# Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem 

David Baqaee Emmanuel Farhi

UCLA Harvard

## Macroeconomic Impact of Shocks

- For economy with efficient equilibrium, Hulten (1978):

$$
\mathrm{d} \log Y / \mathrm{d} \log A_{i}=\text { sales }_{i} / G D P=\lambda_{i}
$$

- First-order approximation (exact for Cobb-Douglas economies).
- Foundation for Domar aggregation:
- Sales approximate sufficient statistics.
- Details of production structure are irrelevant.
- "Bugbear" for production networks literature. (e.g. shocks to Walmart and electricity equally important)


## What We Do

- Extend Hulten to second order to capture nonlinearities.
- General formula: reduced-form GE-elasticities of substitution.
- Mapping from micro to macro using a general structural model:
- structural elasticites of substitution.
- returns to scale.
- factor market reallocation.
- network linkages.
- Nonlinearities lead to asymmetric responses of output to shocks.
- amplification of negative shocks, attenuation of positive shocks.
- lower mean, negative skewness, excess kurtosis.
- Nonlinearities matter quantitatively:
- $\times 10$ welfare costs of shocks from $0.05 \%$ to $0.6 \%$ of GDP.
- $\times 3$ impact of 70 's oil price shocks from $-0.2 \%$ to $-0.6 \%$ of GDP.
- -20 percentage point reduction in aggregate TFP between 1948-2014.


## What We Can Also Do

- Paper focuses on aggregate output, not co-movement, but can be characterized with same GE-elasticities.
- Paper maintains representative agent assumption.
- Paper abstracts away from RBC channels (elastic labor supply, capital accumulation), dynamics (reallocation).


## Broader Agenda

- Nonlinearities in efficient economies. "Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem"
- Inefficient economies.
"Productivity and Misallocation in General Equilibrium"
- Open economies/Heterogeneous Agents.
"Networks, Barriers, and Trade"
- Micro-foundations of aggregate production functions and the Cambridge-Cambridge Capital controversy.
"The Microeconomic Foundations of Aggregate Production Functions"
- Increasing Returns and Entry.
"Cascading Failures in Production Networks"
"Darwinian Returns to Scale"
"Entry versus Rents"


## Related Literature

- Long and Plosser (1983), Horvath (2000), Gomme and Rupert (2007).
- Jovanovic (1987), Durlauf (1993), Scheinkman and Woodford (1994), Horvath (1998), Dupor (1999).
- Gabaix (2011), Carvalho and Gabaix (2013), Acemoglu et al. (2012), Carvalho (2010), Acemoglu et al. (2017), Foerster et al. (2011), Atalay (2016), Bigio and La'O (2016), Baqaee (2016), Di Giovanni et al. (2014).
- Kremer (1993), Jones (2011), Jones (2013).
- Houthakker (1955), Oberfield and Raval (2014), Beraja et al. (2016).


## Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth

Conclusion

Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

## CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth
Conclusion

## General Framework

- Perfectly competitive economy, representative consumer.
- Preferences represented by homothetic preferences

$$
Y=\mathscr{D}\left(c_{1}, \ldots, c_{N}\right)
$$

where $c_{i}$ is consumption of good $i$.

- Consumer budget constraint

$$
\sum_{i} p_{i} c_{i}=\sum_{i=1}^{M} w_{i} l_{i}+\sum_{i=1}^{N} \pi_{i}
$$

where $p_{i}, w_{i}$, and $\pi_{i}$ are prices, wages, and profits.

## General Framework

- Profits earned by the producer of good $i$ :

$$
\pi_{i}=p_{i} y_{i}-\sum_{k=1}^{M} w_{k} l_{i k}-\sum_{j=1}^{N} p_{j} x_{i j}
$$

- Each good $i$ is produced using production function:

$$
y_{i}=A_{i} F_{i}\left(l_{i 1}, \ldots, l_{i M}, x_{i 1}, \ldots, x_{i N}\right)
$$

- $A_{i}$ Hicks-neutral technology (Harrod-neutral as special case).
- $x_{i j}$ intermediate inputs of good $j$ used in the production of good $i$.
- $l_{i k}$ labor of type $k$ used by $i$.


## Hulten's Theorem

Define $Y\left(A_{1}, \ldots, A_{N}\right)$ to be competitive equilibrium aggregate consumption function interpreted as output.

Theorem (Hulten)
Let $\lambda_{i}$ denote industry i's sales as a share of output, then

$$
\frac{\mathrm{d} \log Y}{\mathrm{~d} \log A_{i}}=\lambda_{i} .
$$

## GE Elasticity of Substitution

- For CRS function $f\left(A_{1}, \ldots, A_{N}\right)$ the Morishima elasticity of substitution:

$$
\frac{1}{\rho_{i j}}=-\frac{\mathrm{d} \log \left(M R S_{i j}\right)}{\mathrm{d} \log \left(A_{i} / A_{j}\right)}=-\frac{\mathrm{d} \log \left(f_{i} / f_{j}\right)}{\mathrm{d} \log \left(A_{i} / A_{j}\right)}
$$

- For output function $Y\left(A_{1}, \ldots, A_{N}\right)$, define GE-elasticity of substitution:

$$
\frac{1}{\rho_{i j}} \equiv-\frac{\mathrm{d} \log \left(M R S_{i j}\right)}{\mathrm{d} \log \left(A_{i}\right)}=-\frac{\mathrm{d} \log \left(Y_{i} / Y_{j}\right)}{\mathrm{d} \log \left(A_{i}\right)}
$$

- Hence

$$
\frac{\mathrm{d} \log \left(\lambda_{i} / \lambda_{j}\right)}{\mathrm{d} \log A_{i}}=1-\frac{1}{\rho_{i j}}
$$

## Input-Output Multiplier

## Definition 1.1

Define input-output mutliplier

$$
\sum_{i=1}^{N} \frac{\mathrm{~d} \log Y}{\mathrm{~d} \log A_{i}}=\sum_{i=1}^{N} \lambda_{i}=\xi
$$

- "Macro returns to scale": $\xi>1$ implies reproducibility.
- $\xi$ constant if and only if $C$ homogenous of degree $\xi$.


## Extending Hulten: Idiosyncratic Shocks

## Theorem

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d}\left(\log A_{i}\right)^{2}}=\frac{\lambda_{i}}{\xi} \sum_{j \neq i} \lambda_{j}\left(1-\frac{1}{\rho_{i j}}\right)+\lambda_{i} \frac{\partial \log \xi}{\partial \log A_{i}}
$$

- General formula for second-order terms (nonlinearities) in terms of reduced-form GE-elasticities of substitution.
- Sales distribution not sufficient statistic.
- $\rho_{i j}=1, \xi$ constant, Cobb-Douglas, zero effect (knife-edge).


## Macro Moments

## Proposition

Suppose that $\log A_{i}$ are subject to idiosyncratic shocks with variance $s_{i}^{2}$. Then we have the following formula for the mean of output:

$$
E(\log (Y / \bar{Y})) \approx \frac{1}{\xi} \sum_{i} \frac{s_{i}^{2}}{2 \xi} \lambda_{i} \sum_{j \neq i} \lambda_{j}\left(1-\frac{1}{\rho_{i j}}\right)+\sum_{i} \frac{s_{i}^{2}}{2} \lambda_{i} \frac{\mathrm{~d} \log \xi}{\mathrm{~d} \log A_{i}}
$$

- See paper for:
- more general mean formula for correlated shocks.
- beyond mean, formulas for skewness and excess kurtosis.


## Welfare Costs of Shocks

## Proposition

Let $u: \mathbb{R} \rightarrow \mathbb{R}$ be a CRRA with parameter $\gamma$. Suppose TFP $A$ has idiosyncratic shocks with variance $s_{k}^{2}$. Then the welfare costs of shocks are given by:

$$
\frac{[E(u(Y))-u(\bar{Y})]}{u^{\prime}(\bar{Y}) \bar{Y}} \approx \underbrace{-\frac{1}{2} \gamma \sum_{k}^{N} \lambda_{k}^{2} s_{k}^{2}}_{\text {Consumption nonlinearities }}+\underbrace{\frac{1}{2} \bar{Y} \sum_{k}^{N} \frac{\partial^{2} Y}{\partial A_{k}^{2}} s_{k}^{2}}_{\text {Production nonlinearities }}
$$

where recall $\bar{Y}=Y(\bar{A})$.

- Nonlinearities in consumption: small cost in Lucas (1987).
- Nonlinearities in production: can be order of magnitude larger.


## Agenda

## Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

## CES Networks

## Quantitative Examples

Business Cycles
Oil Shocks
Long-run Growth

## Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth

Conclusion

## GE Elasticities of Substitution

- $N$ goods produced using the production functions

$$
\frac{y_{i}}{\bar{y}_{i}}=A_{i}\left(\frac{I_{i s_{i}}}{\bar{I}_{i s_{i}}}\right)^{1-\omega_{g}}\left(\frac{I_{i g}}{\overline{I_{i g}}}\right)^{\omega_{g}}
$$

specific labor $l_{\text {is }}$ and general labor $l_{i g}$.

- Output

$$
\frac{Y}{\bar{Y}}=\left(\sum_{i=1}^{N} \omega_{0 i}\left(\frac{c_{i}}{\bar{c}_{i}}\right)^{\frac{\theta_{0}-1}{\theta_{0}}}\right)^{\frac{\theta_{0}}{\theta_{0}-1}}
$$

- Budget constraint:

$$
\sum_{k} p_{k} c_{k}=\sum_{k} w L_{k}+\sum_{k} w_{k} l_{k}+\sum_{k} \pi_{k} .
$$

## GE Elasticities of Substitution

- Market-clearing conditions are

$$
c_{i}=y_{i}, \quad \bar{l}_{s_{i}}=l_{i s_{i}}, \quad \text { and } \quad \bar{l}_{g}=\sum_{i=1}^{N} l_{i g} .
$$

- GE-elasticity of substitution is:

$$
\rho_{j i}=\rho=\frac{\theta_{0}\left(1-\omega_{g}\right)+\omega_{g}}{\theta_{0}\left(1-\omega_{g}\right)+\omega_{g}+\left(1-\theta_{0}\right)}
$$

- Hence,

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{i}^{2}}=\frac{\mathrm{d} \lambda_{i}}{\mathrm{~d} \log A_{i}}=\lambda_{i}\left(1-\lambda_{i}\right)\left(1-\frac{1}{\rho}\right) .
$$

- To build intuition, consider polar cases with $\omega_{g}=1$ and $\omega_{g}=0$.


## Lesson \#1: Micro-Elasticity of Substitution Matters



## Lesson \#2: Reallocation Matters



## Varying Reallocation Parameter



- All these economies are equivalent to a first order.

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{i}^{2}}=b_{i}\left(1-b_{i}\right)\left(1-\frac{1}{\rho}\right) .
$$

## Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth

Conclusion

## The role of $\xi$

- So far, $\xi=1$, constant macro returns to scale.
- For most applications, $\xi>1$ : intermediate goods, capital, trade.
- In many applications, $\xi$ restrictted to be constant: Gomme and Rupert (2007), Aghion and Howitt (2008), Jones (2011), Gabaix (2011), Acemoglu et al. (2012), Kim et al. (2013), Bartelme and Gorodnichenko (2015).


## Variable $\xi$

- Assume

$$
\frac{y_{1}}{\bar{y}_{1}}=A_{1}\left(\omega_{1 /}\left(\frac{l_{1}}{\bar{l}_{1}}\right)^{\frac{\theta_{1}-1}{\theta_{1}}}+\left(1-\omega_{1 /}\right)\left(\frac{x_{1}}{\bar{x}_{1}}\right)^{\frac{\theta_{1}-1}{\theta_{1}}}\right)^{\frac{\theta_{1}}{\theta_{1}-1}}
$$

- Market-clearing

$$
y_{1}=c_{1}+x_{1} \quad \text { and } \quad \bar{l}=l_{1} .
$$

- The steady-state input-output multiplier

$$
\xi=1+\left(1-\omega_{1 /}\right)+\left(1-\omega_{1 /}\right)^{2}+\ldots=1 / \omega_{1 /}
$$

decreases with the labor share $\omega_{1 /}$ and increases with the intermediate input share $1-\omega_{1 /}$.

## Variable $\xi$

- Hulten's theorem implies that

$$
\frac{d \log Y}{d \log A_{1}}=\xi
$$

Proposition

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A^{2}}=\left(\frac{1}{\bar{a}}-1\right)(\theta-1)=(\xi-1)(\theta-1)
$$

## Variable input-output multiplier



For this calibration, $\bar{a}=0.1$.

Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth
Conclusion

## Networks

- General nested CES economy.
- "Relabel" each CES nest to be a new sector with elasticity $\theta_{i}$.
- Input-output matrix

$$
\Omega_{i j}=\frac{p_{j} x_{i j}}{p_{i} y_{i}}
$$

- Leontief inverse

$$
\Psi=(I-\Omega)^{-1}=\sum_{n=0}^{\infty} \Omega^{n}
$$

- $\Omega_{i j}$ and $\Psi_{i j}$ direct and total reliance of $i$ on $j$.
- Domar weights are $\lambda=b^{\prime} \Psi$.


## Networks

- To understand these models, two sets of equations are key: Forward and Backward equations.
- Let $\alpha$ denote the factor shares. Then forward equations:

$$
\mathrm{d} \log p_{i}=-\mathrm{d} \log A_{i}+\sum_{j} \Omega_{i j} \mathrm{~d} \log p_{j}+\sum_{f} \alpha_{i f} \mathrm{~d} \log \Lambda_{f},
$$

or

$$
\mathrm{d} \log P=\Psi(\alpha \mathrm{d} \log \Lambda-\mathrm{d} \log A)
$$

This implies Hulten's theorem

$$
\mathrm{d} \log Y=-b^{\prime} \mathrm{d} \log P=\lambda^{\prime} \mathrm{d} \log A+\Lambda^{\prime} \mathrm{d} \log \Lambda
$$

## Networks - Forward Equations

- Next, we need to understand the backward equations:

$$
\mathrm{d} \log \lambda=f(\mathrm{~d} \log P)
$$

To characterize the backward equations, we need input-output covariance operator.
$\operatorname{Cov}_{\Omega^{(j)}}\left(\Psi_{(k)}, \mathrm{d} \log P\right)=\sum_{i} \Omega_{j i} \Psi_{i k} \mathrm{~d} \log p_{i}-\left(\sum_{i} \Omega_{j i} \Psi_{i k}\right)\left(\sum_{i} \Omega_{j i} \mathrm{~d} \log p_{i}\right)$.

## Input-Output Covariance

Input-output variance operator:
$\operatorname{Cov}_{\Omega^{(j)}}\left(\Psi_{(k)}, \mathrm{d} \log P\right)=\sum_{i} \Omega_{j i} \Psi_{i k} \mathrm{~d} \log p_{i}-\left(\sum_{i} \Omega_{j i} \Psi_{i k}\right)\left(\sum_{i} \Omega_{j i} \mathrm{~d} \log p_{i}\right)$.


## Backward Equations

- The backward equations are given by

$$
\mathrm{d} \lambda_{i}=\sum_{k=0}^{N}\left(1-\theta_{k}\right) \lambda_{k} \operatorname{Cov}_{\Omega^{(k)}}\left(\Psi_{(i)}, \mathrm{d} \log P\right),
$$

- Now we can plug in the forward equations and we are done.
- In the one factor world, this is easy

$$
\mathrm{d} \log P=-\Psi \mathrm{d} \log A
$$

## One Factor, Full Reallocation

Proposition

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{j} \mathrm{~d} \log A_{i}}=\frac{\mathrm{d} \lambda_{i}}{\mathrm{~d} \log A_{j}}=\sum_{k=0}^{N}\left(\theta_{k}-1\right) \lambda_{k} \operatorname{Cov}_{\Omega^{(k)}}\left(\Psi_{(i)}, \Psi_{(j)}\right)
$$

and in particular

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{i}^{2}}=\frac{\mathrm{d} \lambda_{i}}{\mathrm{~d} \log A_{i}}=\sum_{k=0}^{N}\left(\theta_{k}-1\right) \lambda_{k} \operatorname{Var}_{\Omega^{(k)}}\left(\Psi_{(i)}\right)
$$

- Centrality measure mixing network and elasticities.
- Can also compute macro elasticities of substitution (see paper).


## Network Irrelevance Result

## Proposition

Suppose a single factor, $\theta_{j}=\theta$ for every $j$, and factor-augmenting shocks. Then

$$
\frac{Y}{\bar{Y}}=\left(\sum_{i=0}^{N} \bar{\lambda}_{i} A_{i}^{\theta-1}\right)^{\frac{1}{\theta-1}}
$$

where $\bar{\lambda}_{i}$ is the steady-state Domar weight of $i$. Then

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{j} \mathrm{~d} \log A_{i}}=\frac{\mathrm{d} \lambda_{i}}{\mathrm{~d} \log A_{j}}=(\theta-1) \lambda_{i}\left(1(i=j)-\lambda_{j}\right)
$$

and in particular

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{i}^{2}}=\frac{\mathrm{d} \lambda_{i}}{\mathrm{~d} \log A_{i}}=\sum_{j=0}^{N}\left(\theta_{j}-1\right) \lambda_{j} \operatorname{Var}_{\Omega^{(j)}}\left(\Psi_{(i)}\right)=(\theta-1) \lambda_{i}\left(1-\lambda_{i}\right) .
$$

- Extends Hulten network irrelevance to second-order.


## "Universal" Input Example

One factor, full reallocation, two elasticities $\theta_{1} \ll \theta_{0}$.


$$
\begin{aligned}
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{E}^{2}} & =\left(\theta_{0}-1\right) \lambda_{E}\left(\frac{N}{M}-1\right) \lambda_{E}+\left(\theta_{1}-1\right) \lambda_{E}\left(1-\frac{N}{M} \lambda_{E}\right), \\
& =\left(\theta_{0}-1\right) \lambda_{E}\left(1-\lambda_{E}\right)-\left(\theta_{0}-\theta_{1}\right) \lambda_{E}\left(1-\frac{N}{M} \lambda_{E}\right)
\end{aligned}
$$

## Direction of Diffusion

## Proposition

Assume that there is one factor and full reallocation. If industries $k$ and I sell the same share to all other industries and the household, then

$$
\frac{\mathrm{d} \log Y}{\mathrm{~d} \log A_{k}}=\frac{\mathrm{d} \log Y}{\mathrm{~d} \log A_{l}}
$$

and

$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{k}^{2}}=\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{I}^{2}}
$$

- Key: downstream diffusion under CRS.
- Limited Re-allocation, multiple factors or DRS breaks it.


## Multiple Factors, Limited Reallocation

## Proposition

$$
\begin{aligned}
& \frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{k}^{2}}=\sum_{j}\left(\theta_{j}-1\right) \lambda_{j} \operatorname{Var}_{\Omega^{(j)}}\left(\Psi_{(k)}\right) \\
& +\sum_{j}\left(\theta_{j}-1\right) \lambda_{j} \operatorname{Cov}_{\Omega^{(j)}}\left(\sum_{f} \Psi_{(f)} \frac{\mathrm{d} \log \Lambda_{f}}{\mathrm{~d} \log A_{k}}, \Psi_{(k)}\right)
\end{aligned}
$$

- New terms arising from changes in factor shares (prices) given by

$$
\begin{gathered}
\frac{\mathrm{d} \log \Lambda}{\mathrm{~d} \log A_{k}}=\Gamma \frac{\mathrm{d} \log \Lambda}{\mathrm{~d} \log A_{k}}+\delta_{(k)}, \\
\Gamma_{f, g}=\sum_{j}\left(\theta_{j}-1\right) \lambda_{j} \operatorname{Cov}_{\Omega^{(j)}}\left(\Psi_{(f)}, \Psi_{(g)}\right), \\
\delta_{f k}=\sum_{j}\left(\theta_{j}-1\right) \lambda_{j} \operatorname{Cov}_{\Omega^{(j)}}\left(\Psi_{(f)}, \Psi_{(k)}\right) .
\end{gathered}
$$

- Can compute macro factor elasticities of substitution (see paper).


## "Universal" Energy Example

- Two factors: electricity and labor.
- Sectors use energy and labor with elasticity $\theta_{1}<1$.
- Final demand uses downstreams sectors with elasticity $\theta_{0} \gg \theta_{1}$.


$$
\frac{\mathrm{d}^{2} \log Y}{\mathrm{~d} \log A_{E}^{2}}=\frac{\mathrm{d} \Lambda_{E}}{\mathrm{~d} \log A_{E}^{2}}=\frac{\left(\theta_{0}-1\right) \Lambda_{E}\left(1-\Lambda_{E}\right)-\left(\theta_{0}-\theta_{1}\right) \Lambda_{E}\left(1-\frac{N}{M} \Lambda_{E}\right)}{\theta_{0}-\left(\theta_{0}-\theta_{1}\right) \frac{\left(1-\frac{N}{M} \Lambda_{E}\right)}{1-\Lambda_{E}}} .
$$

## Beyond CES

- Define the substitution operator for $j$ as

$$
\begin{aligned}
\Phi_{j}\left(\Psi_{(k)}, \Psi_{(\jmath)}\right) & =\left(\sum_{\substack{x, y \\
x \neq y}} \Omega_{j x} \Omega_{j y}\left(1-\sigma^{j}(x, y)\right) \Psi_{x \mid} \Psi_{y k}\right) \\
& =\frac{1}{2} E_{\Omega^{(j)}}\left(\left(1-\sigma^{j}(x, y)\right)\left(\Psi_{k}(x)-\Psi_{k}(y)\right)\left(\Psi_{l}(x)-\Psi_{l}(y)\right)\right)
\end{aligned}
$$

where $\Psi_{k}(x)=\Psi_{x k}$.

- $\Phi_{j}$ similar to covariance:
- symmetric;
- bilinear;
- $\Phi_{j}=0$ if an argument is constant.


## Beyond CES

## Proposition

For a general economy,

$$
\frac{\mathrm{d} \lambda_{i}}{\mathrm{~d} \log A_{k}}=-\sum_{j=0} \Phi_{j}\left(\Psi_{(k)}, \Psi_{(i)}\right)+\sum_{f} \sum_{j} \Phi_{j}\left(\Psi_{(i)}, \Psi_{(f)}\right) \frac{\mathrm{d} \log \Lambda_{l}}{\mathrm{~d} \log A_{k}} .
$$

where

$$
\frac{\mathrm{d} \Lambda_{f}}{\mathrm{~d} \log A_{k}}=-\sum_{j=0} \Phi_{j}\left(\Psi_{(k)}, \Psi_{(f)}\right)+\sum_{l} \sum_{j} \Phi_{j}\left(\Psi_{(I)}, \Psi_{(f)}\right) \frac{\mathrm{d} \log \Lambda_{l}}{\mathrm{~d} \log A_{k}} .
$$

## Agenda

## Framework

## Illustrative Examples

Macro-Substitution
Input-output Mutlipliers

## CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth

## Conclusion

## Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth

Conclusion

## Simulation

- Final demand

$$
\frac{Y}{\bar{Y}}=\left(\sum_{i=1}^{N} \omega_{0 i}\left(\frac{c_{i}}{\bar{c}_{i}}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
$$

- The production function of industry $i$ is

$$
\frac{y_{i}}{\bar{y}_{i}}=A_{i}\left(\omega_{i l}\left(\frac{I_{i}}{\bar{I}_{i}}\right)^{\frac{\theta-1}{\theta}}+\left(1-\omega_{i l}\right)\left(\frac{\hat{X}_{i}}{\bar{X}_{i}}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}
$$

labor inputs $I_{i}$ and intermediate inputs $\hat{X}_{i}$.

- The composite intermediate input $X_{i}$ is given by

$$
\frac{x_{i}}{\bar{X}_{i}}=\left(\sum_{j=1}^{N} \omega_{i j}\left(\frac{x_{i j}}{\bar{x}_{i j}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}
$$

intermediate inputs $x_{i j}$ from industry $j$ used by industry $i$.

## Simulation

- Set $\theta_{j}=\theta=0.5, \varepsilon_{i}=\varepsilon=0.001$, and $\sigma=0.9$ drawing on Atalay (2016), Boehm et al. (2015), Barrot and Sauvagnat (2016), Comin et al. (2015).
- Impose no-movement in labor for benchmark (Acemoglu et al. (2016), Autor et al. (2016), Notowidigdo (2011)).
- Use the 88 -sector US KLEMS annual input-output data from 1960-2005, with sector-level TFP data constructed using Jorgenson et al. (1987) methodology by Carvalho and Gabaix (2013).
- Sectoral TFP (annual or quadrennial) shocks to be $\log \mathscr{N}\left(-\Sigma_{i i} / 2, \Sigma_{i i}\right)$, where $\Sigma_{i i}$ is sample variance of $\Delta \log T F P$ for industry $i$.
- Check that $\sigma_{\lambda}=\sum_{i} \bar{\lambda}_{i} \sigma_{\lambda_{i}}$ matches data.


## Simulation Results

| $(\sigma, \theta, \varepsilon)$ | Mean $\quad$ Std | Skew | Ex-Kurtosis | $\sigma_{\lambda}$ |
| :--- | :--- | :--- | :--- | :--- |

Full Reallocation - Annual

| $(0.7,0.3,0.001)$ | -0.0023 | 0.011 | -0.10 | 0.1 | 0.090 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $(0.9,0.5,0.001)$ | -0.0022 | 0.011 | -0.08 | 0.0 | 0.069 |
| $(0.9,0.6,0.2)$ | -0.0020 | 0.011 | -0.05 | 0.0 | 0.056 |
| $(0.99,0.99,0.99)$ | -0.0013 | 0.011 | 0.01 | 0.0 | 0.001 |

No Reallocation - Annual

| $(0.7,0.3,0.001)$ | -0.0045 | 0.012 | -0.31 | 0.4 | 0.171 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $(0.9,0.5,0.001)$ | -0.0034 | 0.012 | -0.18 | 0.1 | 0.115 |
| $(0.9,0.6,0.2)$ | -0.0024 | 0.011 | -0.11 | 0.1 | 0.068 |
| $(0.99,0.99,0.99)$ | -0.0011 | 0.011 | 0.00 | 0.0 | 0.001 |

$\begin{array}{lllll}\text { Annual Data } & -0.015 & - & - & 0.13\end{array}$

## Simulation Results

| $(\sigma, \theta, \varepsilon)$ | Mean | Std | Skew | Ex-Kurtosis |
| :--- | :--- | :--- | :--- | :--- |

Full Reallocation - Quadrennial

| $(0.7,0.3, .0 .001)$ | -0.0118 | 0.026 | -0.4 | 0.4 | 0.307 |
| :--- | ---: | :--- | ---: | ---: | ---: |
| $(0.9,0.5,0.001)$ | -0.0113 | 0.026 | -0.28 | 0.4 | 0.176 |
| $(0.9,0.6,0.2)$ | -0.0100 | 0.026 | -0.23 | 0.2 | 0.133 |
| $(0.99,0.99,0.99)$ | -0.0058 | 0.025 | 0.01 | 0.0 | 0.003 |

No Reallocation - Quadrennial

| $(0.7,0.3,0.001)$ | -0.0270 | 0.037 | -2.18 | 12.7 | 0.404 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $(0.9,0.5,0.001)$ | -0.0187 | 0.030 | -1.11 | 3.6 | 0.267 |
| $(0.9,0.6,0.2)$ | -0.0129 | 0.027 | -0.44 | 0.7 | 0.154 |
| $(0.99,0.99,0.99)$ | -0.0057 | 0.025 | 0.00 | 0.0 | 0.002 |

Quadrennial Data - 0.030 - 0.27

## Histograms




Figure: The left panel shows the distribution of GDP for the annual model. The right panel shows these for shocks for quadrennial shocks.

## Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples

## Business Cycles

Oil Shocks
Long-run Growth

Conclusion

## Oil v. Retail



- Intuition: low micro-elasticity of substitution, universal input.
- Consistent with large asymmetric effects of oil shocks (Hamilton, 2003), even without frictions.


## Reduced-form Impact of Oil Shocks

## Proposition

Up to the second order in the vector $\Delta$, we have

$$
\log (Y(A+\Delta) / Y(A))=\frac{1}{2}[\lambda(A+\Delta)+\lambda(A)]^{\prime} \log (\Delta)+O\left(\log (\Delta)^{3}\right)
$$

## Reduced-form Impact of Oil Shocks



Figure: Global expenditures on crude oil as a fraction of world GDP.

- First-order effect: $1.8 \% \times-13 \% \approx-0.2 \%$.
- Second-order effect: $\frac{1}{2}(1.8 \%+7.6 \%) \times-13 \% \approx-0.6 \%$.


## Agenda

Framework

Illustrative Examples
Macro-Substitution
Input-output Mutlipliers

CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth

Conclusion

## Nonlinearities and Cost Disease

- "Nonlinear" measure of aggregate TFP growth

$$
\Delta \log T F P^{\text {nonlinear }}=\sum_{i=1}^{N} \sum_{t=1948}^{2013} \lambda_{i, t}\left(\log A_{i, t+1}-\log \left(A_{i, t}\right)\right)
$$

- Approximation, by discrete left Riemann sums, of the exact aggregate TFP growth, given by

$$
\sum_{i=1}^{N} \int_{1948}^{2014} \lambda_{i, t} d \log A_{i, t}
$$

- If economy was log-linear, TFP growth is

$$
\Delta \log T F P^{1 \text { st order }}=\sum_{i=1}^{N} \lambda_{i, 1948}\left(\log A_{i, 2014}-\log A_{i, 1948}\right)
$$

## Baumol's Cost Disease



- Baumol's cost disease: slow growth sectors get big.
- Structural change: non-homothetic preferences.

Agenda

## Framework

## Illustrative Examples

Macro-Substitution
Input-output Mutlipliers

## CES Networks

Quantitative Examples
Business Cycles
Oil Shocks
Long-run Growth
Conclusion

Acemoglu, D., D. Autor, D. Dorn, G. H. Hanson, and B. Price (2016). Import competition and the great US employment sag of the 2000s. Journal of Labor Economics 34(S1), S141-S198.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The network origins of aggregate fluctuations. Econometrica 80(5), 1977-2016.

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2017). Microeconomic origins of macroeconomic tail risks. The American Economic Review 107(1), 54-108.

Aghion, P. and P. W. Howitt (2008). The economics of growth. MIT press.

Atalay, E. (2016). How important are sectoral shocks? Technical report.

Autor, D. H., D. Dorn, and G. H. Hanson (2016). The china shock: Learning from labor-market adjustment to large changes in trade. Annual Review of Economics 8, 205-240.

Baqaee, D. R. (2016). Cascading failures in production networks.
Barrot, J.-N. and J. Sauvagnat (2016). Input specificity and the
propagation of idiosyncratic shocks in production networks. The Quarterly Journal of Economics.
Bartelme, D. and Y. Gorodnichenko (2015). Linkages and economic development. Technical report, National Bureau of Economic Research.

Beraja, M., E. Hurst, and J. Ospina (2016). The aggregate implications of regional business cycles. Technical report, National Bureau of Economic Research.

Bigio, S. and J. La'O (2016). Financial frictions in production networks. Technical report.

Boehm, C., A. Flaaen, and N. Pandalai-Nayar (2015). Input linkages and the transmission of shocks: Firm-level evidence from the 2011 tōhoku earthquake.

Carvalho, V. and X. Gabaix (2013). The great diversification and its undoing. The American Economic Review 103(5), 1697-1727.

Carvalho, V. M. (2010). Aggregate fluctuations and the network structure of intersectoral trade.

Comin, D. A., D. Lashkari, and M. Mestieri (2015). Structural change
with long-run income and price effects. Technical report, National Bureau of Economic Research.
Di Giovanni, J., A. A. Levchenko, and I. Méjean (2014). Firms, destinations, and aggregate fluctuations. Econometrica 82(4), 1303-1340.
Dupor, B. (1999). Aggregation and irrelevance in multi-sector models. Journal of Monetary Economics 43(2), 391-409.
Durlauf, S. N. (1993). Nonergodic economic growth. The Review of Economic Studies 60(2), 349-366.
Foerster, A. T., P.-D. G. Sarte, and M. W. Watson (2011). Sectoral versus aggregate shocks: A structural factor analysis of industrial production. Journal of Political Economy 119(1), 1-38.
Gabaix, X. (2011). The granular origins of aggregate fluctuations. Econometrica 79(3), 733-772.
Gomme, P. and P. Rupert (2007). Theory, measurement and calibration of macroeconomic models. Journal of Monetary Economics 54(2), 460-497.
Hamilton, J. D. (2003). What is an oil shock? Journal of econometrics 113(2), 363-398.

Horvath, M. (1998). Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. Review of Economic Dynamics 1(4), 781-808.

Horvath, M. (2000). Sectoral shocks and aggregate fluctuations.
Journal of Monetary Economics 45(1), 69-106.
Houthakker, H. S. (1955). The pareto distribution and the
Cobb-Douglas production function in activity analysis. The Review of Economic Studies 23(1), 27-31.

Jones, C. I. (2011). Intermediate goods and weak links in the theory of economic development. American Economic Journal: Macroeconomics, 1-28.

Jones, C. I. (2013). Input-Output economics. In Advances in
Economics and Econometrics: Tenth World Congress, Volume 2, pp. 419. Cambridge University Press.

Jorgenson, D. W., F. Gollop, and B. M Fraumeni (1987). Productivity and US economic growth.

Jovanovic, B. (1987). Micro shocks and aggregate risk. The Quarterly Journal of Economics, 395-409.

Kim, S.-J., H. S. Shin, et al. (2013). Working capital, trade and macro fluctuations. Technical report.
Kremer, M. (1993). The O-ring theory of economic development. The Quarterly Journal of Economics, 551-575.
Long, J. B. and C. I. Plosser (1983). Real business cycles. The Journal of Political Economy, 39-69.

Lucas, R. E. (1987). Models of business cycles, Volume 26. Basil Blackwell Oxford.

Notowidigdo, M. J. (2011). The incidence of local labor demand shocks. Technical report, National Bureau of Economic Research.

Oberfield, E. and D. Raval (2014). Micro data and macro technology. Technical report, National Bureau of Economic Research.
Scheinkman, J. A. and M. Woodford (1994). Self-organized criticality and economic fluctuations. The American Economic Review, 417-421.

