Fiscal Unions*

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We study cross-country risk sharing as a second-best problem for members of a currency union using an open economy model with nominal rigidities and provide two key results. First, we show that if financial markets are incomplete, the value of gaining access to any given level of aggregate risk sharing is greater for countries that are members of a currency union. Second, we show that even if financial markets are complete, privately optimal risk sharing is constrained inefficient. A role emerges for government intervention in risk sharing both to guarantee its existence and to influence its operation. The constrained efficient risk sharing arrangement can be implemented by contingent transfers within a fiscal union. We find that the benefits of such a fiscal union are larger, the more asymmetric the shocks affecting the members of the currency union, the more persistent these shocks, and the less open the member economies. Finally we compare the performance of fiscal unions and of other macroeconomic stabilization instruments available in currency unions such as capital controls, government spending, fiscal deficits, and redistribution.

1 Introduction

The benefits of flexible exchange rates were famously argued for by Friedman (1953) and are widely accepted by economists. Countries in a currency union forego the possibility of adjustments to their exchange rates in response to asymmetric shocks. How costly is this loss in flexibility and what can be done to compensate for it? These questions are precisely those tackled by the Optimal Currency Area (OCA) literature (for the pioneering articles, see Mundell, 1961; McKinnon, 1963; Kenen, 1969).

In a seminal contribution, Kenen (1969) argued that fiscal integration was critical to a well-functioning currency union:

> "It is a chief function of fiscal policy, using both sides of the budget, to offset or compensate for regional differences, whether in earned income or in unemployment rates. The

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large-scale transfer payments built into fiscal systems are interregional, not just interpersonal [...]" (pg. 47)

Countries such as the United States, which can be thought as a currency and fiscal union of regions, share federal revenues and transfers—through the unemployment insurance program, federal income and social security taxes and, in extreme cases, direct federal assistance—in a manner that provides macroeconomic stabilization across regions. The ongoing crisis in the Eurozone, where such mechanisms are lacking, is seen by many as a vindication of Kenen's fiscal integration criterion. Going forward, many policy discussions center around the construction of a fiscal union. How should a fiscal union be designed and how effective can we expect it to be?

Unfortunately, the OCA literature is couched in terms of Keynesian models that lack proper micro-foundations. As a result, the treatment of welfare is cursory. Recently, the New Open Economy Macro literature has developed open economy New Keynesian models with explicit micro-foundations and applied them to currency areas (see e.g. Benigno 2004; Beetsma and Jensen 2005; Gali and Monacelli 2008; Ferrero 2009). Our goal is to revisit Kenen's idea using such a model. This allows for a rigorous treatment of optimal policy design. Indeed, we are able to deliver a complete characterization of the required transfers and of their effectiveness as a function of a small number of key characteristics of the economy.

We tackle the design of a fiscal union within a currency union as an optimal international risk sharing arrangement. We begin our analysis with the simplest possible model: a static setting with a traded good, a non-traded good and labor as in Obstfeld and Rogoff (2000). We then extend the analysis to a standard dynamic model featuring non-trivial intra-temporal trade and price adjustment dynamics that builds on Gali and Monacelli (2005, 2008). The key features in both settings are fixed exchange rates, price or wage stickiness, and limited openness in the form of non-traded goods or home bias. In this context, we set up and study the second-best planning problem for constrained efficient risk sharing via international transfers among countries in a currency union.[1]

International transfers have a dual role ex post. First, they help smooth consumption across countries. This is their usual direct microeconomic role. Second, under a fixed exchange rate, in the presence of nominal price or wage rigidities, and with non-traded goods or home bias, international transfers also have an indirect effect by influencing total spending across goods produced by different countries—a mechanism first discussed in the famous Transfer Problem debate involving Keynes (1929) and Ohlin (1929). Transfers from countries in a boom to countries in a bust improve macroeconomic stability in the currency union. We show that this dual role gives rise to an aggregate demand externality ex ante: the social benefits from international risk sharing are greater

[1] We follow the approach of the OCA literature by taking the existence of a currency union as an exogenous constraint and not attempting to model the reasons for its formation in the first place. In other words, we abstract from the potential benefits and focus on the costs of currency unions. We characterize to what extent these costs can be mitigated by the establishment of a fiscal union. Of course, one potential concern is that the factors leading to the formation of currency unions could influence the optimal design of fiscal unions. Unfortunately, there is no consensus among economists on the benefits of currency unions. In addition, at least in the case of the Eurozone, the adoption of the euro was part of a larger political unification project. For all these reasons we believe that treating the existence of a currency union as an exogenous constraint is a useful starting point.

than what is appreciated by private economic agents, since they do not internalize these indirect macroeconomic stability effects and only value the direct microeconomic consumption smoothing role. Indeed, our main result is that even under ideal conditions with complete financial markets to support these international transfers, the competitive equilibrium without government intervention does not provide the constrained Pareto efficient level of international risk sharing.[2]

The constrained inefficiency of private international risk sharing can be addressed by government intervention. Indeed constrained efficient outcomes can be implemented in a number of ways. If individuals do have access to complete financial markets, then constrained efficiency can be ensured by introducing quantity restrictions or tax incentives that distort their individual portfolios choices. A second possibility is for the government to take over intergenerational risk sharing by assuming the necessary positions in financial markets itself. Equivalently, instead of using financial markets, it can arrange ex ante with other union members for state-contingent international fiscal transfers ex post. In either case, it must then also take steps to ensure that the private sector does not undo these arrangements by setting up the aforementioned quantity restrictions or tax incentives, or by employing more extreme measures such as banning financial markets.

We view the complete financial markets paradigm as a useful assumption to highlight that the constrained inefficiency of private risk sharing that we derive does not arise from inefficiencies in financial markets. However, our preferred interpretation is that financial markets are incomplete so that markets for sharing aggregate risk across countries are imperfect or nonexistent. This only strengthens the argument for building a fiscal union to share risks across members within a currency union.[3] Indeed, the constrained efficient risk sharing arrangement can then be implemented through ex-post international fiscal transfers that are contingent on the shocks experienced by each country. Since agents have no access to financial markets, neither restrictions nor taxes on private portfolios are needed. Under this interpretation, our paper can be seen as offering a precise characterization of these ex-post international fiscal transfers and clarifying that for members of a currency union: (i) the value of gaining access to any given level of international risk sharing is greater; and (ii) international fiscal transfers should go beyond emulating the outcome that private risk sharing would reach if financial markets were complete. These two points are distinct but complement each other to motivate the formation of fiscal unions within currency unions.

Importantly, we do not reach the same conclusion for countries outside a currency union with flexible exchange rates. As long as they exercise their independent monetary policy optimally, it is optimal to let agents trade freely in a complete set of financial markets or to replicate this outcome through international fiscal transfers. Our argument for government involvement in international risk sharing relies on membership in a currency union precisely because this constrains monetary policy and prevents the stabilization of asymmetric shocks: fiscal and currency unions go hand in

[2]These aggregate demand externalities are pervasive in New-Keynesian models and their normative implications have recently been analyzed in different contexts in parallel and independent work by Farhi and Werning (2012), Schmitt-Grohe and Uribe (2012), Farhi and Werning (2016), Korinek and Simsek (2016), and Schmitt-Grohe and Uribe (2016).

[3]Atkeson and Bayoumi (1993) examine cross-regional insurance in the United States and conclude that "integrated capital markets are [...] unlikely to provide a substantial degree of insurance against regional economic fluctuations [...] This task will continue to be primarily the business of government."

hand.

Our results qualify a view often presented in the OCA literature that international fiscal transfers and international risk sharing through private financial markets are substitutes since both buffer against asymmetric macroeconomic shocks in a currency union. For example, Mundell (1973) argues that a common currency could help improve international risk sharing by increasing cross holdings of assets or deepening financial markets. While our model is silent on whether a currency union may facilitate the development of financial markets, it shows that the benefits of risk sharing are larger in a currency union and that government intervention is needed to reap the full benefits. Indeed, we establish that private risk sharing is not constrained Pareto efficient in a currency union, so that financial integration alone is not sufficient.

We emphasize three key determinants of the macroeconomic stabilization performance of fiscal unions in currency unions: the asymmetry of the shocks hitting the members of the currency union, the persistence of these shocks, and the openness of the member economies. Indeed, symmetric shocks can be accommodated with union-wide monetary policy so that transfers should be used only in response to asymmetric shocks. Optimal transfers are increasing in the persistence of these shocks but hump-shaped as a function of openness. However, a given transfer is more effective at stabilizing the economy when the economy is more closed. Hence more stabilization is achieved at the optimum both when the economy is more closed and when shocks are more persistent. Indeed, we show that full stabilization is achieved in the limit as shocks become permanent and the economy becomes closed. This contrasts with the ideas in McKinnon (1963), who discusses reasons why openness may mitigate the costs of currency unions.

We extend the model by introducing agent heterogeneity with a fraction of hand-to-mouth consumers and a fraction of permanent-income consumers. This extension allows us to capture a key feature of the data—the high average marginal propensity to consume out of transitory income and its heterogeneity across the population—while preserving enough tractability to lend itself to a full-fledged normative analysis. With hand-to-mouth consumers, the performance of transfers in dealing with transitory shocks is improved. This is because in contrast to the consumption spending of permanent-income consumers, the consumption spending of hand-to-mouth consumers tracks international fiscal transfers and hence can be efficiently targeted over time to stabilize the economy. For example in the case of a transitory recessive shock, international fiscal transfers can be front-loaded to provide stimulus in the short run when it is needed.

We compare international fiscal transfers with other macroeconomic stabilization instruments in currency unions, such as capital controls (Farhi and Werning 2012; Schmitt-Grohe and Uribe 2012) and different variants of domestic fiscal policy ranging from government spending (as analyzed in Beetsma and Jensen 2005; Gali and Monacelli 2008; Ferrero 2009; Farhi and Werning 2012, 2017) to redistribution and budget deficits. Ideally, it is best to jointly use all these different instruments together, to the extent that they are available, and our main results about the optimal design of fiscal unions are robust to the inclusion of these other instruments. But one might also want to assess the relative performance of these instruments. To that end, we discuss theoretically and compare numerically the relative performance of these different instruments depending on a number of important parameters of the economy such as the fraction of hand-to-mouth consumers, the openness of the economy, the rigidity of prices, and the persistence of shocks.

Finally, and although this is not our main focus, we briefly explore the robustness of our results in the presence of agency problems at the national level, such as limited commitment or moral hazard. We show how these incentive problems, together with the forces that we have identified above, jointly influence the optimal design of fiscal unions, and that our main insights carry over.

The rest of the paper is organized as follows. The static model is covered in Sections 2 and 3. The dynamic model is developed in Sections 4 and 5. The numerical illustrations are covered in Section 6. Section 7 contains our conclusions. The online appendix contains all the proofs and derivations as well as several extensions.

Related literature. First and foremost, our paper is related to the Optimal Currency Area (OCA) literature. This literature has emphasized a number of important factors for successful currency unions: factor mobility (Mundell, 1961), openness (McKinnon, 1963), fiscal integration (Kenen, 1969), and financial integration (Mundell, 1973). Our paper formalizes and refines the arguments of Kenen (1969), by seeing fiscal unions as the implementation of an optimal risk sharing arrangement within in a currency union, in a model with explicit micro-foundations. We offer a precise characterization of the size, direction, and effectiveness of fiscal transfers. Our results qualify the view implicit in Mundell (1973) that financial integration is a substitute for fiscal integration. Finally, our work contrasts with the ideas in McKinnon (1963), who discusses reasons why openness may mitigate the costs of currency unions. In our paper, fiscal unions are more effective when member countries are more closed. However, our results are fully compatible with the notion that openness is beneficial in a currency union lacking a fiscal union.

Our modeling approach follows the New Keynesian tradition embraced by the New Open Economy Macro literature. In particular, our static analysis builds on the model of Obstfeld and Rogoff (2000), and our dynamic analysis builds on the model of Gali and Monacelli (2005, 2008). A flexible exchange rate allows the implementation of the flexible price allocation (see e.g. Benigno, 2000; Clarida et al., 2002; Gali and Monacelli, 2005). A fixed exchange rate represents a constraint on macroeconomic stabilization, and raises the question of the optimal use of monetary policy in a currency union. Benigno (2004) analyzes the case of a currency union with complete markets, shows that monetary policy at the union level cannot achieve perfect stabilization with asymmetric shocks, and characterizes optimal monetary policy at the union level.

Our paper explores the optimal use of macroeconomic instruments beyond monetary policy, focusing, in particular, on cross-country transfers or interventions in financial markets. Other studies have focused on different policy instruments. Beetsma and Jensen (2005) and Gali and Monacelli (2008) analyze optimal fiscal policy in a currency union by characterizing how government purchases of domestic goods can help stabilize the economy in response to asymmetric shocks. Adao et al. (2009) and Farhi et al. (2014) show that with a rich enough set of distortionary taxes, the flexible price allocation can be achieved.[4] In our view, however, there are important practical limitations that constrain the extent to which these tax incentives can be used, leaving considerable room for other instruments. Ferrero (2009) analyzes another dimension of fiscal policy, focusing on distortionary taxes and government debt. Farhi and Werning (2012), Schmitt-Grohe and Uribe (2012), and Farhi and Werning (2016) analyze capital controls. None of these papers considers fiscal transfers across union members and most assume complete private financial markets. Our work complements these contributions by analyzing fiscal transfers as another macroeconomic tool.

Few papers consider optimal policy with incomplete financial markets. An exception is Benigno (2009) who analyzes optimal monetary policy in the case of incomplete markets and flexible exchange rates. Nominal rigidities create a tradeoff between completing markets and stabilizing the economy. On the one hand, if prices were flexible, the optimum would imitate complete markets by tailoring the real returns of international bonds. On the other hand, if markets could be completed or if transfers imitated complete markets, the optimum would be fully efficient. Our modeling assumptions and results are essentially the polar opposite. Our analysis assumes that the exchange rate is fixed, so that the aforementioned tradeoff is not considered. Furthermore, in the presence of non-traded goods or home bias, our main result is that complete markets, or transfers that imitate complete markets, lead to a suboptimal outcome.[5]

Auray and Eyquem (2013) consider a currency union with sticky prices. They do not attempt to study optimal policy but using a set of numerical calibrations, they show that it is possible in some cases for the laissez faire complete markets equilibrium with a full set state-contingent bonds and no government interventions in financial markets to deliver lower welfare than the incomplete markets laissez faire equilibrium with only non-state-contingent bonds or with no bonds. This possibility is also present in our model because of the market failure that opens up a wedge between privately and socially optimal portfolios.[6]

The key ingredient of the New Open Economy Macro literature is the presence of nominal rigidities. Another important ingredient, present in some but not all papers in that literature, is the as-

[4]Kehoe and Pastorino (2017) builds on our environment but include such flexible instruments in the form of flexible country-specific and state-contingent taxes on non-traded goods, ensuring that the first best can be achieved (as if exchange rates were flexible) without transfers if markets are complete, that optimal transfers simply replicate the complete markets allocation if markets are incomplete, and also negating the need for any other macroeconomic stabilization instrument such as government spending or capital controls.

[5]Other features could lead to a similar result even in the absence of home bias or non-traded goods. For example, in the dynamic model of Sections 4-5, when prices are sticky but not fully rigid, complete markets, or transfers that imitate complete markets are suboptimal. This is because even in the absence of home bias, transfers still influence the domestic wage through a wealth effect, and hence also influence inflation and therefore macroeconomic stabilization. This effect, which arises only when prices are sticky but not fully rigid, would not be internalized by private agents when they form their portfolios under complete markets. This justifies interventions in financial markets if markets are complete, or transfers that do not replicate complete markets if markets are incomplete. See Section 6 and in particular footnote 38 for a discussion.

[6]This result, while interesting, is neither our main focus nor our main contribution, which is to study: (i) the socially optimal use of fiscal transfers to share risk between countries when markets are incomplete; (ii) the socially optimal use of state-contingent bonds to share risk between countries when markets are complete with optimal government interventions in financial markets; (iii) how effective these transfers may be in mitigating the inefficiencies from nominal rigidities and fixed exchange rates. We fully characterize how to optimally design a fiscal union within a monetary union decentralized as in (i) or as in (ii), and to understand how effective such fiscal unions are as in (iii).

sumption of home bias or non-traded goods. This ingredient is absolutely central for our theory, and it is also at the core of all analyses of the Transfer Problem. Given these ingredients, we study a policy instrument that has not been considered before in the literature.

2 A Static Model of a Currency Union

We start with a simple static model that illustrates our main idea most transparently. Later we show that the same effects are present in standard dynamic open economy models. The model builds on the model with traded and non-traded goods presented in Obstfeld and Rogoff (2000). There is a continuum of countries in a currency union. There is a traded good, a non-traded good and labor. The traded good is supplied inelastically and traded competitively. The non-traded good is supplied from labor by monopolistic firms. The prices set by these monopolistic firms are sticky.[7]

We offer two market settings and associated policy interventions for the same model environment. The first assumes complete financial markets and features portfolio taxes as the policy instrument to influence equilibrium risk sharing across countries. The second assumes incomplete markets, so that private agents have no opportunities to share risk internationally. In this case we focus on government arranged international fiscal transfers across countries to provide international risk sharing. Importantly, we show that both settings lead to the same set of implementable allocations. This allows us to characterize efficient allocations using the same second-best Ramsey planning problems for both settings in Section 3.

In our view, the first setting offers several conceptual advantages even though is it less realistic. First, it allows us to make the point that constrained efficient allocations require government intervention *even if* financial markets are complete. By implication if markets are incomplete, government intervention should not simply mimic the complete-markets outcome. Second, we can provide simple formulas for the interventions in the form of portfolio taxes. The incomplete markets setting, on the other hand, seems more realistic and the implementation of constrained efficient allocations involves cross-country risk sharing through international fiscal transfers, providing a foundation for fiscal unions. In any case, although we favor the incomplete-market setting and its implementation in practical terms, the characterization using complete markets sheds light on both.

We first present the model with complete markets in Sections 2.1-2.4. We then introduce the incomplete markets in Section 2.5.

2.1 Households

There is a single period and a continuum of countries indexed by $i \in [0, 1]$. We start by assuming that all countries belong to a currency union, but will relax this later. Uncertainty affects preferences and technology: the state of the world $s \in S$ has density $\pi(s)$ and determines preferences and technology,

[7] In Appendix B.1, we show that all our results go through if wages are nominally rigid instead of prices. In particular, Propositions 1–12 are still valid.

possibly asymmetrically, in all countries.

In each country $i \in I$, there is a representative agent with preferences over non-traded goods, traded goods and labor given by the expected utility

$$\int U^i(C^i_{NT}(s), C^i_T(s), N^i(s); s)\pi(s)ds.$$

Below we make some further assumptions on preferences.

Agents can trade in a complete set of financial markets before the realization of the state of the world $s \in S$. Households are subject to the following budget constraints

$$\int D^i(s)Q(s)\pi(s)ds \leq 0, \tag{1}$$

$$P^i_{NT}C^i_{NT}(s) + P_T(s)C^i_T(s) \leq W^i(s)N^i(s) + P_T(s)E^i_T(s) + \Pi^i(s) + T^i(s) + (1 + \tau^i_D(s))D^i(s), \tag{2}$$

where P^i_{NT} is the price of non-traded goods which as we will see shortly, does not depend on s due to the assumed price stickiness; $P_T(s)$ is the price of traded goods in state s; $W^i(s)$ is the nominal wage in state s; $E^i_T(s)$ is country i's endowment of traded goods in state s; $\Pi^i(s)$ represents aggregate profits in state s; $T^i(s)$ is a lump sum rebate; $D^i(s)$ is the nominal payoff of the household portfolio in state s; $Q(s)$ is the price of one unit of currency in state s in world markets, normalized by the probability of state s; and $\tau^i_D(s)$ is a state-contingent portfolio return subsidy.[8] The lump sum rebate $T^i(s)$ is used to rebate the proceeds from the tax on financial transactions to households. We sometimes also consider lump-sum transfers over and above such rebates to redistribute wealth across countries. Note that the nominal price of traded goods is assumed to be the same across countries, reflecting the law of one price and the fact that all countries in the union share the same currency.

The households' first-order conditions can be written as

$$\frac{U^i_{C_T}(s)[1 + \tau^i_D(s)]}{Q(s)P_T(s)} = \frac{U^i_{C_T}(s')[1 + \tau^i_D(s')]}{Q(s')P_T(s')}, \tag{3}$$

$$\frac{U^i_{C_T}(s)}{P_T(s)} = \frac{U^i_{C_{NT}}(s)}{P^i_{NT}}, \tag{4}$$

$$-\frac{U^i_N(s)}{W^i(s)} = \frac{U^i_{C_{NT}}(s)}{P^i_{NT}}. \tag{5}$$

2.2 Firms

We assume that the traded good is in inelastic supply: each country is endowed with a quantity $E^i_T(s)$ of traded goods. These goods are traded competitively in international markets.

Non-traded goods are produced in each country by competitive firms that combine a continuum

[8]Above we assumed that the returns from firms are not subsidized. Another possibility is to subsidize profits $\Pi^i(s)$ at the same rate $\tau^i_D(s)$ as financial returns. None of our analysis or conclusions are affected by this modeling choice.

of non-traded varieties indexed by $j \in [0,1]$ using the constant returns to scale CES technology

$$Y^i_{NT}(s) = \left(\int_0^1 Y^{i,j}_{NT}(s)^{1-\frac{1}{\varepsilon}} dj \right)^{\frac{1}{1-\frac{1}{\varepsilon}}},$$

with elasticity $\varepsilon > 1$.

Each variety is produced by a monopolist using a linear technology

$$Y^{i,j}_{NT}(s) = A^i(s) N^{i,j}(s).$$

Each monopolist hires labor in a competitive market with wage $W^i(s)$, but pays $W^i(s)(1+\tau^i_L)$ net of a country specific tax on labor. Monopolists must set prices in advance, at the beginning of the period, before the realization of uncertainty. The demand for each variety is given by $C^i_{NT}(s)(P^{i,j}_{NT}/P^i_{NT})^{-\varepsilon}$ where $P^i_{NT} = (\int (P^{i,j}_{NT})^{1-\varepsilon} dj)^{1/(1-\varepsilon)}$ is the price of non-traded goods. They solve

$$\max_{P^{i,j}_{NT}} \int \frac{Q(s)}{1+\tau^i_D(s)} \Pi^{i,j}(s) \pi(s) ds,$$

where

$$\Pi^{i,j}(s) = [P^{i,j}_{NT} - \frac{1+\tau^i_L}{A^i(s)} W^i(s)] C^i_{NT}(s) \left(\frac{P^{i,j}_{NT}}{P^i_{NT}} \right)^{-\varepsilon}.$$

Aggregate profits are given by $\Pi^i(s) = \int \Pi^{i,j}(s) dj$.

In a symmetric equilibrium, all monopolists in country i set the same profit maximizing price. Rearranging the first-order condition yields the familiar expression for the price as a markup over a weighted average of the marginal cost across states

$$P^i_{NT} = (1+\tau^i_L) \frac{\varepsilon}{\varepsilon - 1} \frac{\int \frac{Q(s)}{1+\tau^i_D(s)} \frac{W^i(s)}{A^i(s)} C^i_{NT}(s) \pi(s) ds}{\int \frac{Q(s)}{1+\tau^i_D(s)} C^i_{NT}(s) \pi(s) ds}. \tag{6}$$

2.3 Government

The government is subject to the budget constraint

$$T^i(s) = \tau^i_L W^i(s) N^i(s) - \tau^i_D(s) D^i(s) + \hat{T}^i(s). \tag{7}$$

Here $\hat{T}^i(s)$ are net international fiscal transfers which redistribute resources across countries subject to the constraint

$$\int \hat{T}^i(s) di = 0, \tag{8}$$

for all $s \in S$.

2.4 Equilibrium with Complete Markets

An equilibrium is such that households and firms maximize, the government's budget constraint is satisfied, and markets clear[9]:

$$C^i_{NT}(s) = A^i(s)N^i(s), \tag{9}$$

$$\int C^i_T(s)di = \int E^i_T(s)di. \tag{10}$$

These conditions imply that the financial markets clear so that $\int D^i(s)di = 0$ for all $s \in S$.

The conditions for an equilibrium (1)–(10) act as constraints on the planning problem we study next in Section 3.[10] In a spirit similar to Lucas and Stokey (1983), we seek to drop variables and constraints as follows. Given quantities, equations (3), (5) and (6) can be used to back out certain prices, wages and taxes. Since these variables do not enter the welfare function they can be dispensed with from our planning problem, along with equations (1), (2), (3), (5), (6), (7), and (8). We summarize these arguments in the following proposition.

Proposition 1 (Implementability, Complete Markets)**.** *An allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ together with prices $\{P_T(s), P^i_{NT}\}$ form part of an equilibrium with complete markets if and only if equations* (4) *and* (9) *hold for all $i \in I, s \in S$ and equation* (10) *holds for all $s \in S$.*

Importantly, we cannot dispense with equation (4). This equation summarizes the restriction imposed by a currency union—that the price of traded goods cannot vary across countries—and price stickiness—that the price of non-traded goods cannot vary across states of the world. Consider attempting to use equation (4) as a residual to back out the prices that support a given allocation, as we did with equations (3), (5) and (6). Equation (4) requires that the relative price of traded to non-traded goods equal $U^i_{C_T}(s)/U^i_{C_{NT}}(s)$. For any arbitrary allocation, this required relative price can be computed, but the problem is that it may not be possible to express it as a ratio of a price that is independent of i and a price that is independent of s, i.e. as a ratio $P_T(s)/P^i_{NT}$. This is why we must keep equation (4) as a constraint.

Our constructive proof shows that an allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ and prices $\{P_T(s), P^i_{NT}\}$ that satisfy the conditions in the propositions are actually part of several equilibria. We emphasize two dimensions of indeterminacy. First, we can choose any set of state prices $Q(s)$. Second, we can choose different transfers $\hat{T}^i(s)$. These two dimensions are actually related in the sense that different state prices require different ex-post fiscal transfers.

The first dimension of indeterminacy can be intuitively understood as follows. The relevant state prices for households are adjusted for portfolio taxes $\frac{Q(s)}{1+\tau^i_D(s)}$. Scaling up state prices $Q(s)$ and the corresponding portfolio taxes $1+\tau^i_D(s)$ by a function $\lambda(s)$ leaves these tax-adjusted state prices

[9]Our notation already takes into account the symmetry of prices, output, and labor across varieties j within each country i.

[10]In addition, the budget constraints (1) and (2) must hold as an equality.

unchanged. However this change indirectly transfers resources across countries and states. These indirect transfers need to be compensated by adjusting transfers $\hat{T}^i(s)$.

The second dimension of indeterminacy can be intuitively understood as follows. To what extent transfers across countries actually operate through financial markets $D^i(s)$ or transfers $\hat{T}^i(s)$ is not pinned down. This can easily be seen by starting with the household's budget constraint, holding with equality, and substituting out profits $\Pi^i(s)$ and transfers $T^i(s)$ to arrive at the following country budget constraint

$$\int Q(s)\left[P_T(s)(C_T^i(s) - E_T^i(s))\right]\pi(s) = \int Q(s)\hat{T}^i(s)\pi(s)ds,$$

which states that the expected value of the trade balance must be covered by the expected value of international fiscal transfers. Indeed, this is the only constraint on fiscal transfers. For example, one possibility is to constrain transfers to be non-state-contingent $\hat{T}^i(s) = \hat{T}^i$. All risk sharing is then being delivered through financial markets, and portfolio taxes are required to make sure that private agents secure the right amount of insurance $D^i(s)$. Another possibility is to set $\hat{T}^i(s) = P_T(s)(C_T^i(s) - E_T^i(s))$. In that case, all risk sharing is being delivered through transfers. Portfolio taxes are then required to ensure that private agents do not "undo" these transfers.

2.5 Equilibrium with Incomplete Markets

We also consider an alternative setup where markets are incomplete, in the sense that there are no financial markets before the realization of the state of the world $s \in S$. We split the representative agent in country i into a continuum of households $j \in [0,1]$. Household j is assumed to own the firm of variety j. Household j maximizes utility

$$\int U^i(C_{NT}^i(s), C_T^i(s), N^i(s); s)\pi(s)ds,$$

by choosing $\{C_T^i(s), C_{NT}^i(s), N^i(s)\}$ and the prices set by its own firm $P_{NT}^{i,j}$, taking aggregate prices and wages $\{P_T(s), P_{NT}^i, W^i(s)\}$ and aggregate demand $\{\bar{C}_{NT}^i(s)\}$ as given, subject to

$$P_{NT}^i C_{NT}^i(s) + P_T(s)C_T^i(s) \leq W^i(s)N^i(s) + P_T(s)E_T^i(s) + \Pi^{i,j}(s) + T^i(s), \tag{11}$$

where

$$\Pi^{i,j}(s) = [P_{NT}^{i,j} - \frac{1+\tau_L^i}{A^i(s)}W^i(s)]\bar{C}_{NT}^i(s)\left(\frac{P_{NT}^{i,j}}{P_{NT}^i}\right)^{-\varepsilon}.$$

The corresponding first-order conditions are symmetric across j and given by (4) and (5) and the price setting condition

$$P^i_{NT} = (1+\tau^i_L)\frac{\varepsilon}{\varepsilon-1}\frac{\int \frac{U^i_{C_T}(s)}{P_T(s)}\frac{W^i(s)}{A^i(s)}\bar{C}^i_{NT}(s)\pi(s)ds}{\int \frac{U^i_{C_T}(s)}{P_T(s)}\bar{C}^i_{NT}(s)\pi(s)ds}. \tag{12}$$

In equilibrium we impose the consistency condition that $\bar{C}^i_{NT}(s) = C^i_{NT}(s)$ for all i and s.

The government budget constraint simplifies to

$$T^i(s) = \tau^i_L W^i(s)N^i(s) + \hat{T}^i(s). \tag{13}$$

We can now define an equilibrium with incomplete markets. An equilibrium specifies quantities $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$, prices and wages $\{P_T(s), P^i_{NT}, w^i(s)\}$, taxes $\{\tau^i_L, T^i(s)\}$ and international fiscal transfers $\{\hat{T}^i(s)\}$ such that households and firms maximize, the government's budget constraint is satisfied, and markets clear. More formally, the conditions for an equilibrium are given by (4), (5), (8), (11) holding with equality, (12) with $\bar{C}^i(s) = C^i(s)$, and (13).

As in the complete markets implementation, we can drop variables and constraints as follows. Given quantities, equations (5) and (12) can be used to back out certain prices, wages and taxes. Since these variables do not enter the welfare function they can be dispensed with from our planning problem, along with equations (5), (8), (11), (12), and (13). We summarize these arguments in the following proposition.

Proposition 2 (Implementability, Incomplete Markets)**.** *An allocation $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ together with prices $\{P_T(s), P^i_{NT}\}$ form part of an equilibrium with incomplete markets if and only if equations (4) and (9) hold for all $i \in I, s \in S$ and equation (10) holds for all $s \in S$.*

Propositions 1 and 2 reach the same implementability conditions for the complete- and incomplete-market settings. Although the set of implementable quantities $\{C^i_T(s), C^i_{NT}(s), N^i(s)\}$ and prices $\{P_T(s), P^i_{NT}\}$ is the same, the required policy instruments are of course different.

Under complete markets, portfolio taxes $\{\tau^i_D(s)\}$ are needed, and transfers $\{\hat{T}^i(s)\}$ are largely indeterminate. In contrast, in the incomplete market setting no restriction on private portfolios are introduced since no assets are available to private agents. In this case, transfers $\{\hat{T}^i(s)\}$ are uniquely determined as $\hat{T}^i(s) = P_T(s)(C^i_T(s) - E^i_T(s))$ and are typically state-contingent.

2.6 Homothetic Preferences

We characterize the key condition (4) further by making some weak assumptions on preferences: (i) preferences over consumption goods are weakly separable from labor; and (ii) preferences over consumption goods are homothetic. Denoting by $p^i(s) = \frac{P_T(s)}{P^i_{NT}}$ the relative price of traded goods in state s in country i, these assumptions imply that

$$C^i_{NT}(s) = \alpha^i(p^i(s); s) C^i_T(s),$$

for some function $\alpha^i(p; s)$ that is increasing and differentiable in its first argument. This conveniently encapsulates the restriction implied by the first-order condition (4). This condition is crucial because the stickiness of non-traded prices, together with the lack of monetary independence, places restrictions on the possible variability across $i \in I$, for any state of the world s, in the relative price $p^i(s)$.

3 Constrained Efficient Risk Sharing in the Static Model

In this section we characterize constrained Pareto efficient allocations. Before doing so, it is useful to briefly describe first-best allocations. They are characterized by two requirements. First, the marginal rate of substitution between labor and non-traded goods must be equal to the corresponding marginal rate of transformation[11]

$$\frac{-U^i_N(s)}{U^i_{C_{NT}}(s)} = A^i(s),$$

for every sate s and country i. Second, the standard optimal risk-sharing conditions

$$\frac{U^i_{C_T}(s)}{U^i_{C_T}(s')} = \frac{U^{i'}_{C_T}(s)}{U^{i'}_{C_T}(s')},$$

must hold for every pair of states (s, s') and countries (i, i'). For every first-best allocation, we can find Pareto weights λ^i such that $U^i_{C_T}(s) = \frac{\lambda^i}{\lambda^{i'}} U^{i'}_{C_T}(s)$ for every state s and pair of countries (i, i') and conversely, to any set of Pareto weights λ^i corresponds a first-best allocation, so that we can think of the set of first-best allocations as being indexed by these Pareto weights.

As we shall see, first-best allocations are typically not achievable in a currency union with nominal rigidities under our competitive equilibrium notion with or without complete markets. Instead we seek to characterize second best allocations that are constrained Pareto efficient among the allocations that are the outcomes of a competitive equilibrium. For this purpose, it is useful to first introduce the labor wedge

$$\tau^i(s) = 1 + \frac{1}{A^i(s)} \frac{U^i_N(s)}{U^i_{C_{NT}}(s)},$$

for every state s and country i. Labor wedges are zero at first-best allocations, but are typically not zero at equilibrium allocations. The optimal risk sharing condition above holds at allocations which are the outcomes of competitive equilibria with complete markets when there are no interventions in financial markets. They typically do not hold at allocations which are the outcomes of competitive

[11] In this and other expressions and functions we streamline the notation by leaving the dependence on some of the arguments implicit.

equilibria with complete markets when there are interventions in financial markets, or which are the outcome of competitive equilibria with incomplete markets. A fundamental result of our analysis below is that they typically do not hold at second-best allocations.

3.1 Indirect Utility Function for Transfers

Define the indirect utility function

$$V^i(C_T, p; s) = U^i\left(\alpha^i(p;s)C_T, C_T, \frac{\alpha^i(p;s)}{A^i(s)}C_T; s\right).$$

In an equilibrium with $C_T^i(s)$ and $p^i(s)$, ex-post welfare in state s in country i is then given by $V^i(C_T^i(s), p^i(s); s)$. The derivatives of the indirect utility function will prove useful for our analysis.

Proposition 3. *The derivatives of the indirect utility function are*

$$V_p^i(s) = \frac{\alpha_p^i(s)}{p^i(s)} C_T^i(s) U_{C_T}^i(s)\, \tau^i(s) \quad \textit{and} \quad V_{C_T}^i(s) = U_{C_T}^i(s)[1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)].$$

These observations about the derivatives and their connection to the labor wedge will be key to our results. A private agent values a transfer in traded goods according to its private marginal utility $U_{C_T}^i(s)$, but the actual social marginal value in equilibrium is $V_{C_T}^i(s)$. The wedge between the two equals $\frac{\alpha^i(s)}{p^i(s)}\tau^i(s) = \frac{P_{NT}^i C_{NT}(s)}{P_T(s)C_T(s)}\tau^i(s)$, the labor wedge weighted by the relative expenditure share of non-traded goods relative to traded goods. We will sometimes refer to it as the *weighted labor wedge* for short.

In particular, a private agent undervalues transfers $V_{C_T}^i(s) > U_{C_T}^i(s)$ whenever the economy is experiencing a recession, in the sense of having a positive labor wedge $\tau^i(s) > 0$. Conversely, private agents overvalue the costs of making transfers $V_{C_T}^i(s) < U_{C_T}^i(s)$ whenever the economy is booming, in the sense of having a negative labor wedge $\tau^i(s) < 0$. These effects are magnified when the economy is relatively closed, so that the relative expenditure share of non-traded goods is large.[12]

When country i receives a transfer, its consumers feel richer and increase their spending on both traded and non-traded goods in equal proportions. Since prices are fixed, the resulting increased demand for non-traded goods translates one-for-one into an increase in output. This in turn generates more income, further raising spending etc. This mechanism is at the core of the famous Transfer Problem controversy between Keynes (1929) and Ohlin (1929). These equilibrium effects, which are not internalized by private agents, open up a wedge between the social and private marginal values of transfers.

[12]Note that this definition of boom and bust is anchored in the difference between output and the potential level of output, in the sense that a boom is defined to be a situation where output is above potential. These Keynesian notions of boom and bust are the relevant ones for the allocation of transfers across countries. It is perfectly possible for output to be below potential but for potential output to be high, so that the country could be growing and experiencing a bust at the same time.

Since the increase in demand for both goods is proportional, the "dollar-for-dollar" output multiplier of transfers is precisely given by the relative expenditure share of non-traded to traded goods $\frac{P^i_{NT} C^i_{NT}(s)}{P_T(s) C^i_T(s)}$. The labor wedge $\tau^i(s)$ summarizes the net calculation for utility of the increase in non-traded consumption and the increase in labor that accompany the increase in output. This explains why the wedge between the social and private marginal valuations is precisely $\frac{P^i_{NT} C^i_{NT}(s)}{P_T(s) C^i_T(s)} \tau^i(s)$.[13]

3.2 Second-Best Ramsey Planning Problem

We consider a planning problem that allows us to characterize the set of constrained Pareto efficient allocations. The planning problem is indexed by a set of nonnegative Pareto weights λ^i. By varying these Pareto weights, we can trace out the entire constrained Pareto frontier. The second-best planning problem is

$$\max_{P_T(s), P^i_{NT}, C^i_T(s)} \int \int \lambda^i V^i \left(C^i_T(s), \frac{P_T(s)}{P^i_{NT}}; s \right) \pi(s) \, di \, ds \tag{14}$$

subject to

$$\int C^i_T(s) di = \int E^i_T(s) di.$$

Let $\mu(s)\pi(s)$ be the multiplier on the resource constraint in state $s \in S$. The first-order conditions for $C^i_T(s)$, $P_T(s)$ and P^i_{NT} are respectively

$$\lambda^i V^i_{C_T}(s) = \mu(s),$$
$$\int V^i_p(s) \frac{1}{P^i_{NT}} \lambda^i di = 0,$$
$$\int V^i_p(s) p^i(s) \pi(s) ds = 0.$$

These first-order conditions tightly characterize the solution. The first-order condition for P^i_{NT} implies our first proposition.

Proposition 4 (Optimal Price Setting)**.** *At a constrained Pareto efficient equilibrium, for every country i, a weighted average of labor wedges across states is zero*

$$\int \alpha^i_p(s) \, C^i_T(s) \, U^i_{C_T}(s) \, \tau^i(s) \, \pi(s) \, ds = 0.$$

In the absence of uncertainty this proposition implies a zero labor wedge in each state. With uncertainty, in general the labor wedge for a given country takes on both signs across states with a

[13]It is theoretically possible for the marginal value of a transfer to be negative $V^i_{C_T}(s) < 0$ if the labor wedge is sufficiently negative, especially if the share of non-traded goods, relative to traded goods, is large enough. In this extreme case a country can improve welfare by making gift transfers, without any counterpart transfer in the opposite direction. If τ^i is sufficiently negative then unilateral gift transfers to other countries are welfare enhancing for country i. This extreme case will not be our focus and is not employed in any of our results below. However, it is a stark example of just how divergent public and private valuations of transfers can become.

weighted average of zero.[14]

The first-order condition for $P_T(s)$ implies the following proposition.

Proposition 5 (Optimal Monetary Policy)**.** *At a constrained Pareto efficient equilibrium, in every state s, a weighted average of labor wedges across countries is zero*

$$\int \alpha_p^i(s) C_T^i(s) U_{C_T}^i(s)\, \tau^i(s) \lambda^i di = 0.$$

This proposition establishes that optimal monetary policy targets a weighted average across countries for the labor wedge. It sets this target to zero in each state of the world. The intuition for the result is that monetary policy can be chosen at the union level, and can adapt across states to the average condition. If all countries are identical and the shock is symmetric, then we obtain perfect stabilization in each country: $\tau^i(s) = 0$ for all $i \in I, s \in S$. By contrast, when shocks across countries are not symmetric then perfect stabilization is impossible. However, at the union level the economy is stabilized in the sense that the weighted average for the labor wedge across countries is set to zero for all states of the world $s \in S$.[15]

Finally, the first-order condition for $C_T(s)$ says that the marginal utility of transfers in traded goods adjusted for the Pareto weight $\lambda^i V_{C_T}^i(s)$ should be equalized across countries for every state s, or in other words that for every pair of states (s, s'), and pair of countries (i, i'), we have

$$\frac{V_{C_T}^i(s)}{V_{C_T}^i(s')} = \frac{V_{C_T}^{i'}(s)}{V_{C_T}^{i'}(s')}.$$

It is more revealing to rewrite this condition using our expressions for the derivative of $V_{C_T}^i(s)$.

Proposition 6 (Constrained Efficient Risk Sharing)**.** *At a constrained Pareto efficient equilibrium, for every pair of states (s, s'), and pair of countries (i, i'), risk sharing takes the following form*

$$\frac{U_{C_T}^i(s)[1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)]}{U_{C_T}^i(s')[1 + \frac{\alpha^i(s')}{p^i(s')}\tau^i(s')]} = \frac{U_{C_T}^{i'}(s)[1 + \frac{\alpha^{i'}(s)}{p^{i'}(s)}\tau^{i'}(s)]}{U_{C_T}^{i'}(s')[1 + \frac{\alpha^{i'}(s')}{p^{i'}(s')}\tau^{i'}(s')]}. \tag{15}$$

If markets are complete and portfolio taxes are not employed, then the risk sharing condition (3) imposes the additional constraint that for every pair of states (s, s') and countries (i, i'), we have

$$\frac{U_{C_T}^i(s)}{U_{C_T}^i(s')} = \frac{U_{C_T}^{i'}(s)}{U_{C_T}^{i'}(s')}. \tag{16}$$

[14] In the absence of uncertainty, the labor tax to cancel the monopolistic markup $\tau_L^i = -1/\varepsilon$. With uncertainty, in general $\tau_L^i \neq -1/\varepsilon$. When the sub-utility function between C_{NT} and C_T is a CES so that $\alpha(\cdot; s)$ has constant elasticity, independent of s, then $\tau_L^i = -1/\varepsilon$ is optimal even with uncertainty. The proof is contained in the Appendix A.3.

[15] The result is related to the result in Benigno (2004) and Gali and Monacelli (2008) that optimal monetary policy in a currency union ensures that the union average output gap, in a linearized version of the model, is zero in every period. Here the result is obtained without linearizing the model and it is expressed in terms of the labor wedge, instead of the output gap.

Comparing these conditions, one may expect the private risk sharing condition (16) to be incompatible with the constrained efficient risk sharing condition (15) except in special cases. Indeed, we next show that because labor wedges must average to zero across states and countries according to Propositions 4 and 5, they are indeed incompatible unless the first best is attainable. This implies that equilibria with privately optimal risk sharing are constrained Pareto inefficient.[16]

Proposition 7 (Inefficiency of Private Risk Sharing)**.** *An equilibrium with complete markets and no portfolio taxes ($\tau_D^i(s) = 0$ for all $i \in I, s \in S$) is constrained Pareto inefficient unless $\tau^i(s) = 0$ for all $i \in I, s \in S$, in which case it is first best.*

With complete markets but without interventions in financial markets, private agents do not secure the constrained efficient amount of risk sharing in financial markets. They do not fully internalize the macroeconomic stability consequences of their portfolio decisions, opening a role for government intervention in financial markets.[17]

3.3 Implementation

We now turn to the implementation of constrained Pareto efficient allocations. With complete markets, constrained Pareto efficient equilibria can be decentralized with appropriate labor taxes τ_L^i and corrective portfolio taxes $\tau_D^i(s)$. Proposition 6 leads to a neat characterization of the required taxes.

Proposition 8 (Complete Markets and Portfolio Taxes)**.** *If private asset markets are complete, constrained Pareto efficient allocations can be implemented by the following portfolio return subsidy/taxes*

$$\tau_D^i(s) = \frac{\alpha^i(s)}{p^i(s)} \tau^i(s).$$

Insurance for bad states of the world, where the weighted labor wedge is high, should be relatively subsidized. Government intervention secures additional transfers from low weighted labor wedge countries ("boom" countries) to high weighted labor wedge countries ("bust" countries). This reduces the demand for non-traded goods in the boom countries and increases it in the bust countries, stabilizing output and income. These stabilization benefits are not internalized by private agents, hence the need for government intervention.

It is interesting to note that the taxes do not depend directly on the Pareto weights λ^i, but only indirectly through the relative expenditure share of non-traded goods and the labor wedge. This

[16]Note that our definition of constrained efficient risk sharing is a second best concept which is different from the concept of first best risk sharing. At the first best, both the constrained efficient risk sharing condition (15) and the privately optimal risk sharing condition (16) hold, because the labor wedges are all equal to zero. At the second best, when it is different from the first best, condition (15) holds but (16) does not. Another way of stating our result is that at the second best, it is optimal to deviate from the target of first-best risk sharing in order to help achieve the other target of stabilizing the economy.

[17]We should also point out that the Propositions 5 and 6 go through if non-traded goods prices are entirely predetermined (i.e. are exogenously fixed).

underscores the fact that they are imposed to correct a macroeconomic aggregate demand externality and not to redistribute.

The implementation of the social optimum with corrective portfolio taxes is only one interesting possibility. Another equally interesting interpretation of our results assumes that private asset markets are nonexistent, so that private opportunities for risk sharing are unavailable. The same optimum can then be implemented through state-contingent transfers.[18]

Proposition 9 (Incomplete Markets and Ex-Post Transfers)**.** *If private asset markets are incomplete so that state-contingent assets are unavailable, constrained Pareto efficient allocations can also be implemented through state-contingent transfers*

$$\hat{T}^i(s) = P_T(s)(C_T^i(s) - E^i(s)).$$

Under this alternative implementation, no restriction on private portfolios are needed since no assets are available to private agents. Our results can then be seen as offering a precise characterization of the required transfers. A key conclusion of our analysis is that these transfers would go beyond replicating the outcome that private risk sharing decisions would achieve if markets were complete.

It is also possible to imagine implementations that are in between the two polar cases of corrective portfolio taxes with complete markets and state-contingent transfers with incomplete markets. In general, government positions in asset markets, or state-contingent transfers, combined with restrictions or tax incentives on private portfolios are required.

It is important to stress that our results hold for any constrained Pareto efficient allocation, or equivalently, for any set of Pareto weights λ^i. They are not driven by a desire to redistribute across countries and instead reflect an insurance motive. One way to see this is that in general it is always possible to find Pareto weights λ^i such that $\int \hat{T}^i(s)\pi(s)ds = 0$ for every country i. For example, in the special case where countries are ex-ante identical these Pareto weights are equal $\lambda^i = \lambda^{i'}$ for all pair of countries (i, i'). Our results hold for this particular set of Pareto weights. Except in knife edge circumstances where the corresponding allocation is first best, portfolio taxes will be required for its implementation if markets are complete, and transfers will go beyond replicating complete markets if markets are incomplete.

[18] We could also envision intermediate cases of incomplete markets, in which case both portfolio taxes and transfers will be required in conjunction. In these cases, transfers are useful and required to the extent that they expand risk-sharing possibilities beyond those available through financial markets, and portfolio taxes are required to influence risk-sharing through financial markets. Assume for simplicity that S is discrete, that there are N aggregate states of nature, and that the space $\mathcal{A}$ of asset payoffs that is available through traded securities is of dimension $M \leq N$. Then the decentralization requires both portfolio taxes and fiscal transfers. In general, to implement the optimum in that way requires access to a set state-contingent fiscal transfers $\mathcal{F}$ such that $\mathcal{A} + \mathcal{F} = \mathcal{R}^N$, which implies that $\dim(\mathcal{F}) = N - \dim(\mathcal{A}) + \dim(A \cap \mathcal{F}) \geq N - M$.

3.4 Countries Outside the Currency Union

Up to this point we have assumed that all countries belong to the currency union. Now, imagine that only a subset of countries $I \subseteq [0,1]$ are members. The rest manage monetary policy independently as follows. Country $i \notin I$ sets its own local nominal price for the traded good $P^i_T(s) = E^i(s)P_T(s)$ in its home currency by manipulating the level of its exchange rate $E^i(s)$ against the union's currency.[19] The planning problem becomes

$$\max \int_{i\in I} \lambda^i V^i \left(C^i_T(s), \frac{P_T(s)}{P^i_{NT}}; s \right) di + \int_{i\notin I} \lambda^i V^i \left(C^i_T(s), \frac{P^i_T(s)}{P^i_{NT}}; s \right) di \tag{17}$$

subject to

$$\int C^i_T(s) di = \int E^i_T(s) di.$$

For a country $i \notin I$ outside the union, the first-order condition for $P^i_T(s)$ is

$$V^i_p(C^i_T(s), p^i(s); s) = 0.$$

Since $V^i_p(C^i_T(s), p^i(s); s) = \frac{\alpha^i_p(s)}{p^i(s)} C^i_T(s) U^i_{C_T}(s)\, \tau^i(s)$, it follows that

$$\tau^i(s) = 0 \qquad \text{for all } s \in S,\ i \notin I.$$

A flexible exchange rate leads to perfect stabilization, in the sense that the labor wedge is set to zero for all states of the world. This result is reminiscent of the arguments set forth by Friedman (1953) and Mundell (1961) in favor of flexible exchange rates. For countries in the currency union, optimal monetary policy is still imperfect and characterized by the average condition for the labor wedge in Proposition 5.

The constrained efficient risk sharing condition in Proposition 6 still applies to all countries, inside or outside the currency union. However, since $\tau^i(s) = 0$ for $s \in S, i \notin I$, this condition coincides with the privately optimal risk sharing condition for countries outside the currency union. As a result, there is no need to upset private risk sharing.

Proposition 10 (Countries Outside the Currency Union)**.** *None of the results are affected by considering countries outside the union. Countries that have independent monetary policy manage to obtain a zero labor wedge $\tau^i(s) = 0$. If markets are complete, they should not intervene in financial markets $\tau^i_D(s) = 0$. If markets are incomplete, they should seek to secure ex-post transfers $\hat{T}^i(s)$ that replicate privately optimal risk sharing outcomes.*

If markets are incomplete, then transfers might be required even outside a currency union. Interestingly, we will show that, in the dynamic version of the model with only traded goods and

[19] Since the price of traded goods is modeled as flexible here, we do not require assumptions about Producer Currency Pricing (PCP) versus Local Currency Pricing (LCP).

home bias in preferences, there are cases (the Cole-Obstfeld case) where transfers are not required for countries outside a currency union, whereas they are required for countries inside a currency union.

Crucially, our results establish that inside a currency union, transfers should go beyond replicating the outcome that would arise if markets were complete. In this sense, our results yield two important insights. First, currency unions and fiscal unions go hand in hand. Second, fiscal integration and financial integration are not perfect substitutes.

How are attitudes towards risk affected by membership in a union? We show that members are more risk averse in the following sense. Suppose country i belongs to the currency union with equilibrium relative price $p^i(s)$. The advantage of leaving the union is that the relative price p^i is not constrained and welfare attains the first best level conditional on C_T^i. Define $V^{i*}(C_T^i; s) = \max_p V^i(C_T^i, p; s)$. It follows that

$$V^i(C_T^i, p^i(s); s) \leq V^{i*}(C_T^i; s), \tag{18}$$

with equality if and only if $p^i(s) \in \arg\max_p V^i(C_T^i, p; s)$, in which case the labor wedge is zero, $\tau(s) = 0$. Thus, for every state s, the function V^{i*} is the upper envelope over $V^i(C_T^i, p; s)$ for different values of p and is tangent to it precisely at a level of p that implies a zero labor wedge. In this sense, $V^i(C_T^i, p^i(s); s)$ is more concave than V^{i*} and member countries are more risk averse. We shall put this inequality to use in the next section.

3.5 Value of Risk Sharing

Our simple model allows for three random disturbances: (i) shocks to productivity of labor in the production of non-traded goods; (ii) shocks to preferences (demand); and (iii) shocks to the endowment of traded goods. Proposition 7 shows that if the equilibrium with complete markets but without portfolio taxes does not attain the first best, then it is constrained inefficient. As we show next, this is the case except in knife-edge cases. Examining these knife-edge cases turns out to be interesting, because even when the equilibria coincide with the first best we find that the planner values the availability of insurance strictly more than private agents do, and do in that sense, risk sharing is of greater social than private value. Extrapolating beyond our model, this could help explain why macro insurance markets may be missing, even if their social value is significant.

To engineer an example where the first best is attainable it is useful to specialize the model by using the utility function

$$U^i(C_T, C_{NT}, N; s) = \log(C_T) + \alpha^i(s)\log(C_{NT}) - \frac{1}{1+\phi}N^{1+\phi}, \tag{19}$$

with $\phi \geq 0$.

Proposition 11. *Suppose that the utility function is given by* (19)*, then the equilibrium with complete markets but without portfolio taxes is constrained efficient if and only if it is first best, which occurs if and only if* *productivity shocks and preference shocks are such for all pairs of countries (i, i'), the ratio*

$$\frac{A^i(s)}{A^{i'}(s)}\left(\frac{\alpha^i(s)}{\alpha^{i'}(s)}\right)^{\frac{-\phi}{1+\phi}}$$

is constant for all $s \in S$.

The key constraint imposed by nominal rigidities and a single monetary policy is condition (4), rewritten here for convenience as

$$\frac{U^i_{C_{NT}}(s)}{U^i_{C_T}(s)} = \frac{P^i_{NT}}{P_T(s)}$$

where $P_T(s)$ is only allowed to vary with s not i, while P^i_{NT} is allowed to vary with i but not s. One can handle fixed differences across countries and union-wide shocks to this marginal rate of substitution, but not individual variations.

Proposition 11 defines a precise notion of symmetric shocks to productivity and preferences for which the first best allocation is attainable without portfolio taxes. For example, if the only shocks are to productivity, then this condition requires that productivity vary proportionally across countries. A currency union can handle such a shock using union-wide monetary policy. A similar point applies to taste shocks.

This discussion highlights just how special these circumstances are. Note, however, that the proposition implies that traded good endowment shocks can be properly insured with complete markets and without portfolio taxes. To understand this result, suppose we only have shocks to the endowment of traded goods. Then the first best features perfect risk sharing in the consumption of traded goods so that only aggregate fluctuations in the endowment of traded goods affect the consumption of traded goods. Due to the separability of preferences, the first best allocation for non-traded goods and labor is not affected by these shocks. It follows that the marginal rate of substitution only varies with union-wide shocks, and the first best is implementable as an equilibrium.[20,21]

It is useful to have a case, however artificial, where private risk sharing is constrained efficient so that we can isolate a separate result. We show that members of a currency union value this risk sharing more than countries outside it. Moreover, this is is not true of the value placed on risk sharing by private individuals. This highlights the role of the aggregate demand externality from risk sharing decisions, which is not internalized by private agents.

[20] In more detail, suppose $A^i(s) = A^i$ and $\alpha^i(s) = \alpha^i$. The first best allocation features $C^i_T(s) = \frac{1}{\lambda^i}\int_0^1 E^i(s)di$, $N^i(s) = (\alpha^i)^{\frac{1}{1+\phi}}$, and $C^i_{NT}(s) = A^i(\alpha^i)^{\frac{1}{1+\phi}}$. This allocation is supported as an equilibrium without portfolio taxes by $P^i_{NT} = (\alpha^i)^{\frac{\phi}{1+\phi}}/(\lambda^i A^i)$, $P_T(s) = (\int_0^1 E^i(s)di)^{-1}$, $W^i(s) = (\alpha^i)^{\frac{\phi}{1+\phi}}/\lambda^i$, $Q(s) = 1$ and $1+\tau^i_L = \frac{\epsilon-1}{\epsilon}$.

[21] Of course, the case of traded good endowment shocks is somewhat artificial, relying on the modeling asymmetry that non-traded goods are produced but traded goods are not. If instead traded goods were produced from labor and another fixed input (capital or land) subject to (industry specific) productivity shocks, then these shocks would also have to satisfy the restriction of being symmetric to attain the first best—just as in the case of productivity shocks in the non-traded goods.

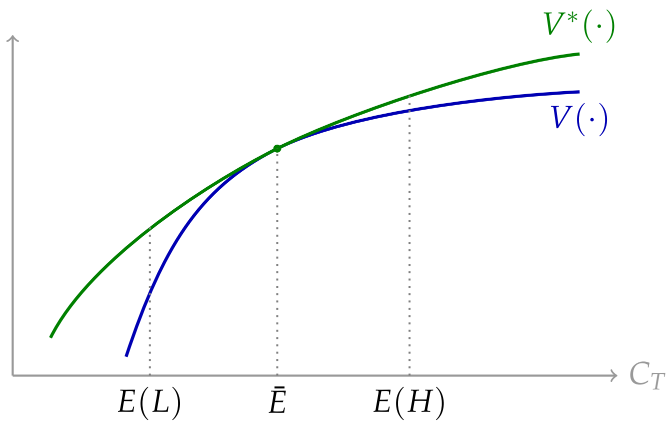

Figure 1: Welfare as perceived by individual agents (upper green curve) and country as a whole (lower blue curve).

Proposition 12. *Suppose that there are only traded good endowment shocks and that all risk is idiosyncratic, so that the aggregate endowment of traded goods is constant across states. If we exclude a country from risk sharing, then its utility loss is greater if it belongs to a currency union. If we exclude a single individual within a country from risk sharing, then the utility loss is the same whether or not his country belongs to a currency union.*

Figure 1 illustrates the basic logic behind the first part this proposition for an example with two equiprobable endowment values. Since the aggregate endowment is constant, the price of traded goods is constant and complete markets without portfolio taxes offer fair insurance. The resulting equilibrium features constant consumption of the traded good at the average value of the endowment and constant prices and wages. This is true for both members and non members. When the country is excluded from risk sharing its consumption of the traded good must now fluctuate with its endowment, creating a mean-preserving spread in consumption of traded goods and a loss in expected utility. The crucial point is that the loss is greater for union members because they are more risk averse, according to inequality (18). Indeed, given that prices are constant and the utility function is independent of the state s this inequality simplifies to $V^i(C^i_T, \bar{p}) \leq \max_p V^i(C^i_T, p) = V^{i*}(C^i_T)$. These two indirect utility functions are depicted in the figure. They are tangent at the average value of the endowment $\bar{E}$ because this represents the equilibrium consumption level with risk sharing.

As to the second part of the proposition, it follows easily from the observation that in the context of this specific example, the equilibrium with risk sharing is the same whether or not the country belongs to the currency union. In both cases the first best allocation is attained. Therefore, if an individual is excluded from risk sharing, he faces the same prices whether the country is a member or not. Thus, the drop in utility is the same.

3.6 Insurance vs. Incentives: Limited Commitment and Moral Hazard

Explicit or implicit insurance arrangements inevitably raise concerns of incentives. We have abstracted from these considerations, not because we believe them to be unimportant, but in order to isolate the effects of the aggregate demand externality on optimal risk sharing. Modeling them requires making specific choices regarding the underlying agency problems.The possibilities are vast and exploring them all is beyond the scope of this paper, but we believe that the main insights of our analysis would carry over.[22]

In the appendix, we analyze two examples, one where countries have a limited ability to commit to paying international transfers to other countries (Appendix B.2) and one where countries can engage in a form of moral hazard which influences the distribution of states of the world (Appendix B.3). In both cases, there is a meaningful tradeoff between insurance and incentives, and the optimality condition characterizing the optimal resolution of this tradeoff involves the social marginal utility of transfers, a key sufficient statistic which is different for countries in a currency union where it is given by $U^i_{C_T}(s)[1 + \frac{\alpha^i(s)}{p^i(s)}\tau^i(s)]$ than for countries with a flexible exchange rate where it is given by $U^i_{C_T}(s)$.

4 A Dynamic Model

The static model reveals some key results in a simple and transparent manner. However, it is perhaps too simple to explore the issues in greater depth, and in particular to think about two key determinants of fiscal unions: price adjustment dynamics and the persistence of shocks. We now build a richer, dynamic model similar to Farhi and Werning (2012) which in turn builds on Gali and Monacelli (2005, 2008). We present the model with incomplete markets where agents can only trade short-term risk free bonds as in Farhi and Werning (2012).[23] Just as in the static model, we tackle the optimal design of international state-contingent fiscal transfers.

In Section 5, we analyze the model using a log-linear approximation of the nonlinear model around its perfect foresight symmetric steady state. We derive an impulse response characterization of optimal transfers following a shock. It is well understood that in log-linearized models, agents behave according to a certainty equivalence principle, and that the impulse response to a particular shock is independent of the probability distribution of shocks in the nonlinear model. For these reasons, and to simplify the exposition, we focus our presentation of the nonlinear model in this section on the perfect foresight case where one-time unanticipated shocks to the path of productivity

[22] In practice of course, institutional mechanisms exist to mitigate these agency problems. For example, most fiscal unions such as the US channel a large part of their transfers through more or less rules-based automatic stabilizers (through the unemployment insurance program, federal income and social security taxes, bailout funds), probably for reasons of political acceptability and transparency, but also to mitigate the difficulties associated with collecting and distributing discretionary transfers ex post. Other examples are state debt limits in the US, and collective budget procedures and enforcement mechanisms that already exist in Europe.

[23] We will also compare it to the complete financial market case when we turn to the log-linearized version of the model in Section 5.

in each country are realized at $t = 0$, with no further uncertainty.[24] The assumption of incomplete markets translates into the requirement that absent international transfers, the initial net foreign asset position of any given country be independent of the particular realization of the shock. International transfers between countries can be arranged contingent on the realization of shocks and it is precisely the optimal arrangement of these transfers in response to shocks that we seek to characterize.[25]

4.1 Households

There is a continuum of measure one of countries $i \in [0,1]$. We focus attention on a single country, which we call Home, and which can be thought of as a particular value $H \in [0,1]$. In every country, there is a representative household with preferences represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right],$$

where N_t is labor, and C_t is a consumption index defined by $C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$. Here $C_{H,t}$ is an index of consumption of domestic goods given by $C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$, where $j \in [0,1]$ denotes an individual good variety. Similarly, $C_{F,t}$ is a consumption index of imported goods given by $C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$, where $C_{i,t}$ is, in turn, an index of the consumption of varieties of goods imported from country i, given by $C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$.

Thus ϵ is the elasticity between varieties produced within a given country, η the elasticity between domestic and foreign goods, and γ the elasticity between goods produced in different foreign countries. An important special case is the Cole-Obstfeld case which obtains when $\sigma = \eta = \gamma = 1$. This case, which was first studied by Cole and Obstfeld (1991), is more tractable and has some special implications which we will highlight later in Sections (5)-(6).

The parameter α indexes the degree of home bias and can be interpreted as a measure of openness. As α approaches zero, the share of foreign goods vanishes. As α approaches one, the share of home goods vanishes. Since the country is infinitesimal, the latter captures a very open economy without home bias, while the former captures a closed economy barely trading with the outside world.

[24] Consider a fully stochastic version of the nonlinear model where shocks are drawn from a given distribution in every period starting at $t = 0$. Take the point-wise limit when the variance of the distribution of shocks goes to zero of the impulse response to a given shock of the log-linearized version of this model. It coincides with the impulse response of the log-linearized version of the perfect foresight model to a one-time unanticipated shock. Because markets are incomplete, the performance of the log-linear approximation of the perfect foresight model as an approximation of the fully stochastic version of the nonlinear deteriorates in the long run as the economy moves away from the steady state in response to successive shocks, but because of discounting, this does not affect the validity of our characterization of optimal transfers and of their benefits in response to shocks at $t = 0$. These issues do not arise in the perfect foresight version of the model with one-time unanticipated shocks.

[25] As should be clear from this description, our notion of market incompleteness is that markets are incomplete across states of the world but not over time.

Households seek to maximize their utility subject to the sequence of budget constraints

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + D_{t+1} + \int_0^1 E_{i,t}D^i_{t+1}di$$
$$\leq W_t N_t + \Pi_t + T_t + (1+i_{t-1})D_t + \int_0^1 E_{i,t}(1+i^i_{t-1})D^i_t di$$

for $t = 0, 1, 2, \ldots$ In this inequality, $P_{H,t}(j)$ is the price of domestic variety j, $P_{i,t}(j)$ is the price of variety j imported from country i, W_t is the nominal wage, Π_t represents nominal profits, and T_t is a nominal lump sum transfer. All these variables are expressed in domestic currency. The portfolio of home agents is composed of home and foreign bond holding: D_t is home bond holdings of home agents, D^i_t is bond holdings of country i of home agents. The returns on these bonds are determined by the nominal interest rate in the home country i_t, the nominal interest rate i^i_t in country i, and the evolution of the nominal exchange rate $E_{i,t}$ between Home and country i.

4.2 Firms

Technology. A typical firm in the home economy produces a differentiated good with a linear technology given by

$$Y_t(j) = A_{H,t}N_t(j),$$

where $A_{H,t}$ is productivity in the home country. We denote productivity in country i by $A_{i,t}$.

We allow for a constant employment tax $1 + \tau^L$, so that real marginal cost deflated by Home PPI is given by

$$MC_t = \frac{1+\tau^L}{A_{H,t}} \frac{W_t}{P_{H,t}}.$$

We take this employment tax to be constant in our model. We pin down this tax rate by assuming that it is set optimally and cooperatively set at a symmetric steady state with flexible prices. The tax rate is simply set to offset the monopoly distortion so that $\tau^L = -\frac{1}{\varepsilon}$.

Price-setting assumptions. As in Gali and Monacelli (2005), we maintain the assumption that the Law of One Price (LOP) holds so that at all times, the price of a given variety in different countries is identical once expressed in the same currency. This assumption is known as Producer Currency Pricing (PCP) and is sometimes contrasted with the assumption of Local Currency Pricing (LCP), where each variety's price is set separately for each country and quoted (and potentially sticky) in that country's local currency. Thus, LOP does not necessarily hold. It has been shown by Devereux and Engel (2003) that LCP and PCP may have different implications for monetary policy. However, for our purposes, these two polar cases are equivalent since, for the most part, we will study the model assuming fixed exchange rates.

We consider Calvo price setting, where in every period, a randomly selected fraction $1 - \delta$ of firms can reset their prices. Those firms that get to reset their price choose a reset price P_t^r to solve

$$\max_{P_t^r} \sum_{k=0}^{\infty} \delta^k \left(\prod_{h=1}^{k} \frac{1}{1+i_{t+h}} \right) \left(P_t^r Y_{t+k|t} - P_{H,t} MC_t Y_{t+k|t} \right)$$

where $Y_{t+k|t} = \left(\frac{P_t^r}{P_{H,t+k}} \right)^{-\epsilon} C_{t+k}$, taking the sequences for MC_t, Y_t, and $P_{H,t}$ as given.

4.3 Government

The government is subject to the budget constraint

$$T_t = \tau_L W_t N_t + \hat{T}_t. \tag{20}$$

Here $\hat{T}_t$ are net international fiscal transfers. The transfers $\hat{T}_t$ are the focus of our analysis. More precisely, we allow for international fiscal transfers across countries, contingent on the shocks experienced by these countries.

4.4 Terms of Trade and Real Exchange Rate

It is useful to define the following price indices: home's Consumer Price Index (CPI) $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$, home's Producer Price Index (PPI) $P_{H,t} = [\int_0^1 P_{H,t}(j)^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$, and the index for imported goods $P_{F,t} = [\int_0^1 P_{i,t}^{1-\gamma} di]^{\frac{1}{1-\gamma}}$, where $P_{i,t} = [\int_0^1 P_{i,t}(j)^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$ is country i's PPI.

Let $E_{i,t}$ be the nominal exchange rate between home and i (an increase in $E_{i,t}$ is a depreciation of the home currency). Because the Law of One Price holds, we can write $P_{i,t}(j) = E_{i,t} P_{i,t}^i(j)$ where $P_{i,t}^i(j)$ is country i's price of variety j expressed in its own currency. Similarly, $P_{i,t} = E_{i,t} P_{i,t}^i$ where $P_{i,t}^i = [\int_0^1 P_{i,t}^i(j)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$ is country i's domestic PPI in terms of country i's own currency. We therefore have $P_{F,t} = E_t P_t^*$, where $P_t^* = [\int_0^1 P_{i,t}^{i\,1-\gamma} di]^{\frac{1}{1-\gamma}}$ is the world price index and E_t is the effective nominal exchange rate.[26]

The effective terms of trade are defined by

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}},$$

where $S_{i,t} = P_{i,t}/P_{H,t}$ is the terms of trade of home versus i. The terms of trade can be used to rewrite the home CPI as $P_t = P_{H,t}[1-\alpha+\alpha S_t^{1-\eta}]^{\frac{1}{1-\eta}}$.

Finally we can define the real exchange rate between home and i as $\mathcal{Q}_{i,t} = E_{i,t} P_t^i / P_t$ where P_t^i is country'i's CPI. We define the effective real exchange rate to be

$$\mathcal{Q}_t = \frac{E_t P_t^*}{P_t}.$$

[26]The effective nominal exchange rate is defined as $E_t = [\int_0^1 E_{i,t}^{1-\gamma} P_{i,t}^{i\,1-\gamma} di]^{\frac{1}{1-\gamma}} / [\int_0^1 P_{i,t}^{i\,1-\gamma} di]^{\frac{1}{1-\gamma}}$.

4.5 Equilibrium Conditions

We now summarize the equilibrium conditions. Equilibrium in the home country can be described by the following equations. We find it convenient to group these equations into two blocks, which we refer to as the demand block and the supply block.

The demand block is independent of the nature of price setting. It is composed of the Backus-Smith condition

$$C_t = \Theta^i C_t^i \mathcal{Q}_{i,t}^{\frac{1}{\sigma}},$$

where Θ^i is a relative Pareto weight which depends on the realization of the shocks, the goods market clearing condition

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1 C_t^i (S_t^i S_{i,t})^{\gamma-\eta}\mathcal{Q}_{i,t}^{\eta} di\right],$$

were S_t^i is denotes the effective terms of trade of country i, the labor market clearing condition

$$N_t = \frac{Y_t}{A_{H,t}}\Delta_t$$

where Δ_t is an index of price dispersion $\Delta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}$, the Euler equation

$$1 + i_t = \beta^{-1}\frac{C_{t+1}^{\sigma}}{C_t^{\sigma}}\Pi_{t+1}$$

where $\Pi_t = \frac{P_{t+1}}{P_t}$ is CPI inflation, the arbitrage condition between home and foreign bonds

$$1 + i_t = (1 + i_t^i)\frac{E_{i,t+1}}{E_{i,t}},$$

for all $i \in [0,1]$, and the country budget constraint

$$\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1}{1+i_s}\right)\hat{T}_t = -\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1}{1+i_s}\right)(P_{H,t}Y_t - P_t C_t)\,. \tag{21}$$

Equation (21) is central to our analysis and it is useful to unpack it. We define NFA_0 as the net present value of transfers received by Home expressed in terms of the initial home CPI

$$NFA_0 = \frac{1}{P_0}\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1}{1+i_s}\right)\hat{T}_t.$$

Equation (21) can be restated as equating the net present value of transfers to the net present value of trade deficits

$$NFA_0 = -\frac{1}{P_0} \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \frac{1}{1+i_s} \right) (P_{H,t} Y_t - P_t C_t) .$$

This makes clear that the net present value of transfers NFA_0 acts like a de facto like initial net foreign asset position which depends on the shock. With ex-ante identical countries, absent international fiscal transfers, we would have $NFA_0 = 0$. Characterizing the optimal value of NFA_0 depending on the shocks is one the main focuses of our analysis below. We will also compare the optimal value of NFA_0 to the value that would obtain if private agents could engage in risk sharing through a complete set of financial markets. One of our main results will establish that these values differ, and to characterize how they differ.

Finally with Calvo price setting, the supply block is composed of the equations summarizing the first-order condition for optimal price setting. These conditions are provided in Appendix C.1. We will only analyze a log-linearized version of the model with Calvo price setting (see Section 5).

For most of the paper, we will be concerned with fixed exchange rate regimes (either pegs or currency unions) in which case we have the additional restriction that $E_t = E_0$ for all $t \geq 0$ where E_0 is predetermined.

5 Optimal Transfers in the Dynamic Model

As is standard in the literature, we work with a log-linearized approximation of the model around a symmetric steady state where productivity (and hence output) in all countries is equal to one. As before, at $t = 0$, the economy is hit with an unanticipated shock. It is convenient to work with a continuous time version of the model. This does not affect our results, but it is useful because it implies that no price index can jump at $t = 0$ and this simplifies the derivation of initial conditions characterizing the equilibrium. We denote the instantaneous discount rate by ρ, and the instantaneous arrival rate for price changes by ρ_δ.

From now on we focus on the Cole-Obstfeld case $\sigma = \eta = \gamma = 1$. This case is attractive for the following reason. Even when prices are sticky, the allocation with incomplete markets and no transfers coincides with the equilibrium with complete markets and no interventions in financial markets. Risk sharing is delivered with balanced trade. This means that we can interpret any deviation from balanced trade at the optimum with transfers as an indication that private risk sharing through complete financial markets, if they were available, would be constrained inefficient. Third, it is possible to derive a simple second-order approximation of the welfare function around the symmetric deterministic steady state. Away from the Cole-Obstfeld case, this derivation is much more involved.

We start by considering the case where all countries are members of the same currency union. We then consider the case where some countries are in a currency union, while others remain outside, with a flexible exchange rate and independent monetary policy.

5.1 The Log-Linearized Economy

We denote with lowercase variables the log deviations from the symmetric steady state of the corresponding uppercase variable introduced in Section 4. We denote with a star the union average of a given variable. For example $\tilde{y}_t^* = \int_0^1 \tilde{y}_t^i di$, $\tilde{c}_t^* = \int_0^1 \tilde{c}_t^i di$, and $a_t^* = \int_0^1 a_{i,t} di$.

Shocks. The shocks hitting the model are $a_{i,t}$. They have both an aggregate component a_t^* and an idiosyncratic component $a_{i,t} - a_t^*$. We can think of the aggregate component as reflecting symmetric shocks and of the idiosyncratic component as capturing asymmetric shocks.

The natural allocation. We define the natural allocation as the flexible price allocation when there are no transfers across countries. We describe this allocation in log deviations from the symmetric steady state with a lower case, and tilde.

At the natural allocation, output in country i is given by $\tilde{y}_t^i = a_{i,t}$, consumption is given by $\tilde{c}_t^i = \alpha a_t^* + (1-\alpha) a_{i,t}$, labor is given by $\tilde{n}_t^i = 0$, and the terms of trade are given by $\tilde{s}_t{}^i = a_{i,t} - a_t^*$, where by construction $\int_0^1 \tilde{s}_t^i di = \int_0^1 \tilde{y}_t^i di = \int_0^1 \tilde{a}_{i,t} di = 0$.[27] In addition, trade is balanced. Finally, aggregate output is equal to aggregate consumption and is given by $\tilde{y}_t^* = \tilde{c}_t^* = a_t^*$.

Summarizing the system in gaps. We denote by $\hat{y}_t^i$ and $\hat{\theta}^i$ the deviations of y_t^i and θ^i from their natural counterparts. We further define the gaps of these variables to their respective union average as $\hat{\hat{y}}_t^i = \hat{y}_t^i - \hat{y}_t^*$, $\hat{\hat{\theta}}^i = \hat{\theta}^i - \hat{\theta}^*$, and $\hat{\pi}_{H,t}^i = \pi_{H,t}^i - \pi_t^*$. We have $\int \hat{\hat{y}}_t^i di = \int \hat{\hat{\theta}}^i di = \int \hat{\pi}_{H,t}^i di = 0$. Note that $\hat{\theta}^i$ is already a normalized variable so that $\hat{\theta}^* = 0$ and $\hat{\theta}^i = \hat{\hat{\theta}}^i$.

The deviation of the trade balance from its natural counterpart of zero is constant and equal to $-\alpha \hat{\hat{\theta}}^i$. The net present value of transfers must pay for the net present value of the trade deficits, so that $N\hat{\hat{F}}A_0^i = \frac{\alpha}{\rho} \hat{\hat{\theta}}^i$.

The disaggregated variables solve two ordinary differential equations, namely the Phillips curve

$$\dot{\hat{\pi}}_{H,t}^i = \rho \hat{\pi}_{H,t}^i - \kappa_y \hat{\hat{y}}_t^i - \lambda \alpha \hat{\hat{\theta}}^i$$

and the Euler equation

$$\dot{\hat{\hat{y}}}_t^i = -\hat{\pi}_{H,t}^i - \dot{\tilde{s}}_t^i,$$

with the initial condition

$$\hat{\hat{y}}_0^i = (1-\alpha) \hat{\hat{\theta}}^i - \tilde{s}_0^i,$$

where $\lambda = \rho_\delta(\rho + \rho_\delta)$ and $\kappa_y = \lambda(1+\phi)$ index price flexibility. We assume that the zero lower bound on the nominal interest is not binding. Then the only constraint on the aggregates is that they must satisfy the aggregate New Keynesian Philips Curve

$$\dot{\pi}_t^* = \rho \pi_t^* - \kappa_y \hat{y}_t^*.$$

[27] Although we do not need it for our analysis, note that the natural interest rate is given by $\tilde{r}_t^i = \dot{a}_{i,t}$.

Thus, there are many possible paths for the aggregate variables, depending on the stance of monetary policy at the union level.

From these equations we can infer aggregate consumption $\hat{c}^*_t = \hat{y}^*_t$. We can also infer the disaggregated variables for country i as follows. The terms of trade gap $\hat{s}^i_t$ can be backed out from $\hat{y}^i_t = (1-\alpha)\hat{\tilde{\theta}}^i + \hat{s}^i_t$, which combines the market clearing condition with the Backus-Smith condition. Similarly, we can back out the employment gap $\hat{n}^i_t$ and the consumption gap $\hat{c}^i_t$ from technology $\hat{y}^i_t = \hat{n}^i_t$ and market clearing $\hat{y}^i_t = \hat{c}^i_t + \alpha\hat{s}^i_t - \alpha\hat{\tilde{\theta}}^i$.

Loss function. We are interested in characterizing the symmetric constrained Pareto efficient allocation that provides optimal ex-ante risk sharing behind the veil of ignorance before shocks are realized. To do so, we maximize an unweighted utilitarian welfare function. A simple representation of the loss function associated with this welfare criterion is as follows (see Farhi and Werning, 2012):

$$\frac{(1+\phi)}{2}\int_0^\infty \int_0^1 e^{-\rho t}\left[\alpha_\pi(\hat{\pi}^i_{H,t}+\pi^*_t)^2 + (\hat{y}^i_t + \hat{y}^*_t)^2 + \alpha_\theta(\hat{\tilde{\theta}}^i)^2\right] di\, dt,$$

where $\alpha_\pi = \frac{\epsilon}{\lambda(1+\phi)}$ and $\alpha_\theta = \frac{\alpha(2-\alpha)}{1+\phi}$.

The first two terms in the loss function are familiar in New Keynesian models and are identical to those obtained by Gali and Monacelli (2005, 2008). The third term captures the direct welfare effects of transfers and penalizes deviations from privately optimal risk sharing. This term disappears in the closed-economy limit as $\alpha \to 0$.[28]

5.2 Optimal Transfers in a Currency Union

Before we begin the analysis of optimal transfers, it is useful to make the following observation.[29] Throughout Sections 4-6, we assume that markets are incomplete. Suppose for a moment that markets are complete. Then the best equilibrium with complete markets and no intervention in financial markets has $\tilde{\theta}^i = 0$. It coincides with the best equilibrium with incomplete markets and no transfers.[30] This equivalence is due to our Cole-Obstfeld specification. As a result, we can interpret any nonzero transfer at the optimum with incomplete markets as an indication that private risk sharing through complete financial markets, if those were available, would be constrained inefficient. See Appendix C.3 for details.

We set up the planning problem for optimal transfers under incomplete markets. We use the fact that $\int_0^1 \hat{y}^i_t di = \int_0^1 \hat{\pi}^i_{H,t} di = 0$. We are led to the following second-best planning problem

$$\min \frac{1}{2}\int_0^\infty \int_0^1 e^{-\rho t}\left[\alpha_\pi(\hat{\pi}^i_{H,t})^2 + (\hat{y}^i_t)^2 + \alpha_\theta(\hat{\tilde{\theta}}^i)^2 + \alpha_\pi(\pi^*_t)^2 + (\hat{y}^*_t)^2\right] di\, dt \tag{22}$$

[28]Note that from the perspective of an individual country i, transfers also have a first-order effect on welfare. The loss function of an individual country inherits a term $-\frac{1}{2}\int_0^\infty e^{-\rho t}\frac{2\alpha(2-\alpha)}{1+\phi}\hat{\tilde{\theta}}^i dt$. This term represents the pure distributional aspect of transfers. These distributional concerns are zero-sum and wash out in the aggregate since $\int_0^1 \hat{\tilde{\theta}}^i di = 0$.

[29]In Appendix C.4, we solve for the positive effects of transfers in a currency union.

[30]In both cases, optimal monetary policy ensures that the aggregate output gap and inflation are zero $\hat{y}^*_t = \pi^*_t = 0$.

subject to

$$\dot{\hat{\tilde{\pi}}}^i_{H,t} = \rho\hat{\tilde{\pi}}^i_{H,t} - \kappa_y\hat{\tilde{y}}^i_t - \lambda\alpha\hat{\tilde{\theta}}^i, \tag{23}$$

$$\dot{\hat{\tilde{y}}}^i_t = -\hat{\tilde{\pi}}^i_{H,t} - \dot{\tilde{s}}^i_t, \tag{24}$$

$$\hat{\tilde{y}}^i_0 = (1-\alpha)\hat{\tilde{\theta}}^i - \tilde{s}^i_0, \tag{25}$$

$$\int_0^1 \hat{\tilde{\theta}}^i di = 0, \tag{26}$$

$$\dot{\pi}^*_t = \rho\pi^*_t - \kappa_y\hat{y}^*_t, \tag{27}$$

where the minimization is over the variables $\hat{\tilde{\pi}}^i_{H,t}$, π^*_t, $\hat{\tilde{y}}^i_t$, $\hat{y}^*_t$, $\hat{\tilde{\theta}}^i$.

In Appendix C.2, we show how to decompose this planning problem into an aggregate planning problem that determines π^*_t and $\hat{y}^*_t$, and a disaggregated planning problem that determines $\hat{\tilde{\pi}}^i_{H,t}$, $\hat{\tilde{y}}^i_t$, and $\hat{\tilde{\theta}}^i$ for all i. Furthermore, we show that the disaggregated planning problem can be relaxed and broken down into separate component planning problems indexed by i. The aggregate planning problem does not depend on the shocks. The disaggregated planning problem depends only on the asymmetric component of shocks captured by the natural terms of trade $\tilde{s}^i_t = a_{i,t} - a^*_t$. The symmetric component of shocks a^*_t does not explicitly show up in the planning problem expressed in gaps.

Monetary policy can be chosen at the union level so that monetary conditions are tailored to the average country, a result which echoes Proposition 5 from the static model and is reminiscent of the results in Benigno (2004) and Gali and Monacelli (2008).

Proposition 13 (Optimal Monetary Policy)**.** *At the second-best optimum, union-wide aggregates are zero* $\hat{y}^*_t = \pi^*_t = 0$*.*

This means that monetary policy at the union level can perfectly stabilize the symmetric component of shocks. Monetary policy is powerless, however, in dealing with the asymmetric component of shocks.

We now characterize disaggregated variables at the second-best optimum and study how transfers are optimally used to stabilize the asymmetric component of shocks $\tilde{s}^i_t$. We provide closed-form solutions for two enlightening special cases: rigid prices and the closed-economy limit. We then explore the general case using extensive numerical simulations in Section 6.

The case of rigid prices. We first treat the case of rigid prices $\kappa_y = 0$.

Proposition 14 (Rigid Prices)**.** *Suppose that prices are rigid, then at the second-best optimum we have*

$$N\hat{\tilde{F}}A^i_0 = \frac{\alpha(1-\alpha)}{(1-\alpha)^2 + \alpha_\theta}\int_0^\infty e^{-\rho t}\tilde{s}^i_t dt \quad \text{and} \quad \hat{\tilde{\theta}}^i = \frac{\rho(1-\alpha)}{(1-\alpha)^2 + \alpha_\theta}\int_0^\infty e^{-\rho t}\tilde{s}^i_t dt.$$

Importantly, we find that the net present value of transfers $N\hat{\tilde{F}}A^i_0$ and $\hat{\tilde{\theta}}^i$ and are nonzero, so that the optimal solution *does not* coincide with the solution that would have arisen with complete markets and no interventions in financial markets.

Countries experiencing shocks that depreciate their natural terms of trade $\tilde{s}_t^i$ receive positive transfers. With positive home bias ($\alpha < 1$), these transfers increase the demand for home goods and help alleviate the temporary recession resulting from the inability of the terms of trade to immediately depreciate to their natural level in the short run.

Optimal transfers are increasing in the size of the shocks. They are also increasing in the persistence of the shocks. This is intuitive since consumers permanently increase their consumption in response to the transfers. As a result, the more transitory the shock, the less the increase in spending is concentrated when the economy is depressed in the short run. In other words, transfers affect the economy permanently and are therefore better suited to deal with persistent shocks.

Optimal transfers depend crucially on the openness of the economy as captured by the degree of home bias α. They are non-monotonic in the degree of openness. Indeed, the net present value of transfers is zero for $\alpha = 0$ (closed economy) and for $\alpha = 1$ (fully open economy).

The reasons for zero transfers with $\alpha = 0$ and with $\alpha = 1$ are very different. For α close to 0, transfers have large expenditure switching effects across goods from different countries and therefore large effects on output. For α close to 1, transfers have small expenditure switching effects and therefore small effects on output. So for α close to 0, we get small transfers because transfers are very effective, and indeed we get $\hat{\hat{\theta}}^i \neq 0$ so that the allocation remains significantly different from the one arising with no transfers. By contrast for α close to 1, we get small transfers because transfers are very ineffective, and we get $\hat{\hat{\theta}}^i = 0$ so that the allocation approaches the one prevailing with no transfers.

The case of the closed-economy limit. We now return to the case where prices are not entirely rigid, $\kappa_y > 0$, but we place ourselves in the closed-economy limit $\alpha \to 0$.

Proposition 15 (Closed-Economy Limit)**.** *In the closed-economy limit and when $\tilde{s}_t^i = \tilde{s}_0^i e^{-\psi t}$, at the second-best optimum we have*

$$N\hat{\hat{F}}A_0^i = 0 \quad and \quad \hat{\hat{\theta}}^i = \tilde{s}_0^i \left[\frac{\kappa_y}{\kappa_y - (\rho + \psi)\psi} + \frac{\rho - 2\nu}{\alpha_\pi \nu^2 + 1} \frac{(\alpha_\pi \nu \kappa_y - \rho - \psi)\psi}{(\rho + \psi - \nu)[\kappa_y - (\rho + \psi)\psi]} \right].$$

This proposition shows that the intuition given above in the case of rigid prices for the increased effectiveness of transfers as the economy becomes more closed generalizes to the case where prices are not entirely rigid. As we approach the closed-economy limit, optimal transfers become vanishingly small ($N\hat{\hat{F}}A_0^i = 0$ in the limit) but the allocation remains significantly different from the allocation with no transfers ($\hat{\hat{\theta}}^i \neq 0$ in the limit).

Proposition 16 (Closed-Economy Limit, Permanent Shocks)**.** *In the closed-economy limit, in response to a permanent shock $\tilde{s}_t^i = \tilde{s}_0^i$, the second-best optimum achieves the first best $\hat{\tilde{y}}_t^i = \hat{\pi}_{H,t}^i = 0$ with*

$$N\hat{\hat{F}}A_0^i = 0 \quad and \quad \hat{\hat{\theta}}^i = \tilde{s}_0^i.$$

Transfers are particularly useful in the case where shocks are permanent so that $\psi = 0$. In this case we have $\hat{\hat{\theta}}^i = \tilde{s}_0^i$ and we get perfect stabilization of output and inflation. This striking result illustrates that for rather closed economies in a currency union, even modest transfers can achieve large stabilization benefits. It offers an interesting contrast to the arguments presented by McKinnon (1963) that common currencies are more costly for economies that are more closed, but did not consider transfers. Our result shows that this matters since transfers are more potent in more closed economies.

5.3 The Role of Fixed Exchange Rates

We now clarify the role of fixed exchange rates for our results. We assume that only a subset of countries $I \subseteq [0,1]$ are in the currency union. Countries outside the currency union have flexible exchange rates and PCP.[31]

We can write down the corresponding planning problem as follows

$$\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[\alpha_\pi (\hat{\hat{\pi}}_{H,t}^i)^2 + (\hat{\hat{y}}_t^i)^2 + \alpha_\theta (\hat{\hat{\theta}}^i)^2 + \alpha_\pi (\pi_t^*)^2 + (\hat{y}_t^*)^2 \right] di\, dt$$

subject to

$$\dot{\pi}_t^* = \rho \pi_t^* - \kappa_y \hat{y}_t^*,$$

$$\int_0^1 \hat{\hat{\theta}}^i di = 0,$$

for $i \in I$,

$$\dot{\hat{\hat{\pi}}}_{H,t}^i = \rho \hat{\hat{\pi}}_{H,t}^i - \kappa_y \hat{\hat{y}}_t^i - \lambda \alpha \hat{\hat{\theta}}^i,$$
$$\dot{\hat{\hat{y}}}_t^i = -\hat{\hat{\pi}}_{H,t}^i - \dot{\tilde{s}}_t^i,$$

$$\hat{\hat{y}}_0^i = (1-\alpha) \hat{\hat{\theta}}^i - \tilde{s}_0^i,$$

and for $i \notin I$,

$$\dot{\hat{\hat{\pi}}}_{H,t}^i = \rho \hat{\hat{\pi}}_{H,t}^i - \kappa_y \hat{\hat{y}}_t^i - \lambda \alpha \hat{\hat{\theta}}^i.$$

For countries outside the currency union the only constraint is the Phillips curve. The Euler equation and the initial condition do not appear as constraints because with a flexible exchange rate $\hat{\hat{e}}_t^i$, they become

$$\dot{\hat{\hat{y}}}_t^i = \dot{\hat{\hat{e}}}_t^i - \hat{\hat{\pi}}_{H,t}^i - \dot{\tilde{s}}_t^i,$$

$$\hat{\hat{y}}_0^i = \hat{\hat{e}}_0^i + (1-\alpha) \hat{\hat{\theta}}^i - \tilde{s}_0^i.$$

Thus, these equations simply define the required value for the path of the exchange rate $\hat{\hat{e}}_t^i$. As a

[31] For countries in a currency union the distinction between LCP and PCP is irrelevant. Here for countries outside a currency union, we have assumed PCP. Under LCP, fluctuations in the exchange rate would not be enough to deliver the first best, and there would remain a role for transfers for countries outside of the currency union, but our analysis of optimal transfers in a currency union would be unchanged.

result, the solution entails $\hat{\pi}^i_{H,t} = \hat{y}^i_t = \hat{\bar{\theta}}^i = 0$ for $i \notin I$. These countries do not send or receive transfers. They reach the same allocation as under complete markets and without interventions in financial markets.

Proposition 17 (Countries Outside the Currency Union)**.** *For countries outside the currency union, second-best optimal transfers are zero and $\hat{\bar{\theta}}^i = 0$. They achieve perfect stabilization $\hat{\pi}^i_{H,t} = \hat{y}^i_t = 0$.*

This proposition echoes Proposition 10 and shows that in our model, the role of transfers hinges on the fixed exchange rates prevailing in a currency union.

6 Numerical Illustration

In Section 5 we set up a framework to analyze optimal transfers in a currency union and provided complete analytical characterizations for two extreme but enlightening special cases: rigid prices and the closed-economy limit. In this section, we undertake a systematic numerical exploration away from these special cases. We also compare the relative effectiveness of transfers and of other macroeconomic stabilization instruments available in currency unions such as domestic fiscal policy and capital controls. We focus throughout on asymmetric shocks, since symmetric shocks can be perfectly stabilized by union-wide monetary policy.

6.1 Extending and Calibrating the Model

We start by extending the dynamic model of Sections 4-5. First, we incorporate hand-to-mouth agents. Second, we allow for other instruments such as domestic fiscal policy as well as capital controls. Given space constraints, we only give high-level descriptions of our extensions here. All the details and derivations can be found in a comprehensive and entirely self-contained appendix (Appendix D).

Hand-to-mouth consumers. In the dynamic model of Sections 4-5, agents have access to perfect financial markets and behave like permanent-income consumers. There is ample evidence that this is counterfactual, and that a substantial fraction of consumers act as if they are borrowing constrained or don't have access to financial markets. As we shall see below, accounting for this feature is important to understand the role of transfers.

We therefore enrich the model by introducing a fraction χ of hand-to-mouth consumers with no access to financial markets who simply consume their income in every period with a high marginal propensity to consume out of transitory income of one. The remaining fraction $1-\chi$ of consumers has perfect access to financial markets and acts in accordance with the permanent-income hypothesis with a low marginal propensity to consume out of transitory income. The baseline model of Sections 4-5 corresponds to the particular case $\chi = 0$.

This modeling device strikes an attractive balance between on the one hand capturing consumer heterogeneity and borrowing constraints and on the other hand maintaining enough tractability to perform an extensive quantitative normative analysis. A substantial technical difficulty is the derivation of the loss function, which unlike in the baseline model with only permanent-income consumers, requires a second-order approximation of several equilibrium constraints to substitute out the linear terms in the second-order approximation of welfare. This is required to obtain an expression with only second-order terms which can then be maximized subject to a first-order approximation of the equilibrium conditions.

Permanent-income consumers are exactly like in the baseline model. Hand-to-mouth consumers consume their income in every period. This income is composed of their labor income and their share of the profits of domestic firms net of a lump-sum tax (which is negative in the case of a rebate). We assume that profits are distributed uniformly in the population. In every period, international transfers are assumed to flow through the government which receives them and rebates them uniformly across the population.

The optimal policy problem for a given shock then consists in finding the temporal sequence of transfers that minimizes the loss function subject to the log-linearized equilibrium constraints of the economy. The presence of hand-to-mouth consumers requires solving not only for the overall level of transfers, as captured by their net present value as in the case where there are only permanent-income consumers, but also for their timing and hence for the entire path of transfers.

Alternative macroeconomic instruments. In the dynamic model of Sections 4-5, international transfers are the only policy instrument. We extend the model to allow for other policy instruments: capital controls and domestic fiscal policy. In Farhi and Werning (2012), we used the same model presented in Sections 4-5 without hand-to-mouth consumers to study time-varying capital controls as well as time-varying domestic government spending on domestic goods.[32,33,34] The appendix extends this analysis to allow for hand-to-mouth consumers. While Ricardian equivalence holds in the domestic economy in the model without hand-to-mouth consumers, it does not in the extended model with hand-to-mouth consumers. In the latter case, other dimensions of domestic fiscal policy pertaining to the timing and distribution of domestic taxes become relevant. We consider time-varying government spending financed by uniform taxes on all consumers, time-varying deficits

[32]For government spending, our analysis replicates Gali and Monacelli (2008). See Section 4.6 of the NBER working paper version of Farhi and Werning (2012).

[33]It is important to clarify the difference between international transfers and our interpretation of capital controls. Whereas international transfers affect the allocation of consumption across states of the world, capital controls affect the allocation of consumption over time in each state. In our rendition, capital controls are therefore different from the state-contingent portfolio taxes that we analyzed in the complete markets implementation of the static model in Sections 2-3 where this dynamic temporal dimension is entirely absent. The optimal use of capital controls can be derived by studying a planning problem related to (22). The differences are as follows. First, in the objective function (22) and in the New Keynesian Philips curve (23), the constant variable $\hat{\hat{\theta}}^i$ is now replaced by the time-dependent variable $\hat{\hat{\theta}}^i_t$. Second, the initial condition (25) now features $\hat{\hat{\theta}}^i_0$ instead of $\hat{\hat{\theta}}^i$. Third, there is an additional set of constraints $\int e^{-\rho t}\hat{\hat{\theta}}^i_t dt = 0$ if international transfers are not available but only capital controls are available.

[34]We refer the reader to Appendix B.4 for a treatment of government spending in the context of the static model of Sections 2-3. We characterize the jointly optimal use of international transfers and government spending. Our analysis underscores that both instruments should be used in conjunction. Moreover, we show that our characterization of fiscal unions is robust to the availability of government spending as an additional instrument. We also compare their relative performance depending on a number of deep economic parameters.

in the form of debt-financed tax cuts combined with future tax increases falling uniformly on consumers, time-varying redistribution between consumers, and various combinations of these different options.

Calibration. To allow for maximal comparability between our different instruments, we perform all our numerical simulations within the same extended model with a steady-state share of domestic government spending of 0.2 as in Gali et al. (2007).[35] We set standard values for $\phi = 3$ corresponding to a Frisch elasticity of labor supply of 0.33, for the elasticity of substitution between varieties $\epsilon = 11$, and for the discount rate $\rho = 0.06$. In our benchmark calibration we set $\rho_\delta = 0.79$ ("sticky") corresponding to an average price duration of 5.6 quarters following Altig et al. (2011); we also experiment with $\rho_\delta = 1.66$ ("more flexible") corresponding to an average price duration of 3 quarters and with fully rigid prices $\rho_\delta = 0$ ("rigid"). We set the degree of openness to $\alpha = 0.4$ ("open") corresponding to a relatively open economy as in Gali and Monacelli (2008); we also experiment with a more closed economy $\alpha = 0.1$ ("closed") and with a very closed economy $\alpha = 0.01$ ("very closed"). We consider both the version of the model without hand-to-mouth consumers $\chi = 0$ ("no HtM"), and with hand-to-mouth consumers. When we consider hand-to-mouth consumers, we follow Gali et al. (2007) and set $\chi = 0.5$ ("HtM") which matches the range of estimated values for the proportion of rule-of-thumb consumers documented in Mankiw (2000).

6.2 Transfers

In this section, we focus on international transfers. To do so, we assume that all the other instruments are passive or equivalently that they are at their natural level, by which we mean that they are given by their optimal values under flexible prices given the shock. We refer to the corresponding allocation as the natural allocation which is first-best efficient. In our model, the natural level of capital controls is zero as in steady state. By contrast, the natural levels of government spending and taxes are different from their steady state values, depend on the shock, and are not constant over time. However, they do not reflect any macroeconomic stabilization motive and hence represent a passive form of fiscal policy.[36,37] We start with the case without hand-to-mouth consumers and then

[35]Following Gali and Monacelli (2008), we take the utility function to be $(1-v)\frac{C_t^{1-\sigma}}{1-\sigma} + v\frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$, and we calibrate v to match a steady state government spending share of 0.2.

[36]That optimal capital controls are zero with flexible prices as in the steady state reflects our focus on coordinated welfare from the perspective of the currency union as opposed to the individual perspective of the country, which neutralizes terms of trade manipulations motives for capital taxation. With flexible prices, optimal government spending and taxes are different from their steady state values, but taxes are uniform across agents and the government budge is balanced in every period.

[37]What constitutes passive fiscal policy is debatable. An alternative to our definition, which we do not pursue, would be to broaden the notion of passive fiscal policy to encompass the "opportunistic" dimension of fiscal policy, which captures the notion that in the presence of nominal rigidities, the microeconomic opportunity cost of fiscal resources might differ from its natural level, and so that on purely microeconomic grounds, there is a case for deviating from the natural fiscal policy. Only the "stimulus" dimension of fiscal policy, which recognizes that with nominal rigidities, there might be a case for fiscal policy to deviate from what would be warranted on purely microeconomic grounds, would then be defined as active. See Werning (2012) for an elaboration of these concepts in the case of government spending. Our

consider the case with hand-to-mouth consumers.

No hand-to-mouth consumers. Figure 2 displays the behavior of the economy with optimal transfers and with no transfers in response to a permanent shock with $\tilde{s}^i_t = 0.05$. The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. In this figure, time is measured in years, output is measured in gaps from the natural allocation, and inflation is annualized. The allocation without transfers features deflation and a recession in the short run which vanishes in the long run as prices adjust: the output gap increases from -5% to 0 and the inflation rate from -6% to 0. The allocation with transfers features less deflation and smaller recession in the short run, but lower output in the long run. For example, with $\alpha = 0.1$, the output gap at impact is only -1.2% and the inflation rate -1.4%. The allocation without transfers is independent of openness α. By contrast, the solution with optimal transfers is more stable the more closed the economy (the lower α). Optimal transfers stabilize the economy more effectively when the economy is more closed.

Figure 3 displays a measure of macroeconomic stabilization due to transfers. As a measure of stabilization, we compute the fraction of the welfare loss without transfers that is eliminated by the introduction of optimal transfers, where welfare losses are computed with respect to the baseline of the natural allocation. We feed in exponentially decaying shocks $\tilde{s}^i_t = e^{-\psi t}\tilde{s}^i_0$. We then plot our stabilization measure as a function of openness α and the persistence of the shock as measured by its half life $-\log(0.5)/\psi$. The measure of stabilization is invariant with respect to the size of the shock $\tilde{s}^i_0$.

Using the same shock, Figure 4 displays $N\hat{\hat{F}}A^i_0$, which can be interpreted as the net present value of transfers as a fraction of GDP, as a function of the same two parameters. These numbers scale in proportion the size of the shock which we normalize to $\tilde{s}^i_0 = 0.05$.

Stabilization is increasing in the persistence of the shock and decreasing in openness. Optimal transfers are increasing in the persistence of the shock starting at zero for fully transitory shocks, but hump-shaped as a function of openness, starting at zero in the closed-economy limit. Significant stabilization is achieved with relatively modest transfers when the economy is relatively closed and shocks are relatively permanent.[38]

Hand-to-mouth consumers. With only permanent-income consumers, transfers are not a well targeted instrument to deal with transitory shocks. For example in the case of a transitory recessive shock, permanent-income consumers spend the permanent value of the transfers in every period,

definition of passive fiscal policy, which identifies as active any deviation from the natural fiscal policy, excludes both the "opportunistic" and the "stimulus" dimensions of fiscal policy. It should be clear that compared to this alternative, our choices only increase the benefits of active fiscal policy.

[38]When prices are perfectly rigid, we saw earlier in Proposition 14 that transfers were zero both when $\alpha = 0$ (but stabilization is not zero) and when $\alpha = 1$ (and stabilization is zero). When prices are not perfectly rigid, transfers are still exactly zero when $\alpha = 0$ but not when $\alpha = 1$ (and stabilization is not zero). When the economy is completely open with $\alpha = 1$, transfers still influence the domestic wage through a wealth effect and hence inflation as long as prices are not entirely rigid. Therefore, transfers help mitigate the deflation and remain useful for stabilization.

both in the short run, when their extra spending is needed to stimulate output, and in the long run, when it is not. This changes in the presence of hand-to-mouth consumers with high marginal propensities to consume out of transitory income. The spending of these consumers can be front-loaded by front-loading transfers. As a result, transfers become a more effective instrument to deal with transitory shocks and, in other words, the performance of transfers depends less on the persistence of shocks. A similar intuition applies to the dependence of the performance of transfers on the degree of price rigidity, since greater price flexibility makes the impact of a given shock on macroeconomic stability more transitory given that prices adjust faster. What matters is the persistence of the effects of the shock on macroeconomic stability, either because the shocks themselves are persistent or because price are somewhat sticky. We will make repeated use of this terminology in the discussion below and refer to a shock with transitory effects as a situation where either the shock itself is transitory or where prices are somewhat flexible.

Figure 5 displays the behavior of the economy with optimal transfers and with no transfers in response to a permanent shock with $\tilde{s}_t^i = 0.05$. The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. The figure also documents the optimal path of transfers. Because prices are sticky but not perfectly rigid, the effects of the shock on macroeconomic stability are transitory even though the shock is permanent. In the absence of transfers, the economy experiences a recession with a negative output gap and deflation in the short run but the economy equilibrates in the long run returning to zero output gap and no inflation. The equilibrium allocation without transfers is actually identical to that which obtains with no hand-to-mouth consumers. This is due to our Cole-Obstfeld specification: In the absence of transfers and without hand-to-mouth consumers, trade remains balanced and permanent-income consumers end up consuming their income in every period just like hand-to-mouth consumers. This invariance with respect to the fraction of hand-to-mouth consumers breaks down with transfers. As is intuitive, the figure shows that optimal transfers are significantly front-loaded. This contrasts with the baseline case with only permanent-income consumers where the timing of transfers is irrelevant.

Figure 6 displays the amount of stabilization (defined as above) achieved by transfers as a function of openness α and the persistence of the shock as measured by its half life $-\log(0.5)/\psi$. Figure 7 displays the net present value of transfers as a fraction of GDP as a function of the same two parameters. The main differences from the baseline case with only permanent-income consumers $\chi = 0$ discussed above are that: Stabilization is much less dependent on the persistence of the shock and the level of transfers is higher. This is because with hand-to-mouth consumers, transfers become a more effective stabilization tool for shocks with transitory effects on the economy.[39]

[39]That stabilization increases is intuitive since transfers become more effective. That the net present value of transfers increases is less obvious because in general, making an instrument more effective could either increase or decrease its optimal use. That it could increase its use is intuitive since the instrument becomes more attractive—a form of policy substitution effect. That it could reduce it is due to the fact that a given improvement in the objective can be achieved with a lesser use of the tool—a form of policy income effect. In the case at hand regarding the effectiveness of transfers as the fraction of hand-to-mouth consumers in the economy increases, the policy substitution effect dominates the policy wealth effect, and as a result transfers are used more.

6.3 Transfers vs. Other Macroeconomic Instruments

In this section we compare the effectiveness of international transfers to that of other macroeconomic stabilization instruments: capital controls and domestic fiscal policy. Of course international transfers, capital controls, and domestic fiscal policy are different instruments. In general no single instrument can achieve perfect macroeconomic stabilization, and it is best to use all three in conjunction. This is not what interests us here. Instead, we seek to characterize the relative effectiveness of these different instruments. We shall see that their relative ranking varies in intuitive ways depending on the fraction of hand-to-mouth consumers, the openness of the economy, the persistence of shocks, and the degree of nominal rigidity.

		PERMANENT													
		No HtM agents							HtM agents						
NOMINAL RIGIDITIES	OPEN-NESS	*No policy*	*Transfers*	*Capital controls*	*Gov. spending*	*Redistri-bution*	*Deficits*	*Joint fiscal policy*	*No policy*	*Transfers*	*Capital controls*	*Gov. spending*	*Redistri-bution*	*Deficits*	*Joint fiscal policy*
More flexible	Open	0%	25%	67%	36%	0%	0%	36%	0%	63%	66%	36%	58%	58%	62%
	Closed	0%	68%	85%	36%	0%	0%	36%	0%	83%	85%	36%	73%	73%	74%
Sticky	Open	0%	41%	65%	36%	0%	0%	36%	0%	66%	64%	36%	55%	55%	61%
	Closed	0%	81%	82%	36%	0%	0%	36%	0%	87%	82%	36%	71%	71%	72%
Rigid	Open	0%	66%	0%	26%	0%	0%	26%	0%	66%	0%	26%	0%	0%	26%
	Closed	0%	94%	0%	26%	0%	0%	26%	0%	94%	0%	26%	0%	0%	26%

		TRANSITORY													
		No HtM agents							HtM agents						
NOMINAL RIGIDITIES	OPEN-NESS	*No policy*	*Transfers*	*Capital controls*	*Gov. spending*	*Redistri-bution*	*Deficits*	*Joint fiscal policy*	*No policy*	*Transfers*	*Capital controls*	*Gov. spending*	*Redistri-bution*	*Deficits*	*Joint fiscal policy*
More flexible	Open	0%	21%	83%	47%	0%	0%	47%	0%	76%	82%	47%	73%	73%	78%
	Closed	0%	57%	96%	47%	0%	0%	47%	0%	91%	96%	47%	88%	88%	89%
Sticky	Open	0%	29%	84%	49%	0%	0%	49%	0%	78%	83%	49%	75%	75%	80%
	Closed	0%	58%	97%	49%	0%	0%	49%	0%	92%	97%	49%	89%	89%	90%
Rigid	Open	0%	10%	56%	26%	0%	0%	26%	0%	53%	54%	26%	43%	43%	53%
	Closed	0%	14%	79%	26%	0%	0%	26%	0%	81%	79%	26%	67%	67%	72%

Table 1: Stabilization achieved by different macroeconomic instruments, for permanent shocks (top panel) and transitory shocks with a half life of one year (bottom panel). The measure of the stabilization of a given instrument is the fraction of the welfare loss which is eliminated by allowing for the optimal use of this instrument, where welfare losses are computed with respect to the baseline of the natural allocation. The labels "more flexible", "sticky", and "rigid", correspond to $\rho_\delta = 1.66$, $\rho_\delta = 0.79$, and $\rho_\delta = 0$. The labels "open" and "closed" correspond to $\alpha = 0.4$ and $\alpha = 0.1$. The labels "No HtM" and "HtM" correspond to $\chi = 0$ and $\chi = 0.5$.

We compute the amount of stabilization (defined as above) achieved by these different instruments. The results are reported in the top and bottom panels of Table 1. The panels cover not only our benchmark calibration with $\rho_\delta = 0.79$ ("sticky") and $\alpha = 0.4$ ("open"), but also variants with

$\rho_\delta = 1.66$ ("more flexible") and $\rho_\delta = 0$ ("rigid") as well as with $\alpha = 0.1$ ("closed"), for an economy without hand-to-mouth consumers ("No HtM") and with hand-to-mouth consumers ("HtM"). We feed in exponentially decaying shocks $\tilde{s}^i_t = e^{-\psi t}\tilde{s}^i_0$ with size $\tilde{s}^i_0$ and half life $-\log(0.5)/\psi$. The top panel focuses on permanent shocks with an infinite half life ("permanent") and the bottom panel on transitory shocks with a half life of one year ("transitory"). We do not need to specify the size of shocks since our stabilization measure is invariant to it.

For each of these cases, we report the stabilization measure obtained for the optimal use of different macroeconomic stabilization instruments. "No policy" corresponds to international transfers at their natural values of zero, capital controls at their natural values of zero, and government spending and taxes at their natural values, i.e. the natural allocation. This constitutes the baseline from which we consider one-at-a-time deviations by allowing for a specific instrument to be optimized at a level different from its natural value. "Transfers" corresponds to time-varying international transfers. "Capital controls" corresponds to time-varying capital controls. "Government spending" corresponds to time-varying deviations of government spending from its natural value financed by contemporaneous and uniform taxes on all consumers. "Redistribution" corresponds to time-varying budget-balanced redistribution between permanent-income consumers and hand-to-mouth consumers. "Deficits" corresponds to time-varying uniform taxes (and government debt) on all consumers. "Joint fiscal policy" corresponds to the combination of all the aforementioned domestic fiscal policy instruments with time-varying government spending and time-varying taxes that are differentiated across consumers.

We start with the case where there are no hand-to-mouth consumers. In this case, Ricardian equivalence holds in the domestic economy and so the timing and distribution of taxes over time are irrelevant. The relative performance of these instruments depends on the parameters of the economy, and in particular on two key parameters: the degree of openness, and the persistence of the effects of the shocks, where as explained above the latter reflects both the persistence of the shock itself and the rigidity of prices. In general, international transfers and capital controls are more potent when the economies are relatively closed.[40] By contrast, the effectiveness of government spending is independent of the degree of openness—an exact result which relies on our Cole-Obstfeld specification.[41] International transfers are better suited to deal with shocks with persistent effects whereas capital controls are better suited to deal with transitory shocks. This is because international transfers permanently shift the level of spending whereas capital controls influence the timing of spending. The effectiveness of government spending is less dependent on the persistence of the effects of shocks.[42]

[40]For capital controls, we have made this point in formal propositions and simulations in this paper and in Farhi and Werning (2012).

[41]We showed in Farhi and Werning (2012) and we confirm here that under our Cole-Obstfeld specification, optimal government spending, the output gap, and inflation are independent of openness.

[42]The intuition is also supported by the following formal results. In the closed-economy limit, when shocks $\tilde{s}_t{}^i = a_{i,t} - a^*_t$ are permanent, international transfers achieve the first best, while capital controls and government spending do not. In this case, there is no residual macroeconomic stabilization role for capital controls or government spending once international transfers are available. Conversely, in the limit where productivity shocks $\tilde{s}_t{}^i = a_{i,t} - a^*_t$ are purely transitory, optimal international transfers are zero and do not perform any macroeconomic stabilization role, while optimal capital controls and government spending do help stabilize the economy.

We now turn to the case with hand-to-mouth consumers. As explained above, the presence of hand-to-mouth consumers improves the performance of international transfers in dealing with shocks with transitory effects. By contrast, the presence of hand-to-mouth consumers leaves the performance of capital controls and government spending almost unchanged. In fact, in the case of government spending, under our Cole-Obstfeld specification, hand-to-mouth consumers are irrelevant in the sense that their presence does not change the equilibrium allocations, simply because trade remains balanced for any path of government spending, and so permanent-income consumers end up simply consuming their income in every period just like hand-to-mouth consumers. This exact irrelevance does not hold in the case of capital controls, but it remains a good approximation under our various calibrations.

The presence of hand-to-mouth consumers also opens up other dimensions of domestic fiscal policy. For example when facing a recessive shock with transitory effects on the economy, it is optimal to redistribute towards hand-to-mouth consumers in the short run and towards permanent-income consumers in the long run. This is because hand-to-mouth consumers spend the extra revenue when they get it whereas permanent-income consumers spread their loss in revenue by reducing spending uniformly over time. Redistribution towards hand-to-mouth consumers therefore increases spending in the short run when it is needed. Similarly, redistribution towards permanent-income consumers in the long run mitigates the decrease in their spending due to redistribution away from them in the short run. Furthermore, a given amount of transitory redistribution towards hand-to-mouth consumers has a more stimulative effect in the short run and a more depressive effect in the long run when the economy is more closed because the corresponding changes in spending fall more on domestic goods. These intuitions explain why the performance of redistribution is higher when the economy is more closed or when the shocks have more transitory effects.

Perhaps surprisingly, optimal deficits achieve exactly the same outcomes as optimal redistribution. This is because under our calibrations, optimal redistribution has the property that the redistribution towards hand-to-mouth consumers in the short run and towards permanent-income consumers in the long run exactly cancel out in a net present value sense, a consequence, we conjecture, of our Cole-Obstfeld specification. As a result, an alternative implementation of the corresponding allocation can be found in the form uniform (across consumers) tax cuts in the short run, financed by debt which is purchased by permanent-income consumers and repaid in the long run by uniform (across consumers) tax increases.

7 Conclusion

In this paper, we have characterized fiscal unions as optimal risk sharing arrangements. We have shown that they play a special role within currency unions. We have given a precise description of the effectiveness of such a fiscal union and of the size of the underlying transfers as a function of a small numbers of key characteristics of the currency union, such as the rigidity of prices, the openness of member countries, the asymmetry of shocks, and their persistence. Finally we have compared the performance of fiscal unions to that of other macroeconomic stabilization instruments available in currency unions such as capital controls, government spending, fiscal deficits, and redistribution.

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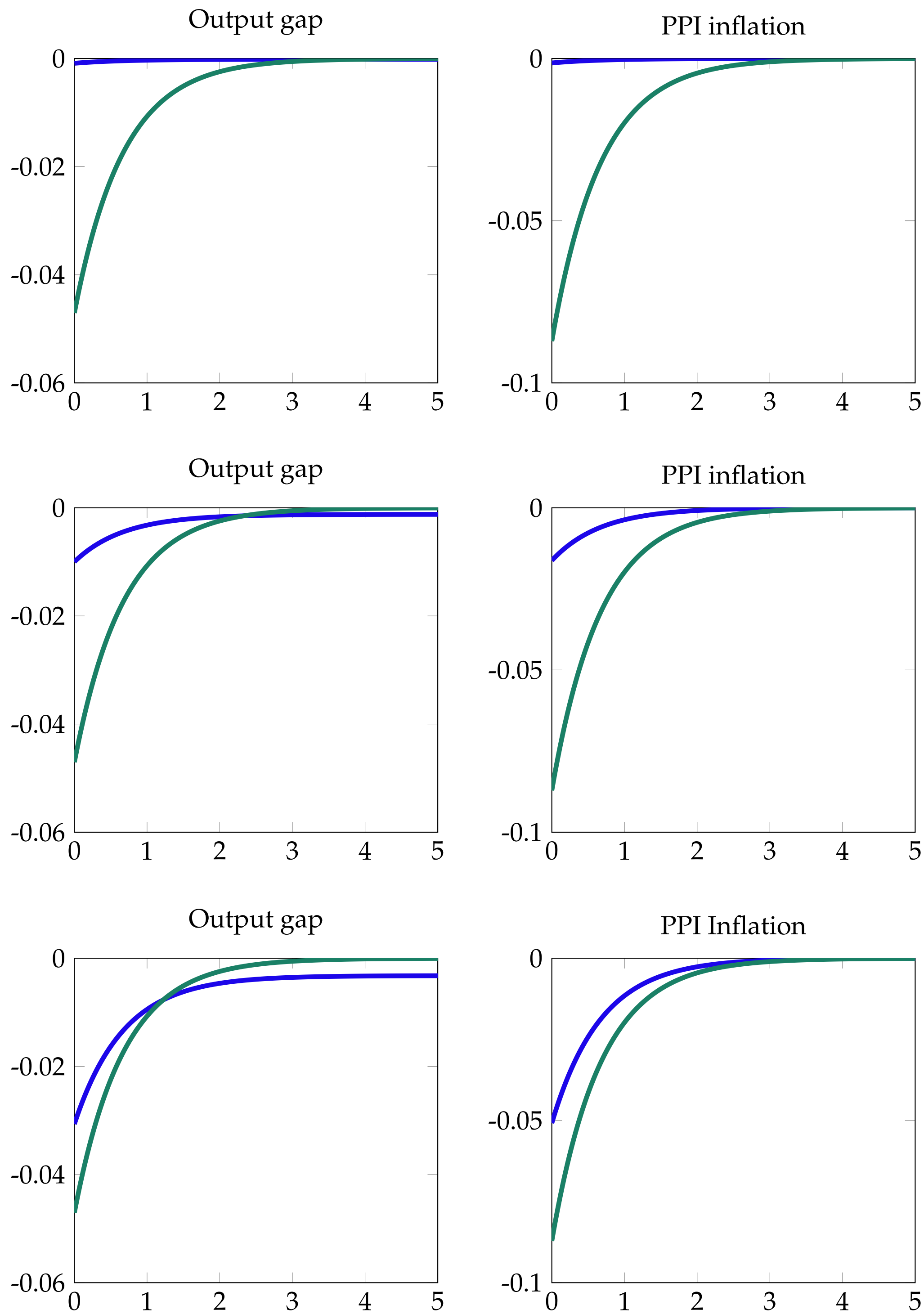

Figure 2: Allocations with optimal transfers (blue) and no transfers (green). The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. Inflation is annualized and time is measured in years. Calibration with no hand-to-mouth consumers $\chi = 0$.

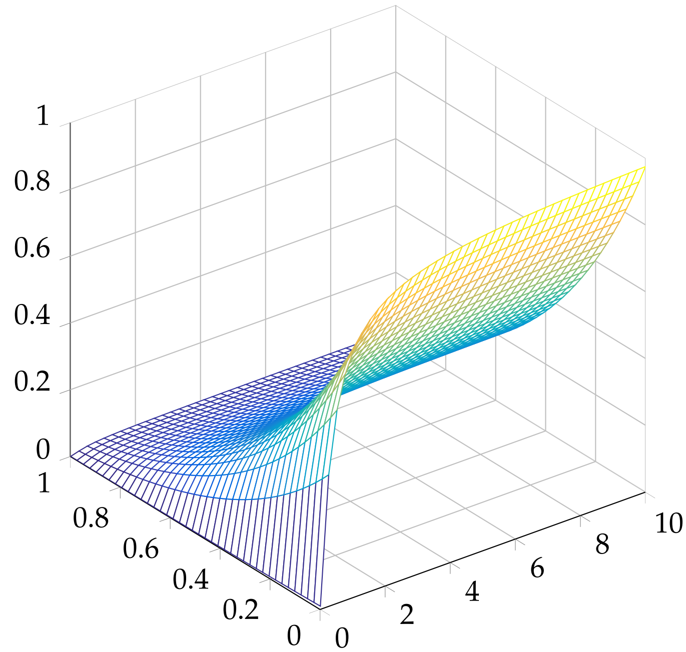

Figure 3: Stabilization effect of transfers as a function of openness measured by $\alpha \in (0, 1)$ and persistence measured by the half-life of the shock $-\frac{\log(0.5)}{\psi} \in (0, 10)$. The measure of the stabilization of transfers is the fraction of the welfare loss without transfers which is eliminated by the introduction of optimal transfers, where welfare losses are computed with respect to the baseline of the natural allocation. Calibration with no hand-to-mouth consumers $\chi = 0$.

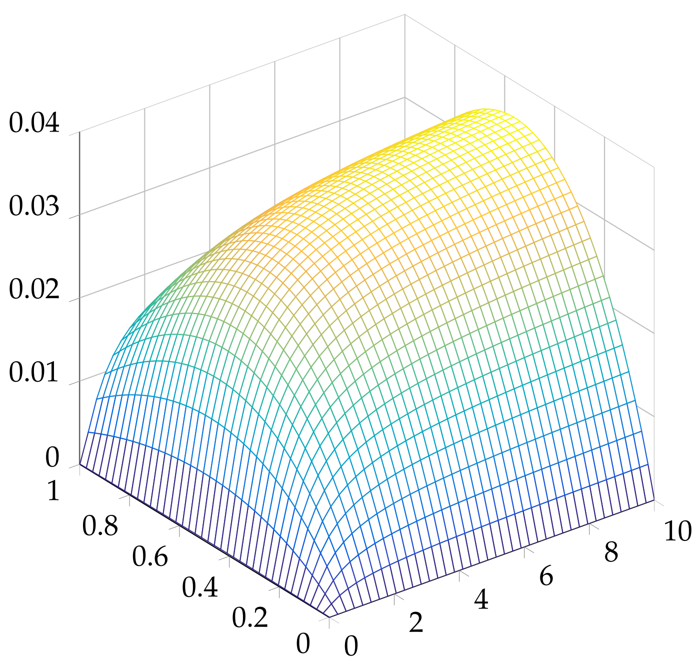

Figure 4: Net present value of transfers as fraction of GDP for a 5% shock to the terms of trade as a function of openness measured by $\alpha \in (0, 1)$ and persistence measured by the half-life of the shock $-\frac{\log(0.5)}{\psi} \in (0, 10)$. Calibration with no hand-to-mouth consumers $\chi = 0$.

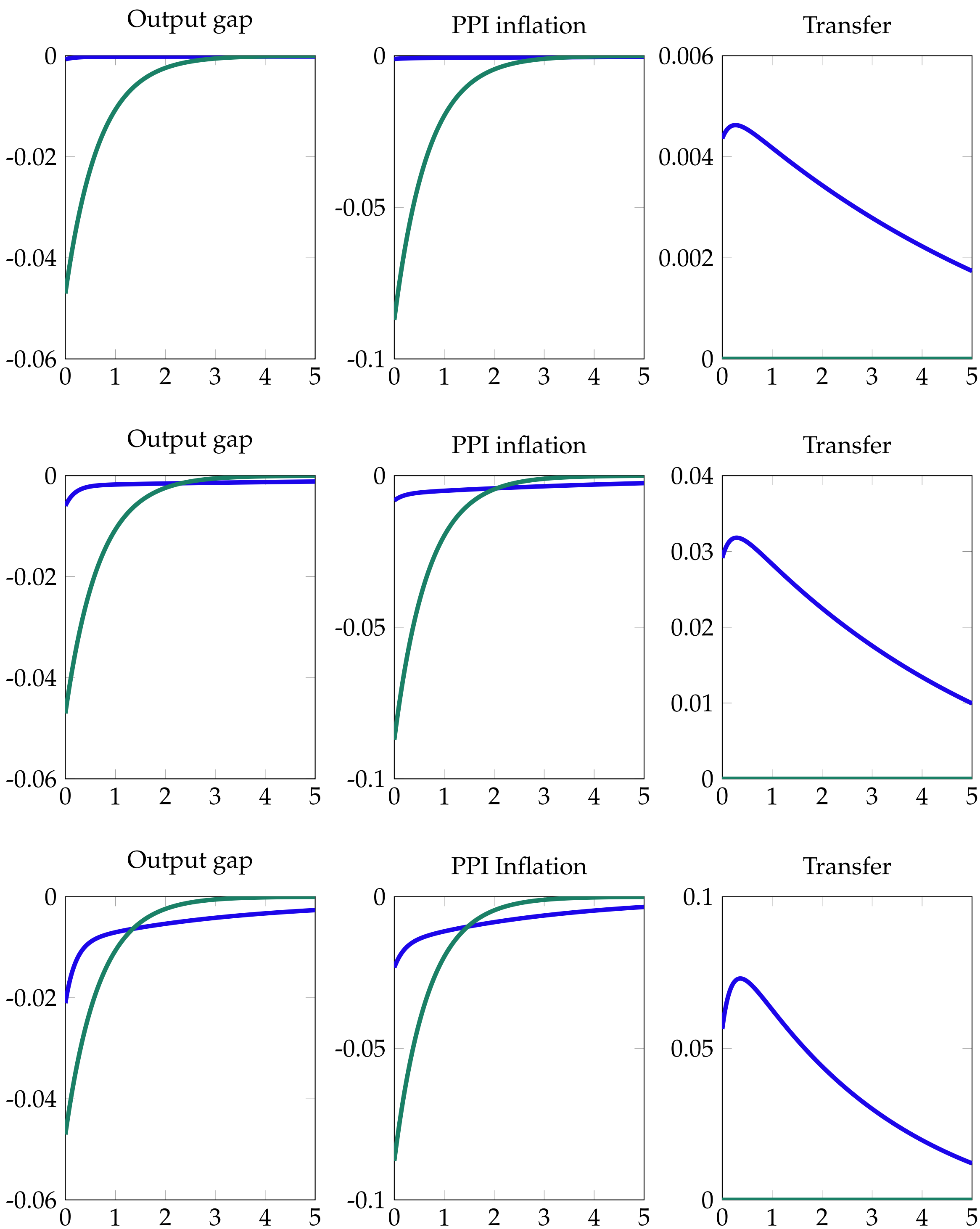

Figure 5: Allocations with optimal transfers (blue) and no transfers (green). The top panel corresponds to $\alpha = 0.01$, the middle panel to $\alpha = 0.1$ and the bottom panel to $\alpha = 0.4$. Inflation is annualized and time is measured in years. Calibration with fraction of hand-to-mouth consumers $\chi = 0.5$.

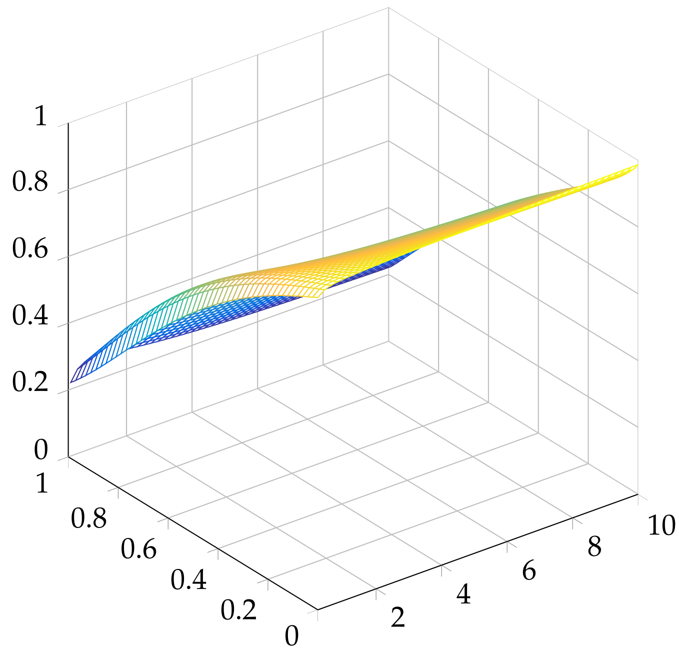

Figure 6: Stabilization effect of transfers as a function of openness measured by $\alpha \in (0,1)$ and persistence measured by the half-life of the shock $-\frac{\log(0.5)}{\psi} \in (0,10)$. The measure of the stabilization of transfers is the fraction of the welfare loss without transfers which is eliminated by the introduction of optimal transfers, where welfare losses are computed with respect to the baseline of the natural allocation. Calibration with fraction of hand-to-mouth consumers $\chi = 0.5$.

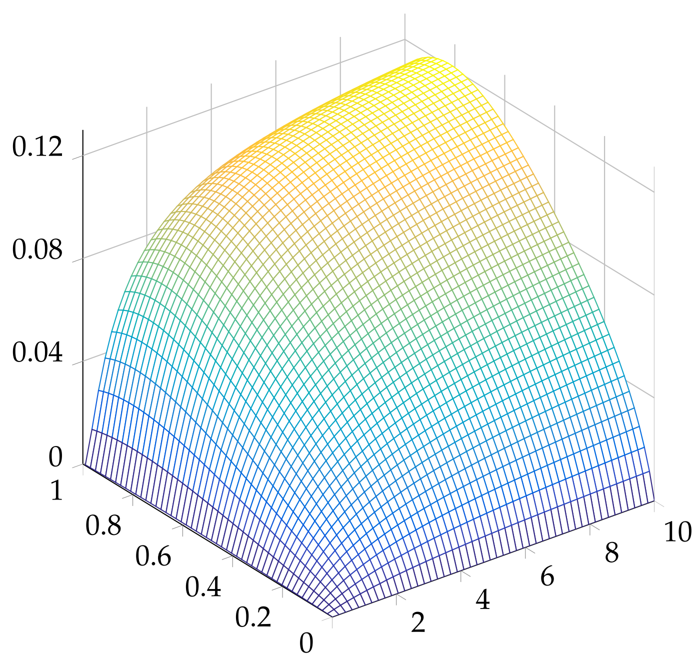

Figure 7: Net present value of optimal transfers as fraction of GDP for a 5% shock to the terms of trade as a function of openness measured by $\alpha \in (0,1)$ and persistence measured by the half-life of the shock $-\frac{\log(0.5)}{\psi} \in (0,10)$. Calibration with fraction of hand-to-mouth consumers $\chi = 0.5$.