap is lower under the cost-push shock. Therefore for a given output gap the cost-push shock generates higher inflation. Correspondingly, an additive "aggregate" cost-push term appears in the divine coincidence Phillips-curve.

Lemma (2) solves for the optimal policy response.

Lemma 2. *The optimal monetary policy response to a cost-push shock* $d\log\mu^D$ *implements the output gap*

$$\tilde{y}^*_{CP} = \frac{\mathcal{B}^T\left(\mathcal{D}\left(\overbrace{\frac{\mathcal{B}\lambda^T}{\gamma+\varphi}}^{\text{inflation-output trade-off}} - \overbrace{\mathcal{V}}^{\text{propagation}}\right) - \overbrace{\mathcal{D}_2\Delta\left(I-\Delta\right)^{-1}}^{\text{direct effect}}\right)}{(\gamma+\varphi)+\mathcal{B}^T\mathcal{D}\mathcal{B}} d\log\mu^D \tag{50}$$

Under the optimal policy the inflation target derived in Proposition 6 *takes value*

$$\pi_\phi = \left(\lambda^T - \mathcal{B}^T\mathcal{D}_2\Delta\left(I-\Delta\right)^{-1}\right) d\log\mu^D \tag{51}$$

Comparing (50) with (26) above, we see that the optimal response to productivity and cost-push shocks has a common component, given by the "propagation" term in (50). Monetary policy seeks to address the relative price distortions that arise as the shock propagates through the input-output network. These are the same regardless of whether the inflation response is triggered by fluctuations in productivity or desired markups.

While productivity shocks cause misallocation only through the propagation channel, by definition cost-push shocks directly distort relative markups. The response of monetary policy to this direct effect is captured by the third term in (50). Implementing a positive output gap is optimal if it raises marginal costs more in the sectors which faced a larger increase in their desired markup. Whenever this is the case the policy target is positive under the optimal policy, as reflected in the second term of (51).

Finally, the first term in equation (50) comes from the fact that monetary policy faces a "worse" trade-off under the cost-push shock than under the productivity shock, because natural output has not fallen. This is also captured by the first term of the policy target (51). The intuition is the same as in the one-sector model: in the face of a cost-push shock the central bank trades off the output loss with the increase in inflation. Therefore the optimal output gap is lower than after an equally-sized negative productivity shock, while the output level and inflation should be higher.[2]

2.2: Dynamics - Main results

This section presents the setup and main results for the dynamic version of the model. Additional details and all the proofs can be found in Section C2 of the Supplemental Material.

Consumers

Consumers' preferences are given by

$$U = \sum_{t=0}^{\infty} \rho^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right)$$

where ρ is the discount factor, and C_t and L_t are defined as in Section 2.2.

In each period consumers are subject to the budget constraint

$$P_t C_t + B_{t+1} \leq w_t L_t + \Pi_t - T_t + (1 + i_t) B_t$$

where $w_t L_t$ is labor income, Π_t are firm profits (rebated lump-sum to households), T_t is a lump-sum transfer (which the government uses to finance input subsidies to firms), B_t is the quantity of risk-free bonds paying off in period t owned by the household and i_t are nominal interest rates.

Consumer optimization yields the Euler equation

$$U_{ct} = \rho(1 + i_{t+1})\mathbb{E} \left[U_{ct+1} \frac{P_{ct}}{P_{ct+1}} \right] \tag{52}$$

[2]However note that in the multi-sector model this channel is potentially counteracted by the response to the “direct” effect.

where P_{ct} is the consumer price index at time t. Log-linearizing equation (52) and imposing market clearing for final goods we find

$$y_t = \mathbb{E}\left[y_{t+1}\right] - \frac{1}{\gamma}\left(i_{t+1} - \mathbb{E}\left[\pi^c_{t+1}\right] - \log\rho\right) \tag{53}$$

Re-writing equation (53) in gaps yields

$$\tilde{y}_t = \mathbb{E}\left[\tilde{y}_{t+1}\right] - \frac{1}{\gamma}\left(i_{t+1} - \mathbb{E}\left[\pi^c_{t+1}\right] - r^n_{t+1}\right) \tag{54}$$

where r^n_{t+1} is the natural interest rate, satisfying

$$r^n_{t+1} = \log\rho + \gamma\lambda^T\mathbb{E}\left[\log A_{t+1} - \log A_t\right]$$

Policy instruments

I consider a cashless economy, in which interest rates are the only policy instrument. At each period t the central bank sets the risk-free rate i_{t+1}.

Production

Within each period the production technology is as described in Section 2.3. Sectoral productivity shifters A_{it} vary across periods.

As in the one-period model, I assume that the government sets input subsidies to offset the markup distortions arising from monopolistic competition. Sectoral subsidies are constant over time, and given by

$$1 - \tau_i = \frac{\epsilon^*_{it} - 1}{\epsilon^*_{it}}$$

as in Section 2.3.

All producers minimize costs given wages and input prices. At every time t producers in sector i solve

$$c_{it}(\bar{y}) = min_{\{x_{ijt}\},L_{it}}\ w_t L_{it} + \sum_j p_{jt}x_{ijt}\ \ s.t.\ A_{it}F_i\left(L_{it}, \{x_{ijt}\}\right) = \bar{y}$$

With constant returns to scale marginal costs are the same for all firms, and all firms use inputs in the same proportions. However not all of them can adjust prices, so that firms within the same sector end up charging different markups.

Sector-level inflation dynamics

The firms who can update their price solve

$$p^*_{it} = max_{p_i} \ \mathbb{E}\left[\sum_t SDF_t \left(1-\delta_i\right)^t Y_{it}(p_i) \left(p_i - \left(1-\tau_i\right) mc_{it}\right)\right] \tag{55}$$

The optimal reset price is

$$p^*_{it} = \frac{\mathbb{E}\sum_t \left[\frac{\epsilon_{it}}{\epsilon^*_i} SDF_t \left(1-\delta_i\right)^t Y_{it}(p_i) mc_{it}\right]}{\mathbb{E}\sum_t \frac{\epsilon_{it}-1}{\epsilon^*_i-1}\left[SDF_t \left(1-\delta_i\right)^t Y_{it}(p_i)\right]} \tag{56}$$

Log-linearizing equation (56) yields the following expression for sector-level inflation rates:

$$\pi_{it} = \frac{\delta_i \left(1-\rho(1-\delta_i)\right)}{1-\delta_i}\left(d\log\mu^D_{it} - \log\mu_{it}\right) + \rho\mathbb{E}_t\left[\pi_{it+1}\right] \tag{57}$$

where μ_{it} is the "sector-level" markup, satisfying:

$$\log\mu_{it} = \log p_{it} - \log mc_{it}$$

and μ^D_{it} is the desired markup of firms in sector i.

Equilibrium

Equilibrium is defined in a similar way as in section 2.5.

For given sectoral probabilities of price adjustment δ_i, sectoral productivity shifters A_{it} and interest rates i_t for each period t, general equilibrium is given by a vector of firm-level markups μ_{fit}, a vector of prices p_{it}, a nominal wage w_t, labor supply L_t, a vector of sectoral outputs y_{it}, a matrix of intermediate input quantities x_{ijt}, and a vector of final demands c_{it} for each period t such that: a fraction δ_i of firms in each sector i can adjust their price in every period; markups are chosen optimally by adjusting firms (see problem (55)), while they are such that prices stay constant for the non-adjusting firms; consumers maximize intertemporal utility subject to the budget constraints; producers in each sector i minimize costs and charge the relevant markup; and markets for all goods and labor clear.

The sales-based Phillips curve

Proposition 7 shows that the "divine coincidence" Phillips curve inherits the properties of the Phillips curve in the one-sector model: it has constant slope, and does not depend on the realization of sectoral productivity shocks; the aggregate cost-push shock enters as an additive residual.[3]

Proposition 7. *It holds that*

$$DC_t \equiv \lambda^T \left(I - \hat{\Delta}\right) \hat{\Delta}^{-1} \pi_t = \rho \mathbb{E}\left(DC_{t+1}\right) + \kappa \tilde{y}_t + \lambda^T d \log \mu_t^D \tag{58}$$

where

$$\kappa = \gamma + \varphi$$

and $\hat{\Delta}$ is a diagonal matrix with elements

$$\hat{\Delta}_{ii} = \frac{\delta_i \left(1 - \rho(1 - \delta_i)\right)}{1 - \rho \delta_i (1 - \delta_i)}$$

Response of inflation rates and markups to productivity and monetary shocks

Proposition (8) characterizes the evolution of sector-level inflation, inflation expectations and markups for given productivity shocks $\log A_t - \log A_{t-1}$, monetary policy $\tilde{y}_t$, and past markups. Note that, different from the one-sector model and the sales-based Phillips curve in Section , the vector of sector-level past markups is a state variable.[4]

[3] Note that the sales-based Phillips curve in (16) does not depend on past markups. This is a consequence of Lemma 12 in Section B2 of the Supplemental Material.

[4] The actual state variables are "relative" past markups $\mathcal{V} d \log \mu_{t-1}$. Given these, the system is invariant to the "aggregate" past markup $\lambda^T d \log \mu_{t-1}$.

Proposition 8. *Denote by*

$$\mathcal{M} \equiv \left(\frac{\hat{\mathcal{B}}\lambda^T}{\gamma+\varphi} - \hat{\mathcal{V}} \right) \left(I - \hat{\Delta} \right) \hat{\Delta}^{-1}$$

The evolution of sectoral markups and inflation rates is given by the following system of difference equations:

$$\begin{pmatrix} \rho\mathbb{E}\pi_{t+1} \\ \log\mu_t \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{M}^{-1}\hat{\mathcal{V}} \\ -\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\left(I-\mathcal{M}^{-1}\right) & -\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\mathcal{M}^{-1}\hat{\mathcal{V}} \end{pmatrix} \begin{pmatrix} \pi_t \\ \log\mu_{t-1} \end{pmatrix} +$$

$$+ \begin{pmatrix} -\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t + \hat{\mathcal{V}}\left(\log A_t - \log A_{t-1}\right)\right) - \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D \\ -\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t + \hat{\mathcal{V}}\left(\log A_t - \log A_{t-1}\right)\right) \end{pmatrix} \tag{59}$$

Proposition 8 extends the results in Sections 4.1.2 and 4.1.3 to the dynamic setup. We can re-write the first equation in (59) as

$$\pi_t = \underbrace{\hat{\mathcal{B}}\tilde{y}_t + \hat{\mathcal{V}}\left(\log A_t - \log A_{t-1} + \log\mu_{t-1}\right)}_{\text{productivity and past markups}} +$$

$$+ \underbrace{\left(\frac{\hat{\mathcal{B}}\lambda^T}{\gamma+\varphi} - \hat{\mathcal{V}} \right) \left(d\log\mu_t^D + \left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\rho\mathbb{E}\pi_{t+1} \right)}_{\text{cost-push shock}} \tag{60}$$

The first term contains the elasticities of sectoral inflation rates with respect to productivity and monetary shocks, which are the same as in the static setup (see Propositions 1 and 2). In addition, we now have to account for inherited markup distortions, due to the fact that some producers could not adjust their price in past periods. Lemma 12 in Section B2 of the Supplemental Material shows that in the static setting inflation responds in the same way to productivity shocks and to initial markups. This happens because both induce the same desired price change under zero output gap. The last term in equation (60) is the response to a "cost-push" shock, which can have an exogenous component ($d\log\mu_t^D$) and an endogenous component coming from expected future inflation. Inflation expectations act as a "cost-push" shock because they change the desired amount of price adjustment for given output gaps and productivity. The cost-push shock implied by inflation expectations is given by:

$$d \log \mu_{\mathbb{E}}^{D} = (I - \Delta) \Delta^{-1} \rho \mathbb{E} \pi_{t+1} \tag{61}$$

Consumer price Phillips curve

We can aggregate sectoral inflation rates in equation (60) into the CPI-based Phillips curve, obtaining

$$\pi_t^C = \kappa \tilde{y}_t + \rho \mathbb{E} \pi_{t+1}^C + u_t + v_t \tag{62}$$

where

$$u_t = \frac{\bar{\delta}_w - \bar{\delta}_A}{1 - \bar{\delta}_w} \lambda^T (d \log A_t - d \log A_{t-1}) + \frac{\bar{\delta}_w - \bar{\delta}_{\mu_{-1}}}{1 - \bar{\delta}_w} \lambda^T d \log \mu_{t-1}$$

$$v_t = \frac{\bar{\delta}_w - \bar{\delta}_{\pi^C}}{1 - \bar{\delta}_w} \rho \mathbb{E} \pi_{t+1}^C + \frac{\bar{\delta}_{\mu^D}}{1 - \bar{\delta}_w} \lambda^T d \log \mu_t^D$$

$$\bar{\delta}_{\mu_{-1}} = \frac{\beta^T \Delta (I - \Omega\Delta)^{-1} d \log \mu_{t-1}}{\lambda^T d \log \mu_{t-1}}$$

$$\bar{\delta}_{\pi^C} = \frac{\beta^T \Delta (I - \Omega\Delta)^{-1} (I - \Omega) \mathbb{E} \pi_{t+1}}{\mathbb{E} \pi_{t+1}^C}$$

$$\bar{\delta}_{\mu^D} = \frac{\beta^T \Delta (I - \Omega\Delta)^{-1} d \log \mu^D}{\lambda^T d \log \mu^D}$$

Equation (62) highlights that past markups and inflation expectations create an endogenous cost-push term in the Phillips curve, in the same way as productivity shocks.

Closing the output gap

Lemma 3 shows that the central bank can implement zero output gap in all periods by targeting it directly in the Taylor rule, as long as the policy rule is "reactive enough".

Lemma 3. *Assume that the productivity shocks follow an AR1 process:*

$$\log A_{t+1} - \log A_t = \eta (\log A_t - \log A_{t-1}) + u_{t+1}$$

with $\mathbb{E} u_{t+1} = 0$ and $\eta < 1$. Then there is a unique path of inflation rates such that the output gap is constantly zero. This equilibrium can be implemented with the interest *rate rule*

$$i_t = \underbrace{r_t^n + \beta^T \mathbb{E}\pi_{t+1}^{zg}}_{\textit{nominal rate under zero output gap}} + \zeta \tilde{y}_t \tag{63}$$

with $\zeta > 0$.

Optimal policy

Propositions 9 and 10 characterize the dynamic welfare loss and the central bank's policy problem.

Proposition 9. *Given a path* $\{y_t, \pi_t, z_{t+1}\}_{t=0}^{\infty}$ *for the output gap, sectoral inflation rates and markups, the expected second-order welfare loss is given by*

$$\sum_{t=0}^{\infty} \rho^s \mathbb{E}\left[(\gamma + \varphi)\,\tilde{y}_t^2 + \pi_t^T \mathcal{D}_1 \pi_t + z_{t+1}^T \mathcal{D}_2 z_{t+1}\right]$$

where

$$z_t \equiv -\left(I - \hat{\Delta}\right)^{-1} \hat{\Delta} \log \mu_{t-1}$$

Proposition 10. *Consider the optimal policy problem without commitment, where the central bank solves*

$$min_{\{\tilde{y}_t, \pi_t, z_{t+1}\}}\,(\gamma + \varphi)\,\tilde{y}_t^2 + \pi_t^T \mathcal{D}_1 \pi_t + z_{t+1}^T \mathcal{D}_2 z_{t+1}$$

subject to (59)*. The FOCs yield*

$$\tilde{y}_t^* = -\frac{\mathcal{B}^T \mathcal{D} \mathcal{V}}{(\gamma + \varphi) + \mathcal{B}^T \mathcal{D} \mathcal{B}}\left[(d\log A_t - d\log A_{t-1}) - d\log \mu_{t-1} + (I - \Omega)\,\rho \mathbb{E}\pi_{t+1}\right] \tag{64}$$

In the same spirit as Proposition 8, Proposition 9 decomposes the optimal output gap into the response to productivity shocks and past markups, which is the same as in the static setup (see Proposition 5), and the response to inflation expectations, which have a similar effect as a "cost-push" shock (see Appendix 2.1).[5] Lemma 4 and Lemma 4 below show that the optimal policy can be implemented with a targeting rule, in the same way as in the static setup (see Proposition 6).

[5]To facilitate the comparison, use the equality $\mathcal{M} = \left(\frac{\mathcal{B}\lambda^T}{\gamma+\varphi} - \mathcal{V}\right)(I - \Delta)\,\Delta^{-1} = I + \mathcal{V}\,(I - \Omega)$.

Lemma 4. *Assume that productivity shocks follow an AR1 process:*

$$\log A_{t+1} - \log A_t = \eta\left(\log A_t - \log A_{t-1}\right) + u_{t+1}$$

with $\mathbb{E}u_{t+1} = 0$ *and* $\eta < 1$.

Then there is a unique path of inflation rates such that the optimal output gap (64) is implemented in each period.

Denote by

$$\phi^T \equiv \frac{\lambda^T\left(I - \hat{\Delta}\right)\hat{\Delta}^{-1} + \hat{\mathcal{B}}^T\mathcal{D}}{\gamma + \varphi} \tag{65}$$

For $\zeta > \gamma$ *the interest rate rule*

$$i_{t+1} = \underbrace{\overbrace{r^n_{t+1} + \gamma\left[\mathbb{E}\widetilde{y}^*_{t+1} - \widetilde{y}^*_t\right]}^{\textit{real rate under opt policy}} + \beta^T\mathbb{E}\pi^*_{t+1}}_{\textit{nominal rate under optimal policy}} + \zeta\phi^T_t\underbrace{\left(\pi_t - \rho\mathbb{E}\pi_{t+1}\right)}_{\textit{inflation target}} \tag{66}$$

implements the optimal policy (64).

Comparing equation (44) from the static setup with equation (65) we see that they yield a very similar inflation target. The key difference is that in the dynamic setup the central bank should not just target current inflation, but also inflation expectations. The intuition is that the welfare-relevant variable are sector-level markups, and the mapping between current inflation and markups depends of inflation expectations. Specifically, from the pricing equation (57) we have $d\log\mu = (I-\Delta)\Delta^{-1}(\pi_t - \rho\mathbb{E}\pi_{t+1})$.