Leverage and the Central Banker's Put

By EMMANUEL FARHI AND JEAN TIROLE*

Among the many unusual aspects of the ongoing financial crisis is the unprecedented provision of backstop liquidity by central banks around the world. The Federal Reserve alone had committed $4,400 billion by mid-November 2008. The Fed funds rate is almost equal to zero. These are extraordinary numbers.

This paper establishes a formal relationship between the recent monetary developments and the trends in private leverage and its structure. Over the last few years, some traditional institutions, for example broker-dealers, have relied more and more on markets (securitization, money market) for their funding. Some banks have also increased their dependence on markets; the standard illustration is Northern Rock, a UK mortgage bank, which prior to its bailout relied on short-term wholesale markets for 75 percent of its funding.

A second factor contributing to the reliance on wholesale markets is the overall shift from a bank-based system to a market-based one. The expanding so-called "shadow banking system" (conduits, hedge funds, investment banks, monolines) has engaged in substantial transformation and, unlike commercial banks, could not avail itself of stable insured deposits. Mutual funds are under the threat of redemptions and may well face liquidity shortages as well.

Adding subprime borrowers, who are heavily dependent on high housing prices and, for those with adjustable-rate mortgages (ARMs), on low short-term interest rates, and highly leveraged corporations, the overall picture is one of a fragile economic environment that has become overly sensitive to interest rate risk.

The paper's key insight is that private leverage choices exhibit strategic complementarities through the policy reaction. Monetary policy, defined here as the public sector exerting a downward pressure on interest rates, is a prototypical nontargeted public policy. It rescues those who depend on low interest rates, but its other benefits and costs apply to the entire economy. As a consequence, the more economic actors exhibit a substantial interest-rate vulnerability, the more the state has to engage in active monetary policy. The lack of targeting implies that one is more likely to be rescued by monetary policy, the higher the overall economy's sensitivity to interest rate conditions.

This central insight has four immediate corollaries. First, private interest-rate exposure is highly sensitive to macroeconomic conditions. Second, private borrowers may deliberately choose to *increase* their interest-rate sensitivity following bad news about future needs for liquidity, a conclusion that runs afoul of the pattern predicted by standard modeling focusing on the microeconomics of corporate finance. Third, optimal monetary policy is time inconsistent, but not for the standard, inflation-bias reason; the central bank does not want to commit to lower the interest rates, but may ex post face the fait accompli of excessive short-term wholesale market exposure. Fourth, and related to the previous point, macro-prudential supervision is called for.

I. The Model

The following stylized model illustrates the basic points. There are three periods, $t = 0, 1, 2$, and two groups of economic agents, of mass 1 each: entrepreneurs and consumers (investors).

Consumers.—Consumers derive utility from consumption path $\{c_0, c_1, c_2\}$:

$$V = c_0 + u(c_1) + c_2,$$

where u is increasing and concave. They have "large" endowments e_0, e_1 at dates 0 and 1.

*Farhi: Department of Economics, Harvard University, Littauer 318, 1875 Cambridge Street, Cambridge MA 02138 (e-mail: efarhi@fas.harvard.edu); Tirole: Toulouse School of Economics, Manufacture des Tabacs, 21 allees de Brienne, F-31000 Toulouse (e-mail: tirole@cict.fr). We thank Arvind Krishnamurthy and participants in the 2009 AEA meetings for useful comments.

Entrepreneurs.—Entrepreneurs have utility function

$$U = c_0 + c_1 + c_2,$$

where c_t is their date-t consumption. Their only endowment is their wealth A at date 0. Their technology set exhibits constant returns to scale. At date 0 they choose their investment scale I. If still productive at date 2 (see below), this investment then delivers $\rho_1 I$, of which $\rho_0 I$ is pledgeable to investors where $\rho_0 < 1$.[1]

In practice, an exposure to funding liquidity risk can stem from multiple factors: a reliance on securitization, a lack of hedging, or the failure to hoard liquid assets or to secure lines of credit. We capture these various possibilities through a metaphor: The entrepreneur chooses at date 0 between a costly but safe technology, which never requires additional funds at date 1, and a cheaper but risky technology, which is vulnerable to liquidity shocks.

The cost of investing at scale i in the safe technology is Ki where $K > 1$. The risky technology is cheaper. The date-0 investment cost is I for investment scale I. However, with probability of "distress" $1 - \alpha$, one unit of reinvestment is needed per unit of initial investment in order for investment to be productive at date 2; otherwise the investment is discarded and there is no liquidation value. With probability α, the firm is "intact" and needs no reinvestment at date 1. For simplicity, we will assume that the liquidity-need realizations are independent across firms choosing the risky technology. Finally we make the following assumption.

ASSUMPTION 1: $1/\alpha > K > 1 + (1 - \alpha)\rho_0$.

This assumption will ensure that entrepreneurs find it preferable to opt for the safe technology if they anticipate that no monetary bailout will take place, and to opt for the risky technology if they anticipate that a monetary bailout will take place.

Storage Technologies.—There exists a linear storage technology that allows resources to be

transferred between date 1 and date 2, with rate of return normalized to one. There is no storage technology between date 0 and date 1.

Markets and Contracts.—The only trades consumers and entrepreneurs can engage in are spot loan contracts. Both the technology choice and the level of investment of each entrepreneur are observable. By so restricting the set of contracts, we implicitly make the following two assumptions. First, because there is no storage technology between date 0 and date 1, and because consumers cannot pledge at date 0 their endowment at date 1, there are no stores of value in the economy to carry wealth from date 0 to date 1. We could introduce stores of value, the price of which would be determined by a date-0 clearing condition in the market for liquidity. Second, firms cannot pledge at date 0 funds for date 1 to other firms, contingent on their being intact (by design or by chance). That is, our model follows Ricardo J. Caballero and Arvind Krishnamurthy (2003, 2004) in assuming that there are no cross-firm insurance schemes. Liquidity is not coordinated, and is therefore wasted. The results are robust to the relaxation of these two assumptions, but we deliberately focus on the simplest possible environment here.

Central Bank.—The central bank sets monetary policy by controlling the real interest rate in the economy. In our environment, this amounts to assuming that investment in the linear storage technology between date 1 and date 2 is observable. Usage of this technology can then be taxed or subsidized, leading to a real (after-tax) interest rate equal to R. The proceeds are rebated via lump sum to the consumers. The central bank maximizes a weighted average W of consumer and entrepreneur welfare V and U. The relative weight of entrepreneurs is $\beta \le 1$:

$$W = V + \beta U,$$

where the following assumption holds. It guarantees that reinvesting in distressed firms is socially optimal in the absence of bailout costs.

ASSUMPTION 2: $\beta(\rho_1 - \rho_0) > 1 - \rho_0$.

The potential costs and benefits of accommodative monetary policy can be understood as follows. On the one hand, lowering the real

---

[1] As usual, the "agency wedge" $\rho_1 - \rho_0$ can be motivated in multiple ways, including incentives to counter moral hazard (see e.g., Bengt Holmström and Tirole 2008 for a discussion).

interest rate below one introduces a wedge between the intertemporal rate of substitution of consumers and the rate of return on the storage technology. On the other hand, it makes both investment in distressed firms at date 1 and investment in the risky technology at date 0 more attractive.[2]

II. Monetary Policy under Commitment

In this section, we analyze the equilibrium under commitment where monetary policy is passive. The interest rate will never fall below the interest rate ρ_0, which allows distressed firms to be refinanced. For this reason, we refer to the event $\{R = \rho_0\}$ as a monetary bailout. We denote by $y \equiv \Pr(R = \rho_0)$ the exogenous probability of such a monetary bailout. Let x denote the fraction of entrepreneurs who choose the risky technology.

Safe Technology.—Consider the case of an entrepreneur investing in the safe technology at date 0. The entrepreneur's borrowing capacity is determined by

$$Ki - A = \rho_0 i.$$

Letting

$$i^* \equiv \frac{A}{K - \rho_0},$$

the entrepreneur's net utility is

$$(1) \qquad U = (\rho_1 - \rho_0)i^*.$$

Risky Technology.—Consider now the case of an entrepreneur investing in the risky technology. A firm in distress at date 1 is fully dependent on funding liquidity, namely its ability to

raise new funds on the market. Such funds can be raised only if (one plus) the interest rate, R, between dates 1 and 2 is low enough, namely $R \leq \rho_0$. A risky firm's borrowing capacity is determined by

$$I - A = \alpha \rho_0 I,$$

since $R \geq \rho_0$ implies that initial investors never make a return when the firm is in distress. Letting

$$I^* \equiv \frac{A}{1 - \alpha \rho_0},$$

the entrepreneur has utility

$$(2) \qquad U = (\rho_1 - \rho_0)[\alpha + (1 - \alpha)y]I^*$$

when choosing the risky technology.

Equilibrium Technology Choice.—Entrepreneurs therefore prefer to invest in the safe technology if and only if

$$(3) \quad (\rho_1 - \rho_0)i^* \geq (\rho_1 - \rho_0)[\alpha + (1 - \alpha)y]I^*.$$

The second inequality in Assumption 1 ensures that choosing the risky technology allows for a larger-scale $I^* > i^*$. The higher the cost disadvantage $K - 1 - (1 - \alpha)\rho_0$ of the safe technology, the larger is the investment scale disparity. The entrepreneurs trade off this larger scale against the lower probability of success $\alpha + (1 - \alpha)y$ involved in the risky technology. The latter depends on the stance of monetary policy. The higher the probability y of a monetary bailout, the larger is the probability of success of a risky project, and the more attractive the risky technology.

The equilibrium is entirely pinned down by this condition. If (3) holds with a strict inequality, then entrepreneurs invest in the safe technology. If (3) is violated, then entrepreneurs invest in the risky technology, and distressed projects are continued only in the case of a monetary bailout.

Optimal Policy under Commitment.—The case of a passive, laissez-faire monetary policy is an important benchmark. The rate of interest between dates 1 and 2 is then

$$R = 1 > \rho_0,$$

---

[2] Note that we are ruling out other forms of policy intervention. For example, we do not consider subsidies to reinvestment. This extreme assumption is meant to rule out direct bailouts and thus to focus on monetary policy. This restriction could be derived from explicit but extreme information constraints, assuming that types—consumer versus investor—are private information and that technology choice, investment by entrepreneurs, and trades between consumers and entrepreneurs are hidden from the central bank, which observes only aggregate variables and investment in the storage technology.

so that $y = 0$. Conditions (1) and (2), together with the first inequality in Assumption 1, imply that the entrepreneurs opt for the safe technology. Thus the equilibrium features $x = 0$.

Another important benchmark is the case where a monetary policy bailout occurs with probability $y = 1$. The second inequality in Assumption 1 then ensures that entrepreneurs choose the risky technology so that $x = 1$.

Optimal monetary policy under commitment involves either laissez-faire or systematic monetary bailouts. To rescue distressed firms, the state must bring the interest rate R down to ρ_0. The following objects are useful to compare welfare under both policies. Let I_1 the amount of reinvestment in distressed firms desired at date 1. Define

$$\hat{V}(R) \equiv u(e_1 + \hat{X}) - \hat{X} \text{ with } u'(e_1 + \hat{X}) = R.$$

The optimal policy under commitment is laissez-faire if and only if the welfare loss resulting from the distortion of the interest rate $\hat{V}(1) - \hat{V}(\rho_0)$ is greater than the net gain from a greater investment scale $\beta(\rho_1 - \rho_0) \times (I^* - i^*)$ due to the choice of the risky technology, net of the reinvestment cost $(1 - \rho_0) \times (1 - \alpha)I^*$ from rescuing distressed firms:

$$(4) \quad \hat{V}(1) - \hat{V}(\rho_0) > \beta(\rho_1 - \rho_0)(I^* - i^*)$$
$$- (1 - \rho_0)(1 - \alpha)I^*$$

III. Limited Commitment and Monetary Bailouts

In the previous section, we assumed that the central bank committed to monetary policy at date 0. In this section, we depart from this assumption. Rather, we assume that monetary policy is set at date 1, without commitment.

The Central Bank's Decision.—The relevant state variable for the date-1 decision problem of the central bank is the aggregate reinvestment need of distressed firms, $I_1 \equiv x(1 - \alpha)I^*$. The central bank either sets the interest rate at the laissez-faire level of one or at the monetary bailout level $\rho_0 < 1$. Ex post, it is optimal to set $R = \rho_0$ if and only if

$$(5)\ \hat{V}(1) - \hat{V}(\rho_0) \leq [\beta(\rho_1 - \rho_0) - (1 - \rho_0)]I_1.$$

The left-hand side of condition (5) corresponds to the "fixed cost" of the nontargeted policy: the interest rate is distorted, creating a welfare loss equal to $\hat{V}(1) - \hat{V}(\rho_0)$. The right-hand side is the net gain (Assumption 2 guarantees that this term is positive) of rescuing distressed firms. This monetary bailout costs $1 - \rho_0$ per unit to consumers, but yields per unit benefit $\rho_1 - \rho_0$ to entrepreneurs. The higher the aggregate reinvestment need I_1 of distressed firms, the more likely is a monetary bailout. Therefore, leverage decisions exhibit strategic complementarities. The higher the fraction of entrepreneurs investing in risky technologies, the higher is the aggregate reinvestment need I_1 of distressed firms, and the more likely a monetary bailout. This in turn makes the risky technology more desirable for entrepreneurs.

An intuition that accords with our title is that the central banker's put is closer to the money the more leveraged the economy. As a result, the entrepreneurs' payoff is more convex, reinforcing the incentives for risk-taking and leverage.

Equilibrium.—Note, first, that if the entrepreneurs opt for the safe technology, then $I_1 = 0$ and it is optimal for the central bank to set $R = 1$. Hence the laissez-faire equilibrium with $y = 0$ and $x = 0$ analyzed in Section II is still an equilibrium under no commitment. Under a stronger condition than Assumption 2, namely,

$$(6) \quad \hat{V}(1) - \hat{V}(\rho_0) \leq$$
$$[\beta(\rho_1 - \rho_0) - (1 - \rho_0)](1 - \alpha)I^*,$$

the systematic monetary bailout equilibrium with $y = 1$ and $x = 1$ analyzed in Section II is also an equilibrium.[3] Furthermore, this equilibrium *Pareto-dominates* the safe strategy equilibrium from the point of view of entrepreneurs. We will henceforth assume that when multiple equilibria coexist, the (entrepreneurs') Pareto superior one prevails.

Sensitivity to Macroeconomic Conditions.— The possibility of multiple equilibria, one with $y = 0$ and $x = 0$, and one with $y = 1$ and $x = 1$, underscores that equilibrium risk-taking,

---

[3] There is also, then, an unstable, mixed equilibrium with $0 < x < 1$.

leverage, and monetary policy can be very sensitive to aggregate macroeconomic conditions (α).[4]

Exposure to funding liquidity risk and monetary bailouts arises if condition (6) is satisfied. We therefore conclude that they are more likely when: (i) the corporate sector receives more weight in the state's objective function (β high), and (ii) liquidity shocks are more likely (α low). While (i) is rather obvious, (ii) deserves further comments. Note that under commitment to laissez-faire, the safe technology is more likely to be chosen $(1 > \alpha K)$ if α is small. Nonetheless, without commitment, the policy reaction implies that the firms may take on more risk $(x = 1)$ when bad news accrues (α decreases). When macroeconomic condition deteriorate (α decreases), the central banker's put is closer to the money. As a result, the entrepreneurs' payoff is more convex, inducing further risk-taking.

Time Inconsistency.—We can also comment on time consistency. When both (4) and (6) hold, the optimal policy under commitment is laissez-faire, while under no commitment, systematic monetary bailouts occur. In this case, the optimal policy features a form of time inconsistency. Welfare is higher when the central bank commits at date 0 not to lower interest rates at date 1. This deters entrepreneurs from engaging in risky projects, which is socially optimal as long as (4) holds. When, instead, the central bank lacks commitment, entrepreneurs anticipate a monetary bailout and invest in the risky technology at date 0. Many firms are then distressed at date 1, and the central bank finds it optimal to lower interest rates.

Regulating Leverage.—The perverse effects from leverage on the likelihood of monetary bailouts can be neutralized if technology choice can be regulated. Thus, there is a role for macroprudential supervision. Note that, in our model, there is no role for such supervision under commitment.[5] Indeed, it is enough for supervision to

ensure that the aggregate amount of investment in the risky technology is capped by $\bar{I}$, where

$$\bar{I} \equiv \frac{\hat{V}(1) - \hat{V}(\rho_0)}{[\beta(\rho_1 - \rho_0) - (1 - \rho_0)](1 - \alpha)}.$$

This ensures that the benefits of implementing a monetary bailout are always less than the costs, which in turn induces the entrepreneurs to limit their leverage by opting for the safe technology.

IV. Conclusion

The insights of this paper are not specific to interest rate policy. The less targeted the policy under consideration, the more relevant is our analysis. For example, some of the various facilities recently introduced by the Fed can be seen as forms of subsidies, which are not targeted to the extent that they can be partly appropriated by agents who are not distressed or who carry a lower weight in the central bank's welfare function. An important empirical question to interpret the recent events in light of our model is whether these facilities are more targeted than the federal funds rate. On the theoretical front, it is worth enriching the model to allow for a finer determination of the trade-offs underlying the choice between different policy instruments. In the same vein, building an explicit monetary model would allow us to analyze a trade-off, much debated during the fall of 2008, between creating inflation and propping up asset values to prevent waves of liquidation.

REFERENCES

Caballero, Ricardo J., and Arvind Krishnamurthy. 2003. "Excessive Dollar Debt: Financial Development and Underinsurance." *Journal of Finance*, 58(2): 867–93.

Caballero, Ricardo J., and Arvind Krishnamurthy. 2004. "Smoothing Sudden Stops." *Journal of Economic Theory*, 119(1): 104–27.

Holmström, Bengt, and Jean Tirole. 2008. *Inside and Outside Liquidity.* Unpublished.

---

[4] Multiple equilibria can be seen as a convenient exposition tool to illustrate the general conclusion that the equilibrium can be very sensitive to parameters.

[5] The reason is twofold. First, it is suboptimal, both from the entrepreneurs' private perspective and from the social perspective of the central bank, to undertake a risky project that will be discontinued in case of distress. Second, in case a monetary bailout takes place, it is always optimal, both privately and socially, to undertake a risky project.