Supplement to "Networks, Phillips Curves, and Monetary Policy"

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A: Positive analysis

A1: Natural output and output gap

This Appendix presents two basic results: it derives the elasticity of efficient output with respect to productivity (Lemma 6), and it shows that in the sticky-price economy there is no first-order loss in aggregate productivity due to misallocation (Lemma 7).

Lemma 7 and Equation (77) imply that the output gap $\tilde{y}$ can be interpreted equivalently as a deviation of total output or of total labor supply from the efficient level:

$$\widetilde{y} = d\log Y - d\log Y^{nat} = d\log L - d\log L^{nat} \tag{67}$$

Lemma 6. *The change in efficient output after a productivity shock $d\log A$ is given by*

$$y^{nat} = \frac{1+\varphi}{\gamma+\varphi}\lambda^T d\log A \tag{68}$$

Proof. The flex-price equilibrium allocation is efficient. Therefore it can be derived as the solution of the planning problem

$$\begin{aligned} &max_{L,\{L_i,y_i,\{x_{ij}\}\}} \frac{C\left(\{y_i\}_{i=1}^N\right)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi} \\ s.t. \quad & y_i + \textstyle\sum_j x_{ij} = A_i F_i\left(\{x_{ij}\}, L_i\right) \ \forall i \\ & \textstyle\sum_i L_i = L \end{aligned} \tag{69}$$

The change in natural output is then given by

$$y^{nat} = \frac{d\log C^*}{d\log A_i}$$

where

$$C^* \equiv C(\{y_i^*\}_{i=1}^N)$$

is aggregate output under the optimal allocation.

The optimization problem in (69) can be solved in two steps: first, we choose $\{L_i, y_i, \{x_{ij}\}\}$ for given L; then we choose the optimal L. Formally, solving problem (69) is equivalent to solving

$$\frac{C^*\left(L;A\right)^{1-\gamma}}{1-\gamma} = max_{\{L_i,y_i,\{x_{ij}\}\}}\ \frac{C\left(\{y_i\}\right)^{1-\gamma}}{1-\gamma}$$

$$s.t.\quad \begin{array}{c} y_i + \sum_j x_{ij} = A_i F_i\left(\{x_{ij}\}, L_i\right)\ \forall i \\ \sum_i L_i = L \end{array} \tag{70}$$

and

$$max_L \frac{C^*(L;A)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi} \tag{71}$$

The solution of (71) must satisfy

$$C^*(L;A)^{\gamma} L^{\varphi} = \frac{\partial C^*}{\partial L}$$

Using the envelope theorem in problem (70) we have that

$$\frac{\partial C^*}{\partial L} = C^{*\gamma} \nu_L(A)$$

where ν_L is the Lagrange multiplier associated to the constraint $\sum_i L_i = L$. Moreover, from the first order condition

$$L^{\varphi} = \nu_L(A)$$

we have

$$\frac{d\log L}{d\log A_i} = \frac{1}{\varphi}\frac{d\log \nu_L}{d\log A_i}$$

Applying again the envelope theorem to problem (70) we have

$$\frac{d\log C^*}{d\log A_i} = C^{*\gamma}\left(\frac{\nu_L L}{\varphi C^*}\frac{d\log \nu_L}{d\log A_i} + \frac{\nu_i F_i\left(\{x_{ij}\}, L_i\right)}{C^*}\right) \tag{72}$$

We now re-write the two elements on the right hand side of equation (72). First, we show that

$$C^{*\gamma}\frac{\nu_i F_i\left(\{x_{ij}\}, L_i\right)}{C^*} = \lambda_i \tag{73}$$

where λ_i is the share of i's sales in GDP; second, we show that

$$C^{*\gamma}\frac{\nu_L L}{\varphi C^*}\frac{d\log\nu_L}{d\log A_i} = \frac{1}{\varphi}\lambda_i - \frac{\gamma}{\varphi}\frac{d\log C^*}{d\log A_i} \tag{74}$$

Putting these two results together in turn implies that

$$\frac{d\log C^*}{d\log A_i} = \frac{1+\varphi}{\gamma+\varphi}\lambda_i$$

which is the result that we set out to demonstrate.

We first prove (73). To do this, we show that in the competitive equilibrium $C^{*\gamma}\nu_i$ is equal to the price of good i relative to the CPI. It then follows from the definition of the sales share λ_i that

$$C^{*\gamma}\frac{\nu_i F_i\left(\{x_{ij}\}, L_i\right)}{C^*} = \frac{p_i F_i\left(\{x_{ij}\}, L_i\right)}{PC^*} = \lambda_i$$

From the FOCs of problem (70) we have that $C_i = C^{\gamma}\nu_i$, and from consumer optimization in the competitive equilibrium we have $\frac{C_j}{C_i} = \frac{p_j}{p_i}$. Thus

$$\frac{C_j}{C_i} = \frac{\nu_j}{\nu_i} = \frac{p_j}{p_i}$$

Using the fact that C is homogeneous of degree one, and normalizing the CPI to 1 ($\sum_j \frac{p_j y_j}{C} = 1$), we have

$$1 = \frac{\sum C_j y_j}{C_i} = \frac{C}{C_i} = \frac{C}{p_i} \Rightarrow p_i = C_i$$

The FOCs for (70) in turn imply that $p_i = C^{\gamma}\nu_i$.

Let's now derive equation (74). From the FOCs of (70) it holds that $C^{\gamma}\nu_L = C^{\gamma}\nu_i A_i F_{iL} = p_i A_i F_{iL} = w \;\forall i$, where the last equality follows from firm optimization in the competitive equilibrium. Moreover, from the consumers' budget constraint we have that $w = \frac{C^*}{L}$. Thus

$$C^{*\gamma}\frac{\nu_L L}{\varphi C^*}\frac{d\log\nu_L}{d\log A_i} = \frac{1}{\varphi}\left(\frac{d\log w}{d\log A_i} - \gamma\frac{d\log C^*}{d\log A_i}\right)$$

To conclude the proof we need to show that

$$\frac{d\log w}{d\log A_i} = \lambda_i$$

Using again the consumers' budget constraint we have

$$\frac{d\log w}{d\log A_i} = \frac{\partial \log C^*}{\partial \log A_i} + \left(\frac{\partial \log C^*}{\partial \log L} - 1\right)\frac{d\log L}{d\log A_i} = \lambda_i$$

□

The intuition for this result is simple. From Hulten's theorem, under flexible prices the first-order change in aggregate productivity is a weighted sum of sector-level productivity shocks, with weights given by sales shares λ:

$$d\log A_{AGG} = \lambda^T d\log A \tag{75}$$

In the efficient (flex-price) economy, the equilibrium change in labor supply can be derived from the optimal consumption-leisure trade-off. It is equal to

$$d\log L^{nat} = \frac{1-\gamma}{\gamma+\varphi}\lambda^T d\log A \tag{76}$$

Finally, aggregate output can be derived as a function of aggregate labor supply and aggregate productivity:

$$Y = A_{AGG}L \tag{77}$$

Log-linearizing equation (77) we obtain

$$y^{nat} = d\log L^{nat} + d\log A_{AGG} \tag{78}$$

Equation (68) follows immediately from (75), (76) and (78).

Lemma 7. *Around the undistorted steady-state, the first order change in aggregate productivity in the economy with price rigidities is the same as in the economy with flexible prices.*[1]

Proof. The flex-price allocation is efficient. This implies that productivity is maximized by optimally allocating labor both within and across sectors. With sticky prices, instead, after a productivity shock the labor allocation is distorted. This happens because the firms who cannot adjust their price absorb cost changes into their markup. Formally, we can derive the efficient equilibrium as the solution of the problem

$$max_{L,\{L_{i,f},y_{if},\{x_{ijf}\},\{\mu_{if}\}\}}\ \frac{C\left(\{y_i\}_{i=1}^N\right)^{1-\gamma}}{1-\gamma} - \frac{L^{1+\varphi}}{1+\varphi}$$

[1]There is a second order productivity loss due to incomplete price adjustment. See Section 4.2

$$
s.t. \quad \begin{array}{c} y_i + \sum_j x_{ij} = A_i \left[\int \left(F_i \left(\{x_{ijf}\}, L_{if} \right) \right)^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right]^{\frac{\epsilon_i}{\epsilon_i - 1}} \ \forall i \\ \frac{F_i(\{x_{ijf}\}, L_{if})}{F_i(\{x_{ijg}\}, L_{ig})} = \left(\frac{\mu_{if}}{\mu_{jg}} \right)^{-\epsilon_i} \ \forall i, f, g \\ \sum_{if} L_{if} = L \end{array} \tag{79}
$$

where μ_{if} is the markup of firm f in sector i. In the efficient equilibrium we have $\mu^*_{if} = 1 \ \forall i, f$. The sticky-price allocation instead solves a modified version of (79), where the markups of non-adjusting firms are constrained to be equal to their value in the sticky-price equilibrium. Applying the envelope theorem to problem (79) we find that, around the efficient equilibrium, the first-order productivity loss induced by these markup distortions is zero. □

A2: Sector-level inflation

Definitions I first introduce two definitions which will be useful in the proofs to follow.

Definition 1. The cost-based input-output matrix $\widetilde{\Omega}$ is an $N \times N$ matrix with element i, j given by the expenditure share on input j in i's cost:

$$
\widetilde{\omega}_{ij} = \frac{p_j x_{ij}}{mc_i y_i}
$$

Definition 2. The sector-level steady-state labor shares in marginal costs are encoded in the $N \times 1$ vector $\tilde{\alpha}$ with components

$$
\tilde{\alpha}_i = \frac{wL_i}{mc_i y_i}
$$

In a steady-state with optimal subsidies it holds that $\Omega = \widetilde{\Omega}$ and $\alpha = \tilde{\alpha}$.

Proof of Propositions 1 **and** 2

The proofs of these two propositions rely on the same algebra, therefore I present them together.

Our objective is to derive the elasticities of sector-level prices with respect to productivity and the output gap. To do this, we first solve for the change in marginal costs as a function of the change in prices, wages and productivity. We will then solve for the endogenous response of prices and wages to productivity shocks and the output gap.

The change in marginal costs is given by:

$$
d \log mc_i = \widetilde{\alpha}_i d \log w + \sum_j \widetilde{\omega}_{ij} d \log p_j - d \log A_i
$$

We can then write the change in sectoral prices as function of the change in marginal costs using the Calvo assumption:

$$
d \log p_i = \delta_i d \log mc_i \tag{80}
$$

so that

$$d\log mc_i = \widetilde{\alpha}_i d\log w - d\log A_i + \sum_j \widetilde{\omega}_{ij}\delta_j d\log mc_j$$

This allows to solve for the change in marginal cost as a function of the change in wages and productivity:

$$d\log mc = \left(I - \widetilde{\Omega}\Delta\right)^{-1}(\widetilde{\alpha} d\log w - d\log A) \tag{81}$$

The change in consumer prices is

$$d\log P = \beta^T d\log p = \beta^T \Delta d\log mc = \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}(\widetilde{\alpha} d\log w - d\log A) \tag{82}$$

From the consumption-leisure trade-off we have

$$d\log w = d\log P + (\varphi d\log L + \gamma d\log y) =$$

$$= \left(\varphi d\log L + \gamma\tilde{y} + \gamma y^{nat} + d\log P\right) =$$

$$= \left((\gamma+\varphi)\,\widetilde{y} + \lambda^T d\log A + d\log P\right)$$

We can then use (82) to solve for the change in wages as a function of the output gap and productivity shocks. We have:

$$d\log w - d\log P = \left(1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}\right) d\log w + \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} d\log A =$$

$$= (\gamma+\varphi)\,\widetilde{y} + \lambda^T d\log A$$

so that

$$d\log w = \frac{(\gamma+\varphi)\,\widetilde{y} + \beta^T\left[\left(I-\widetilde{\Omega}\right)^{-1} - \Delta\left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] d\log A}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}} \tag{83}$$

Lemma 8 below shows that the denominator in (83) is always well defined.

To find marginal costs as function of the output gap and productivity shocks, we plug plug (83) into (81):

$$d\log mc = \frac{(\gamma+\varphi)\left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}\widetilde{y} +$$

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}\left(\frac{\widetilde{\alpha}\left[\lambda^{T}-\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]}{1-\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}-I\right)d\log A$$

From the Calvo assumption (80), the price response is

$$\pi=(\gamma+\varphi)\frac{\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}{1-\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}\widetilde{y}+$$

$$\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\left(\frac{\widetilde{\alpha}\left[\lambda^{T}-\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]}{1-\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}-I\right)d\log A \tag{84}$$

The expressions for the elasticities $\mathcal{B}$ and $\mathcal{V}$ in Section 3.2 follow immediately from (84).

Lemma 8. $1-\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}>0.$

Proof. First note that, by definition of labor and input shares, it holds that $\tilde{\alpha}=(I-\Omega)\,\mathbf{1}$, where $\mathbf{1}$ is a $N\times 1$ vector with all entries equal to 1. Thus we have that

$$\beta^{T}\left(I-\widetilde{\Omega}\right)^{-1}\widetilde{\alpha}=\beta^{T}\left(I-\widetilde{\Omega}\right)^{-1}(I-\Omega)\,\mathbf{1}=$$

$$\beta^{T}\mathbf{1}=\sum_{j}\beta_{j}=1$$

To prove Lemma 8 it is enough to show that

$$\beta^{T}\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}<\beta^{T}\left(I-\widetilde{\Omega}\right)^{-1}\widetilde{\alpha}$$

A sufficient condition for this to hold is that

$$\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}<(I-\Omega)^{-1}_{ij}\ \ \forall i,j$$

Note that

$$\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}=\delta_{i}\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}<\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}$$

therefore it is sufficient to prove that

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}<(I-\Omega)^{-1}_{ij}\ \ \forall i,j$$

We can do so using the relations

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}=I+\widetilde{\Omega}\Delta+\left(\widetilde{\Omega}\Delta\right)^{2}+...$$

$$\left(I-\widetilde{\Omega}\right)^{-1}=I+\widetilde{\Omega}+\widetilde{\Omega}^{2}+...$$

This yields

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}_{ij}=\mathbb{I}(i=j)+\omega_{ij}\delta_j+\sum_k\omega_{ik}\omega_{kj}\delta_j\delta_k+...<$$

$$\mathbb{I}(i=j)+\omega_{ij}+\sum_k\omega_{ik}\omega_{kj}+...=\left(I-\widetilde{\Omega}\right)^{-1}_{ij}$$

which proves our result. □

Corollary 1. *As long as some sector uses an intermediate input with sticky prices, the pass-through of wages into marginal costs is less than one:*

$$\exists i,j \text{ such that } \omega_{ij}\delta_j<\omega_{ij}\Rightarrow (I-\Omega\Delta)^{-1}\alpha<\mathbf{1} \tag{85}$$

As a result, sectoral price pass-throughs are smaller than the corresponding adjustment frequencies, and the aggregate price pass-through $\bar{\delta}_w$ *is less than the average price rigidity* $\mathbb{E}_\beta(\delta)$*:*

$$\exists i,j \text{ such that } \omega_{ij}\delta_j<\omega_{ij}\Rightarrow \begin{cases}\Delta\left((I-\Omega\Delta)^{-1}\alpha\right)<diag(\Delta)\\ \bar{\delta}_w<\mathbb{E}_\beta(\delta)\end{cases} \tag{86}$$

A reduction in labor shares compensated by a uniform increase in input shares reduces $\bar{\delta}_w$ *:*

$$d\alpha_i<0,\ d\omega_{ij}=d\omega_{ik}\ \forall j,k,\ \exists j \text{ such that } \omega_{ij}\delta_j<\omega_{ij}\Rightarrow d\bar{\delta}_w<0 \tag{87}$$

Proof. In our setup labor is the only factor of production. Therefore labor and input shares must sum to one:

$$\alpha+\Omega\mathbf{1}=\mathbf{1}$$

so that $(I-\Omega)^{-1}\alpha=\mathbf{1}$. The result

$$\exists i,j \text{ such that } \omega_{ij}\delta_j<\omega_{ij}\Longrightarrow (I-\Omega\Delta)^{-1}\alpha<\mathbf{1}$$

follows immediately from the fact that each term in the geometric sum

$$(I-\Omega\Delta)^{-1}\alpha=\left(I+\Omega\Delta+(\Omega\Delta)^2+...\right)\alpha$$

has at least one component that is smaller than in the corresponding term of

$$(I-\Omega)^{-1}\alpha = \left(I+\Omega+\Omega^2+...\right)\alpha$$

It then follows that

$$\bar{\delta}_w = \sum_i \beta_i\delta_i\left[(I-\Omega\Delta)^{-1}\alpha\right]_i < \sum_i \beta_i\delta_i \equiv \mathbb{E}_\beta(\delta)$$

Equation 87 is obtained by differentiating (9). □

Corollary 2. *It holds that $\mathcal{V}\alpha = \mathbf{0}$, and α is the only vector with this property.*

Proof. We first show that $\mathcal{V}\alpha = \mathbf{0}$, that is, α belongs to $ker(\mathcal{V})$.

Recall the expression for $\mathcal{V}$:

$$\mathcal{V} = \Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\left[\frac{\widetilde{\alpha}\left[\lambda^T-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}-I\right]$$

Thus we have

$$\mathcal{V}\alpha = \left(I-\widetilde{\Omega}\Delta\right)^{-1}\left[\widetilde{\alpha}\frac{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\alpha}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}}-\widetilde{\alpha}\right]=0$$

We then prove that $\widetilde{\alpha}$ is the only element of $ker(\mathcal{V})$. Note that for every vector $x \neq 0$ such that $\mathcal{V}x = 0$ it must hold that

$$\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}\frac{\left[\lambda^T-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]x}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}} = \left(I-\widetilde{\Omega}\Delta\right)^{-1}x \iff$$

$$\widetilde{\alpha}\frac{\left[\lambda^T-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]x}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}} = x \iff$$

$$\widetilde{\alpha}_i\frac{\left[\lambda^T-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\right]x}{1-\beta^T\Delta\left(I-\widetilde{\Omega}\Delta\right)^{-1}\widetilde{\alpha}} = x_i\ \forall i \tag{88}$$

where

$$\frac{\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] x}{1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}} \in \mathbb{R} \neq 0$$

otherwise we would have $x = 0$. From (88) we then have that

$$\frac{\alpha_i}{\alpha_j} = \frac{x_i}{x_j} \ \forall i, j$$

so that x is proportional to the vector of labor shares α. □

A3: Output gap and aggregate inflation

Proof of Proposition 3:

The pricing equation (2) allows to infer markup changes from inflation rates and price adjustment probabilities:

$$-\, d \log \mu = (I - \Delta)\, \Delta^{-1} \pi \tag{89}$$

Lemma 9 below then relates the output gap with sector-level markups:

$$(\gamma + \varphi)\, \tilde{y} = -\lambda^T d \log \mu \tag{90}$$

Together, Equations (90) and (89) yield the sales-weighted Phillips curve:

$$\lambda^T (I - \Delta)\, \Delta^{-1} \pi = -\lambda^T d \log \mu = (\gamma + \varphi)\, \tilde{y}$$

Finally, Lemma 10 below implies that $DC = \lambda^T (I - \Delta)\, \Delta^{-1} \pi$ is the only aggregate inflation statistic which yields a Phillips curve with no endogenous cost-push term.

Lemma 9. *The output gap is proportional to a notion of "aggregate" markup, which weights sector level markups according to sales shares:*

$$(\gamma + \varphi)\, \tilde{y} = -\lambda^T d \log \mu \tag{91}$$

Proof. From the consumers' optimal labor supply decision we have:

$$\left(\log w - \log w^{nat}\right) - \left(\log P - \log P^{nat}\right) = \gamma \left(\log C - \log C^{nat}\right) + \varphi \left(\log L - \log L^{nat}\right)$$

From the definition of output gap we have

$$\tilde{y} = d \log C - d \log C^{nat}$$

while Lemma 7 implies that

$$\log L - \log L^{nat} = d\log C - d\log C^{nat}$$

Therefore we have

$$\gamma\left(\log C - \log C^{nat}\right) + \varphi\left(\log L - \log L^{nat}\right) = (\gamma+\varphi)\,\tilde{y}$$

so that

$$\left(\log w - \log w^{nat}\right) - \left(\log P - \log P^{nat}\right) = (\gamma+\varphi)\,\tilde{y} \tag{92}$$

We next need to compute the left hand side of (92), which corresponds to the change in real wages induced by markup distortions. To solve for real wages as a function of sector-level markups we first consider how nominal wages w impact marginal costs and prices. We have:

$$d\log mc_i = \widetilde{\alpha}_i d\log w + \sum_j \widetilde{\omega}_{ij} d\log p_j - d\log A_i$$

and

$$d\log p_i = d\log mc_i + d\log \mu_i \tag{93}$$

$$\Rightarrow d\log mc = \left(I-\widetilde{\Omega}\right)^{-1}\left(\widetilde{\alpha} d\log w - d\log A + \widetilde{\Omega} d\log\mu\right) \tag{94}$$

$$\Rightarrow d\log P = \beta^T(d\log mc + d\log\mu) = d\log w + \tilde{\lambda}^T\left(d\log\mu - d\log A\right)$$

It follows that

$$d\log w - d\log P = \tilde{\lambda}^T\left(d\log A - d\log\mu\right) \tag{95}$$

In the natural outcome the productivity change is the same as in the economy with sticky prices, while markups are constant ($d\log\mu = 0$). Therefore we have

$$\left(\log w - \log w^{nat}\right) - \left(\log P - \log P^{nat}\right) = -\lambda^T d\log\mu \tag{96}$$

Equations (92) and (96) together give the result. □

Lemma 10. *If $\Delta \neq I$ then $\lambda^T\left(I-\Delta\right)\Delta^{-1}$ is the only vector ν that satisfies*

$$\nu^T\mathcal{V} = 0$$

Proof. We need to prove that all the vectors $x \neq 0$ satisfying $x^T\mathcal{V} = \mathbf{0}$ are proportional to $\left(I-\Delta\right)\Delta^{-1}\lambda$. Proposition 3 implies that $\lambda^T\left(I-\Delta\right)\Delta^{-1}\mathcal{V} = \mathbf{0}$.

Consider then all vectors x such that $x^T\mathcal{V} = \mathbf{0}$. Note that

$$x^T\mathcal{V} = \mathbf{0} \iff$$

$$x^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \left[\widetilde{\alpha}\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] - \left(1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}\right) I\right] = \mathbf{0}$$

$$\iff \tilde{x}^T \left[\widetilde{\alpha}\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] - \left(1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}\right) I\right] = \mathbf{0} \tag{97}$$

where $\tilde{x}^T \equiv x^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}$.

To prove the Lemma we need to show that all vectors $\tilde{x}$ satisfying (97) are proportional to $\lambda^T (I - \Delta) \left(I - \widetilde{\Omega}\Delta\right)^{-1}$.

From (97) we have the relation

$$\left(1 - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} \widetilde{\alpha}\right) \tilde{x}_j = \tilde{x}^T \widetilde{\alpha} \left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_j \quad \forall j \tag{98}$$

The product $\tilde{x}^T \widetilde{\alpha}$ is a scalar, and we must have $\tilde{x}^T \widetilde{\alpha} \neq 0$, otherwise we would get $\tilde{x}^T = \mathbf{0}$ (while we imposed that $\tilde{x} \neq 0$). Therefore (98) implies the condition

$$\frac{\tilde{x}_i}{\tilde{x}_j} = \frac{\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_i}{\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_j}$$

The ratio on the RHS is well defined, because

$$\left[\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right]_j > \left[\lambda^T - \beta^T \left(I - \widetilde{\Omega}\right)^{-1}\right]_j = 0 \ \forall j$$

(see Lemma 8).

Thus, $\tilde{x}^T$ must be proportional to the vector

$$\lambda^T - \beta^T \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1} = \beta^T \left[(I - \Omega)^{-1} - \Delta \left(I - \widetilde{\Omega}\Delta\right)^{-1}\right] =$$

$$= \beta^T \left[(I - \Omega)^{-1} (I - \Omega\Delta) - \Delta\right] (I - \Omega\Delta)^{-1} =$$

$$= \beta^T (I - \Omega)^{-1} (I - \Delta) (I - \Omega\Delta)^{-1} = \lambda^T (I - \Delta) (I - \Omega\Delta)^{-1}$$

□

B: Optimal policy

B1: Welfare function

Lemma 11. *The distortion in sectoral relative prices with respect to the flex-price outcome is given by*

$$d\log p - d\log w = (I-\Omega)^{-1}(I-\Delta)\Delta^{-1}\pi \tag{99}$$

Proof. From equation (94) we have

$$d\log mc = \left(I-\widetilde{\Omega}\right)^{-1}\left(\widetilde{\alpha} d\log w - d\log A + \widetilde{\Omega} d\log \mu\right)$$

so that

$$d\log p = d\log w + \left(I-\widetilde{\Omega}\right)^{-1}(d\log \mu - d\log A)$$

Therefore for each sector i we have

$$\left(d\log p_i - d\log p_i^{nat}\right) - \left(d\log w - d\log w^{nat}\right) = (I-\Omega)^{-1} d\log \mu$$

We can then use the pricing equation (2) to substitute for markups as a function of inflation rates. □

Proof of Proposition 4**:**

In what follows, I will use the second-order approximation

$$\frac{Z-Z^*}{Z} \simeq \log\left(\frac{Z}{Z^*}\right) + \frac{1}{2}\log\left(\frac{Z}{Z^*}\right)^2$$

I denote by

$$\hat{z} = \log\left(\frac{Z}{Z^*}\right)$$

I will prove below that, to the second-order, the log change in output with respect to the efficient equilibrium is given by

$$\hat{y} = \hat{l} - d$$

where d is a second order term.

Using this result we can approximate the utility function around the efficient outcome as

$$\frac{U-U^*}{U_cC} \simeq \hat{y} + \frac{1}{2}\hat{y}^2 + \frac{1}{2}\frac{U_{cc}C}{U_c}\hat{y}^2 + \frac{U_lL}{U_cC}\left(\hat{l} + \frac{1}{2}\frac{U_{ll}N}{U_l}\hat{l}^2\right) =$$

$$= \hat{y} + \frac{1-\gamma}{2}\hat{y}^2 - \left(\hat{l} + \frac{1+\varphi}{2}\hat{l}^2\right) =$$

$$= \hat{y} + \frac{1-\gamma}{2}\hat{y}^2 - \left(\hat{y} + d + \frac{1+\varphi}{2}\hat{y}^2\right) =$$

$$= -\frac{\gamma+\varphi}{2}\tilde{y}^2 - d$$

where the last equality follows from the fact that, to the second order, $\hat{y}^2 = \tilde{y}^2$ and $d^2 = \hat{y}d = 0$.

I will now derive the approximation

$$\hat{y} = \hat{l} - d$$

and the explicit expression for the second order component d.

Lemma 7 proves that $d\log y = d\log L$ to a first order. Therefore we have

$$\hat{y} = \underbrace{\hat{l}}_{\text{first order}} - \underbrace{d}_{\text{second order}} + \text{higher order terms}$$

Intuitively, the second order term is a productivity loss induced by markup distortions. These markup distortions endogenously arise from productivity shocks when prices are sticky, and have two effects. First, the relative price of different firms within the same sector is distorted with respect to the efficient equilibrium, therefore sector-level productivities are lower (i.e. more labor is required to produce one unit of sectoral output). I will denote the productivity loss from within-sector price distortions by the vector a, with components

$$a_i \equiv \log\left(\frac{Y_i}{F\left(\{x_{ij}\}, L_i\right)}\right) - \log A_i$$

where

$$x_{ij} = \int x_{ij}(t)dt$$

$$L_i = \int L_i(t)dt$$

and A_i is the TFP of sector i. Second, sector-level markups are also distorted, so that the relative price indexes of different sectors are different from the efficient equilibrium. Cross-sector price distortions result in lower aggregate productivity.

I define sector-level markups as

$$\mu_i = \frac{p_i}{mc_i}$$

where p_i is the sectoral price index (note that the marginal cost is the same for all producers in sector i). I derive a first-order approximation of the "within-sector" and the "cross-sector" component of the productivity loss, and then compute the second order approximation around the efficient steady-state.

Note that aggregate productivity $\frac{Y}{L}$ can be expressed as a function of real wages and labor shares. Denoting the aggregate labor share by $\Lambda = \frac{wL}{GDP} = \frac{wL}{PY}$, by definition we can write aggregate output as

$$Y = \frac{1}{\Lambda}\frac{w}{P}L$$

In log-deviations from steady-state we have:

$$\hat{Y} = \hat{w} - \hat{P} - \hat{\Lambda} + \hat{l} \tag{100}$$

The first order change in real wages $d\log w - d\log P$ is derived in the proof of Lemma 9 (see equation (96)). Combining (96) with (100) we obtain the first-order approximation

$$d\log Y - d\log L = \tilde{\lambda}^T \left(a - d\log\mu\right) - d\log\Lambda \tag{101}$$

We then need to compute $d\log\Lambda$ as function of the change in sectoral markups and productivities.

The consumers' budget constraint is

$$PC = wL + \Pi - T$$

where Π are aggregate profits and T is a lump-sum tax used to finance input subsidies. Dividing both sides by PC we find

$$1 = \Lambda + \frac{\Pi - T}{PY} = \Lambda + \lambda^T\left(1 - \frac{1}{\mu}\right)$$

where μ is the vector of sector-level markups defined above. Therefore we have

$$d\log\Lambda = -\frac{1}{\Lambda}\left(\sum_i d\lambda_i\left(1 - \frac{1}{\mu_i}\right) + \sum_i \lambda_i \frac{d\log\mu_i}{\mu_i}\right)$$

Using (101) we find that, around the efficient steady state (where $\mu_i = 1\ \forall i$)

$$d\log Y - d\log L = \underbrace{\tilde{\lambda}^T a}_{\text{within sector}} + \underbrace{\left(\frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T\right) d\log\mu}_{\text{cross-sector}} \tag{102}$$

As $\frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T = 0$ around $\mu = \mathbf{1}$, the first-order productivity loss from cross-sector misallocation is zero. To compute the second-order loss we need to take the second derivative of the cross-sector component in equation (102) .

Note that, since the first order effect on both cross-sector misallocation and sector-level productivities is zero, the second-order terms in $(d\log A)(d\log\mu)$ are also going to be zero. Therefore we only need to derive the cross-sector component with respect to sector-level markups. We have:

$$D^2\left(\left(\frac{\lambda^T}{\Lambda} - \tilde{\lambda}^T\right) d\log\mu\right) =$$

$$= \frac{1}{\Lambda}\left(-\left(\sum_i \lambda_i \frac{d\log\mu_i}{\mu_i}\right)^2 + 2\sum_i d\lambda_i \frac{d\log\mu_i}{\mu_i} + \sum_i \frac{\lambda_i}{\mu_i}(d\log\mu_i)^2\right) - \sum_i d\tilde{\lambda}_i d\log\mu_i =$$

$$= -\frac{1}{2}\sum_i\sum_j \widetilde{d}^2_{ij} d\log\mu_i d\log\mu_j \tag{103}$$

where

$$\widetilde{d}^2_{ij} = \sum_h\sum_k \beta_h\beta_k\sigma_{hk}\left[(I-\Omega)^{-1}_{hi} - (I-\Omega)^{-1}_{ki}\right]\left[(I-\Omega)^{-1}_{hj} - (I-\Omega)^{-1}_{kj}\right] +$$

$$+\sum_t \lambda_t \sum_h\sum_k \omega_{th}\omega_{tk}\theta^t_{hk}\left[(I-\Omega)^{-1}_{hi} - (I-\Omega)^{-1}_{ki}\right]\left[(I-\Omega)^{-1}_{hj} - (I-\Omega)^{-1}_{kj}\right] +$$

$$+\sum_t \lambda_t\alpha_t \sum_h \omega_{th}\theta^t_{hL}(I-\Omega)^{-1}_{hi}(I-\Omega)^{-1}_{hj} =$$

$$= \Phi_C\left((I-\Omega)^{-1}_{(i)}, (I-\Omega)^{-1}_{(j)}\right) + \sum_t \lambda_t\Phi_t\left((I-\Omega)^{-1}_{(i)}, (I-\Omega)^{-1}_{(j)}\right) \tag{104}$$

To derive the welfare loss as a function of sector-level inflation rates we need to solve for the endogenous change in sector-level markups due to price rigidities. The mapping between the two is given by equation (2):

$$d\log\mu = -(I-\Delta)\, d\log mc = -(I-\Delta)\,\Delta^{-1}\pi$$

Therefore we can re-write (103) as

$$d^2\log Y - d^2\log L = \tilde{\lambda}^T a - \frac{1}{2}\pi^T\mathcal{D}_2\pi$$

with

$$d^2_{ij} = \frac{1-\delta_i}{\delta_i}\frac{1-\delta_j}{\delta_j}\widetilde{d}^2_{ij}$$

It remains to compute the "within-sector" component $\lambda^T a$.

Index by t the different varieties of product i and note that, given the CES assumption, sectoral output can be written as

$$Y_i = A_i F\left(\{x_{ij}\}, L_i\right) \frac{p_i^{-\epsilon_i}}{\int p_{it}^{-\epsilon_i} dt} \tag{105}$$

where

$$x_{ij} = \int x_{ij}(t) dt$$

$$L_i = \int L_i(t) dt$$

as above. Using the definition of a we have

$$a_i = \log\left(\frac{p_i^{-\epsilon_i}}{\int p_{it}^{-\epsilon_i} dt}\right)$$

A first order approximation of a_i is given by

$$da_i = \epsilon_i \left[\frac{\int p_{it}^{-\epsilon_i} d\log p_{it} dt}{\int p_{it}^{-\epsilon_i} dt} - \frac{\int p_{it}^{1-\epsilon_i} d\log p_{it} dt}{\int p_{it}^{1-\epsilon_i} dt}\right] \tag{106}$$

Given the Calvo assumption, around the efficient steady state we have that

$$\frac{\int p_{it}^{-\epsilon_i} d\log p_{it} dt}{\int p_{it}^{-\epsilon_i} dt} = \frac{\int p_{it}^{1-\epsilon_i} d\log p_{it} dt}{\int p_{it}^{1-\epsilon_i} dt} = \delta d \log mc_i$$

so that $da_i = 0$.

Let's now compute the second-order loss by deriving (106) a second time with respect to $\{d\log p_{it}\}$. We find[2]

$$d^2 a_i = \epsilon_i \left[\int \left(\log p_{it} - \log p_i\right)^2 dt - \left(\int \left(\log p_{it} - \log p_i\right) dt\right)^2\right] =$$

$$= \epsilon_i \frac{1-\delta_i}{\delta_i} \pi_i^2$$

We can thus express the second-order welfare loss from within-sector misallocation as

$$\frac{1}{2} \pi \mathcal{D}_1 \pi$$

where

$$d_{ij}^1 = \begin{cases} 0 & if\ i \neq j \\ \lambda_i \epsilon_i \frac{1-\delta_i}{\delta_i} & if\ i = j \end{cases}$$

[2]This is the same as in the traditional NK model (Gali (2008) Ch.4)

B2: Policy target, past markups

Proof of Proposition 6:

We look for weights ϕ such that

$$\phi^T \pi = \phi^T \left(\mathcal{B}\widetilde{y} + \mathcal{V} d\log A\right) > 0 \Longleftrightarrow \widetilde{y} > \widetilde{y}^* \tag{107}$$

I will first construct a vector ϕ that satisfies the condition

$$\phi^T \left(\mathcal{B}\widetilde{y} + \mathcal{V} d\log A\right) = 0 \Longleftrightarrow \widetilde{y} = \widetilde{y}^* \tag{108}$$

and then argue that this vector also satisfies (107).

Note that, as long as $\phi^T \mathcal{B} \neq 0$, we have

$$\phi^T \left(\mathcal{B}\widetilde{y} + \mathcal{V} d\log A\right) = 0 \Longleftrightarrow \widetilde{y} = -\frac{\phi^T \mathcal{V} d\log A}{\phi^T \mathcal{B}}$$

while the optimal output gap is

$$\widetilde{y}^* = -\frac{\mathcal{B}^T \mathcal{D} \mathcal{V} d\log A}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}}$$

Thus (108) is satisfied for all realizations of $d\log A$ if and only if ϕ is such that

$$\frac{\phi^T \mathcal{V} d\log A}{\phi^T \mathcal{B}} = \frac{\mathcal{B}^T \mathcal{D} \mathcal{V} d\log A}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}} \quad \forall d\log A$$

In turn, this is true if and only if

$$\phi^T \left[I - \frac{\mathcal{B}\mathcal{B}^T \mathcal{D}}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}}\right] \mathcal{V} = 0 \tag{109}$$

that is, if and only if ϕ is a left eigenvector of the matrix $\left[I - \frac{\mathcal{B}\mathcal{B}^T \Delta\Xi\Delta}{\gamma+\varphi+\mathcal{B}^T \Delta\Xi\Delta\mathcal{B}}\right]\mathcal{V}$, relative to the eigenvalue 0.

We already proved in Lemma 9 that $\lambda^T (I - \Delta)\Delta^{-1}$ is a left eigenvector of the matrix $\mathcal{V}$ relative to the eigenvalue 0 (and it is the only such eigenvector). Therefore, as long as $\left[I - \frac{\mathcal{B}\mathcal{B}^T \mathcal{D}}{\gamma+\varphi+\mathcal{B}^T \mathcal{D}\mathcal{B}}\right]$ is invertible,

$$\phi^T = \lambda^T (I - \Delta)\Delta^{-1} \left[I - \frac{\mathcal{B}\mathcal{B}^T \mathcal{D}}{\gamma + \varphi + \mathcal{B}^T \mathcal{D} \mathcal{B}}\right]^{-1}$$

is the (unique) desired eigenvector of the matrix $\left[I - \frac{\mathcal{B}\mathcal{B}^T \mathcal{D}}{\gamma+\varphi+\mathcal{B}^T \mathcal{D}\mathcal{B}}\right]\mathcal{V}$.

The matrix $\left[I - \frac{\mathcal{B}\mathcal{B}^T \mathcal{D}}{\gamma+\varphi+\mathcal{B}^T \mathcal{D}\mathcal{B}}\right]$ is indeed invertible: it is immediate to see that $\frac{\mathcal{B}\mathcal{B}^T \mathcal{D}}{\gamma+\varphi+\mathcal{B}^T \mathcal{D}\mathcal{B}}$ has only one non-zero eigenvalue, $\frac{\mathcal{B}^T \mathcal{D}\mathcal{B}}{\gamma+\varphi+\mathcal{B}^T \mathcal{D}\mathcal{B}} < 1$, and $\mathcal{B}$ is the unique corresponding eigenvector.

Next, to satisfy condition (107) we need

$$\phi^T \left(\mathcal{B}\widetilde{y} + \mathcal{V} d\log A\right)$$

to be increasing in the output gap $\widetilde{y}$, which is true if and only if $\phi^T\mathcal{B} > 0$. To prove this we use the fact that $\mathcal{B}$ is an eigenvector of $\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$ relative to the eigenvalue $\frac{\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$. Therefore it is also an eigenvector of $\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1}$, relative to the eigenvalue $\frac{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi} > 1$. Thus we have

$$\phi^T\mathcal{B} = \lambda^T \left(I - \Delta\right)\Delta^{-1}\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1}\mathcal{B} =$$
$$= \gamma + \varphi + \mathcal{B}^T\mathcal{D}\mathcal{B} > 0$$

Finally, to obtain the formulation in (27) we observe that

$$\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1} = I + \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}} + \left(\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right)^2 + ...$$

and

$$\left(\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right)^n = \left(\frac{\mathcal{B}^T\mathcal{D}\mathcal{B}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right)^{n-1}\frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}$$

so that

$$\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1} = I + \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi}$$

Moreover, we have that

$$\frac{\lambda^T\left(I-\Delta\right)\Delta^{-1}\mathcal{B}}{\gamma+\varphi} = \frac{\lambda^T\left(I-\Delta\right)\left(I-\Omega\Delta\right)^{-1}\alpha}{1-\beta^T\Delta\left(I-\Omega\Delta\right)^{-1}\alpha} = 1$$

so that

$$\lambda^T\left(I-\Delta\right)\Delta^{-1}\left[I - \frac{\mathcal{B}\mathcal{B}^T\mathcal{D}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}\right]^{-1} = \lambda^T\left(I-\Delta\right)\Delta^{-1} + \mathcal{B}^T\mathcal{D}$$

Lemma 12 characterizes inflation, welfare and the optimal policy when pre-set prices at the sector-level are not equal to desired prices. This is captured by a deviation of initial markups μ_{-1} from their optimal level $\mu_{-1} = 1$. This result is useful to understand the evolution of inflation in the dynamic version of the model, derived in Appendix D2.

Lemma 12. *Denote the log-deviation of initial sector-level markups by the vector* $d\log\mu_{-1}$. *The elasticity of sectoral prices with respect to* μ_{-1} *is given by the matrix* $\mathcal{V}$. *The optimal monetary policy implements the output gap*

$$\widetilde{y}^* = -\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}d\log\mu_{-1}}{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}} \tag{110}$$

Proof. Sectoral inflation rates are given by

$$\pi_i = \delta_i \left(d\log mc_i - d\log \mu_{i-1}\right)$$

The mapping between sector-level inflation and current period markups is not affected by the presence of past markups, and is still given by (89). We proceed as in the proof of Propositions 2 and 1 to derive

$$\pi = \Delta \left(I - \Omega\Delta\right)^{-1} \left(\alpha d\log w - d\log \mu_{-1}\right)$$

and

$$d\log w = \frac{\gamma+\varphi}{1-\beta^T\left(I-\Omega\Delta\right)^{-1}\alpha}\left(\tilde{y}-\tilde{y}_{-1}\right) - \frac{\beta^T\Delta\left(I-\Omega\Delta\right)^{-1}}{1-\beta^T\left(I-\Omega\Delta\right)^{-1}\alpha} d\log\mu_{-1}$$

We solve for sectoral inflation rates as a function of $\tilde{y}$,$\tilde{y}_{-1}$ and $d\log\mu_{-1}$ following the same steps as in the proof of Propositions 2 and 1

Welfare is the same function of the output gap and sectoral inflation rates as in (18). This is because welfare depends on sector-level markups and on the variance of firm-level prices within sectors, and the mapping between both of these variables and sectoral inflation rates does not change in the presence of past markups. The optimal output gap follows from the first order conditions. □

C: Dynamics - Proofs

Proof of Proposition 8

This lemma characterizes the evolution of sectoral inflation rates and markups as a function of initial markups (which are a state variable), productivity shocks and monetary policy.

Denote by $\hat{\Delta}$ the diagonal matrix with elements

$$\hat{\delta}_i \equiv \frac{\delta_i\left(1-\rho(1-\delta_i)\right)}{1-\rho\delta_i\left(1-\delta_i\right)}$$

The first step is to solve for the growth rate of sector-level markups, remembering that it is given by the log-difference between the growth rates of prices and marginal costs:

$$-\left(log\mu_t - log\mu_{t-1}\right) = logmc_t = logmc_{t-1} - \pi_t =$$

$$= \alpha\left(logw_t - logw_{t-1}\right) - \left(I-\Omega\right)\pi_t - \left(logA_t - logA_{t-1}\right)$$

Using the pricing equation (57) we can rewrite this as

$$
\begin{aligned}
-\left(log\mu_t - log\mu_{t-1}\right) = -\left(I-\Omega\right)\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1}\left(-log\mu_t\right) + \alpha\left(logw_t - logw_{t-1}\right) + \\
-\left[\left(logA_t - logA_{t-1}\right) + \left(I-\Omega\right)\left[\rho\mathbb{E}\pi_{t+1} + \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D\right]\right] \Rightarrow \\
\left(\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1} + \left(I-\Omega\right)\right)\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1}\left(-log\mu_t\right) = \\
= \left(-log\mu_{t-1}\right) + \alpha\left(logw_t - logw_{t-1}\right) + \\
-\left[\left(logA_t - logA_{t-1}\right) + \left(I-\Omega\right)\left[\rho\mathbb{E}\pi_{t+1} + \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D\right]\right]
\end{aligned} \tag{111}
$$

Denote by

$$x_t \equiv -\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} log\mu_t$$

$$x_t^D \equiv \hat{\Delta}\left(I-\hat{\Delta}\right)^{-1} d\log\mu_t^D$$

We can then re-write equation (111) as

$$
\begin{aligned}
\left(\hat{\Delta}^{-1} - \Omega\right)x_t = \left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}x_{t-1} + \\
+\alpha\left(logw_t - logw_{t-1}\right) - \left[\left(logA_t - logA_{t-1}\right) + \left(I-\Omega\right)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right] \Rightarrow \\
x_t = \hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\left[\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}x_{t-1} + \alpha\left(logw_t - logw_{t-1}\right)\right. \\
\left. -\left[\left(logA_t - logA_{t-1}\right) + \left(I-\Omega\right)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right]\right]
\end{aligned}
$$

From the consumers' labor-leisure trade-off, wages evolve according to

$$
\begin{aligned}
logw_t - logw_{t-1} = \left(\gamma+\varphi\right)\left(\tilde{y}_t - \tilde{y}_{t-1}\right) + \lambda^T\left(logA_t - logA_{t-1}\right) + \\
+\beta^T\left(x_t + x_t^D + \rho\mathbb{E}\left(\pi_{t+1}\right)\right)
\end{aligned} \tag{112}
$$

so that

$$
\begin{aligned}
logw_t - logw_{t-1} = \frac{\gamma+\varphi}{1-\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\alpha}\left(\tilde{y}_t - \tilde{y}_{t-1}\right) + \\
+\frac{\lambda^T - \beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}}{1-\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\alpha}\left[\left(logA_t - logA_{t-1}\right) + \left(I-\Omega\right)\left[\rho\mathbb{E}\pi_{t+1} + x_t^D\right]\right] +
\end{aligned}
$$

$$+\frac{\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}}{1-\beta^T\hat{\Delta}\left(I-\Omega\hat{\Delta}\right)^{-1}\alpha}\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}x_{t-1} \tag{113}$$

Combining (112) and (113) we obtain

$$x_t=\hat{\mathcal{B}}\left(\tilde{y}_t-\tilde{y}_{t-1}\right)+\hat{\mathcal{V}}\left[\left(logA_t-logA_{t-1}\right)+\left(I-\Omega\right)\left[\rho\mathbb{E}\pi_{t+1}+x_t^D\right]\right]+\mathcal{M}x_{t-1} \tag{114}$$

Lemma 13 below proves that the matrix $\mathcal{M}$ is invertible. Denoting by $z_t\equiv x_{t-1}$, equations (57) and (114) can then be combined to obtain the following system of difference equations in π_t and z_t:

$$\begin{pmatrix}\rho\mathbb{E}\pi_{t+1}\\ z_{t+1}\end{pmatrix}=\begin{pmatrix}\mathcal{M}^{-1} & -I\\ I-\mathcal{M}^{-1} & I\end{pmatrix}\begin{pmatrix}\pi_t\\ z_t\end{pmatrix}+$$

$$+\begin{pmatrix}-\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\left(\tilde{y}_t-\tilde{y}_{t-1}\right)+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)-x_t^D\\ \mathcal{M}^{-1}\left(\hat{\mathcal{B}}\left(\tilde{y}_t-\tilde{y}_{t-1}\right)+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)\end{pmatrix} \tag{115}$$

Finally, it is useful to re-write (115) substituting out for the past output gap, using Lemma 9:

$$\begin{pmatrix}\rho\mathbb{E}\pi_{t+1}\\ z_{t+1}\end{pmatrix}=\begin{pmatrix}\mathcal{M}^{-1} & -\mathcal{Z}\\ I-\mathcal{M}^{-1} & \mathcal{Z}\end{pmatrix}\begin{pmatrix}\pi_t\\ z_t\end{pmatrix}+$$

$$+\begin{pmatrix}-\mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)-x_t^D\\ \mathcal{M}^{-1}\left(\hat{\mathcal{B}}\tilde{y}_t+\hat{\mathcal{V}}\left(logA_t-logA_{t-1}\right)\right)\end{pmatrix} \tag{116}$$

where

$$\mathcal{Z}\equiv\mathcal{M}^{-1}\hat{\mathcal{V}}\left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}$$

To obtain the system in (59) just use the definition

$$z_t\equiv-\hat{\Delta}\left(I-\hat{\Delta}\right)^{-1}log\mu_{t-1}$$

Lemma 13. *As long as no sector has fully flexible prices ($\delta_i<1\ \forall i$), the matrix $\mathcal{M}$ is invertible. Moreover, all of its eigenvalues have modulus (weakly) smaller than one.*

Proof. It holds that

$$\mathcal{M}=\left(I+\frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Delta\right)\Delta^{-1}$$

The matrix $\left(I+\frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)$ has eigenvalues 1 (and all vectors orthogonal to β are corresponding eigenvectors) and $\frac{1}{1-\beta^T\Delta(I-\Omega\Delta)^{-1}\alpha}>0$, with corresponding eigenvector $\frac{\mathcal{B}}{\gamma+\varphi}$. Therefore it is invertible. The matrix $\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Delta\right)\Delta^{-1}$

is invertible because we assumed that no sector has fully rigid or fully flexible prices. Thus $\mathcal{M}$ is invertible.

To prove that all eigenvalues are (weakly) smaller than one in modulus, note that $\mathcal{M}\mathbf{1} = \mathbf{1}$:

$$\mathcal{M}\mathbf{1} = \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\Delta\left(I-\Omega\Delta\right)^{-1}\left(\Delta^{-1} - \Omega - (I-\Omega)\right)\left(I-\Omega\right)^{-1}\alpha =$$

$$= \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\Delta\left(I-\Omega\Delta\right)^{-1}\left(\left(I-\Omega\Delta\right)\Delta^{-1}\left(I-\Omega\right)^{-1} - I\right)\alpha =$$

$$= \left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)\left(\left(I-\Omega\right)^{-1} - \Delta\left(I-\Omega\Delta\right)^{-1}\right)\alpha =$$

$$= \mathbf{1} - \Delta\left(I-\Omega\Delta\right)^{-1}\alpha + \frac{\mathcal{B}}{\gamma+\varphi}\left(1 - \beta^T\Delta\left(I-\Omega\Delta\right)^{-1}\alpha\right) = \mathbf{1}$$

In addition $\mathcal{M}$ has all positive elements, because both $\left(I + \frac{\mathcal{B}\beta^T}{\gamma+\varphi}\right)$ and

$$\Delta\left(I-\Omega\Delta\right)^{-1}\left(I-\Delta\right)\Delta^{-1}$$

have positive elements. These two properties imply that all of its eigenvalues must be smaller than one in modulus.
□

Proof of Lemma 3

We want to prove that there is a unique path of inflation rates and markups which remains bounded and where the output gap is zero in every period. We start from the system

$$\begin{pmatrix} \mathbb{E}\pi_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho}\mathcal{M}^{-1} & -\frac{1}{\rho}\mathcal{Z} \\ I-\mathcal{M}^{-1} & \mathcal{Z} \end{pmatrix}\begin{pmatrix} \pi_t \\ z_t \end{pmatrix} + \begin{pmatrix} -\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_t - logA_{t-1}\right) - \frac{1}{\rho}x_t^D \\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_t - logA_{t-1}\right) \end{pmatrix} \tag{117}$$

which corresponds to the system (116) with the additional condition that $\tilde{y}_t \equiv 0$. We show that the matrix

$$\mathcal{A} = \begin{pmatrix} \frac{1}{\rho}\mathcal{M}^{-1} & -\frac{1}{\rho}\mathcal{Z} \\ I-\mathcal{M}^{-1} & \mathcal{Z} \end{pmatrix}$$

has N eigenvectors greater than 1, and N smaller than 1.

This is enough to guarantee that the system has a unique bounded solution for any given past markups z_t and productivity/markup shocks $logA_t - logA_{t+1}$ and x_t^D. That is, given an initial condition for z_t, imposing that $|lim_{t\to\infty}\pi_{it}^*| < \infty \ \forall i$ and $|lim_{t\to\infty}z_{it}^*| < \infty \ \forall i$ pins down a unique initial value for π_t^*. We will first prove that having N eigenvectors greater than 1, and N smaller than 1 is sufficient to guarantee a unique solution. Then we will demonstrate that this condition is satisfied.

Given our assumption about the productivity process, we have that

$$\mathbb{E}lim_{t\to\infty}\begin{pmatrix}\pi_t^*\\ z_t^*\end{pmatrix} = lim_{t\to\infty}\mathcal{A}^t\begin{pmatrix}\pi_0^*\\ z_0\end{pmatrix} +$$

$$+lim_{t\to\infty}\left(\sum_{s\leq t}\eta^s\mathcal{A}^{t-s}\right)\begin{pmatrix}-\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) - \frac{1}{\rho}x_0^D\\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right)\end{pmatrix}$$

In turn, we can decompose the as a linear combination of the eigenvectors of $\mathcal{A}$, $\{w_1, ..., w_{2N}\}$:

$$\begin{pmatrix}-\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) - \frac{1}{\rho}x_0^D\\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right)\end{pmatrix} = a_1w_1 + ... + a_{2N}w_{2N}$$

Denote by $\{\nu_1, ..., \nu_{2N}\}$ the eigenvalues corresponding to $\{w_1, ..., w_{2N}\}$. We then have

$$lim_{t\to\infty}\left(\sum_{s\leq t}\eta^s\mathcal{A}^{t-s}\right)\begin{pmatrix}-\frac{1}{\rho}\mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right) - \frac{1}{\rho}x_0^D\\ \mathcal{M}^{-1}\hat{\mathcal{V}}\left(logA_0 - logA_{-1}\right)\end{pmatrix} =$$

$$= \mathcal{C} + lim_{t\to\infty}\mathcal{A}^t\sum_{i/\nu_i>1}\frac{\nu_i}{\nu_i-\eta}a_iw_i$$

where

$$\mathcal{C} < \sum_{i/\nu_i<1}\frac{a_iw_i}{1-\nu_i} < \infty$$

To have a unique bounded solution we need the condition

$$lim_{t\to\infty}\mathcal{A}^t\begin{pmatrix}\pi_0^*\\ z_0\end{pmatrix} = -lim_{t\to\infty}\mathcal{A}^t\sum_{i/\nu_i>1}\frac{\nu_i}{\nu_i-\eta}a_iw_i \tag{118}$$

to yield a unique solution π_0^*. Let's write $\begin{pmatrix}\pi_0^*\\ z_0\end{pmatrix}$ in components with respect to $\{w_1, ..., w_{2N}\}$:

$$\begin{pmatrix}\pi_0^*\\ z_0\end{pmatrix} = \sum_{i=1}^{2N}x_iw_i$$

For condition (118) to be satisfied we need that

$$\begin{cases}x_i = -\frac{\nu_i}{\nu_i-\eta}a_i & \forall i/\nu_i > 1\\ \sum_{i/\nu_i<1}x_iw_{i,N+1:2N} = z_0 + \sum_{i/\nu_i>1}\frac{\nu_i}{\nu_i-\eta}a_iw_{i,N+1:2N}\end{cases} \tag{119}$$

The second line in (119) is a system of N equations, with unknowns the coefficients x_i for i such that $\nu_i < 1$. The system has a unique solution if and only if there are exactly N eigenvalues $\nu_i < 1$, while the remaining N are greater or equal than 1.

Let's then prove that this condition is satisfied. Note that (for $x_t^D \equiv 0$) the two equations in (117) yield the optimal reset price equation

$$\rho \mathbb{E} \pi_{t+1} = \pi_t - z_{t+1}$$

It is convenient to substitute this to the first equation and use it together with the second to look for the eigenvectors of the matrix $\mathcal{A}$. Assume that $\begin{pmatrix} \pi \\ z \end{pmatrix}$ is an eigenvector relative to the eigenvalue ν. From the optimal reset price equation we find

$$\nu z = (1 - \rho \nu) \pi$$

The second equation in (117) yields

$$\nu z = \left(I - \mathcal{M}^{-1}\right) \pi + \mathcal{Z} z$$

For $\nu = 0$ these conditions are satisfied for $\pi = 0$ and $z = \mathcal{M}^{-1} \hat{\mathcal{B}}$.

For $\nu = \frac{1}{\rho}$ the conditions are satisfied for $z = 0$ and $\pi = \begin{pmatrix} 1 \\ ... \\ 1 \end{pmatrix}$.

Otherwise we can merge the two equations above and substitute out for νz, to obtain:

$$\rho \nu \pi = \mathcal{M}^{-1} \pi - \frac{1 - \rho \nu}{\nu} \mathcal{Z} \pi \tag{120}$$

It holds that all eigenvectors of $\mathcal{M}$ except $\begin{pmatrix} 1 \\ ... \\ 1 \end{pmatrix}$ are orthogonal to $\lambda^T (I - \Delta) \Delta^{-1}$. Therefore if $\pi$is an eigenvector of $\mathcal{M}$, with corresponding eigenvalue $\xi \neq 0$, then $\mathcal{Z} \pi = \pi$. Thus equation (120) becomes

$$\frac{\rho \nu^2 - \rho \nu + 1}{\nu} \pi = \frac{1}{\xi} \pi$$

Now we need to have $\pi \neq 0$ (otherwise we would also have $z = 0$, which cannot be an eigenvector). Therefore it must hold that

$$\frac{\rho \nu^2 - \rho \nu + 1}{\nu} = \frac{1}{\xi} \tag{121}$$

Lemma 13 shows that all eigenvalues ξ of $\mathcal{M}$ have modulus in $(0, 1)$. Therefore equation (121) has two solutions, ν^+ and ν^-, with $0 < \nu^- < 1$ and $\nu^+ > 1$. Thus we have $N - 1$ couples of solutions (one smaller than 1 and one greater than 1), plus 0 and $\frac{1}{\rho}$. It follows that the matrix $\mathcal{A}$ has N eigenvalues greater than 1 and N smaller than 1 in absolute value, as we wanted to show.

It remains to prove that the interest rate rule

$$i_t = \underbrace{r_t^n + \beta^T \mathbb{E}\pi_{t+1}^{zg}}_{\text{nominal rate under zero output gap}} + \zeta \tilde{y}_t$$

with $\zeta > 0$ implements zero output gap in every period.

Under this rule the system becomes

$$\begin{pmatrix} \rho \mathbb{E}\pi_{t+1} \\ z_{t+1} \\ \mathbb{E}\tilde{y}_{t+1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1}\hat{\mathcal{B}} \\ I - \mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1}\hat{\mathcal{B}} \\ 0 & 0 & \zeta + 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \\ \tilde{y}_t \end{pmatrix} +$$

$$+ \begin{pmatrix} -\mathcal{M}^{-1}\hat{\mathcal{V}} \\ \mathcal{M}^{-1}\hat{\mathcal{V}} \\ 0 \end{pmatrix} (logA_t - logA_{t-1}) + \begin{pmatrix} -I \\ 0 \\ 0 \end{pmatrix} x_t^D$$

Note that the solution to the previous system is still a solution of the new system. To prove that there are no additional solutions we will show that the matrix

$$\tilde{\mathcal{A}} \equiv \begin{pmatrix} \mathcal{M}^{-1} & -\mathcal{Z} & -\mathcal{M}^{-1}\hat{\mathcal{B}} \\ I - \mathcal{M}^{-1} & \mathcal{Z} & \mathcal{M}^{-1}\hat{\mathcal{B}} \\ 0 & 0 & \zeta + 1 \end{pmatrix}$$

has the same eigenvalues and eigenvectors as $\mathcal{A}$ above, plus the eigenvalue $\nu = \zeta + 1$, with associated eigenvector $\begin{pmatrix} \pi \\ z \\ \tilde{y} \end{pmatrix}$ such that

$$\pi = \left(I + \frac{1 - \rho\nu + \rho\nu^2}{\nu} \mathcal{V} \right)^{-1} \hat{\mathcal{B}}$$

$$z = \frac{1 - \rho\nu}{\nu} \pi$$

$$\tilde{y} = (1 - \rho\nu) \frac{\lambda^T (I - \Delta) \Delta^{-1}}{\gamma + \varphi} \pi$$

This would imply that for $\zeta > 0$ the new system has a unique bounded solution, equal to the solution of the original system.

Let's then study the eigenvalues and eigenvectors of $\tilde{\mathcal{A}}$. Denote the eigenvalues by ν, and the first N components of the corresponding eigenvector by π. From the first two rows and the definition of eigenvector we derive the conditions

$$z = \frac{1-\rho\nu}{\nu}\pi$$

$$\tilde{y} = (1-\rho\nu)\frac{\lambda^T (I-\Delta)\Delta^{-1}}{\gamma+\varphi}\pi$$

$$\left(I - \frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)\pi = \hat{\mathcal{B}}\frac{\lambda^T (I-\Delta)\Delta^{-1}}{\gamma+\varphi}\left(I - \frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)\pi$$

The last condition implies

$$\left(I - \frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)\pi = \hat{\mathcal{B}}$$

From the last row of $\tilde{\mathcal{A}}$ we derive the relation

$$(1+\zeta-\nu)(1-\rho\nu)\frac{\lambda^T (I-\Delta)\Delta^{-1}}{\gamma+\varphi}\pi = 0$$

which we know is satisfied by the eigenvalues/eigenvectors of $\mathcal{A}$. In addition, it is also satisfied for $\nu = 1+\zeta$ and $\pi = \left(I - \frac{1-\rho\nu+\rho\nu^2}{\nu}\mathcal{G}\right)^{-1}\hat{\mathcal{B}}$. This proves the result.

Proof of Proposition 9

Within each period, the cross-sector misallocation loss is the same function of sector-level markups derived in Section 4. It can be written as

$$x_t^T \mathcal{D}_2 x_t$$

where now $\mathcal{D}_2$ is defined as

$$\mathcal{D}_2 = \left(I-\hat{\Delta}\right)\hat{\Delta}^{-1}\tilde{\mathcal{D}}_2\hat{\Delta}^{-1}\left(I-\hat{\Delta}\right)$$

and the elements of $\tilde{\mathcal{D}}_2$ are derived in equation (104) (see the proof of Proposition 4).

The within-sector productivity loss is given by

$$\sum_{i=1}^{N}\lambda_i\epsilon_i\left[\int (logp_{ift}-logp_{it})^2\,df - \left(\int (logp_{ift}-logp_{it})\,df\right)^2\right]$$

as derived in Proposition 4.

The following lemma shows how the discounted sum of within-sector losses in the present and future periods can be written as a function of sectoral inflation rates.

Lemma 14. *It holds that*

$$\sum_{s\geq 0}\rho^s\left(\sum_{i=1}^{N}\lambda_i\epsilon_i\left[\int (logp_{ift+s}-logp_{it+s})^2\,df - \left(\int (logp_{ift+s}-logp_{it+s})\,df\right)^2\right]\right) =$$

$$= \sum_{s \geq 0} \rho^s \pi_{t+s}^T \mathcal{D}_1 \pi_{t+s}$$

where $\mathcal{D}_1$ is a diagonal matrix with elements

$$d_{1ii} = \lambda_i \epsilon_i \frac{1 - \hat{\delta}_i}{\hat{\delta}_i}$$

Proof. To prove the lemma it is enough to show that

$$\sum_{s \geq 0} \rho^s \left[\int \left(logp_{ift+s} - logp_{it+s} \right)^2 df - \left(\int \left(logp_{ift+s} - logp_{it+s} \right) df \right)^2 \right] = \frac{1 - \hat{\delta}_i}{\hat{\delta}_i} \sum_{s \geq 0} \rho^s \pi_{it+s}^2$$

Given the Calvo assumption, in each sector i the fraction δ_i of firms who adjust prices set

$$logp_{ift} - logp_{it} = (1 - \delta_i) \left(logp_{it}^* - logp_{it-1} \right) = \frac{1 - \delta_i}{\delta_i} \pi_{it}$$

For the remaining fraction $(1 - \delta_i)$ of non-adjusting firms we have

$$logp_{ift} - logp_{it} = (-\delta_i) \left(logp_{it}^* - logp_{it-1} \right) + \left(logp_{ift-1} - logp_{it-1} \right) = \left(logp_{ift-1} - logp_{it-1} \right) - \pi_{it}$$

Define

$$D_{it} \equiv \int \left(logp_{ift+s} - logp_{it+s} \right)^2 df - \left(\int \left(logp_{ift+s} - logp_{it+s} \right) df \right)^2$$

Around a steady-state where $logp_{ift} - logp_{it} = 0 \ \forall f$, we have

$$D_{it} = (1 - \delta_i) \left(\frac{1 - \delta_i}{\delta_i} \pi_{it}^2 + D_{it-1} \right)$$

It follows that

$$\sum_s \rho^s D_{it+s} = \sum_s \rho^s \frac{1 - \delta_i}{\delta_i} \pi_{is}^2 \left(\sum_{\tau \geq s} \left(\rho (1 - \delta_i) \right)^{\tau - s} \right) =$$

$$= \frac{1 - \hat{\delta}_i}{\hat{\delta}_i} \sum_s \rho^s \pi_{is}^2$$

□

Proof of Proposition 10

The central bank solves the problem

$$min_{\{\tilde{y}_t, \pi_t, z_{t+1}\}_{t=0}^{\infty}} \sum_t \rho^t \left[(\gamma + \varphi) \tilde{y}_t^2 + \pi_t^T \mathcal{D}_1 \pi + z_{t+1}^T \mathcal{D}_2 z_{t+1} \right]$$

$$s.t. \quad \begin{pmatrix} \mathbb{E}\pi_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{\mathcal{M}^{-1}}{\rho} & -\frac{\mathcal{Z}}{\rho} \\ \left(I - \mathcal{M}^{-1}\right) & \mathcal{Z} \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \end{pmatrix} + \begin{pmatrix} -\frac{\mathcal{M}^{-1}}{\rho}\left(\mathcal{B}y_t + \mathcal{V}\left(logA_t - logA_{t-1}\right)\right) \\ \mathcal{M}^{-1}\left(\mathcal{B}y_t + \mathcal{V}\left(logA_t - logA_{t-1}\right)\right) \end{pmatrix}$$

In the absence of commitment, we can re-write this as

$$v\left(logA_t - logA_{t-1}, z_t\right) = min_{y_t,\pi_t,z_{t+1}}\left(\gamma+\varphi\right)\tilde{y}^2 + \pi_t^T\mathcal{D}_1\pi + z_{t+1}^T\mathcal{D}_2 z_{t+1} + \rho\mathbb{E}\left[v\left(logA_{t+1} - logA_t, z_{t+1}\right)\right]$$

$$s.t. \quad \begin{pmatrix} \mathbb{E}\pi_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{\mathcal{M}^{-1}}{\rho} & -\frac{\mathcal{Z}}{\rho} \\ \left(I - \mathcal{M}^{-1}\right) & \mathcal{Z} \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \end{pmatrix} + \begin{pmatrix} -\frac{\mathcal{M}^{-1}}{\rho}\left(\mathcal{B}y_t + \mathcal{V}\left(logA_t - logA_{t-1}\right)\right) \\ \mathcal{M}^{-1}\left(\mathcal{B}y_t + \mathcal{V}\left(logA_t - logA_{t-1}\right)\right) \end{pmatrix}$$

The first order conditions are

$$\begin{aligned} 2\left(\gamma+\varphi\right)\tilde{y}_t + 2\mathcal{B}^T\mathcal{M}^{-1T}\mathcal{D}_2 z_{t+1} + \rho\mathcal{B}^T\mathcal{M}^{-1T}\mathbb{E}\left[v_z'\left(logA_{t+1} - logA_t; z_{t+1}\right)\right] = 0 \\ 2\mathcal{D}_1\pi_t + 2\left(I - \mathcal{M}^{-1T}\right)\mathcal{D}_2 z_{t+1} + \rho\left(I - \mathcal{M}^{-1T}\right)\mathbb{E}\left[v_z'\left(logA_{t+1} - logA_t; z_{t+1}\right)\right] = 0 \end{aligned}$$

The envelope theorem yields

$$v_z'\left(logA_t - logA_{t-1}; z_t\right) = 2\mathcal{Z}^T\mathcal{D}_2 z_{t+1} + \rho\mathcal{Z}^T\mathbb{E}\left[v_z'\left(logA_{t+1} - logA_t; z_{t+1}\right)\right] \tag{122}$$

Rearranging the first order conditions and noting that

$$\mathcal{B}^T\mathcal{Z} = \mathcal{B}^T\left(I - \mathcal{M}^{-1T}\right)$$

we find the optimality condition

$$\left(\gamma+\varphi\right)\tilde{y}_t + \mathcal{B}^T\mathcal{D}_1\pi_t + \mathcal{B}^T\mathcal{D}_2 z_{t+1} = -\rho\mathcal{B}^T\frac{\mathbb{E}\left[v_z'\left(logA_{t+1} - logA_t; z_{t+1}\right)\right]}{2}$$

We can further use the first order conditions, together with equation (122), to compute

$$\mathcal{B}^T v_z'\left(logA_t - logA_{t-1}; z_t\right) = -2\mathcal{B}^T\mathcal{D}_1\pi_t$$

so that the optimality condition becomes

$$\left(\gamma+\varphi\right)\tilde{y}_t + \mathcal{B}^T\mathcal{D}_1\pi_t + \mathcal{B}^T\mathcal{D}_2 z_{t+1} = \rho\mathcal{B}^T\mathcal{D}_1\mathbb{E}\pi_{t+1}$$

Finally, noting that

$$z_{t+1} = \pi_t - \rho\mathbb{E}\pi_{t+1} \tag{123}$$

the optimality condition becomes

$$\left(\gamma+\varphi\right)\tilde{y}_t^* + \mathcal{B}^T\mathcal{D}z_{t+1}^* = 0 \tag{124}$$

To obtain the expression for the optimal output gap in (64) we subtitute for z_{t+1} in (124) using the pricing equation

$$z_{it+1} = \pi_{it} + \rho \mathbb{E} \pi_{t+1}$$

together with equation (60) and the equality

$$\mathcal{M} = I + \mathcal{V}(I - \Omega)$$

Proof of Proposition 10

D: Complements to the quantitative analysis

D1: Welfare loss from business cycles

Main results

The results for the main calibration are plotted in the left panel of Figure 5. The right panel reports results for an alternative calibration without input-output linkages. The bars correspond to the percentage of per-period GDP that consumers would be willing to forego in exchange of switching from a sticky-price economy to the efficient equilibrium, for a given monetary policy rule. Bars of different colors represent different rules. Each set of bars corresponds to a different assumption about the correlation of sectoral shocks, keeping the variance of aggregate productivity constant across calibrations. In the first set the covariance matrix is calibrated from the data, while in the second set there are only idiosyncratic shocks, and in the third there are only aggregate shocks.

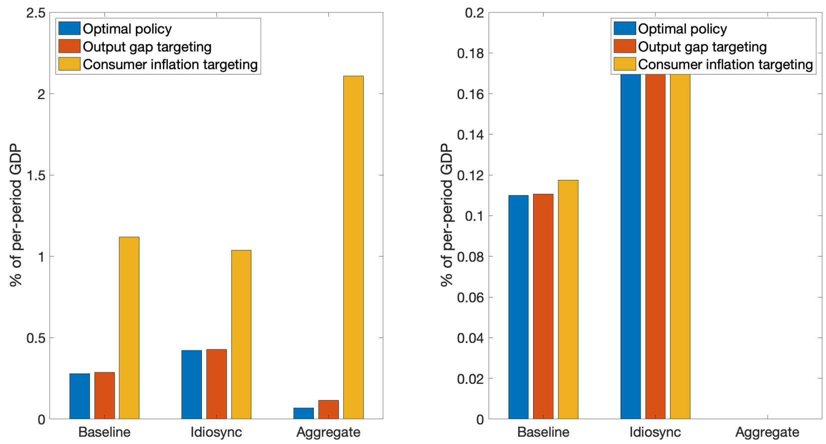


Figure 5: Welfare loss from business cycles

DC index and optimal policy target

Section 6.2 in the main text argues that targeting the output gap almost replicates the optimal policy. We reach a similar conclusion when comparing the behavior over time of the "divine coincidence" index DC -our inflation proxy for the output gap- and the optimal policy target, plotted in Figure 6. The two series move closely together, which means that the optimal target almost coincides with the output gap. The target however is often a few basis points lower than DC, suggesting that the optimal policy should be slightly more expansionary than output gap targeting.

Analytical expressions for the welfare loss under various policy rules

Below I report expressions for the expected welfare loss under different policy rules, as a function of the network primitives (captured by $\mathcal{B}$,$\mathcal{V}$ and $\mathcal{D}$) and of the covariance matrix of sectoral shocks (Σ). I further decompose the loss into deviations from zero output gap and misallocation.

Optimal policy

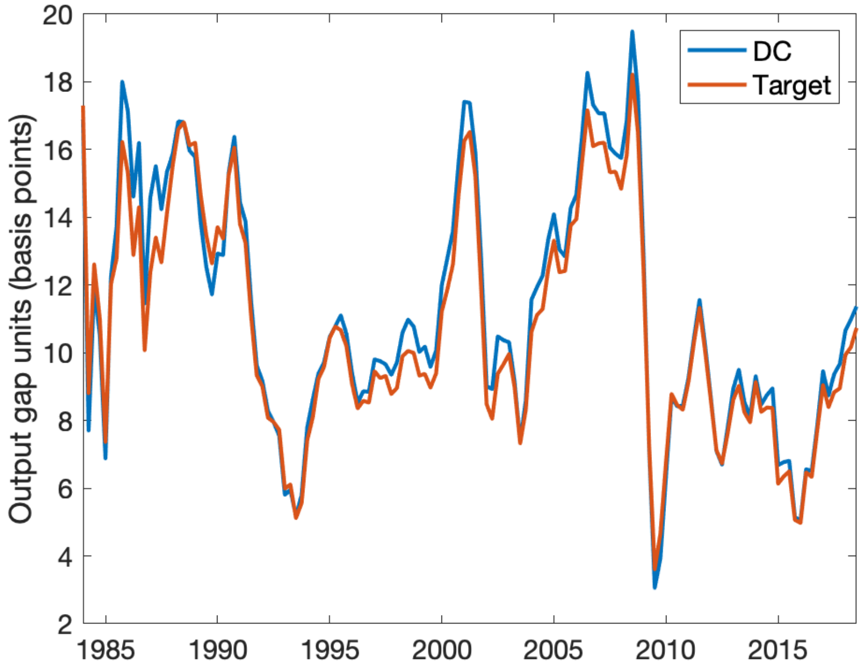


Figure 6: Time series of the DC inflation index and the optimal policy target

The weights on sectoral inflation rates are normalized so that the value of the "divine coincidence" index is equal to the output gap, as in Proposition 3 (note that the weights do not sum to one).

The total welfare loss is

$$\frac{1}{2}\left[\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}-\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}\right]$$

The loss from non-zero output gap is:

$$\frac{1}{2}\left(\gamma+\varphi\right)\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{\left(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}\right)^2}$$

The gain in allocative efficiency from non-zero output gap is:

$$\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}-\frac{1}{2}\mathcal{B}^T\mathcal{D}\mathcal{B}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{\left(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}\right)^2}$$

The net misallocation loss is:

$$\frac{1}{2}\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}-\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}+\frac{1}{2}\mathcal{B}^T\mathcal{D}\mathcal{B}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{\left(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}\right)^2}$$

Loss under zero consumer inflation relative to the optimal policy

The total loss is:

$$\frac{1}{2}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}+$$
$$+\frac{1}{2}\left[\frac{\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B}}{\left(\beta^T\mathcal{B}\right)^2}\beta^T\mathcal{V}-2\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}}{\beta^T\mathcal{B}}\right]\Sigma\mathcal{V}^T\beta$$

The loss from non-zero output gap is:

$$\frac{1}{2}\left(\gamma+\varphi\right)\frac{\beta^T\mathcal{V}\Sigma\mathcal{V}^T\beta}{\left(\beta^T\mathcal{B}\right)^2}$$

The loss from misallocation is:

$$\frac{1}{2}\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}+\left[\frac{1}{2}\mathcal{B}^T\mathcal{D}\mathcal{B}\frac{\beta^T\mathcal{V}}{\left(\beta^T\mathcal{B}\right)^2}-\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}}{\beta^T\mathcal{B}}\right]\Sigma\mathcal{V}^T\beta$$

Loss under zero output gap relative to the optimal policy

The total loss is:

$$\frac{1}{2}\frac{\mathcal{B}^T\mathcal{D}\mathcal{V}\Sigma\mathcal{V}^T\mathcal{D}\mathcal{B}}{(\gamma+\varphi+\mathcal{B}^T\mathcal{D}\mathcal{B})}$$

The total loss from misallocation is:

$$\frac{1}{2}\sum_{i,j}\left(\mathcal{V}^T\mathcal{D}\mathcal{V}\right)_{ij}\Sigma_{ij}$$

Within- versus cross-sector misallocation

Section 5.1 shows that the welfare loss from misallocation has two components, coming from relative price distortions within and across sectors. Figure 7 compares the relative magnitude of these components. The three sets of bars in the figure correspond to different policy rules (optimal policy, output gap targeting and consumer price targeting). Within each group, the bar on the left-hand-side is based on our preferred calibration, which assumes higher substitutability between varieties from the same sector than across goods from different sectors. Unsurprisingly, the within-sector loss dominates in this calibration. The bar on the right-hand-side of each group instead is based on an alternative calibration, which assumes the same elasticity of substitution within and across sectors. In this case we find that the largest contribution to the welfare loss comes from cross-sector misallocation.

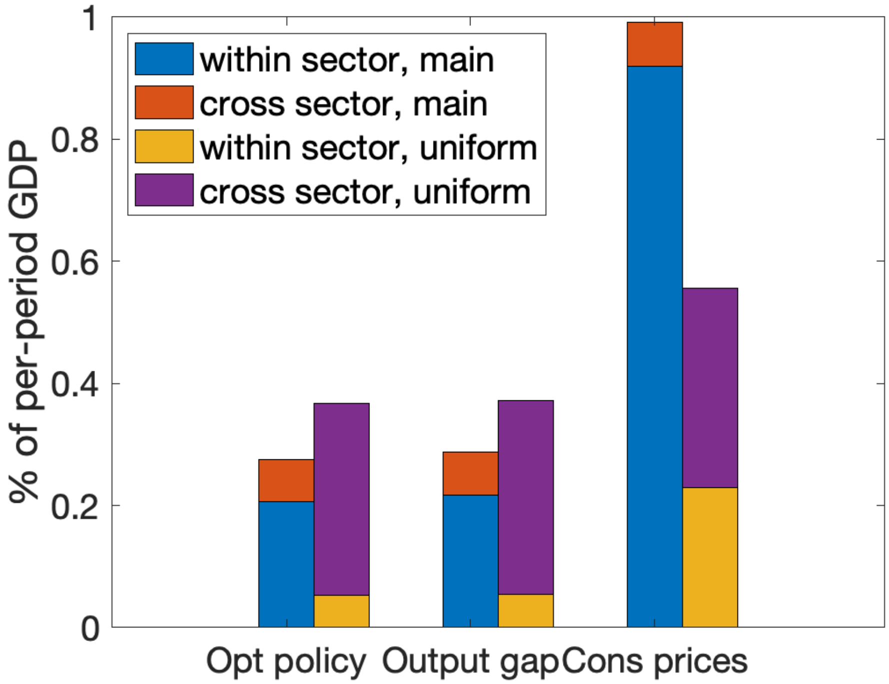


Figure 7: Main calibration: $\epsilon = 8$, $\sigma = 0.9$, $\theta_L = 0.5$, $\theta = 0.001$; uniform elasticities: $\epsilon = \sigma = \theta_L = \theta = 2$

D2: Phillips curve and monetary non-neutrality over time

Slope of the Phillips curve

Section 6.3.1 in the main text shows that the Phillips curve flattened because of changes in the input-output structure and in the composition of the consumption basket. To isolate these two components and evaluate their relative importance we can use the results in Section 6.3.1. The role of consumption and input shares is fully captured by the pass-through of nominal wages into consumer prices, $\bar{\delta}_w$. This pass-through in turn can be decomposed into a term related with consumption shares, and a term related with the input-output structure:

$$\bar{\delta}_w = \underbrace{\beta^T}_{\text{consumption}} \underbrace{\Delta \left(I - \Omega\Delta\right)^{-1} \alpha}_{\text{input-output}}$$

The evolution of the two components is represented by the dashed red and green lines in Figure 1. The red line represents the slope implied by a calibration where the input-output matrix is fixed at its 1947 value, and consumption shares evolve as observed in the data. The green line plots the slope of the Phillips curve implied by an alternative calibration where consumption shares remain constant at their 1947 value, while the input-output matrix changes over time as observed in the data. The shift of consumption from manufacturing towards services contributed to the decline after 1980. Service sectors have more rigid prices, therefore a shift towards these sectors increases average price stickiness and flattens the Phillips curve. Pre-1980, however, all of the decline can be attributed to the evolution of the production structure.

This last effect is driven by a uniform increase in intermediate input purchases, and not by a shift towards rigid sectors. The light blue line depicts the slope implied by a calibration where consumption shares remain constant, and input shares increase uniformly in all sectors.[3] The light blue line tracks the green one closely.

More formally, $\bar{\delta}_w$ is an average of sector-level pass-throughs of monetary shocks, with weights given by consumption shares. Thus we can split the overall change in $\bar{\delta}_w$ into the change in sector-level pass-through for constant consumption shares, and the change in consumption shares for constant pass-through. Sector-level pass-throughs only depend on the production structure, and not on consumption shares. Therefore we obtain the following decomposition:

$$\bar{\delta}_w^{2017} - \bar{\delta}_w^{1947} = \frac{\beta_{1947}^T + \beta_{2017}^T}{2} \left(PT_{2017} - PT_{1947}\right) + \\ + \left(\beta_{2017}^T - \beta_{1947}^T\right) \frac{PT_{1947} + PT_{2017}}{2}$$

where I used the notation

$$PT \equiv \Delta \left(I - \Omega \Delta\right)^{-1} \alpha$$

I find that 79% of the overall decline in $\bar{\delta}_w$ can be attributed to changes in the input-output structure, while the remaining effect comes from changes in the composition of the consumption basket.

I further break down the effect of changes in consumption and input-output shares into their sector-level components. Figure **??** provides a graphical representation.

[3] The change in input shares is calibrated to replicate the change in the aggregate value added to output ratio observed in the data.

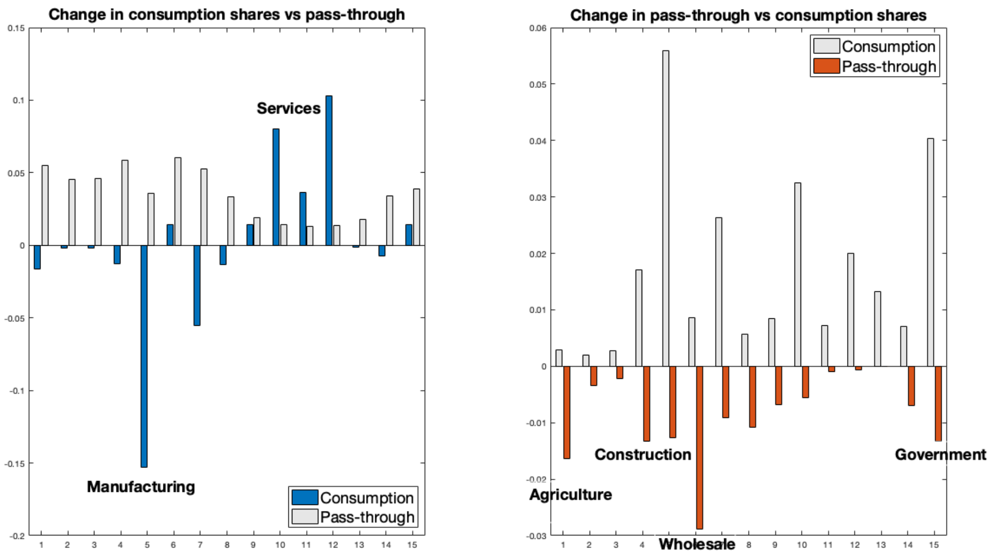


Figure 8: Upper panel: change in consumption shares and average wage pass-through.
Lower panel: change in pass-through and average consumption shares.

The grey bars in the two plots respectively represent the average pass-through $\frac{1}{2}\left(PT_{i,1947} + PT_{i,2017}\right)$ and the average consumption share $\frac{1}{2}\left(\beta_{i,1947} + \beta_{i,2017}\right)$ for each sector. The bars in color represent changes in sectoral consumption shares $\beta^T_{i,2017} - \beta^T_{i,1947}$ and pass-through $PT_{i,2017} - PT_{i,1947}$. From the left plot we see that consumption shifted away from manufacturing (which has high pass-through) towards services (which has lower pass-through). The right plot shows that the pass-through fell in all sectors, and more so in sectors with high consumption share (such as construction, manufacturing and government). Both channels lead to a flatter Phillips curve, although quantitatively the drop in sectoral pass-through (due to larger intermediate input flows) accounts for most of the effect.

Monetary non-neutrality

Figure 9 reports the impact response of inflation to a 1% real rate shock implied by the model, for each year between 1947 and 2017.

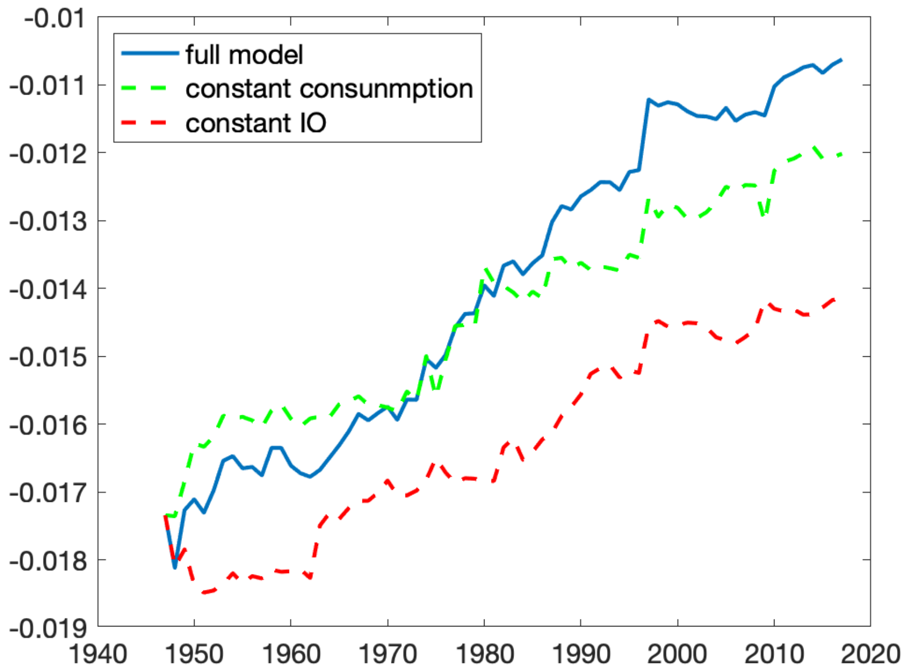


Figure 9: Impact response of consumer inflation to a 1% real rate shock

Mirroring the slope of the Phillips curve, monetary non-neutrality has increased over time (the same output change triggers a smaller inflation response). Most of the effect can be attributed to the increase in intermediate input flows.

E: Phillips curve regressions

E1: The "divine coincidence" index (time series)

I construct a time series for the "divine coincidence" index DC starting in 1984. This requires to aggregate sector-level price series based on the respective sales shares and adjustment frequencies. We compute sales shares from the BEA input-output data, and rely on the price adjustment data collected by Pasten, Schoenle and Weber. The main source for sector-level price series is PPI data from the BLS.

In the BLS dataset the sample period varies across sectors: most manufacturing series are available from the mid-1980s, while most service series are available from 2006 onwards. Out of the 405 sectors in the BEA classification, 172 have an incomplete price series in the BLS dataset, and 67 are missing. Information about the incomplete and missing series (sector names and weights in the DC index) is reported in Appendix F2 in this Supplemental Material.

To extend the incomplete price series further back in time we use sector-level data underlying the PCE, which is available from 1960. We run Lasso regressions of each incomplete PPI series on disaggregated (338 sectors) PCE components for the period in which both are available. Summary statistics for the Lasso regressions are reported in Appendix F2 in this Supplemental Material. We also use PCE components to make up for 40 missing series, using the concordance table between NAICS sectors and PCE series provided by the BEA.

Figure 10 compares the weights assigned to different sectors by the divine coincidence index DC and the PCE (the main indicator used by central banks), at an aggregated 21-sector level. Sectoral weights at a more disaggregated level are reported in Appendix F2 in this Supplemental Material.

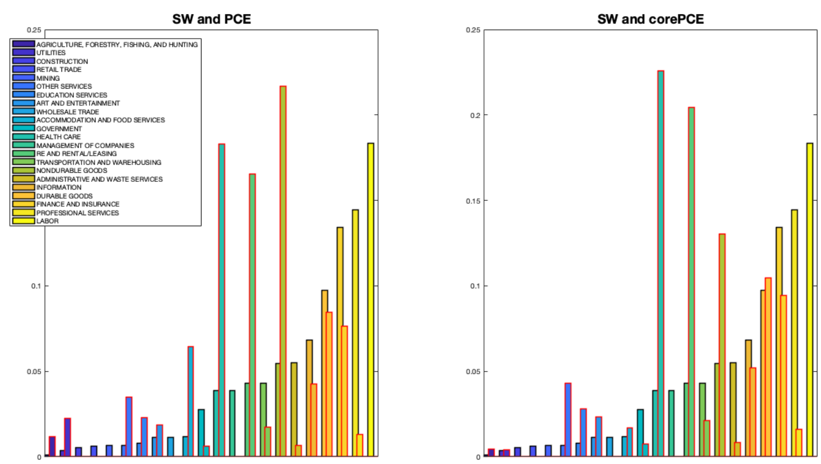


Figure 10: DC and PCE weights (The bars are ordered so that sectoral weights in DC are increasing. Those with red borders correspond to the PCE)

We see from the figure that wages have the highest weight (of 18%) in DC, while they are not part of the PCE. The divine coincidence index also assigns high weight to professional services, durable goods, and IT and administrative services. These sectors have a large input share in production and adjust prices infrequently. By contrast the PCE places the highest weight on healthcare, housing and non-durable goods. These sectors capture a large share of consumer expenditures, but are not important as inputs in production. Therefore their relative consumption share is much larger than their relative sales share, which is why they have a smaller weight in the divine coincidence index relative to the PCE.

Figure 11 plots DC against CPI, PCE and their core versions, and against the PPI.

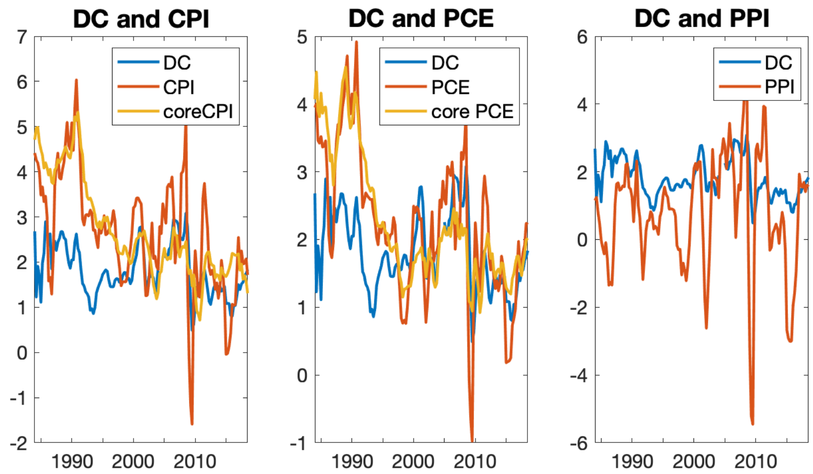


Figure 11: Comparison of DC against consumer and producer prices (1965-2018)

Here the weights on sectoral inflation rates are normalized to sum to one for all inflation indexes.

E2: Summary statistics

Sectoral weights

Table 5 reports the weights of the top-15 sectors in DC in percentage of the total (at the disaggregated 405 sector level).

Industry name	Weight (SW)	Weight (Domar)	Weight (PCE)
Labor	18.3221	27.8648	0
Insurance agencies, brokerages, and related activities	9.23917	1.39786	0
Management of companies and enterprises	3.887	1.68309	0
Architectural, engineering, and related services	2.51957	0.812411	0
Insurance carriers, except direct life	2.13001	1.04094	2.5369
Warehousing and storage	2.12367	0.344483	0.0019132
Accounting, tax preparation, bookkeeping, and payroll services	2.05855	0.53267	0.17815
Other real estate	2.05001	2.87134	0.057851
Legal services	1.87954	0.893466	1.0623
Advertising, public relations, and related services	1.68975	0.415808	0.017779
Hospitals	1.65114	1.17451	9.6864
Employment services	1.63912	0.913483	0.012342
Management consulting services	1.63082	0.569068	0
Wired telecommunications carriers	1.44281	0.78146	2.0335
All other miscellaneous professional, scientific, and technical services	1.31821	0.312412	0

Table 5: Weights of top-15 series in DC (in %)

Missing and incomplete series

Tables (6) and (7) report details of the missing and incomplete series in the PPI dataset. Table (8) presents summary statistics from the Lasso regressions used to extend the incomplete series back in time.

	Weight in SW	Added?
Oilseed farming	4.00	0
Funds, trusts, and other financial vehicles	2.02	1
Management of companies and enterprises	0.29	0
Sound recording industries	0.22	1
Elementary and secondary schools	0.20	1
Monetary authorities and depository credit intermediation	0.18	1
State and local government hospitals and health services	0.13	0
State and local government passenger transit	0.12	0
Other educational services	0.12	1
Motion picture and video industries	0.12	1
Transit and ground passenger transportation	0.10	1
Limited-service restaurants	0.10	1
Federal general government (nondefense)	0.09	0
Full-service restaurants	0.08	1
Promoters of performing arts and sports and agents for public figures	0.07	1

Table 6: Weights of top-15 missing series in DC (in %)

	Weight in SW	Initial date
Employment services	0.85	19940901
Management consulting services	0.55	20060901
Insurance agencies, brokerages, and related activities	0.47	20030301
Architectural, engineering, and related services	0.45	19970301
Automotive equipment rental and leasing	0.45	19920301
Custom computer programming services	0.41	20060901
Specialized design services	0.37	19970301
Nursing and community care facilities	0.36	20040301
Services to buildings and dwellings	0.36	19950301
Environmental and other technical consulting services	0.36	20060901
Wireless telecommunications carriers (except satellite)	0.31	19930901
Office administrative services	0.27	19940901
Satellite, telecommunications resellers, and all other telecommunications	0.23	19930901
Other computer related services, including facilities management	0.22	20060901
Internet publishing and broadcasting and Web search portals	0.21	20100301

Table 7: Weights of top-15 incomplete series in DC (in %)

Mean	**Max**	**Min**
88	127	19

Table 8: Number of series in Lasso approximation

Proxy for inflation expectations

Our preferred regression specification controls for inflation expectations. We construct a proxy for the expectations of each of the inflation indexes which are used as left hand side variables, based on the statistical properties of the inflation series (see Stock and Watson (2007)). Inflation changes $\pi_t - \pi_{t-1}$ are well approximated by an IMA(1,1) model. We estimate the parameters of the model for each inflation index, and use it to construct a prediction for future inflation changes, $\mathbb{E}\left[\pi_{t+1} - \pi_t\right]$. Inflation expectations are then given by $\mathbb{E}\pi_{t+1} = \pi_t + \mathbb{E}\left[\pi_{t+1} - \pi_t\right]$. Figure 12 plots the actual inflation series against the expectations series constructed based on the IMA(1,1) model.

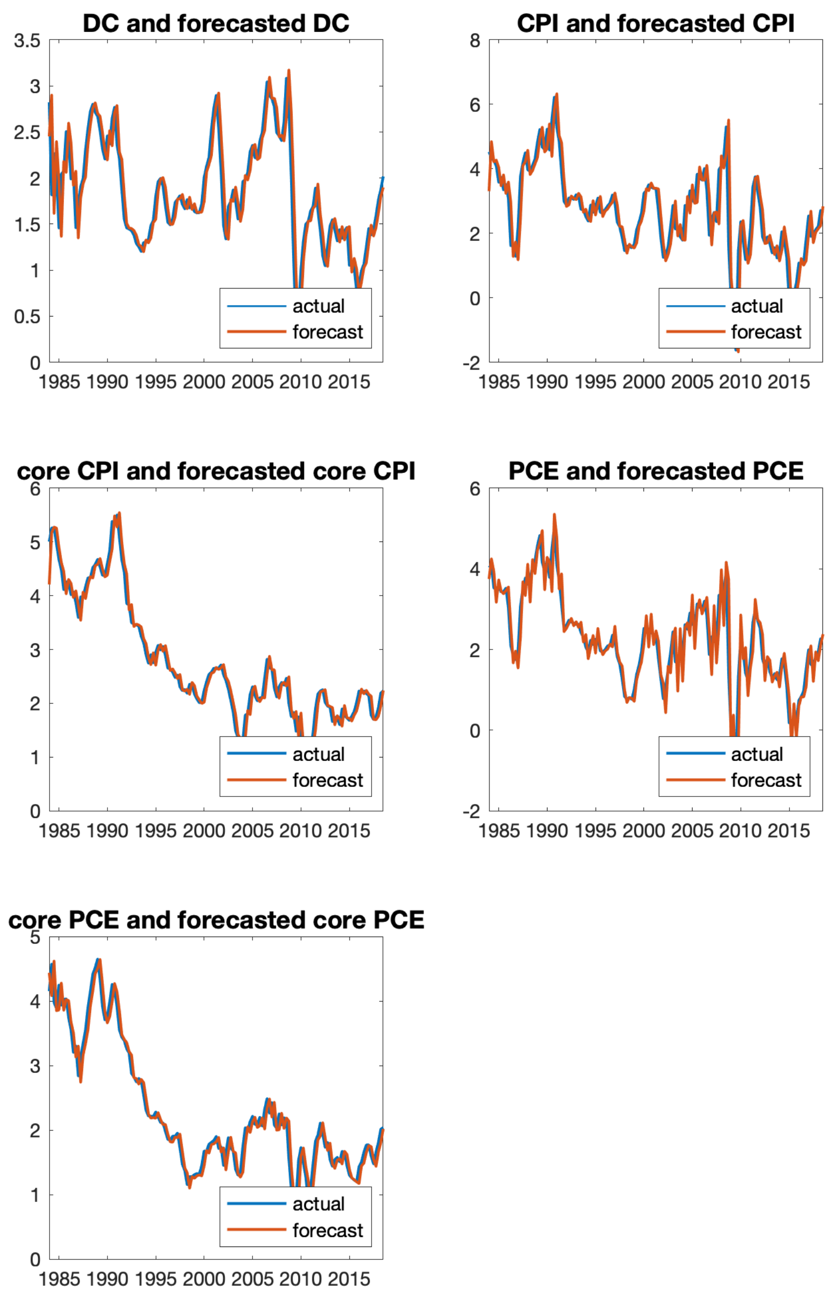


Figure 12: Actual and forecasted inflation

Scatterplots

We report scatterplots of inflation and output gaps for the different inflation and gap measures used in the regressions. Figures (??), (??) and (??) report scatterplots in levels, while Figures (??), (??) and (??) report scatterplots for inflation changes versus gap levels.

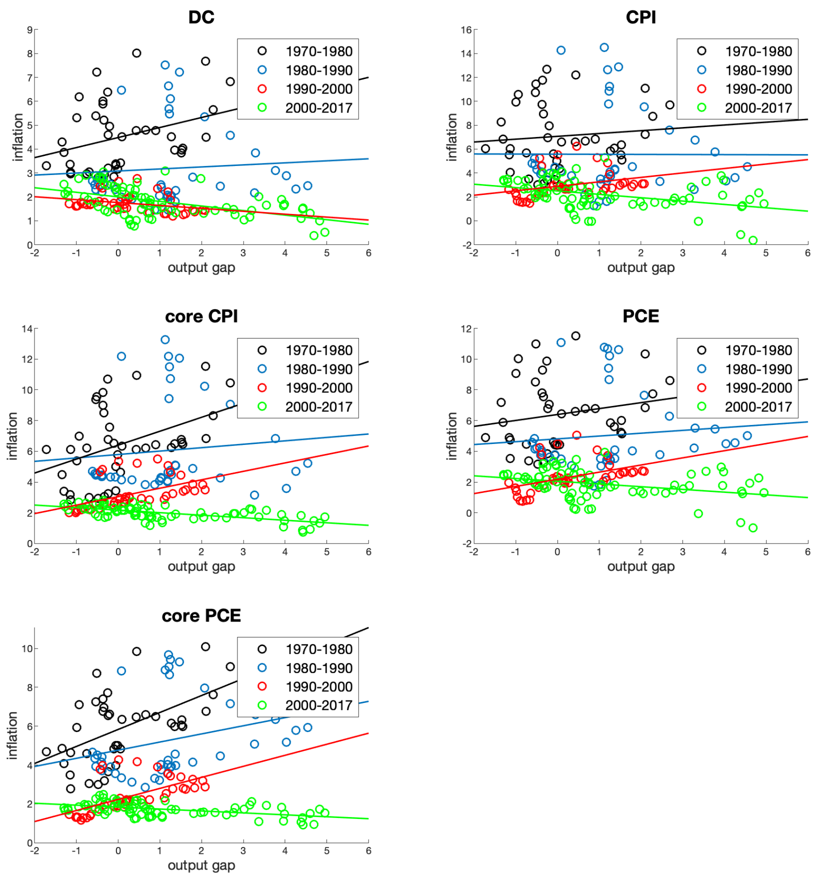


Figure 13: Inflation and unemployment gap

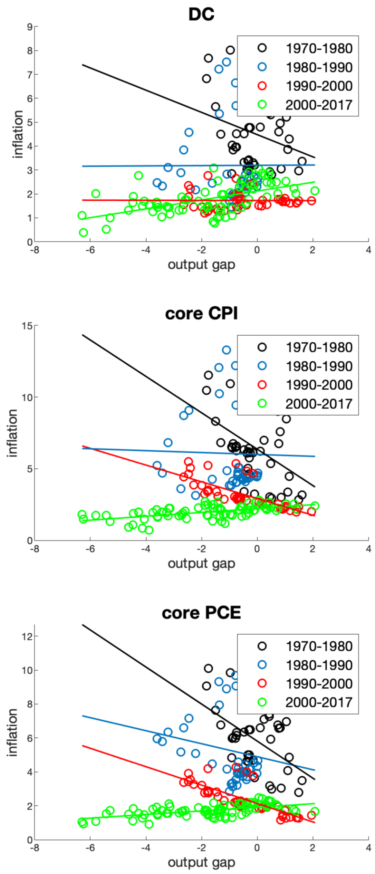


Figure 14: Inflation and output gap

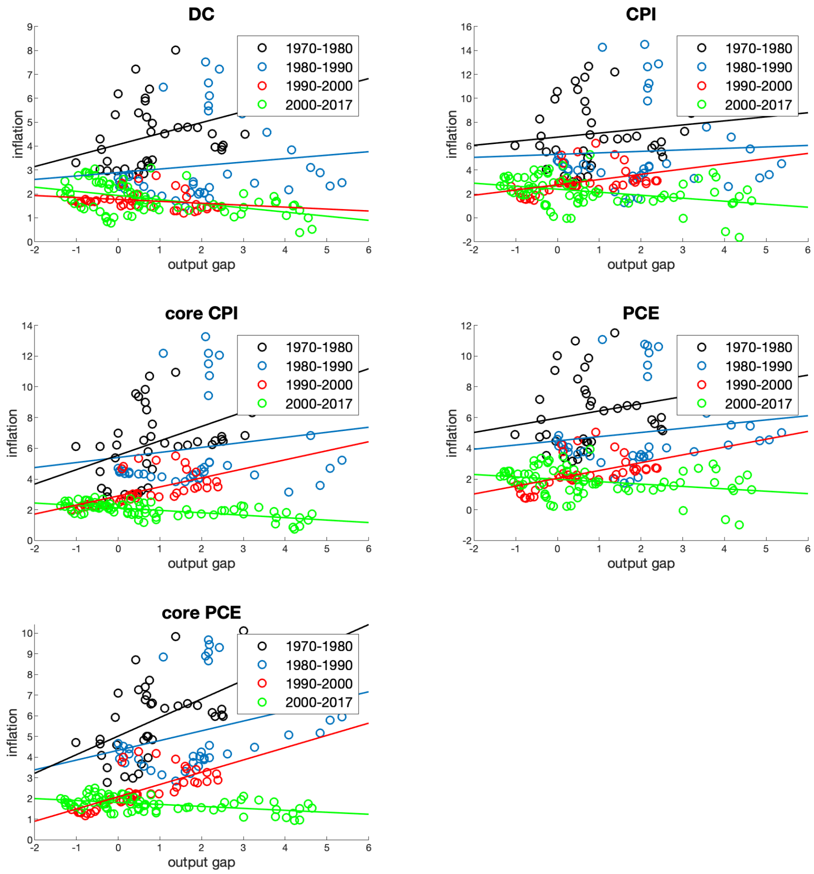


Figure 15: Inflation and unemployment rate

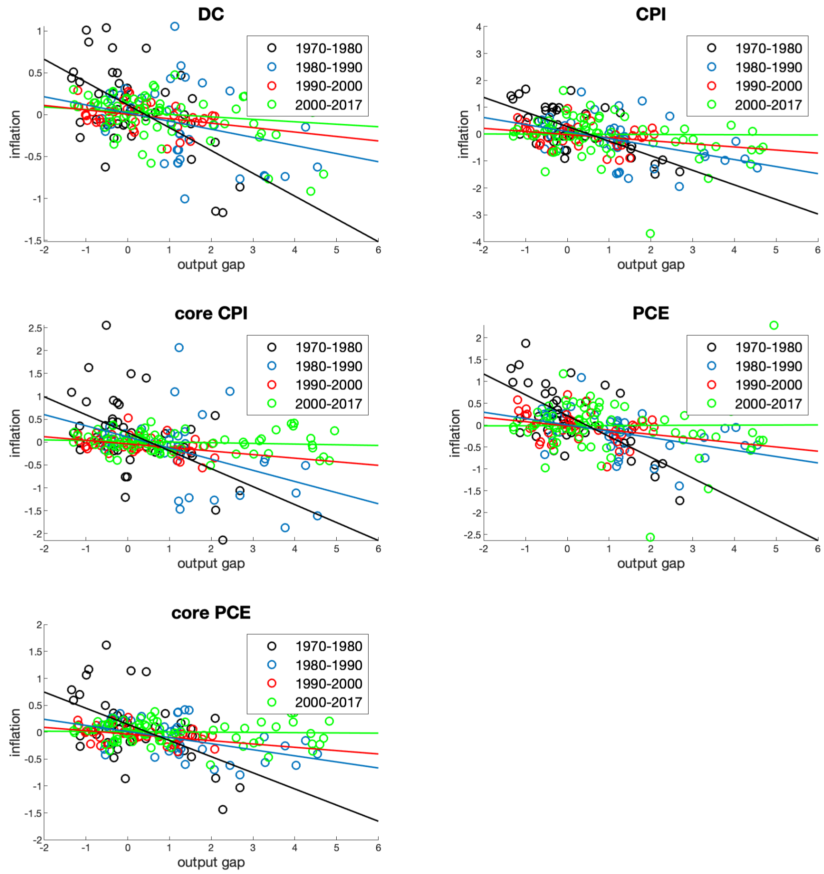


Figure 16: Inflation changes and unemployment gap

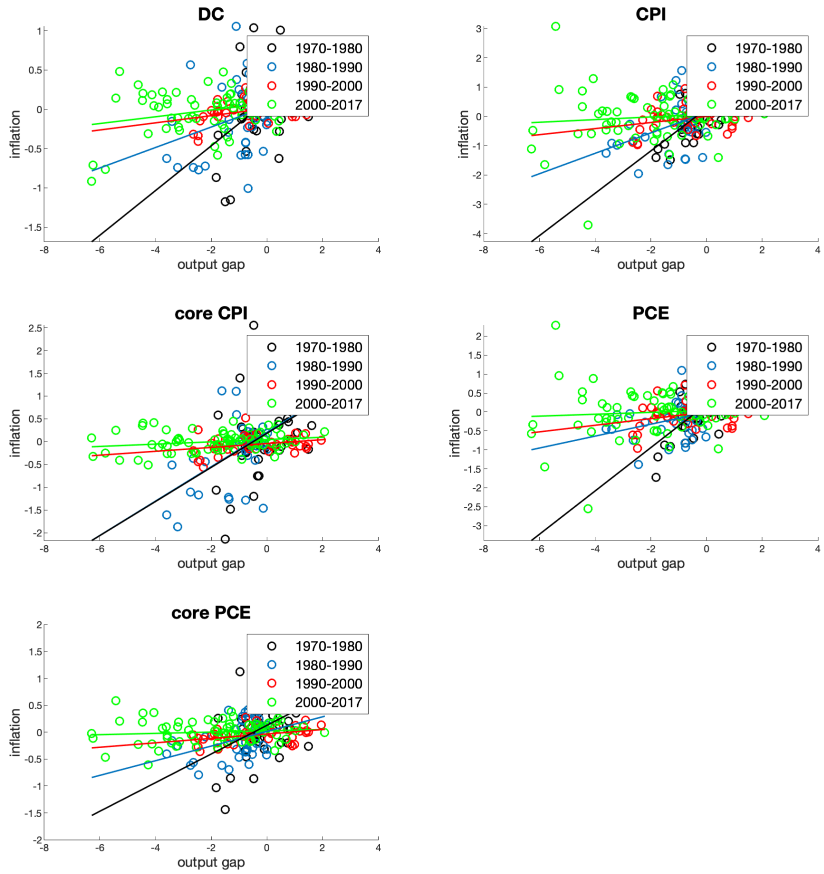


Figure 17: Inflation changes and output gap

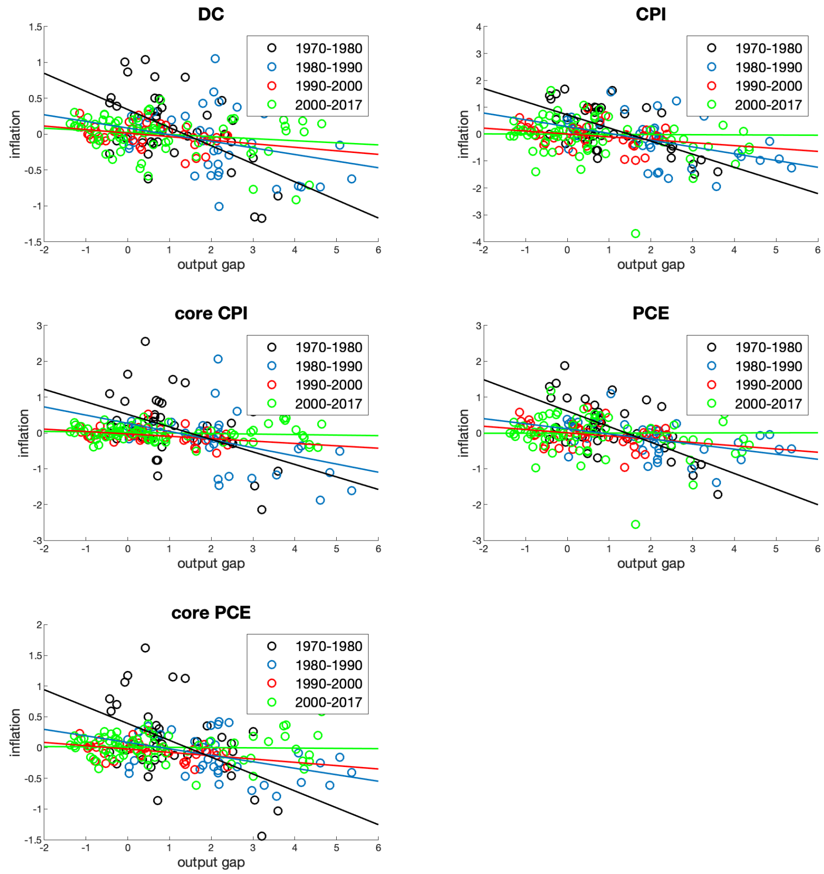


Figure 18: Inflation changes and unemployment rate

E3: Regressions over the full sample period

This section contains robustness checks for the regressions presented in Section 7.2. It shows results for different measures of the output gap on the right hand side, and for different specifications.

Tables 9 and ?? below present results for a plain specification without lags or expectations, as in equation (125). The right hand side variables are the CBO output gap and the unemployment rate respectively.

$$\pi_t = c + \kappa \tilde{y}_t + u_t \tag{125}$$

	SW	CPI	core CPI	PCE	core PCE
gap	3.144**	0.2791**	0.1728**	0.1837**	0.1162**
	(0.5538)	(0.0618)	(0.055)	(0.0532)	(0.0482)
intercept	2.0189**	3.0193**	2.9661**	2.4878**	2.4325**
	(0.0522)	(0.1271)	(0.1131)	(0.1095)	(0.0992)
R-squared	0.1905	0.1297	0.0673	0.08	0.0407

Table 9: CBO output gap

	DC	CPI	core CPI	PCE	core PCE
gap	-3.084**	-0.1405*	0.0028	-0.036	0.0545
	(0.6645)	(0.0759)	(0.0661)	(0.0644)	(0.057)
intercept	1.9621**	2.8021**	2.7595**	2.2996**	2.2514**
	(0.0505)	(0.1259)	(0.1096)	(0.1067)	(0.0945)
R-squared	0.1359	0.0244	0	0.0023	0.0066

Table 10: Regression results for the unemployment rate

Tables **??**, **??** and **??** present result for a specification that includes the proxy for the endogenous component of the residual constructed in Section 6.4.2. The new regression equation is:

$$\pi_t = c + \kappa\tilde{y}_t + u_t^C + v_t$$

where u_t^C is the endogenous component of the residual constructed in Section 6.4.2, and v_t is the exogenous component.

	DC	CPI	core CPI	PCE	core PCE
cost-push	0.5627**	2.5545**	0.4886	2.3948**	1.1224**
	(0.2345)	(0.565)	(0.4768)	(0.4745)	(0.4102)
gap	-3.7586**	-0.1906**	-0.2175**	-0.0783	-0.0886
	(0.6872)	(0.0758)	(0.064)	(0.0637)	(0.0551)
intercept	2.0842**	3.2239**	2.8559**	2.6509**	2.397**
	(0.058)	(0.1398)	(0.118)	(0.1174)	(0.1015)
R-squared	0.3317	0.2782	0.142	0.2558	0.1275

Table 11: Regression results for the CBO unemployment gap , with CP shock

	DC	CPI	core CPI	PCE	core PCE
cost-push	0.6059**	2.5472**	0.6387	2.4715**	1.2896**
	(0.2604)	(0.5964)	(0.5145)	(0.4983)	(0.4333)
gap	2.4282**	0.1363**	0.1176**	0.0369	0.0225
	(0.6496)	(0.0682)	(0.0588)	(0.057)	(0.0495)
intercept	2.0936**	3.2425**	2.8535**	2.6467**	2.3802**
	(0.0633)	(0.145)	(0.1251)	(0.1212)	(0.1054)
R-squared	0.2458	0.2635	0.0852	0.2484	0.1086

Table 12: Regression results for the CBO output gap , with CP shock

	DC	CPI	core CPI	PCE	core PCE
cost-push	0.6321**	2.8683**	0.8598*	2.6413**	1.3999**
	(0.2357)	(0.5706)	(0.4905)	(0.4722)	(0.4102)
gap	-3.6783**	-0.0954	-0.1038	0.006	0.0063
	(0.731)	(0.0811)	(0.0697)	(0.0671)	(0.0583)
intercept	2.0911**	3.1954**	2.8214**	2.6213**	2.3637**
	(0.0594)	(0.1439)	(0.1237)	(0.1191)	(0.1034)
R-squared	0.309	0.2462	0.0706	0.2456	0.1071

Table 13: Regression results for the unemployment rate , with CP shock

Tables 14, 15 and 16 below present results for the baseline specification augmented with oil price inflation, as in equation (126). The gap measures are given by the CBO unemployment gap, the CBO output gap and the unemployment rate respectively.

$$\pi_t = c + \kappa \tilde{y}_t + \pi_{oil} + u_t \tag{126}$$

	SW	CPI	core CPI	PCE	core PCE
gap	-3.6385**	-0.2198**	-0.2038**	-0.1194**	-0.1066*
	(0.6294)	(0.0655)	(0.0643)	(0.0584)	(0.0573)
intercept	1.9532**	2.7286**	2.9576**	2.266**	2.3883**
	(0.0483)	(0.1099)	(0.1078)	(0.0979)	(0.0961)
oil prices	0.0032**	0.0185**	-0.0058*	0.0138**	-0.0017
	(0.0013)	(0.003)	(0.0029)	(0.0027)	(0.0026)
R-squared	0.2488	0.2959	0.0829	0.2049	0.0257

Table 14: Regression results for the CBO unemployment gap , with oil prices

	SW	CPI	core CPI	PCE	core PCE
gap	2.8985**	0.2137**	0.1961**	0.1351**	0.1243**
	(0.5562)	(0.0562)	(0.0553)	(0.0501)	(0.0492)
intercept	1.9843**	2.8179**	3.038**	2.3383**	2.4576**
	(0.0536)	(0.1184)	(0.1164)	(0.1055)	(0.1036)
oil prices	0.0031**	0.0179**	-0.0064**	0.0133**	-0.0022
	(0.0014)	(0.003)	(0.0029)	(0.0027)	(0.0026)
R-squared	0.2199	0.3108	0.0985	0.2221	0.0458

Table 15: Regression results for the CBO output gap , with oil prices

	SW	CPI	core CPI	PCE	core PCE
gap	-2.7813**	-0.0591	-0.0158	0.0259	0.0524
	(0.6655)	(0.0684)	(0.067)	(0.0596)	(0.0582)
intercept	1.9278**	2.601**	2.8056**	2.1465**	2.2564**
	(0.0516)	(0.116)	(0.1136)	(0.101)	(0.0987)
oil prices	0.0033**	0.0196**	-0.0045	0.0149**	-0.0005
	(0.0014)	(0.0031)	(0.0031)	(0.0027)	(0.0027)
R-squared	0.1707	0.2418	0.0155	0.1816	0.0069

Table 16: Regression results for the unemployment rate , with oil prices

Tables IV, 27 and 28 present results for a specification with inflation expectations, using the CBO unemployment gap, the CBO output gap and the unemployment rate as right hand side variables:

$$\pi_t = c + \kappa \tilde{y}_t + \rho \mathbb{E}\pi_{t+1} + \epsilon_t \tag{127}$$

	DC	CPI	core CPI	PCE	core PCE
gap	-1.1054**	-0.1613**	-0.0344	-0.062	0.0047
	(0.3275)	(0.0809)	(0.052)	(0.0487)	(0.0368)
inflation expecations	0.8287**	0.4846**	0.5446**	0.6364**	0.6406**
	(0.0383)	(0.1557)	(0.0559)	(0.0621)	(0.045)
intercept	0.3484**	1.3851**	1.3193**	0.5522**	0.8388**
	(0.0789)	(0.5021)	(0.1818)	(0.196)	(0.1228)
R-squared	0.8234	0.159	0.4425	0.4635	0.6072

Table 17: Regression results for the CBO unemployment gap , with expectations

	DC	CPI	core CPI	PCE	core PCE
gap	1.0861**	0.1881**	0.0412	0.0881**	0.0084
	(0.2714)	(0.0678)	(0.0449)	(0.0417)	(0.032)
inflation expecations	0.8297**	0.4412**	0.5398**	0.6231**	0.6365**
	(0.0368)	(0.1515)	(0.0561)	(0.0617)	(0.0455)
intercept	0.3668**	1.6124**	1.3548**	0.6459**	0.8614**
	(0.0772)	(0.4987)	(0.1892)	(0.2005)	(0.1291)
R-squared	0.8288	0.1808	0.4442	0.4744	0.6073

Table 18: Regression results for the CBO output gap , with expectations

	DC	CPI	core CPI	PCE	core PCE
gap	-0.9404**	-0.0049	0.0781	-0.0505	0.0757**
	(0.3185)	(0.0788)	(0.0499)	(0.0477)	(0.0355)
inflation expecations	0.8468**	0.6312**	0.5668**	0.6549**	0.6432**
	(0.0372)	(0.1518)	(0.0537)	(0.0608)	(0.0434)
intercept	0.3108**	0.8344*	1.1705**	0.4941**	0.7757**
	(0.0762)	(0.4879)	(0.1711)	(0.1851)	(0.1155)
R-squared	0.8202	0.1344	0.4507	0.4616	0.6198

Table 19: Regression results for the unemployment rate , with expectations

Finally, Tables 20, 21 and 22 present results for the specification in equation (128) with inflation changes on the left hand side (instead of inflation levels). We present results for our three usual gap measures (CBO unemployment gap, CBO output gap and unemployment rate). All of them are in levels.

$$\pi_t - \pi_{t-1} = c + \kappa \tilde{y}_t + u_t \tag{128}$$

	SW	CPI	core CPI	PCE	core PCE
gap	-0.6945**	-0.0287	-0.0212*	-0.0169	-0.0105
	(0.3258)	(0.0404)	(0.0119)	(0.0296)	(0.0123)
intercept	0.0182	0.008	-0.003	-0.0006	-0.0078
	(0.0244)	(0.0661)	(0.0195)	(0.0485)	(0.0201)
R-squared	0.0323	0.0037	0.0227	0.0024	0.0054

Table 20: Regression results for the CBO unemployment gap (inflation changes)

	SW	CPI	core CPI	PCE	core PCE
gap	0.7805**	0.0472	0.02*	0.0299	0.0103
	(0.2768)	(0.0345)	(0.0102)	(0.0254)	(0.0106)
intercept	0.0365	0.0422	0.0046	0.022	-0.0035
	(0.0262)	(0.0713)	(0.0211)	(0.0524)	(0.0218)
R-squared	0.0552	0.0136	0.0273	0.0101	0.007

Table 21: Regression results for the CBO output gap (inflation changes)

	SW	CPI	core CPI	PCE	core PCE
gap	-0.7464**	-0.0317	-0.0238**	-0.0199	-0.0133
	(0.3264)	(0.0405)	(0.0119)	(0.0297)	(0.0123)
intercept	0.0212	0.0113	-0.0002	0.0024	-0.0052
	(0.0247)	(0.0668)	(0.0197)	(0.049)	(0.0203)
R-squared	0.037	0.0045	0.0284	0.0033	0.0085

Table 22: Regression results for the unemployment rate (inflation changes)

E4: Full sample period - more detail

Residual plots

Figures (19), (20) and (21) report residual plots for the baseline specification (29) in Section 7.2:

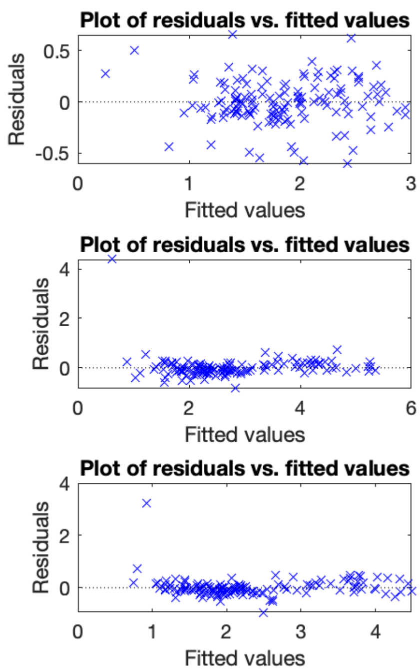


Figure 19

Other regression specifications

The tables below present results for a regression specification that includes for inflation lags:

$$\pi_t = c + \kappa \tilde{y}_t + \sum_{s=1}^{4} \gamma_s \pi_{t-s} + u_t$$

Each table is based on a different measure of the output gap (CBO unemployment gap, CBO output gap or unemployment rate).

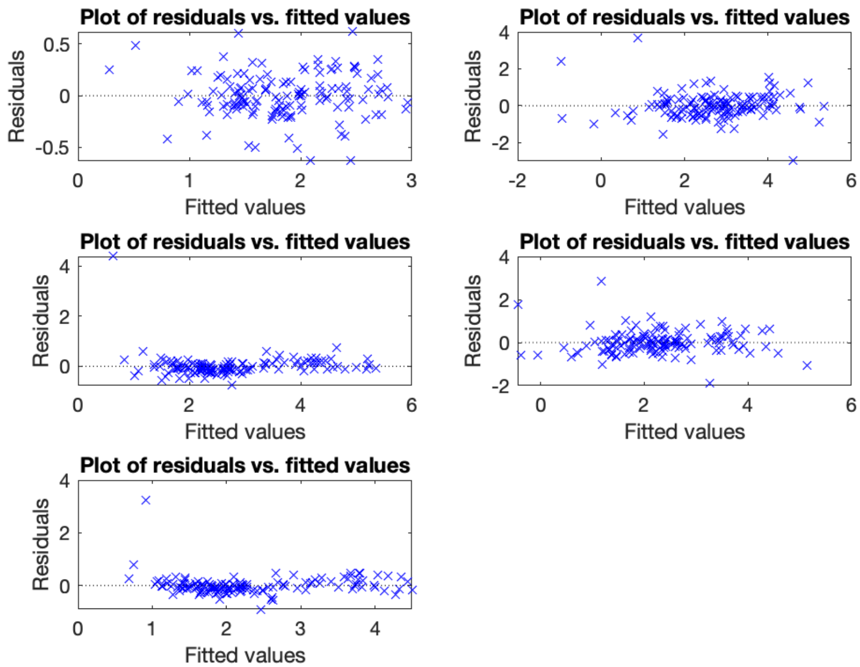


Figure 20

	SW	CPI	core CPI	PCE	core PCE
gap	-1.3803**	-0.0696	-0.0232	-0.0285	-0.0069
	(0.4361)	(0.0444)	(0.027)	(0.0339)	(0.0235)
intercept	0.6573**	0.7961**	0.367**	0.515**	0.2801**
	(0.1085)	(0.1717)	(0.1109)	(0.128)	(0.0928)
lag 1	0.7443**	0.9835**	0.9983**	1.0655**	1.068**
	(0.0858)	(0.0869)	(0.0867)	(0.0867)	(0.0866)
lag 2	0.1224	-0.2051*	-0.0391	-0.3083**	-0.2537**
	(0.1064)	(0.1218)	(0.1225)	(0.1265)	(0.1264)
lag 3	-0.1573	-0.0051	-0.068	0.0912	0.1143
	(0.1065)	(0.1219)	(0.1225)	(0.1265)	(0.1264)
lag 4	-0.0356	-0.0469	-0.0138	-0.0624	-0.0438
	(0.0799)	(0.0843)	(0.0848)	(0.0848)	(0.0849)
R-squared	0.6954	0.705	0.8465	0.7436	0.8396

Table 23: Regression results for the CBO unemployment gap , with lags

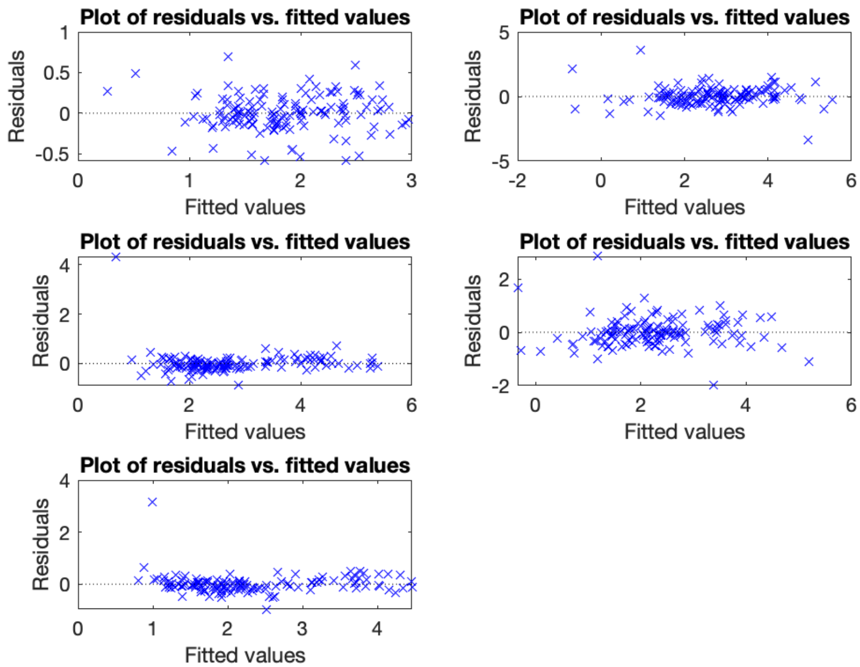


Figure 21

	SW	CPI	core CPI	PCE	core PCE
gap	1.3534**	0.0931**	0.0359	0.0524*	0.0212
	(0.3627)	(0.0378)	(0.0231)	(0.0293)	(0.0203)
intercept	0.6586**	0.8709**	0.4035**	0.5697**	0.3093**
	(0.1043)	(0.1717)	(0.1122)	(0.1294)	(0.0947)
lag 1	0.7359**	0.9662**	0.9875**	1.0502**	1.0595**
	(0.0845)	(0.0861)	(0.0866)	(0.0864)	(0.0867)
lag 2	0.1261	-0.2035*	-0.0362	-0.3064**	-0.2521**
	(0.105)	(0.1202)	(0.1217)	(0.1253)	(0.1259)
lag 3	-0.1512	-0.006	-0.066	0.0883	0.1131
	(0.1051)	(0.1203)	(0.1217)	(0.1253)	(0.1259)
lag 4	-0.0237	-0.0367	-0.012	-0.0522	-0.0397
	(0.0789)	(0.0833)	(0.0842)	(0.0843)	(0.0847)
R-squared	0.7035	0.7126	0.8484	0.7484	0.8408

Table 24: Regression results for the CBO output gap , with lags

	SW	CPI	core CPI	PCE	core PCE
gap	-1.0051**	-0.0248	0.0067	0.0035	0.0172
	(0.4293)	(0.0433)	(0.0263)	(0.0334)	(0.0232)
intercept	0.6012**	0.7115**	0.3251**	0.4737**	0.2612**
	(0.1071)	(0.1648)	(0.105)	(0.1232)	(0.0887)
lag 1	0.773**	1.002**	1.0058**	1.0747**	1.068**
	(0.0861)	(0.0867)	(0.0865)	(0.0864)	(0.0863)
lag 2	0.1231	-0.2067*	-0.04	-0.3094**	-0.2536**
	(0.1081)	(0.1228)	(0.1228)	(0.1268)	(0.1262)
lag 3	-0.1607	-0.0036	-0.0688	0.0937	0.1147
	(0.1082)	(0.1228)	(0.1228)	(0.1268)	(0.1262)
lag 4	-0.0373	-0.0461	-0.0127	-0.0655	-0.0445
	(0.0812)	(0.085)	(0.085)	(0.0851)	(0.0848)
R-squared	0.6854	0.7003	0.8457	0.7423	0.8402

Table 25: Regression results for the unemployment rate , with lags

The tables below present results for a regression specification that includes for inflation lags and inflation expectations:

$$\pi_t = c + \kappa \tilde{y}_t + \rho \mathbb{E}\pi_{t+1} + \sum_{s=1}^{4} \gamma_s \pi_{t-s} + \epsilon_t$$

	SW	CPI	core CPI	PCE	core PCE
gap	-1.1389**	-0.0488	-0.0081	-0.016	0.0073
	(0.3111)	(0.049)	(0.0274)	(0.0327)	(0.022)
inflation expecations	1.0886**	0.0987	0.1086**	0.215**	0.1927**
	(0.0952)	(0.0984)	(0.0378)	(0.0591)	(0.04)
intercept	0.3695**	0.5385*	0.2741**	0.2951**	0.1857**
	(0.0812)	(0.3089)	(0.1166)	(0.1355)	(0.0876)
lag1	-0.3659**	0.9746**	0.8858**	0.9401**	0.8238**
	(0.1147)	(0.0873)	(0.0936)	(0.09)	(0.0948)
lag2	0.2617**	-0.2079*	-0.0063	-0.2927**	-0.1762
	(0.0767)	(0.1219)	(0.1199)	(0.1212)	(0.1181)
lag3	-0.2055**	-0.0078	-0.0721	0.0827	0.0875
	(0.0759)	(0.1219)	(0.1194)	(0.1212)	(0.1171)
lag4	0.0353	-0.0509	-0.0068	-0.1163	-0.0043
	(0.0572)	(0.0844)	(0.0826)	(0.0825)	(0.079)
R-squared	0.8469	0.7072	0.8506	0.7688	0.8645

Table 26: Regression results for the CBO unemployment gap , with expectations

	SW	CPI	core CPI	PCE	core PCE
gap	1.0634**	0.0822*	0.0226	0.0383	0.0076
	(0.2599)	(0.0416)	(0.0236)	(0.0284)	(0.0191)
inflation expecations	1.0744**	0.0618	0.1033**	0.2066**	0.1884**
	(0.0944)	(0.0967)	(0.0378)	(0.0591)	(0.0401)
intercept	0.3678**	0.7088**	0.3114**	0.3494**	0.2103**
	(0.0787)	(0.3064)	(0.1187)	(0.1382)	(0.09)
lag1	-0.3551**	0.961**	0.8831**	0.9324**	0.824**
	(0.1132)	(0.0866)	(0.0933)	(0.0896)	(0.0948)
lag2	0.2629**	-0.2053*	-0.0057	-0.2917**	-0.1771
	(0.0758)	(0.1205)	(0.1195)	(0.1205)	(0.118)
lag3	-0.2004**	-0.0078	-0.0709	0.0806	0.0872
	(0.0751)	(0.1206)	(0.119)	(0.1205)	(0.1171)
lag4	0.0436	-0.0404	-0.0068	-0.1063	-0.0034
	(0.0566)	(0.0837)	(0.0823)	(0.0824)	(0.0791)
R-squared	0.8503	0.7135	0.8515	0.7715	0.8646

Table 27: Regression results for the CBO output gap , with expectations

	SW	CPI	core CPI	PCE	core PCE
gap	-1.0106**	0.0026	0.0214	-0.0048	0.0303
	(0.303)	(0.047)	(0.0265)	(0.032)	(0.0216)
inflation expecations	1.1128**	0.1424	0.1163**	0.2187**	0.198**
	(0.0958)	(0.0976)	(0.0376)	(0.059)	(0.0396)
intercept	0.3354**	0.3572	0.2385**	0.2744**	0.1753**
	(0.079)	(0.2931)	(0.1091)	(0.1283)	(0.083)
lag1	-0.3745**	0.9843**	0.8803**	0.9422**	0.8131**
	(0.1159)	(0.0871)	(0.0936)	(0.0901)	(0.0945)
lag2	0.2653**	-0.2102*	-0.0047	-0.293**	-0.1736
	(0.0773)	(0.1223)	(0.1196)	(0.1213)	(0.1173)
lag3	-0.2082**	-0.0076	-0.0724	0.0835	0.0867
	(0.0765)	(0.1224)	(0.1191)	(0.1213)	(0.1163)
lag4	0.0359	-0.0535	-0.0056	-0.118	-0.0042
	(0.0576)	(0.0848)	(0.0824)	(0.0826)	(0.0785)
R-squared	0.8445	0.705	0.8513	0.7684	0.8664

Table 28: Regression results for the unemployment rate , with expectations

The tables below present results for a regression specification that includes the time series of "endogenous" cost-push shocks constructed in Section 6.4.2 as a control:

$$\pi_t = c + \kappa \tilde{y}_t + CP_t + \epsilon_t$$

	DC	CPI	core CPI	PCE	core PCE
cost-push	0.5627**	2.5545**	0.4886	2.3948**	1.1224**
	(0.2345)	(0.565)	(0.4768)	(0.4745)	(0.4102)
gap	-3.7586**	-0.1906**	-0.2175**	-0.0783	-0.0886
	(0.6872)	(0.0758)	(0.064)	(0.0637)	(0.0551)
intercept	2.0842**	3.2239**	2.8559**	2.6509**	2.397**
	(0.058)	(0.1398)	(0.118)	(0.1174)	(0.1015)
R-squared	0.3317	0.2782	0.142	0.2558	0.1275

Table 29: Regression results for the CBO unemployment gap , with CP shock

	DC	CPI	core CPI	PCE	core PCE
cost-push	0.6059**	2.5472**	0.6387	2.4715**	1.2896**
	(0.2604)	(0.5964)	(0.5145)	(0.4983)	(0.4333)
gap	2.4282**	0.1363**	0.1176**	0.0369	0.0225
	(0.6496)	(0.0682)	(0.0588)	(0.057)	(0.0495)
intercept	2.0936**	3.2425**	2.8535**	2.6467**	2.3802**
	(0.0633)	(0.145)	(0.1251)	(0.1212)	(0.1054)
R-squared	0.2458	0.2635	0.0852	0.2484	0.1086

Table 30: Regression results for the CBO output gap , with CP shock

	DC	CPI	core CPI	PCE	core PCE
cost-push	0.6321**	2.8683**	0.8598*	2.6413**	1.3999**
	(0.2357)	(0.5706)	(0.4905)	(0.4722)	(0.4102)
gap	-3.6783**	-0.0954	-0.1038	0.006	0.0063
	(0.731)	(0.0811)	(0.0697)	(0.0671)	(0.0583)
intercept	2.0911**	3.1954**	2.8214**	2.6213**	2.3637**
	(0.0594)	(0.1439)	(0.1237)	(0.1191)	(0.1034)
R-squared	0.309	0.2462	0.0706	0.2456	0.1071

Table 31: Regression results for the unemployment rate , with CP shock

The tables below present results for a regression specification that includes both the time series of "endogenous" cost-push shocks constructed in Section 6.4.2 and oil price inflation:

$$\pi_t = c + \kappa \tilde{y}_t + CP_t + \pi_t^{oil} + \epsilon_t$$

	SW	CPI	core CPI	PCE	core PCE
cost-push	0.2874	1.1895**	0.99*	1.4128**	1.3585**
	(0.265)	(0.5943)	(0.5409)	(0.5123)	(0.4707)
gap	-3.8932**	-0.2211**	-0.2062**	-0.1003*	-0.0833
	(0.6795)	(0.0698)	(0.0635)	(0.0602)	(0.0553)
intercept	2.0185**	2.8983**	2.9754**	2.4167**	2.4533**
	(0.065)	(0.1458)	(0.1327)	(0.1257)	(0.1155)
oil prices	0.0034**	0.0167**	-0.0062*	0.012**	-0.0029
	(0.0016)	(0.0036)	(0.0033)	(0.0031)	(0.0028)
R-squared	0.3581	0.399	0.1692	0.3472	0.1358

Table 32: Regression results for the CBO unemployment gap (CP shock and oil prices)

	SW	CPI	core CPI	PCE	core PCE
cost-push	0.3912	1.294**	1.1824**	1.5569**	1.5453**
	(0.291)	(0.6217)	(0.5707)	(0.5302)	(0.4871)
gap	2.4623**	0.1454**	0.1137*	0.0436	0.0207
	(0.6454)	(0.0632)	(0.058)	(0.0539)	(0.0495)
intercept	2.0396**	2.9277**	2.99**	2.417**	2.4444**
	(0.0713)	(0.1522)	(0.1397)	(0.1298)	(0.1193)
oil prices	0.0027	0.016**	-0.0069**	0.0116**	-0.0033
	(0.0017)	(0.0036)	(0.0033)	(0.0031)	(0.0028)
R-squared	0.2632	0.3741	0.1199	0.3346	0.1192

Table 33: Regression results for the CBO output gap (CP shock and oil prices)

	SW	CPI	core CPI	PCE	core PCE
cost-push	0.3818	1.5809**	1.4042**	1.7135**	1.6665**
	(0.2666)	(0.6055)	(0.5539)	(0.5115)	(0.4689)
gap	-3.7784**	-0.119	-0.0938	-0.011	0.0112
	(0.724)	(0.0753)	(0.0689)	(0.0636)	(0.0583)
intercept	2.0301**	2.8814**	2.9542**	2.3951**	2.4287**
	(0.0667)	(0.1515)	(0.1386)	(0.128)	(0.1173)
oil prices	0.0031*	0.0161**	-0.0068**	0.0116**	-0.0033
	(0.0016)	(0.0037)	(0.0034)	(0.0031)	(0.0029)
R-squared	0.3318	0.3584	0.1041	0.3308	0.1181

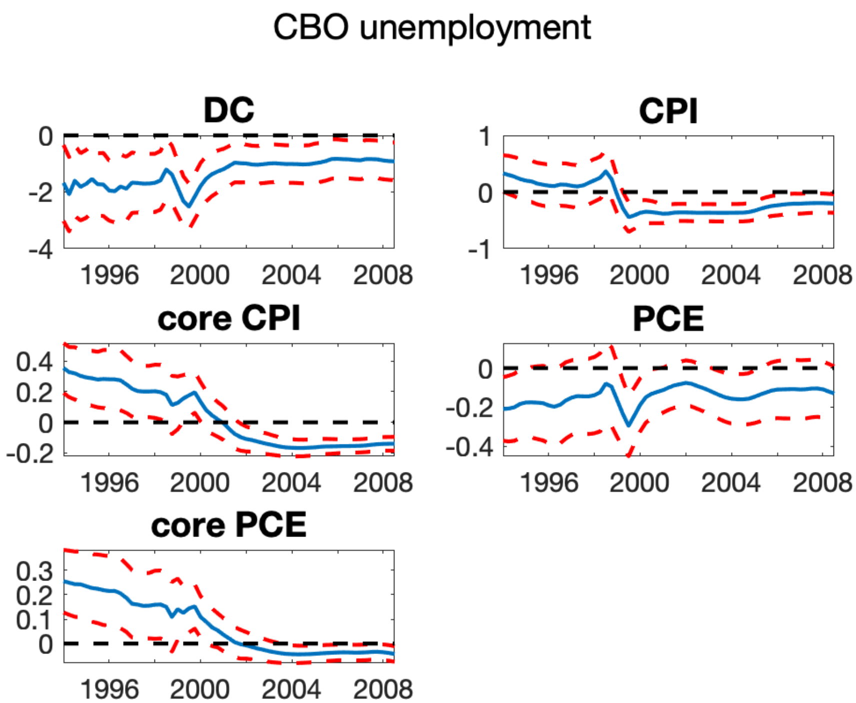
Table 34: Regression results for the unemployment rate (CP shock and oil prices)

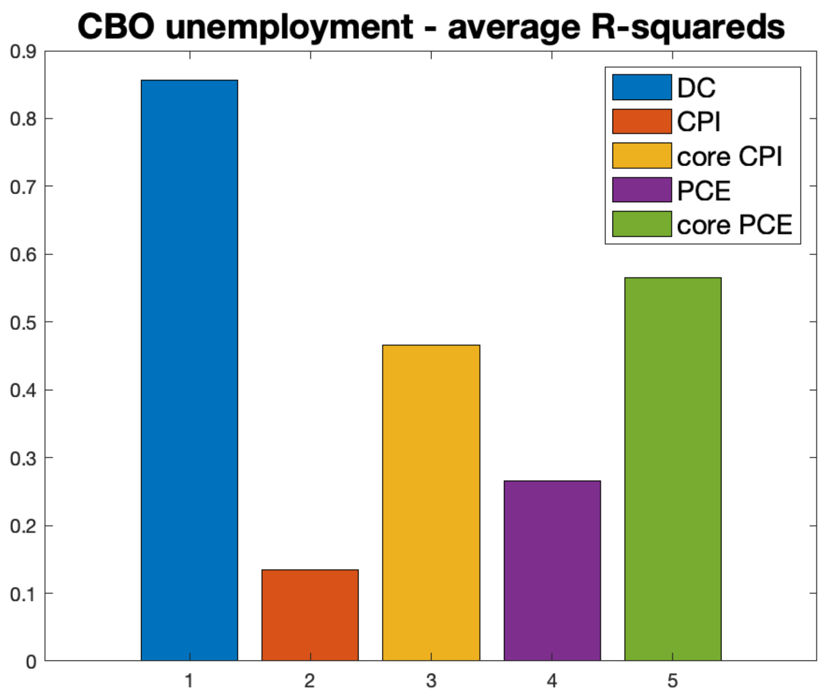
E5: Rolling regressions

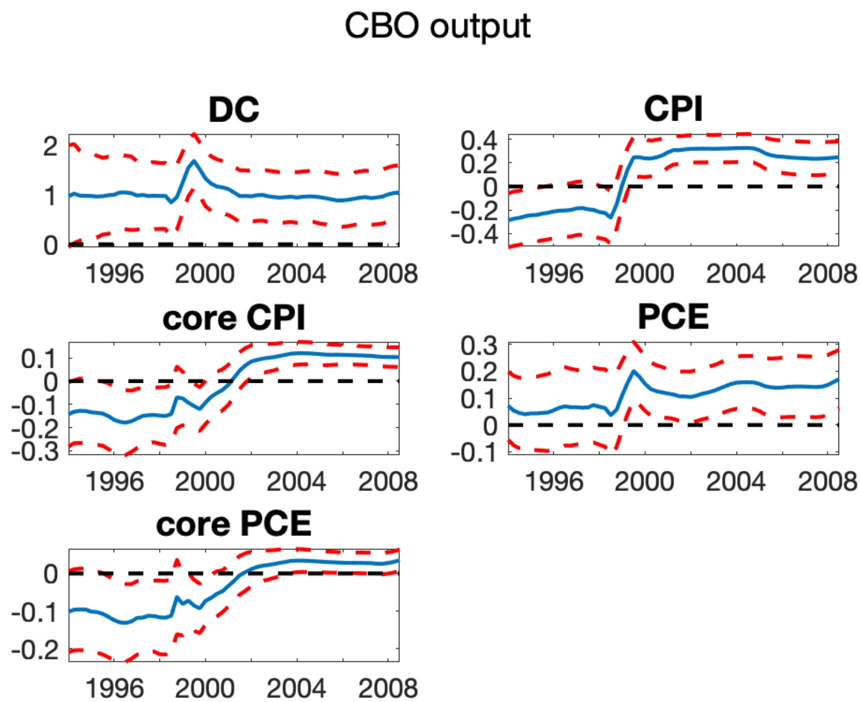
The figures below provide additional detail for the rolling regressions introduced in Section 7.3. They plot estimated coefficients for each 20-year window (with confidence intervals), and average R-squareds over the sample. The years on the x-axis correspond to the middle of the estimation window.

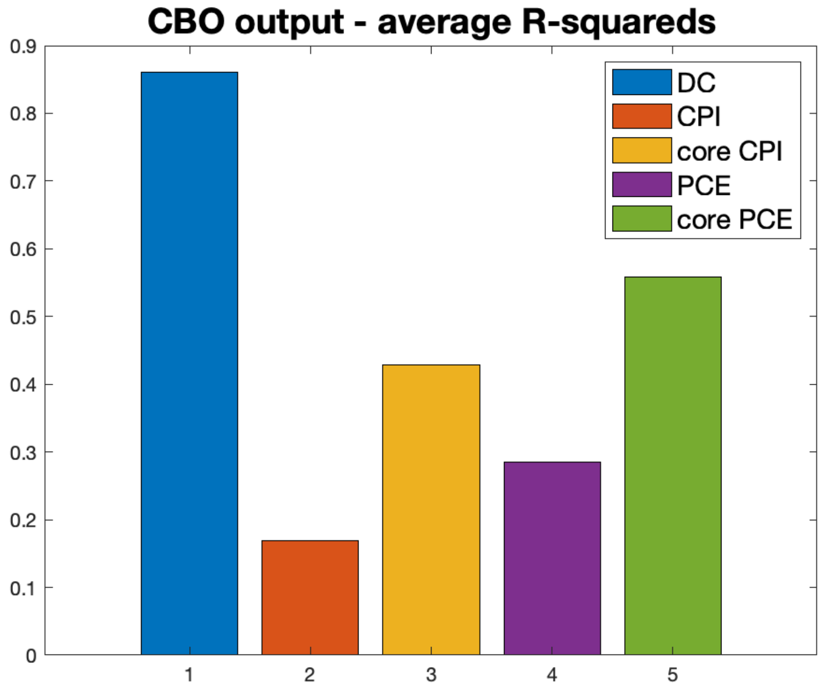
We report results for our preferred specification with inflation expectations, as in equation (129). Appendix F6 in this Supplemental Material reports results for alternative specifications and alternative measures of the gap on the right hand side.

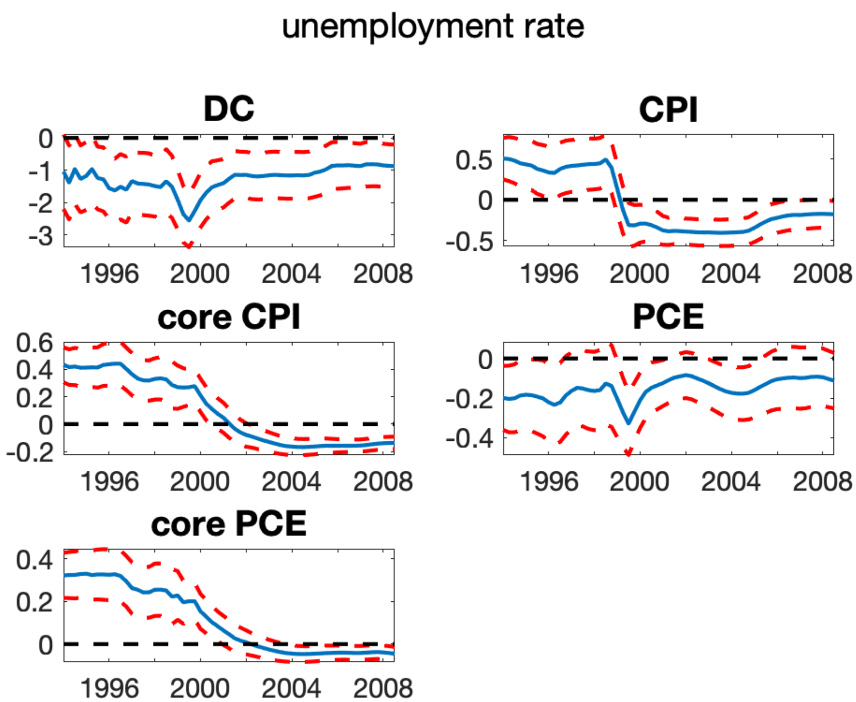
$$\pi_t = \kappa \tilde{y}_t + \rho \mathbb{E} \pi_{t+1} + \epsilon_t \tag{129}$$

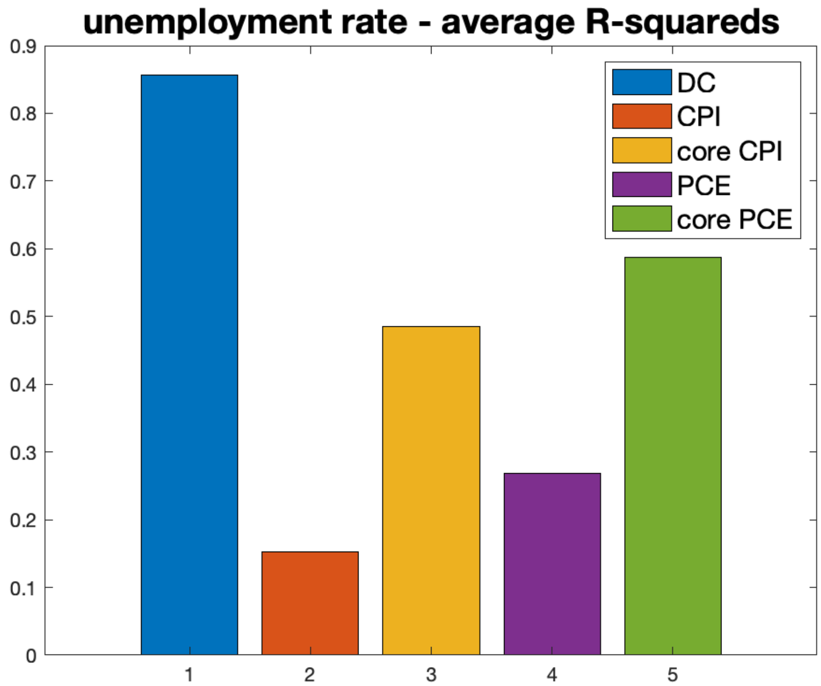



CBO unemployment - average R-squareds
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
DC
CPI
core CPI
PCE
core PCE


CBO output
DC
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008


CBO output - average R-squareds
DC
CPI
core CPI
PCE
core PCE
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
3
4
5


unemployment rate
DC
0
-1
-2
-3
1996
2000
2004
2008
CPI
0.5
0
-0.5
1996
2000
2004
2008
core CPI
0.6
0.4
0.2
0
-0.2
1996
2000
2004
2008
PCE
0
-0.2
-0.4
1996
2000
2004
2008
core PCE
0.4
0.2
0
1996
2000
2004
2008

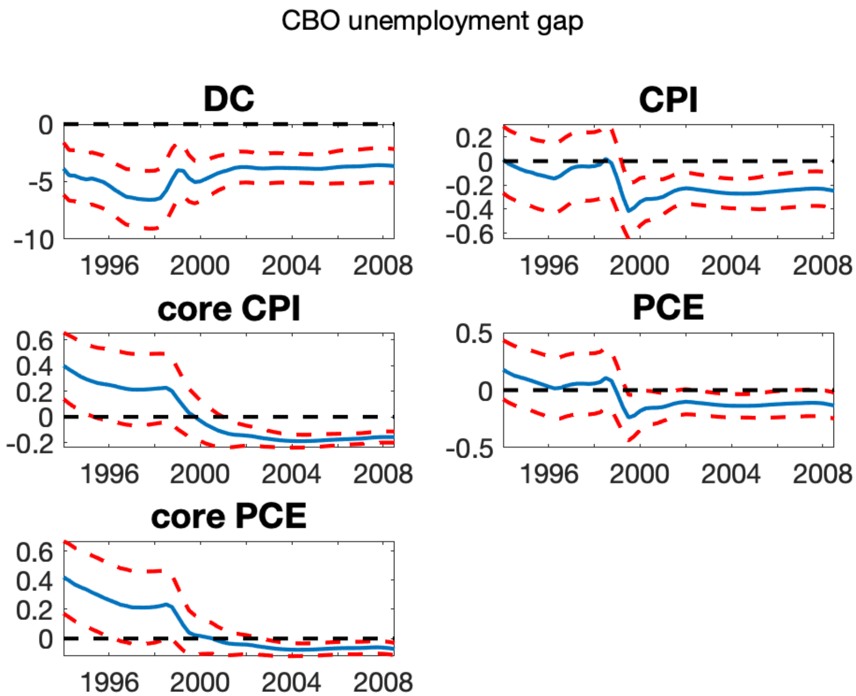


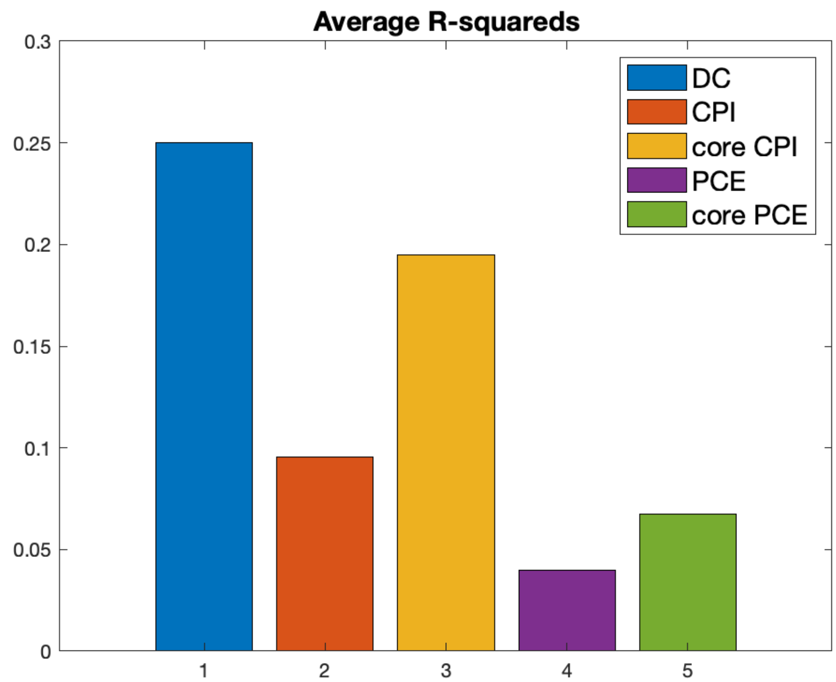
E6: Rolling regressions - more detail

The figures below plot rolling regression coefficients and R-squareds for the baseline specification

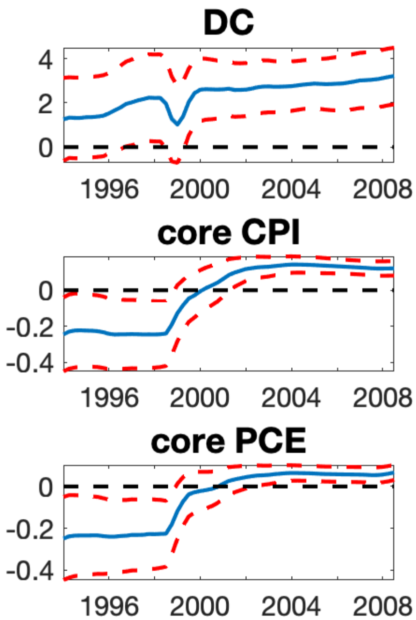
$$\pi_t = \kappa \tilde{y}_t + \epsilon_t$$

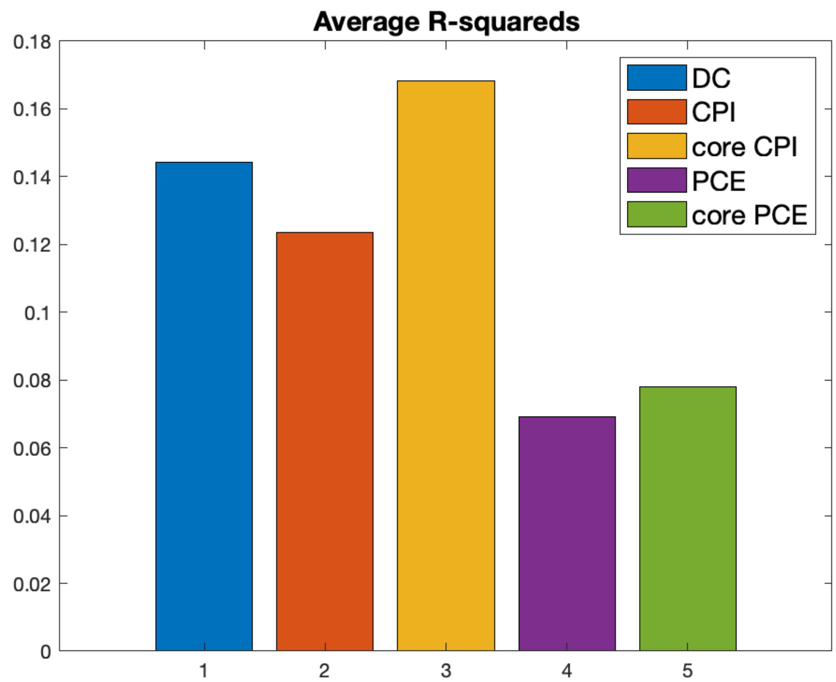
using different measures of the output gap on the right hand side (CBO unemployment gap, CBO output gap and unemployment rate).

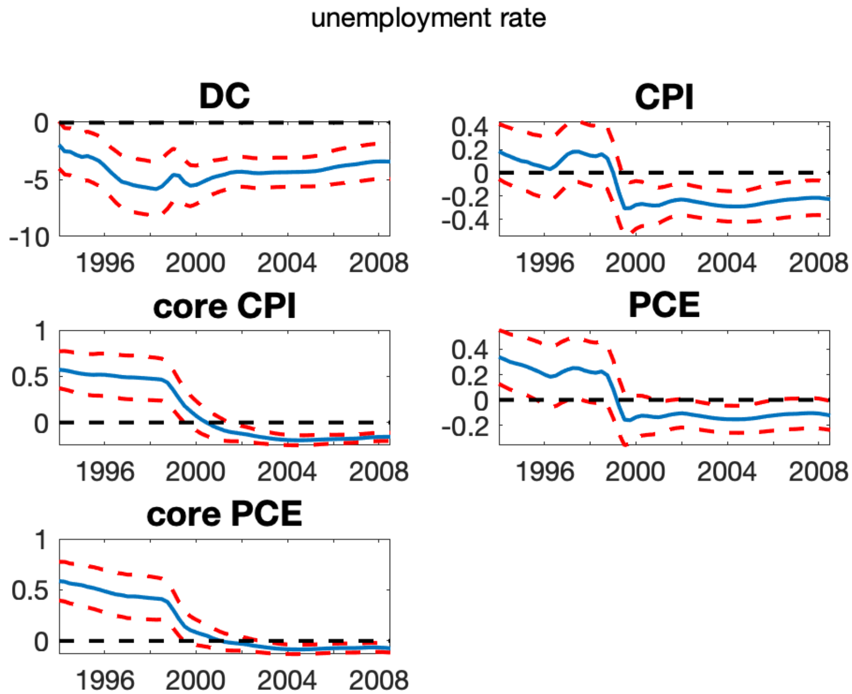

CBO unemployment gap
DC
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008

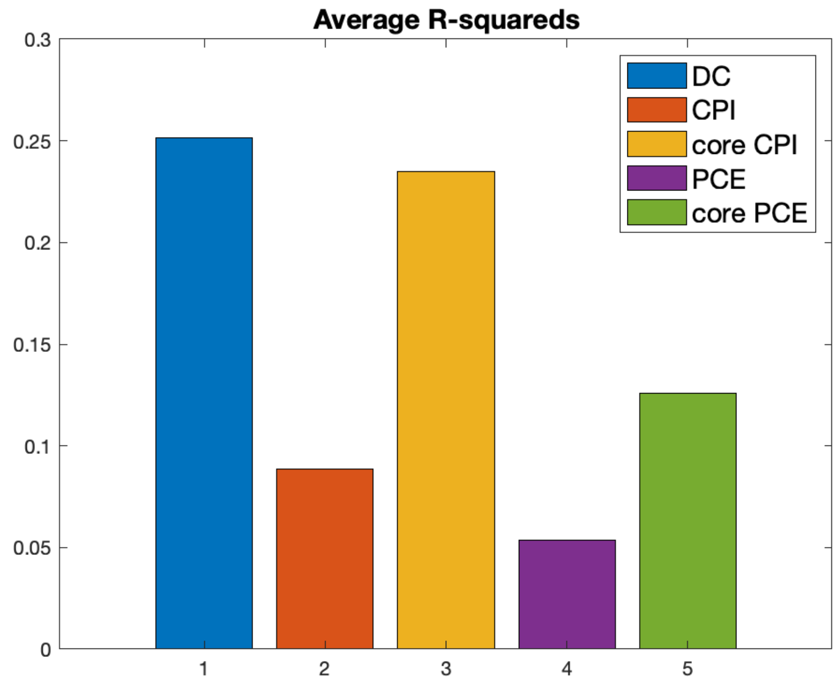

Average R-squareds
DC
CPI
core CPI
PCE
core PCE
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5

CBO output gap




Average R-squareds
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
DC
CPI
core CPI
PCE
core PCE

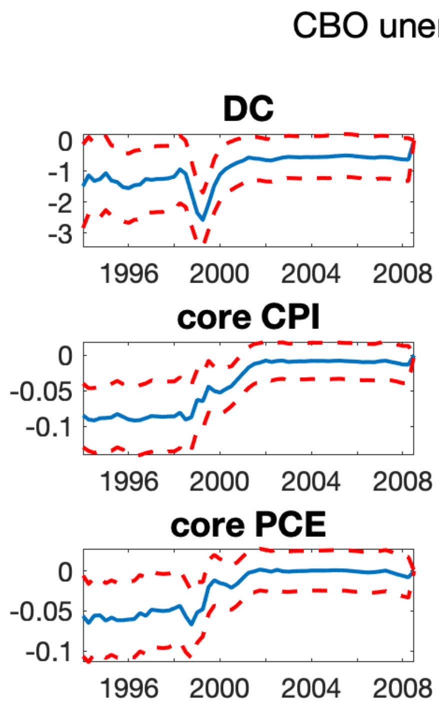

unemployment rate
DC
0
-5
-10
1996
2000
2004
2008
CPI
0.4
0.2
0
-0.2
-0.4
core CPI
1
0.5
PCE
core PCE



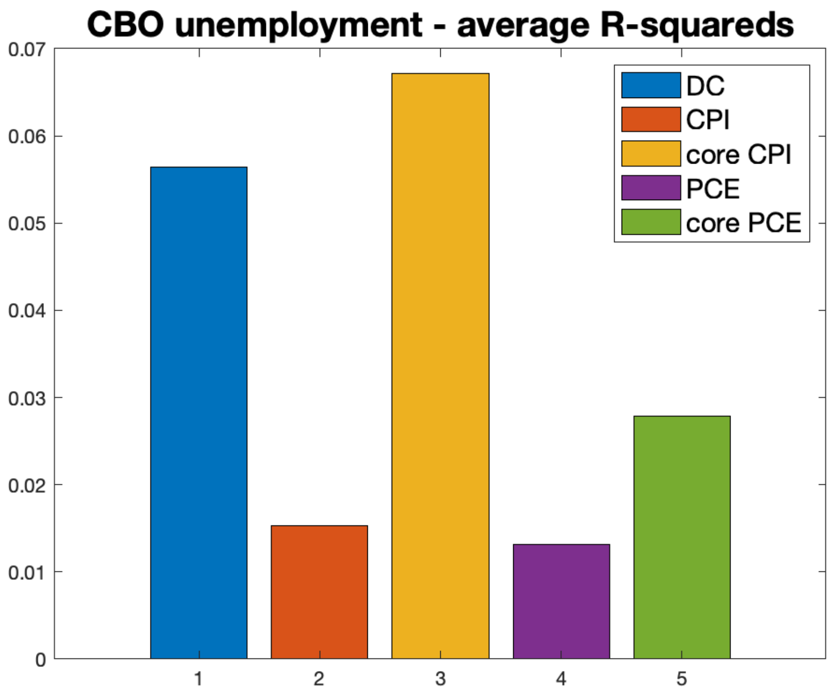
The figures below plot rolling regression coefficients and R-squareds for a regression of output gap levels on inflation changes:

$$\pi_t - \pi_{t-1} = \kappa \tilde{y}_t + \epsilon_t$$

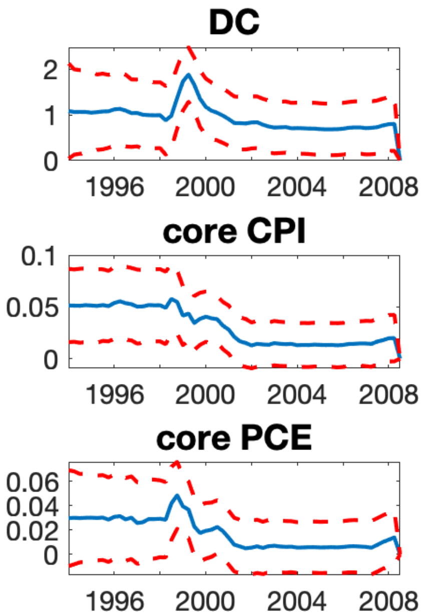
using different measures of the output gap on the right hand side (CBO unemployment gap, CBO output gap and unemployment rate).

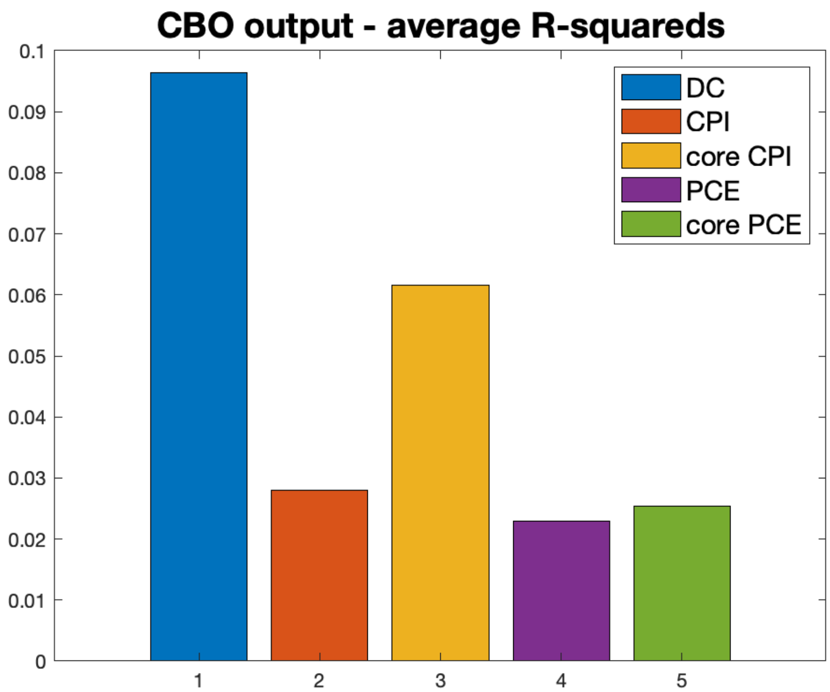


CBO unemployment
DC
CPI
core CPI
PCE
core PCE
1996
2000
2004
2008

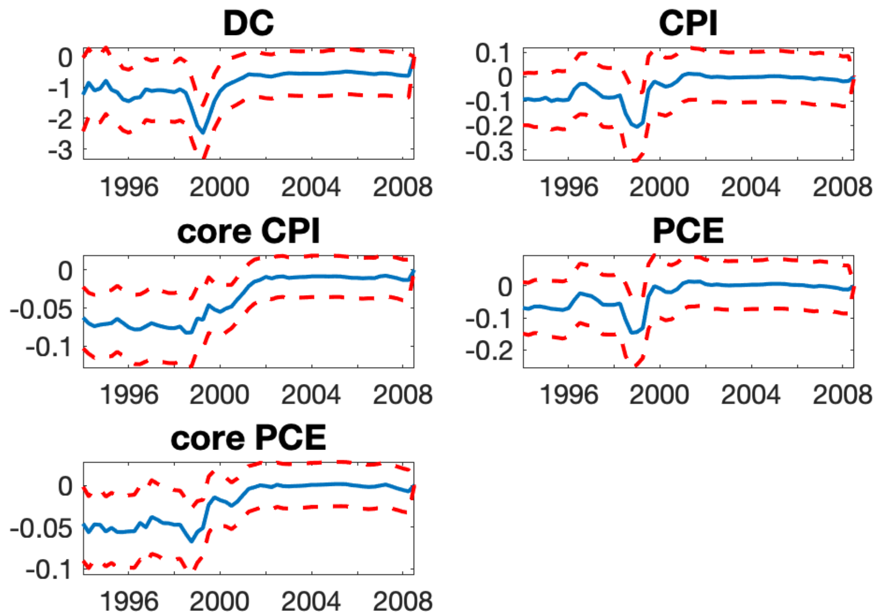

CBO unemployment - average R-squareds
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
DC
CPI
core CPI
PCE
core PCE

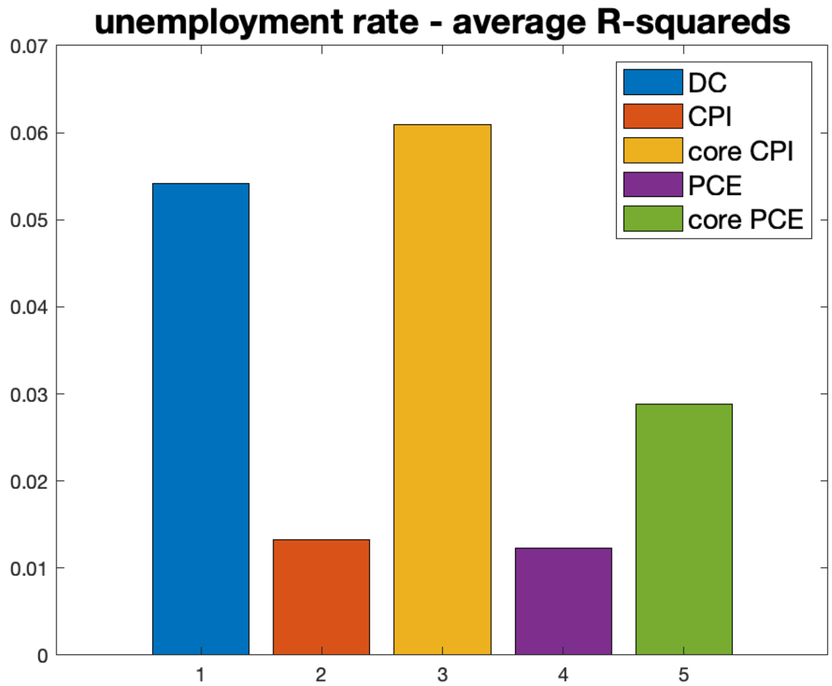
CBO output


DC
2
1
0
1996
2000
2004
2008
CPI
0.2
0.1
0
1996
2000
2004
2008
core CPI
0.1
0.05
0
1996
2000
2004
2008
PCE
0.2
0.1
0
1996
2000
2004
2008
core PCE
0.06
0.04
0.02
0
1996
2000
2004
2008


CBO output - average R-squareds
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
DC
CPI
core CPI
PCE
core PCE

unemployment rate




unemployment rate - average R-squareds
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
DC
CPI
core CPI
PCE
core PCE