Fiscal Devaluations in a Model with Capital

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1 Summary

In this note we demonstrate that even in a model with capital, fiscal devaluation policies contained in Propositions 1–3 of the paper suffice to mimic a nominal devaluation. In particular, an unexpected fiscal devaluation can be replicated using a VAT-payroll tax swap *alone*, without any additional tax instruments. This is a stronger result than what is stated in Proposition 7 of the paper because we now assume that investment goods are exempt from VAT, as is commonly the case in practice. This reinforces the case for the ability of governments to replicate the effects of nominal devaluations with a small set of conventional fiscal instruments.

To be clear, there are two main differences with our original analysis, which allow to streamline the fiscal devaluation policies: First, as mentioned above, we redefine the VAT τ_t^v to include the VAT-exemption for investment goods, which is a more accurate description of the VAT use in practice. In the paper's notation, this is equivalent to automatically setting the investment tax credit to $\varsigma_t^I = \tau_t^v/(1 - \tau_t^v) = \delta_t$, when VAT is adjusted to implement a δ_t-devaluation. This excludes the need in using the additional investment tax credit ς_t^I when implementing a fiscal devaluation.

Second, our original analysis imposed implicitly the requirement to replicate the path of rental rate of capital, R_t, in addition to the paths of real variables. This requirement is re-

dundant given our definition of a fiscal devaluation which only concerns the real allocations.[1] When the path of the rental rate of capital does not need to be replicated, the necessary set of instruments needed to implement a fiscal devaluation is smaller, and in particular, the capital income tax τ_t^K and the capital expenditure subsidy to firms ς_t^R can be dispensed with for both expected and unexpected fiscal devaluations.

Together, these two observations imply that the additional instruments introduced in Section 4.2 of the paper—namely, the capital income tax τ_t^K, the capital expenditure subsidy to firms ς_t^R, and the investment tax credit ς_t^R—are not necessary, and can be left unchanged when implementing a VAT-based fiscal devaluation. That is, the full fiscal devaluation policy (FD″) in Proposition 1 and the reduced policy (FD″$_R$) for a one-time unanticipated fiscal devaluation in Proposition 3 remain robust in economies with capital under VAT-exempt capital expenditures.[2]

2   Model with Capital

Consider the generalized capital evolution equation:

$$K_{t+1} = (1 - d)K_t + \varphi(I_t, K_t),$$

which takes as special cases $\phi(I_t, K_t) = I_t$ of Section 4.2 and $\varphi(I_t, K_t) = I_t - \frac{\phi_I}{2}\left(\frac{I_t}{K_t} - d\right)^2 K_t$ of Section 5.[3] The household budget constraint is:

$$\frac{P_t C_t}{1 + \varsigma_t^c} + (1 - \tau_t^v)\frac{P_t I_t}{1 + \varsigma_t^I} + M_t + \sum_{j \in J_t} Q_t^j B_{t+1}^j$$

$$\leq M_{t-1} + \sum_{j \in J_{t-1}} (Q_t^j + D_t^j)B_t^j + \frac{R_t K_t}{1 + \tau_t^K} + \frac{W_t N_t}{1 + \tau_t^n} + \frac{\Pi_t}{1 + \tau_t^d} + T_t.$$

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[1] The redundancy relies on the assumption that the rental rate of capital is flexible, i.e. there is no stickiness in this market price (since it can be viewed as a return on an asset rather than a factor price). If the rental rate of capital is also sticky, then Proposition 7 outlines the minimal fully general fiscal devaluations in an economy with capital. However, if the investment in capital is done primarily by firms and not households, then the rental rate of capital is only a shadow price at which no market transactions happen, and hence it can be assumed flexible *without loss of generality.*

[2] When fiscal devaluations are implemented using an import tariff-cum export subsidy, the full policy (FD′) in Proposition 1 needs to be complimented with an investment tax credit ($\varsigma_t^I = \varsigma_t^c = \delta_t$), while the reduced policy (FD′$_R$) for a one-time unexpected devaluation in Proposition 3 requires no adjustment (i.e., $\varsigma_t^c = \varsigma_t^I \equiv 0$), and hence is robust to introducing capital in the same way the VAT-payroll-tax swap (FD″$_R$) is. We omit the proofs in this case for brevity.

[3] Note the typo in this equation in Section 5 (on p. 751 of the published version). Also note that in the Appendix (Online Supplement) and in the numerical Section 5 we have a notational inconsistency denoting the depreciation rate with δ, the same letter as a size of the devaluation, while in Section 4.2 it is introduced as d, as we use here.

Note the single difference relative to the case discussed in the paper is that now investment expenditure $P_t I_t$ is VAT-exempt, i.e. the VAT is reimbursed to the consumers on their purchases of the investment goods since the price index P_t is VAT-inclusive. Therefore, it is immediate to see that this extra role of the VAT tax is exactly equivalent in the previous analysis to setting the investment subsidy (tax credit) to

$$\varsigma_t^I = \frac{\tau_t^v}{1 - \tau_t^v}$$

whenever the VAT is used.

Consider now the first order conditions for the choice of I_t and K_{t+1}:

$$\beta^t u_{C,t} = \lambda_t \frac{P_t}{1 + \varsigma_t^c},$$

$$\mu_t = \mathbb{E}_t \left\{ \left[(1 - d) + \varphi_{K,t+1} \right] \mu_{t+1} + \lambda_{t+1} \frac{R_t}{1 + \tau_t^K} \right\},$$

$$\lambda_t \frac{(1 - \tau_t^v) P_t}{1 + \varsigma_t^I} = \mu_t \varphi_{I,t},$$

where $u_{C,t} \equiv u_C(C_t, L_t)$ and by analogy for $\varphi_{I,t}$ and $\varphi_{K,t}$. We combine to obtain:

$$(1 - \tau_t^v) \frac{1 + \varsigma_t^c}{1 + \varsigma_t^I} \frac{u_{C,t}}{\varphi_{I,t}} = \beta \mathbb{E}_t u_{C,t+1} \left[\left((1 - d) + \varphi_{K,t+1} \right) \frac{1 + \varsigma_{t+1}^c}{1 + \varsigma_{t+1}^I} \frac{1 - \tau_{t+1}^v}{\varphi_{I,t+1}} + \frac{1 + \varsigma_t^c}{1 + \tau_t^K} \frac{R_{t+1}}{P_{t+1}} \right], \quad (1)$$

which replaces the corresponding Euler equation in Section 4.2 (on p. 747 of the published version).

With this we can prove the following:

Lemma 1 *Given the allocation $\{C_t, L_t, I_t, K_{t+1}\}$, fiscal devaluation policies (FD'') and (FD_R'') with $\tau_t^K \equiv \varsigma_t^I = 0$ result in the path of rental rate $R_t' = R_t/(1 + \delta_t)$, where $\{R_t\}$ is the path of rental rate under a nominal δ_t-devaluation.*

Proof: Consider first the full policy (FD'') which has $\varsigma_t^c \equiv \delta_t$, $\tau_t^v = \delta_t/(1 + \delta_t)$, and thus $(1 - \tau_t^v)(1 + \varsigma_t^c) \equiv 1$. Imposing additionally $\tau_t^K = \tau_t^I = 0$, we substitute this into (1) to obtain:

$$\frac{u_{C,t}}{\varphi_{I,t}} = \beta \mathbb{E}_t u_{C,t+1} \left[\left((1 - d) + \varphi_{K,t+1} \right) \frac{1}{\varphi_{I,t+1}} + (1 + \varsigma_{t+1}^c) \frac{R_{t+1}}{P_{t+1}} \right].$$

If R_{t+1}/P_{t+1} satisfies this equation under a nominal δ_t-devaluation, then under the (FD'') policy we must have,

$$(1 + \delta_{t+1}) \frac{R_{t+1}'}{P_{t+1}'} = \frac{R_{t+1}}{P_{t+1}},$$

if it constitutes a fiscal devaluation, i.e. if it keeps the real allocation unchanged. Note that fiscal devaluations that we consider also result in $P_{t+1}' = P_{t+1}$, and therefore we conclude that $(1 + \delta_t) R_{t+1}' = R_{t+1}$ must hold in this case.

Consider next the reduced (FD''_R) policy for a one-time unexpected devaluation at $t = 0$. In this case we have $\varsigma_t^c = \varsigma_t^I = \tau_t^K \equiv 0$ and $\tau_t^v = \delta_t/(1 + \delta_t)$, where $\delta_t \equiv \delta \mathbf{1}_{\{t \geq 0\}}$ with $\mathbb{E}_{t-k}\delta_t = 0$ for $k > 0$. We specialize (1) to this case:

$$(1 - \tau_t^v)\frac{u_{C,t}}{\varphi_{I,t}} = \beta\mathbb{E}_t u_{C,t+1}\left[((1 - d) + \varphi_{K,t+1})\frac{1 - \tau_{t+1}^v}{\varphi_{I,t+1}} + \frac{R_{t+1}}{P_{t+1}}\right].$$

Under the (FD''_R)-policy we can rewrite it as:

$$t < 0: \qquad \frac{u_{C,t}}{\varphi_{I,t}} = \beta\mathbb{E}_t u_{C,t+1}\left[((1 - d) + \varphi_{K,t+1})\frac{1}{\varphi_{I,t+1}} + \frac{R'_{t+1}}{P_{t+1}}\right],$$

$$t \geq 0: \qquad \frac{u_{C,t}}{\varphi_{I,t}} = \beta\mathbb{E}_t u_{C,t+1}\left[((1 - d) + \varphi_{K,t+1})\frac{1}{\varphi_{I,t+1}} + (1 + \delta)\frac{R'_{t+1}}{P_{t+1}}\right].$$

Again, if (FD''_R) constitutes a δ-fiscal devaluation, we must have $(1 + \delta_t)R'_{t+1} = R_{t+1}$.[4] ∎

Since the household allocation remains unchanged under our fiscal devaluation policies, it only remains to show that given the new path of $\{R'_{t+1}\}$ the firm allocation is also unchanged, and hence markets clear with these new path of rental rate of capital. Indeed, after-tax firm costs are $(1 - \varsigma_t^p)W_t N_t(i) + (1 - \varsigma_t^R)R_t K_t(i)$, and given its production function $Y_t(i) = A_t Z_t(i)N_t(i)^\alpha K_t(i)^{1-\alpha}$, the resulting cost function is:

$$TC_t^i = \frac{1}{A_t Z_t(i)}\left(\frac{(1 - \varsigma_t^p)W_t}{\alpha}\right)^\alpha\left(\frac{(1 - \varsigma_t^R)R_t}{1 - \alpha}\right)^{1-\alpha}Y_t(i).$$

Therefore, the path of firm costs with unchanged $\{R_t\}$ and $\varsigma_t^R = \delta_t/(1 + \delta_t)$, as in Proposition 7, can be replicated by a path of $\{R'_t\}$ with $R'_t = R_t/(1 + \delta_t)$ and $\varsigma_t^R \equiv 0$. Specifically, in both of these cases the firm allocation of capital and labor, and in particular

$$\frac{N_t(i)}{K_t(i)} = \frac{\alpha}{1 - \alpha}\frac{1 - \varsigma_t^R}{1 - \varsigma_t^p}\frac{R_t}{W_t},$$

are the same as under a nominal devaluation. This completes the argument for why the same allocation that can be implemented using policies in Proposition 7 can be implemented by simpler fiscal devaluations policies (FD'') and (FD''_R) of Proposition 1–3.[5]

We conclude by noting that the (incremental) government revenues in a model with capital under the simple fiscal devaluation policies (FD'') and (FD''_R) are *larger* than in the

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[4]Note that the equilibrium value of R_t at $t = 0$ is not pinned down by this conditions, but since the capital stock at $t = 0$ is predetermined at $t = -1$, R_0 is pinned down by the firm optimality (i.e., demand for capital) which we consider next.

[5]As the final step of the proof note that firm profits in foreign currency are given by:

$$\frac{\Pi_t^i}{\mathcal{E}_t} = \frac{1 - \tau_t^v}{\mathcal{E}_t}P_{Ht}(i)C_{Ht}(i) + P_{Ht}^*(i)C_{Ht}^*(i) - \frac{1 - \varsigma_t^p}{\mathcal{E}_t}W_t N_t(i) - \frac{(1 - \varsigma_t^R)R_t}{\mathcal{E}_t}K_t(i),$$

and hence are the same under both a nominal and a simple fiscal devaluation (with $\varsigma_t^R \equiv 0$) that we consider here, as $R_t'/\mathcal{E}_t' = R_t/\mathcal{E}_t = R_t/(1 + \delta_t) = R'_t$ and $\mathcal{E}'_t \equiv 1$.

model without capital, as long as part or all of capital is held by the private sector, thereby reinforcing our conclusions in Proposition 4. Indeed, VAT τ_t^v is collected from aggregate final consumption, while payroll subsidy ς_t^p is payed only to the labor input, which in the model with capital constitutes a smaller share of aggregate income part of which is payed to capital.[6]

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[6]Formally, one can show, in parallel with (31) in the published version, that in the case with capital the incremental government revenues equal:

$$TR_t = -\frac{\delta_t}{1 + \delta_t} NX_t + \delta_t \left[\Pi_t + R_t K_t \right].$$

(note that under the VAT-based policy dividend (profit) income tax does not need to be adjusted, $\tau_t^d \equiv 0$).