Nonlinearities in Disaggregated Economies with Implications for the Covid-19 Crisis

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The Question We Ask

- Covid-19 unusual aggregate shock.
- Messy mix of big heterogenous supply and demand shocks.
- Nonlinearities (amplification and interactions)?

How We Try to Answer the Question

- Study output, unemployment, inflation.
- Use general disaggregated model and aggregate up.
- Allow for neoclassical and Keynesian channels.
- Find quantitatively large nonlinearities from both channels.
- Explain where they come from.
- Explain why they matter.

Agenda

Neoclassical Nonlinearities
- Setup
- General Results
- Illustrative Examples
- Quantitative Illustration
- Additional Results

Keynesian Nonlinearities
- Setup
- General Results
- Illustrative Examples
- Quantitative Illustration
- Additional Results

Conclusion

Agenda

Model Structure

- $\mathscr{N}$ produced goods.
- $\mathscr{G}$ factors in inelastic supply.
- Homothetic final demand (can generalize).
- Goods produced using other goods and factors.

Final Demand

- Final demand maximizes homothetic aggregator:

$$\mathscr{D}\left(c_1,\ldots,c_{\mathscr{N}};\omega_{\mathscr{D}}\right),$$

with c_i final consumption of good i, $\omega_{\mathscr{D}}$ demand shifter.

- Budget constraint:

$$\sum_{i\in\mathscr{N}} p_i c_i = \sum_{f\in\mathscr{G}} p_f L_f + \sum_{i\in\mathscr{N}} \pi_i,$$

with p_i prices, p_f wages, L_f factors, π_i profits.

Producers and Factors

- Good i produced under constant returns:

$$y_i = A_i F_i \left(x_{i1}, \ldots, x_{i\mathcal{N}}, L_{i1}, \ldots, L_{i\mathcal{G}} \right),$$

with y_i output, x_{ij} input j, L_{if} factor f, A_i total factor productivity.

- Producer i maximizes profits:

$$\pi_i = \max_{\{y_i\},\{x_{ij}\},\{L_{if}\}} p_i y_i - \sum_{j \in \mathcal{N}} p_j x_{ij} - \sum_{f \in \mathcal{G}} p_f L_{if}.$$

- Inelastic factor supplies L_f.

Equilibrium

- Agents optimize.
- Markets clear.

Comparative Statics

- Perturb initial equilibrium with shocks.
- Supply shocks: A_i, L_f.
- Demand shocks: $\omega_{\mathscr{D}}$.
- Preference-driven or policy-induced.
- Could also be equilibrium outcomes in richer model.

Generality

- Captures factor augmenting productivity shocks with relabeling.
- Captures variable returns with fixed quasi-factors.
- Can be applied within a period or intertemporally.

Input-Output Definitions

- Final demand as producer 0, factors as endowment producers.

- Input-output matrix:

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i} = \frac{p_j x_{ij}}{\sum_{k \in \mathcal{N} + \mathcal{G}} p_k x_{ik}}.$$

- Leontief inverse matrix:

$$\Psi \equiv (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \ldots.$$

- Nominal GDP:

$$GDP \equiv \sum_{i \in N} p_i x_{0i}.$$

- Sales shares:

$$\lambda_i \equiv \frac{p_i y_i}{GDP} = \Psi_{0i}.$$

Real GDP (Output) Definition

- Define changes in real GDP $\Delta \log Y$ as in data.
- Deflate changes in nominal GDP by changes in GDP deflator.
- With demand shocks (see paper): exotic path-dependence properties and divergence from welfare.

Nested CES Economies

- General nested-CES economy (can generalize).
- Relabel network so that each node corresponds to one CES nest.
- Demand share-shifter ω_0 in final demand nest 0.

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Aggregation Equation

- Changes in output are approximated by (second order):

$$\Delta \log Y \approx \underbrace{\sum_{k\in\mathcal{N}} \lambda_k \Delta \log A_k + \sum_{f\in\mathcal{G}} \lambda_f \Delta \log L_f}_{\text{first order: Hulten}}$$
$$+ \underbrace{\frac{1}{2}\sum_{k\in\mathcal{N}} \lambda_k \Delta \log \lambda_k \Delta \log A_k + \frac{1}{2}\sum_{f\in\mathcal{G}} \lambda_f \Delta \log \lambda_f \Delta \log L_f}_{\text{second order: nonlinearities}}.$$

- First order (Hulten's theorem): initial shares.
- Second order (nonlinearities): equilibrium changes in shares.
- Demand shocks: no independent impact; matter only via interactions with supply shocks.
- Network and elasticities matter via changes in shares (suff. stat.).

Propagation Equations

- Changes in shares are approximated by (first order):

$$\lambda_i \Delta \log \lambda_i \approx \theta_0 Cov_{\Omega^{(0)}}\left(\Delta \log \omega_0, \Psi_{(i)}\right) + \sum_{j \in 1+\mathcal{N}} \lambda_j (\theta_j - 1) Cov_{\Omega^{(j)}}\left(\sum_{k \in \mathcal{N}} \Psi_{(k)} \Delta \log A_k + \sum_{g \in \mathcal{G}} \Psi_{(g)} \left(\Delta \log \lambda_g - \Delta \log L_g\right), \Psi_{(i)}\right).$$

- Network and elasticities suff. stat.

Cobb-Douglas ($\theta_j = 1$)

- Changes in output are approximated by (second order):

$$\Delta \log Y \approx \underbrace{\sum_{k \in \mathcal{N}} \lambda_k \Delta \log A_k + \sum_{f \in \mathcal{G}} \lambda_f \Delta \log L_f}_{\text{first order: Hulten}}$$

$$\underbrace{+ \frac{1}{2} Cov_{\Omega^{(0)}} \left(\Delta \log \omega_0, \sum_{k \in \mathcal{N}} \Psi_{(k)} \Delta \log A_k + \sum_{f \in \mathcal{G}} \Psi_{(f)} \Delta \log L_f \right)}_{\text{second order: demand shocks} \times \text{supply shocks}}.$$

- Amplification or mitigation.

- Key role of network in propagating demand shocks.

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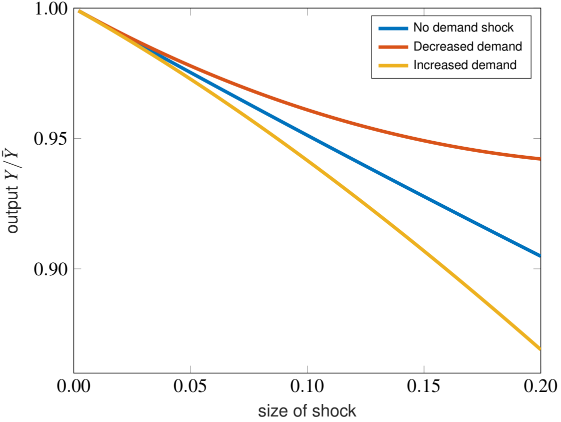
Concrete Examples

- Amplification of negative supply shocks:
  - Amazon's forced warehouse closure in France while consumers shift demand towards Amazon;
  - meat-packing plants (Smithfield, JBS) in U.S. while consumers shift demand towards meat.
- Mitigation of negative supply shocks:
  - restaurant closures while consumers shift demand away from restaurants;
  - cinema closures while consumers shift demand away from cinemas.

Numerical Illustration

- Two sectors of same size.
- Each sector produce with a labor.
- Labor reduction in one sector.
- Compare scenario without simultaneous demand shocks and scenarios with simultaneous demand shocks towards or away from affected sector.

Interaction of Supply and Demand Shocks



Uniform Elasticities ($\theta_j = \theta$) and Labor-Supplied Shocks

- ‘As if” CES aggregate production function:

$$\frac{Y}{\bar{Y}} = \left(\sum_{f \in \mathscr{G}} \bar{\lambda}_f \left(\frac{L_f}{\bar{L}_f} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} .$$

- Irrelevance of the network!

- Conditions for network relevance (see paper for examples): non-uniform elasticities, demand shocks, TFP shocks.

Uniform Elasticities ($\theta_j = \theta$) and Labor-Supplied Shocks

- Changes in output are approximated by (second order):

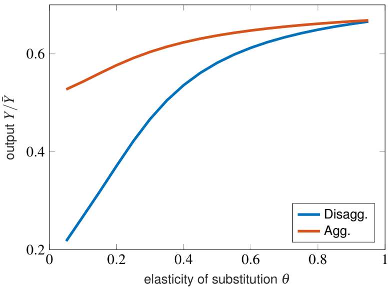
$$\Delta \log Y \approx \underbrace{\sum_{f \in \mathscr{G}} \lambda_f \Delta \log L_f}_{\text{first order: Hulten}} + \underbrace{\frac{1}{2} \frac{\theta - 1}{\theta} Var_{\lambda} \left(\Delta \log L \right)}_{\text{second order: nonlinearities}}.$$

- Amplify negative shocks with complementarities $\times$ heterogeneity.

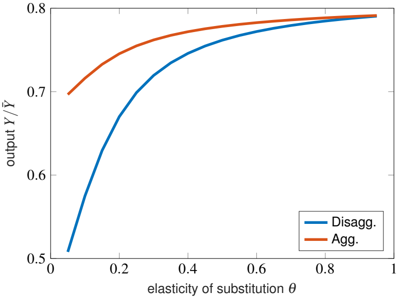
Numerical Illustration

- Stylized version of U.S. economy: 66 sectors, sectoral production using capital, labor, and intermediates.
- Factors cannot be reallocated across sectors (short run).
- Uniform elasticities ($\theta_j = \theta$).
- Labor-supplied shocks: in each sector, remove fraction x of workers in jobs that cannot be done from home (built from Mongey et al. (2020) and Dingel and Neiman (2020)).
- Compare to CES aggregate production function with same elasticity θ, and single labor and capital aggregates.

More Amplification with Heterogenous Shocks



(a) more extreme scenario $x = 1$



(b) less extreme scenario $x = 0.7$

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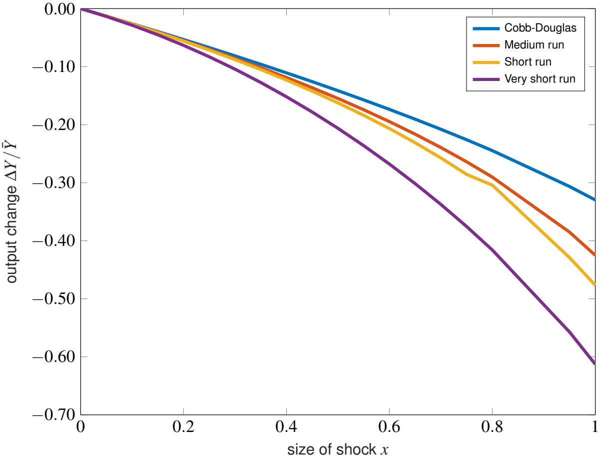
Quantitative Illustration

- Same 66-sector inout-output model as before.
- Elasticities: between labor and capital (η), between intermediates (θ), between value-added and intermediates (ε), and between final consumptions in final demand (σ).
- Three calibrations inspired by estimates in literature:
  - $(\sigma,\theta,\varepsilon,\eta) = (0.7,0.1,0.1,0.1)$ for very short run;
  - $(\sigma,\theta,\varepsilon,\eta) = (0.7,0.1,0.3,0.5)$ for short run;
  - $(\sigma,\theta,\varepsilon,\eta) = (0.95,0.1,0.5,0.5)$ for medium run.
- Same labor-supply shock: fraction x of "can't work from home" removed from workforce.

Amplification from Nonlinearities



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More in Paper

- Elastic factor supplies.
- Reallocation.
- Welfare.

Keynesian Nonlinearities

- So far, second-order neoclassical nonlinearities.
- Now, first-order Keynesian nonlinearities from nominal “kinks”:
  - downward nominal wage rigidity;
  - ZLB constraint on nominal interest rate.

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Keynesian Model

- Two periods: present and future.
- Utility maximization s.t. intertemporal budget constraint.
- Present: same as before + downward nominal wage rigidity.
- Future: full employment + fixed nominal expenditure.
- Monetary policy: full employment s.t. ZLB.

Euler Equation for Intertemporal Problem

- Log-linearized Euler equation:

$$d\log Y = -\rho d\log p^Y + d\log \zeta .$$

- Aggregate demand (AD) shock:

$$d\log \zeta = -\rho \left(d\log(1+i) + d\log \beta - d\log \bar{p}_*^Y \right) + d\log \bar{Y}_* .$$

- No AD shock $\Longrightarrow$ stagflation (recession and inflation).

Euler equation for Nominal Expenditure $E = p^Y Y$

- Euler equation for nominal expenditure:

$$d \log E = (1-\rho) d \log p^Y + d \log \zeta.$$

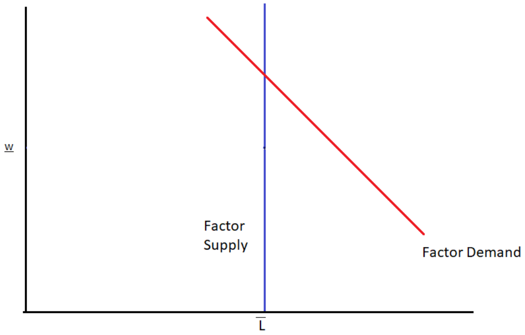
- Focus on $\rho = 1$ nominal expenditure exogenous:

$$d \log E = d \log \zeta.$$

Factor Supply and Factor Demand

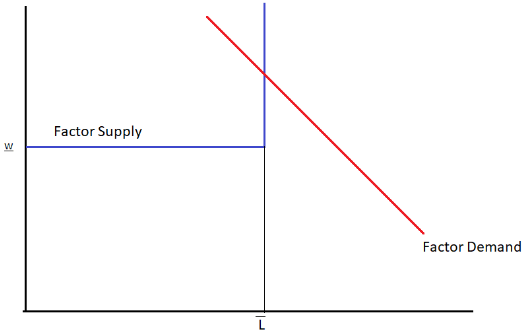
- Different types of factors.
- Always-flexible $f \in \mathscr{K}$ (“capitals”) with vertical supply curves.
- Potentially-rigid $f \in \mathscr{L}$ (“labors”) with L-shaped supply curves:
    - flexible in equilibrium $f \in \mathscr{F}$;
    - rigid in equilibrium $f \in \mathscr{R}$.

Supply and Demand for “Capitals”



- Shocks to supply $d\log\bar{L}_f$ and nominal demand $d\log\lambda_f + d\log E$.
- Flexible wage adjustment $d\log w_f$.
- Factor supplied $d\log L_f = d\log\bar{L}_f$.

Supply and Demand for "Labors"



- Shocks to supply $d\log \bar{L}_f$ and nominal demand $d\log \lambda_f + d\log E$.
- Constrained wage adjustment $d\log w_f \geq 0$.
- Factor supplied $d\log L_f = \min\left\{d\log \lambda_f + d\log E, d\log \bar{L}_f\right\}$.

Agenda

Propagation and Aggregation Equations

- Beware of multiple equilibria!
- Nonlinear determination of rigid-factor set $\mathscr{R}$.
- Linear derivative given set of rigid factors $\mathscr{R}$:
  - same propagation and aggregation equations;
  - apply with $d\log L_f$ from factor-supplied equations.

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Cobb-Douglas ($\theta_j = 1$)

- Changes in output

$$d\log Y = \underbrace{\sum_{i\in\mathcal{N}} \lambda_i d\log A_i + \sum_{f\in\mathcal{G}} \lambda_f d\log \bar{L}_f,}_{\text{Neoclassical effect}}$$

$$+ \underbrace{\sum_{f\in\mathcal{L}} \lambda_f \min\left\{ Cov_{\Omega^{(0)}}\left(d\log\omega_0, \frac{\Psi_{(f)}}{\lambda_f}\right) + d\log\zeta - d\log\bar{L}_f, 0\right\}}_{\text{Keynesian amplification}}.$$

- First-order Keynesian nonlinearities amplify output reductions.
- Keynesian unemployment if demand shifts away from a factor.
- Network key for propagation of shocks to composition of demand.

Uniform Elasticities ($\theta_j = \theta$) and Labor-Supply Shocks

- Network irrelevance once again!
- With substitutes ($\theta > 1$):
  - reduction in labor supplied *increases* demand for other labors;
  - no Keynesian unemployment;
  - unique equilibrium.
- With complements ($\theta < 1$):
  - reduction in labor supplied *reduces* demand for other labors;
  - Keynesian unemployment (factors with smallest supply shocks);
  - possibility of multiple equilibria.
- Generalizes to also include AD shocks.

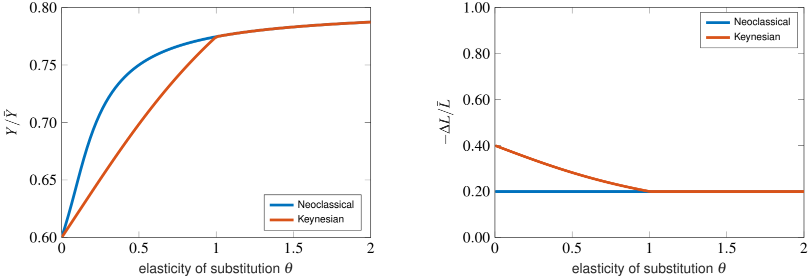
Uniform Complementarities ($\theta_j = \theta < 1$)

- Global comparative statics (Δ instead of d).
- Set of equilibrium vectors $\Delta \log L$ with partial ordering $\leq$ is *lattice*.
- Unique best and worst equilibrium.
- $\Delta \log Y$ and $\Delta \log L$ increasing in $\Delta \log \bar{L}$ and $\Delta \log \zeta$.
- Key: reduction in labor supplied *reduces* demand for other labors.

Numerical Illustration

- Uniform elasticity θ.
- Two labor markets of equal size.
- Reduction of 40% in labor supply of one of the labors.
- Compare Keynesian economy with downward nominal rigidity to neoclassical economy with flexible wages.

Keynesian vs. Neoclassical



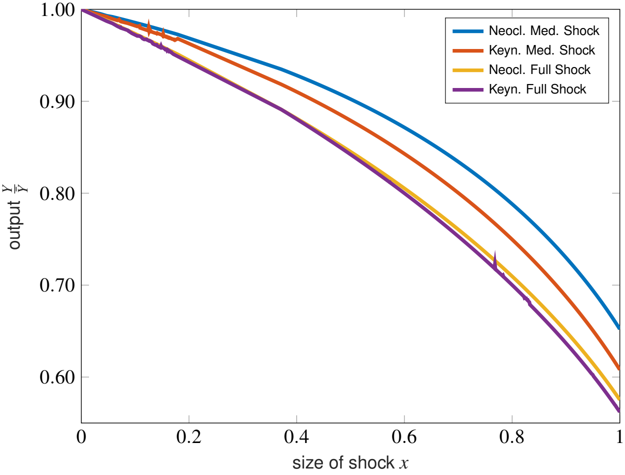
- Keynesian unemployment decreases with elasticity θ.
- Keynesian output amplification hump-shaped in elasticity θ.

Agenda

Quantitative Illustration

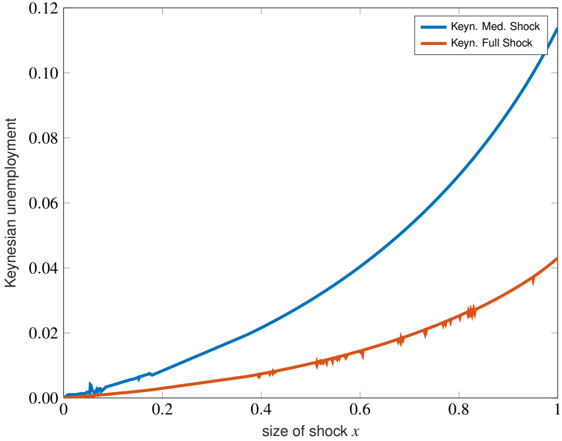
- Same 66-sector inout-output model as before.
- Use higher elasticities $(\sigma, \theta, \varepsilon, \eta) = (0.95, 0.1, 0.5, 0.5)$.
- Same labor-supply shock: fraction x of ”can't work from home” removed from workforce.
- Also consider only shock to sectors with below-median ability to work from home (more heterogenous, milder).
- Compare neoclassical and Keynesian economies

Keynesian Output Amplification



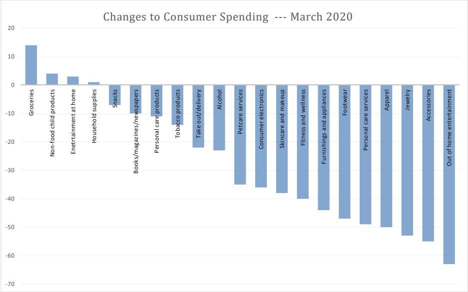
- More Keynesian output amplification with more heterogenous shock even though milder shock.

Keynesian Unemployment



- More Keynesian unemployment with more heterogenous shock even though milder shock.

Negative AD shock?

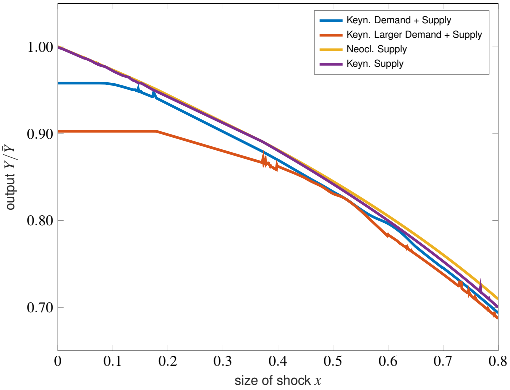


- Survey from McKinsey: "Over the next 2 weeks, do you expect that you will spend more, about the same, or less money on these categories than usual?"
- Histogram bars: fraction of respondents who said "increase" minus fraction of respondents who said "decrease".

Interaction of Labor Supply and AD Shocks

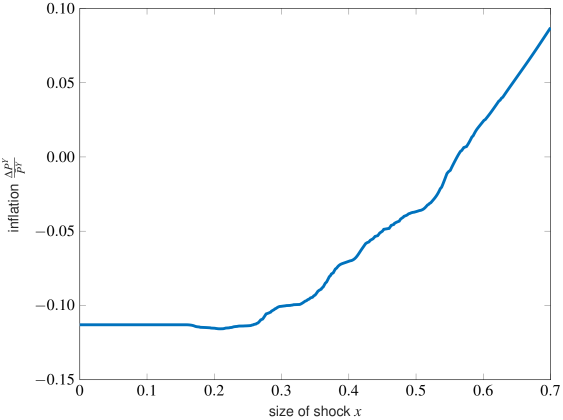
- AD shock and labor-supply shocks *together*.
- Scale negative labor-supply shock x as before.
- Combine with fixed negative AD shock $\Delta \log \zeta \in \{-0.1, -0.2\}$.

Labor Supply and AD Shocks: Output



- A given negative AD shock matters *less* for output with larger negative supply shocks.

Labor Supply and AD Shocks: Inflation



- A given negative AD shock matters *a lot* for inflation even with large negative supply shocks.

Agenda

More in Paper

- Semi-downward-flexible wages.
- Policy: monetary, fiscal...
- More “kinks”: endogenous demand and supply shocks.

Agenda

Conclusion

- Covid-19 large heterogenous shock.
- Supply shocks and demand shocks.
- Triggers neoclassical and Keynesian nonlinearities.
- Important qualitatively and quantitatively.
- Important for designing policy response.
- Revisit numbers as data becomes available.