Online Appendix to

Optimal Policy for Behavioral Financial Crises

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Contents

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E Extensions for the Current-Price Collateral Constraint

E.1 Real Production

E.1.1 A Simple Extension with Production

To incorporate a real side to the model, we allow households to supply labor at $t = 2$. Households have linear utility over consumption, and have a convex disutility for supplying labor in the intermediate period:

$$U^h = \mathbb{E}_1 \left[c_1^h + \beta \left(c_2^h - \nu \frac{l_2^{1+\eta}}{(1+\eta)} \right) + \beta^2 c_3^h \right] \tag{E.1}$$

where l_2 is the amount of labor supplied by households at time $t = 2$.

There is a fringe of competitive firms of measure one, producing from the labor of households. Firms use a decreasing returns to scale technology from labor, with productivity A:

$$Y_2 = A l_2^{\alpha} \tag{E.2}$$

To bridge the gap between Main street and Wall street, I add a financial friction. Firms need to pay a fraction γ of wage bills in advance to workers, which require them to borrow from financial intermediaries. In period 2, firms need to borrow $f_2 = \gamma w_2 l_2$ from financial intermediaries. We assume that the interest rate required by financial intermediaries to advance such funds depends on the size of the loan according to:

$$1 + r_f = \frac{\delta}{f_2} \tag{E.3}$$

This innocuous trick allows the model to say away from corner solutions and preserve financial amplification.[1] The set of budget constraint is now given by:

$$c_1^h + d_1 \leq e_1^h \tag{E.4}$$

$$c_2^h + d_1 \leq e_2^h + w_2 l_2 + d_1(1+r_1) + \pi_2 \tag{E.5}$$

$$c_3^h \leq e_3^h + d_2(1+r_2) \tag{E.6}$$

for households, and:

$$c_1 + c(H) \leq d_1 + e_1 \tag{E.7}$$

$$c_2 + d_1(1+r_1) + f_2 + q_2 m \leq d_2 + (z_2 + q_2)H \tag{E.8}$$

$$c_3 + d_2(1+r_2) \leq z_3 m + f_2(1+r_f) \tag{E.9}$$

[1] Remember that financial amplification comes from the two-way feedback effect between the Stochastic discount factor and the price fo the risky asset. A corner solution with respect to the borrowing of real firms would break this link.

for financial intermediaries. Household optimization then simply yields:

$$w_2 = \nu l_2^{\eta} \tag{E.10}$$

It is also assumed for simplicity that loans made to firms cannot be used as collateral.[2] The specific form assumed in (E.3) simplifies matter since funds allocated to firms verify the following identity:

$$\frac{f_2}{\delta} = \beta c_2 \tag{E.11}$$

so that bankers' consumption and funds allowed to firms are proportional. Intuitively, when collateral constraints are extremely tight, this forces financial intermediaries to cut back on consumption *and* their traditional intermediary activities in the same way.[3] Thus the amount of labor used for production verifies:

$$l_2 = \left(\frac{z_2 H - d_1(1+r_1) + \phi H q_2}{\gamma \nu \left(1 + \frac{1}{\beta\delta}\right)} \right)^{\frac{1}{1+\eta}} \tag{E.12}$$

which translates into a production level at time $t = 2$ of:

$$Y_2 = A \left(\frac{z_2 H - d_1(1+r_1) + \phi H q_2}{\gamma \nu \left(1 + \frac{1}{\beta\delta}\right)} \right)^{\frac{\alpha}{1+\eta}} \tag{E.13}$$

Hence, a drop in the price of the risky asset q_2 directly impacts output, as well as a fall in financial intermediaries' net worth $z_2 H - d_1(1 + r_1)$. Hence, looking at q_2 inside a crisis is a sufficient statistics even in this extended model with real production.

E.1.2 Welfare Analysis with Real Production

The planner maximizes:

$$\mathcal{W}_1 = \Phi^h \mathbb{E}_1^{SP} \left(c_1^h + \beta \left[c_2^h - \nu \frac{l_2^{1+\eta}}{1+\eta} \right] + \beta^2 c_3^h \right) + \Phi^b \mathbb{E}_1^{SP} \left(\ln(c_1) + \beta \ln(c_2) + \beta^2 c_3 \right) \tag{E.14}$$

where Φ^h and Φ^b are the Pareto weights attached to each group by the planner. I denote by V_2^h and V_2^h the value functions of each group at time $t = 2$.

[2] A more complete formulation of the collateral constraint would be:

$$d_2 \leq \phi H q_2 + \psi f_2$$

whereby assuming that a fraction of the amount lent to firms can be recovered by depositors in the (non-equilibrium) possibility of default. I am here analyzing the limiting case where $\psi \to 0$. The general case complexifies matters without bringing any new intuition. Analytical derivations of the general case are thus relegated to Appendix E.1.3.

[3] Consumption is needed for the SDF to generate financial amplification: a risk-neutral valuation pricing kernel breaks the feedback loop between the price of the asset and marginal utility. But one could think of c_2 as dividends or compensation.

Leverage: We are interested in the derivatives of these value functions at time $t = 2$ with respect to the amount of short-term debt (or savings) chosen at time $t = 1$. Because funds allocated to firms (the f_2) chosen optimally without a constraint (see equation E.11), an infinitesimal change in f_2 will not have a first-order impact on the welfare of bankers:

$$\frac{dV_2^b}{dd_1} = \phi H(\lambda_2 - 1)\frac{dq_2}{dd_1} + \underbrace{\beta\frac{\delta}{f_2} - \lambda_2}_{=0}. \tag{E.15}$$

For households, however, there is a new term coming from the expansion of bank lending to firms in the real sector:

$$\frac{dV_2^h}{dd_1} = \phi H\underbrace{\left(\lambda_3^h - \lambda_2^h\right)}_{=0}\frac{dq_2}{dd_1} + \max\left(\underbrace{A\alpha\left(\frac{z_2 H - d_1(1+r_1) + \phi H q_2}{\gamma\nu\left(1+\frac{1}{\delta}\right)}\right)^{\frac{\alpha}{1+\eta}-1} - \nu}_{\to 0 \text{ when unconstrained}}, 0\right)\frac{dc_2}{dd_1}. \tag{E.16}$$

To understand why this second term is 0 when firms are unconstrained, notice that when firms are able to perfectly maximize profits they hire an amount of labor corresponding to:

$$\alpha A l_2^{\alpha-1} = w \tag{E.17}$$

which iself implies, when combined with households first-order condition for labor/leisure:

$$\alpha A l^{\alpha-1-\eta} = \nu. \tag{E.18}$$

Similarly, the derivative dc_2/dd_1 is also 0 when financial intermediaries are unconstrained. To conclude, the planner's optimality condition for short-term debt is given by:

$$\begin{aligned} 0 = \Phi^h \mathbb{E}_1^{SP}\left[\left(\nu - \alpha A l_2^{\alpha-1}\right)\left(\beta\phi H\frac{dq_2}{dd_1} - (1+r_1)\right)\right] + \\ \Phi^b\left\{\mathbb{E}_1\left[\lambda_2\right] - \mathbb{E}_1^{SP}\left[\lambda_2\right] - \mathbb{E}_1^{SP}\left[\phi H\kappa_2\frac{\partial q_2}{\partial n_2}\right]\right\} \end{aligned} \tag{E.19}$$

where $\nu - \alpha A l_2^{\alpha-1}$ plays the role of a "capacity wedge:" it measures how far firms are from their first-best production level. When this wedge is negative (there is underemployment, since $\alpha < 1$) a reduction in the leverage of financial intermediaries is beneficial for households, since it will increase the production of real goods in a crisis.

Collateral Asset Investment: The same analysis applies to the externalities created by investing in H, keeping q_1 fixed. Similarly, a supplementary term will appear because a marginal change in H will cause a marginal change in c_2, and thus a change in real output in a financial crisis. We thus have, following the same derivations as just above, that the planner's optimality condition for the creation of collateral assets is given by:

$$0 = \Phi^h \mathbb{E}_1^{SP}\left[(\nu - \alpha A l_2^{\alpha-1})\left(\beta\phi H \frac{dq_2}{dH} + z_2 + \phi q_2\right)\right] + \\ \Phi^b\left\{\lambda_1 q_1 - \beta \mathbb{E}_1^{SP}\left[\lambda_2(z_2+q_2)\right] - \beta\mathbb{E}_1^{SP}\left[\kappa_2 \phi H\left(\frac{\partial q_2}{\partial n_2} z_2 + \frac{dq_2}{dH}\right)\right]\right\} \tag{E.20}$$

Current Prices: The reversal externality, similar to the collateral externality, also enters in production. The welfare effects of changing marginally equilibrium prices q_1 are given by:

$$\mathcal{W}_q = \Phi^h \mathbb{E}_1^{SP}\left[(\nu - \alpha A l_2^{\alpha-1})\left(\beta\phi H \frac{\partial q_2}{\partial \Omega_3}\frac{\partial \Omega_3}{\partial q_1}\right)\right] + \Phi^b\left\{\beta\mathbb{E}_1^{SP}\left[\kappa_2\phi H \frac{\partial q_2}{\partial \Omega_3}\frac{\partial \Omega_3}{\partial q_1}\right]\right\} \tag{E.21}$$

E.1.3 Pledgeable Private Sector Loans

The previous section assumed that loans f_2 could not be used as collateral by financial intermediaries. Here, I look at the complete formulation of the collateral constraint, given by:

$$d_2 \leq \phi H q_2 + \psi f_2$$

whereby assuming that a fraction of the amount lent to firms can be recovered by depositors in the (non-equilibrium) possibility of default. The first-order condition for loans to real firms is now given by;

$$\lambda_2 = (1 + r_f) + \kappa_2 \psi \tag{E.22}$$

since lending to firms also expand the borrowing capacity of financial institutions *vis-à-vis* households. Since $\kappa_2 = \lambda_2 - 1$ as usual, this yields:

$$\lambda_2 = \frac{1 + r_f - \psi}{1 - \psi} \tag{E.23}$$

$$\implies \frac{1}{c_2} = \frac{\frac{\delta}{f_2} - \psi}{1 - \psi} \tag{E.24}$$

$$\implies \frac{1-\psi}{c_2} = \frac{\delta}{f_2} - \psi \tag{E.25}$$

$$\implies f_2 = \frac{\delta c_2}{1 - \psi + \phi c_2} \tag{E.26}$$

where it is clear that the relation between c_2 and f_2 is not linear anymore. Using the budget constraint since financial intermediaries are constrained:

$$c_2 + f_2 = n_2 + \phi H q_2 \tag{E.27}$$

$$\implies c_2 + \frac{\delta c_2}{1 - \psi + \phi c_2} = n_2 + \phi H q_2. \tag{E.28}$$

The fixed-point problem corresponding to financial amplification is now complexified by this additional non-linearity:

$$c_2 + \frac{\delta c_2}{1 - \psi + \phi c_2} = n_2 + \phi H q_2 \tag{E.29}$$

$$q_2 = \beta c_2 \mathbb{E}_1[z_3] + \phi q_2 (1 - c_2). \tag{E.30}$$

As in Section 2.3, we can represent this equilibrium graphically. This is depicted in Figure 1. This modification clearly amplifies financial amplification by making the budget constraint a convex function instead of a linear one inside a crisis. The assumption made that $\psi \to 0$ in the previous section were thus conservative in terms of spillovers from the banking sector to real production in terms of welfare.

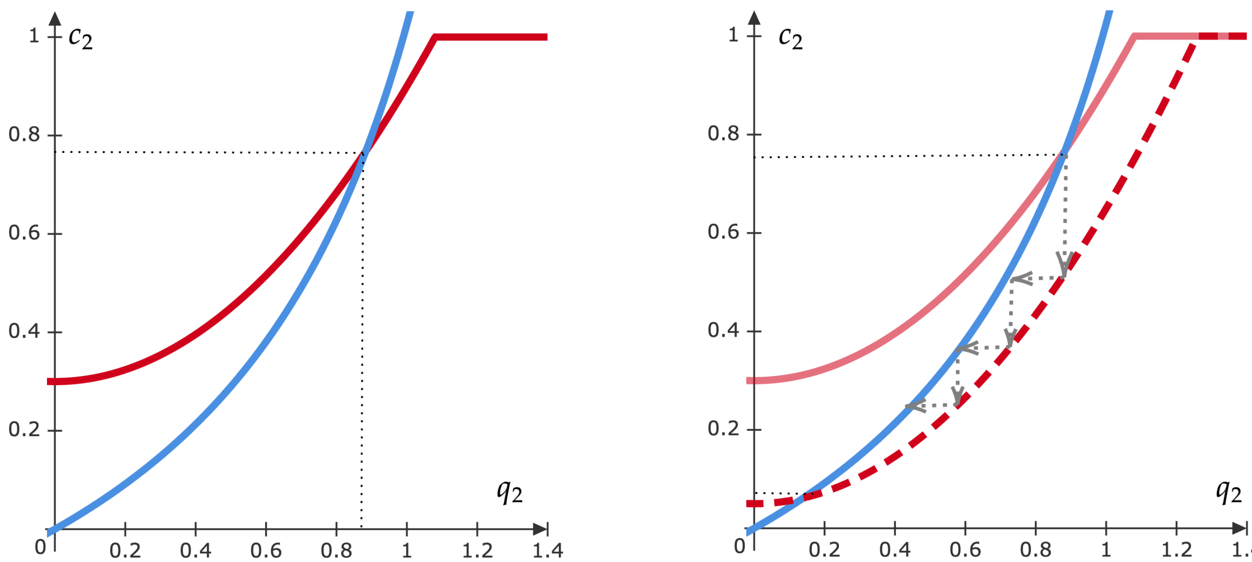

Figure 1: Graphical Illustration of Equilibrium Determination at $t = 2$ with pledgeable private sector loans. The red line represents the budget constraint, and the blue line represents the pricing condition. The right panel illustrates the phenomenon of financial amplification after a fall in net worth n_2. The arrows indicate the fixed-point problem that leads consumption to fall more than the size of the shock because of the tightening of the collateral constraint.

E.2 Bailouts

Similarly to the baseline model in the main paper, the costs of bailouts are modeled in reduced-form as:

$$g(b) = \frac{b^2}{2\xi} \tag{E.31}$$

Outside of a financial crisis, there is no point in providing liquidity to financial intermediaries. In a crisis, welfare at $t = 2$ with this collateral constraint becomes:[4]

$$\mathcal{W}_2 = \ln\left(z_2 H - d_1(1+r_1) + b + \phi H q_2\right) + \beta\left(\mathbb{E}^{SP}[z_3]H - \phi H q_2/\beta - b/\beta\right) - g(b). \tag{E.32}$$

This leads to the following expression for the optimal bailout size:

$$b^*(d_1, H, z_2, \Omega_3) = \xi\left(\frac{\partial \mathcal{W}_2}{dn_2}(d_1, H, z_2, \Omega_3) - 1\right). \tag{E.33}$$

Intuitively, the optimal bailout size takes the same form as in the paper, but there will be an additional effect because of financial amplification. By providing liquidity to the financial sector, bailouts support asset prices and thus increase the borrowing capacity of the financial sector, an effect present even in the rational benchmark.

The behavioral wedge of equation takes the exact same form in both cases of collateral constraints:

$$\mathcal{B}_{d,b^*} = \mathbb{E}_1[\lambda_2(b^*)] - \mathbb{E}_1^{SP}[\lambda_2(b^*)] \tag{E.34}$$

Harginal utility during a crisis always depends on the level of bailouts b^*. But if agents recognize that bailouts will be determined optimally, according to equation (E.33), their expected bailout size state-by-state will differ from the planner's. The insight of the moral hazard consequences in Appendix D are thus preserved here.

Similarly the insight about the interaction of endogenous sentiment and bailouts survives, since it only requires the *marginal welfare* functions to be impacted by price extrapolation and bailouts in the same way:

$$u'(c_1) = -\mathbb{E}_1\left[\frac{\partial \mathcal{W}_2}{\partial d_1}\left(d_1, b^*(d_1, H, z_2 + \Omega_2(\boldsymbol{q}_1 - q_0)), H, z_2 + \Omega_2(\boldsymbol{q}_1 - q_0)\right)\right] \tag{E.35}$$

$$\boldsymbol{q}_1 = \mathbb{E}_1\left[\frac{\partial \mathcal{W}_2}{\partial H}\left(d_1, b^*(d_1, H, z_2 + \Omega_2(\boldsymbol{q}_1 - q_0)), H, z_2 + \Omega_2(\boldsymbol{q}_1 - q_0)\right)\right] \tag{E.36}$$

E.3 Monetary Policy

The only difference yields in the form of the reversal externality: changes in future sentiment now impact welfare *indirectly* by changing asset prices. This leads to the following expression for monetary policy:

$$\frac{d\mathcal{W}_1}{dr_1} = \underbrace{\frac{dY_1}{dr_1}\mu_1}_{(i)} + \underbrace{\frac{dd_1}{dr_1}\mathcal{W}_d}_{(ii)} + \underbrace{\frac{dH}{dr_1}\mathcal{W}_H}_{(iii)}$$

[4] The welfare of households is still irrelevant here since the loan is made at the market rate.

$$+ \underbrace{\frac{d\Omega_2}{dq_1}\frac{dq_1}{dr_1}\left(\frac{dd_1}{d\Omega_2}\mathcal{W}_d + \frac{dH}{d\Omega_2}\mathcal{W}_H\right)}_{(iv)} + \underbrace{\beta\mathbb{E}_1\left[\frac{dq_2}{d\Omega_3}\frac{d\Omega_3}{dq_1}\frac{dq_1}{dr_1}\kappa_2\phi H\right]}_{(v)} \tag{E.37}$$

F Heterogenous Beliefs

A fully general treatment of heterogeneity in beliefs inside the framework presented previously lies outside the scope of this paper. I thus focus on a stylized version of heterogeneity where all financial intermediaries are over-optimistic, but differ in their degree of over-optimism.[5] Financial intermediaries are indexed by $i \in [0,1]$, and bank i holds a belief distortion of $\Omega_{2,i}$, with:

$$\Omega_{2,i} = \Omega_2 + \epsilon_2(2i-1) \tag{F.1}$$

such that the most pessimistic bank is bank 0 with a bias of $\Omega_2 - \epsilon_2 > 0$, bank 1/2 holds an average bias Ω_2 and bank $t = 1$ is the most optimistic with a bias of $\Omega_2 + \epsilon_2$. Put simply, financial intermediaries' beliefs are distributed uniformly around a value of Ω_2. Furthermore, I assume that this heterogeneity is common knowledge, and everyone agrees that there is no more heterogeneity in beliefs at time $t = 2$ onwards, and that the social planner can only impose a uniform tax or leverage limit (i.e., the planner imposes a uniform regulation). Last, I assume that the risky asset is in a fixed supply H to focus on leverage decisions. I start, as usual, by backward induction.

Financial Intermediaries at $t = 2$ Financial intermediaries enter the period with heterogeneous net worth $n_{2,i}$ (coming from heterogeneous leverage and heterogeneous holdings of the risky asset), and they hold homogeneous beliefs. Start with the following lemma:

Lemma 1. *In a crisis equilibrium, financial intermediaries have the same consumption level at $t = 2$, irrespective of the heterogeneity in net worth. This consumption level is given by:*

$$c_2 = \int_0^1 n_{2,i}di + \phi H q_2 \tag{F.2}$$

and the price of the risky asset in equilibrium is implicitly defined by:

$$q_2 = \beta\bar{c}_2\mathbb{E}_2[z_3] + \phi q_2(1-\bar{c}_2) \tag{F.3}$$

Proof. An individual bank's optimality condition, in a crisis, for holding the risky asset is given by:

$$q_2 = \beta c_{2,i}\mathbb{E}_2[z_3] + \phi q_2(1-c_{2,i}) \tag{F.4}$$

[5] Accordingly, the planner will use beliefs that are outside the convex combination of agents' beliefs. See Brunnermeier, Simsek and Xiong (2014) for an analysis of a welfare criterion with heterogeneous beliefs and when the planner does not take a stand on whose belief is correct.

which is a linear function of $c_{2,i}$, while all other variables are common to all agents. Thus, $c_{2,i} = c_{2,j}$ for all i and j in $[0,1]$. Integrating over gives:

$$\int_0^1 c_{2,i} = \int_0^1 n_{2,i} + \phi q_2 \int_0^1 m_{2,i} \tag{F.5}$$

and by market clearing $m_{2,i} = H$, while $c_{2,i} = \bar{c}_2$ by what precedes. □

This lemma also implies that individual's holdings of the risky asset are given by:

$$m_{2,i} = \frac{\bar{c}_2 - n_{2,i}}{\phi q_2}. \tag{F.6}$$

Note that this means that financial intermediaries entering with higher net worth end up holding *less* of the assets. This is because they need to borrow less: indeed, the level of borrowing of bank i in equilibrium at $t = 2$ is $d_{2,i} = \phi m2, i q_2$. This level of consumption, however, is not what is expected by agents since they believe that the realization of the dividend z_2 will be higher on average. In other words, we have $\mathbb{E}_1^{SP}[\bar{\lambda}_2] > \mathbb{E}_{1,i}[\bar{\lambda}_2]$ for all i.

Welfare Analysis at $t = 1$: Taking this into account, the social planner first-order condition is given by:[6]

$$0 = \int_0^i \lambda_{1,i} - \mathbb{E}_1^{SP}\left[\bar{\lambda}_2\right] - \mathbb{E}_1^{SP}\left[\phi H \bar{\kappa}_2 \frac{\partial q_2}{\partial \bar{n}_2}\right]. \tag{F.7}$$

The utilitarian social planner can thus maximize welfare by imposing a uniform tax on leverage equal to:

$$\tau_d = \frac{\mathbb{E}^{SP}[\bar{\lambda}_2] - \int_0^i \mathbb{E}_{1,i}[\bar{\lambda}_2] + \mathbb{E}_1^{SP}\left[\phi H \bar{\kappa}_2 \frac{\partial q_2}{\partial \bar{n}_2}\right]}{\int_0^i \lambda_{1,i}} \tag{F.8}$$

which is, again, showing the robustness of the formulation in Proposition 1. And here again, intuitively, a leverage limit is robust to heterogeneity, whereas the tax is not. Since the planner's beliefs are outside the convex set of agents' beliefs, the required leverage is below the decentralized outcome for each financial intermediary, hence a leverage limit will be binding for every financial intermediary, and will bring back this margin to the second-best.

Impact of Heterogeneity on the Optimal Tax: A natural question that arises is whether heterogeneity in beliefs has a detrimental effect on the behavioral wedge and the collateral externality.

Bank i with beliefs $\Omega_{2,i}$ believes that the net worth of bank j in period $t = 2$ will be:

$$\mathbb{E}_{1,i}[n_{2,j}] = (z_2 + \Omega_{2,i}) m_{1,j} - (1 + r_1) d_{1,j} \tag{F.9}$$

[6]The aggregation made on the collateral externality part is made possible by the linearity of preferences at $t = 3$, also responsible for the fact that marginal utility is homogeneous at $t = 2$.

and so it believes that the aggregate net worth of the financial system will be:

$$\mathbb{E}_{1,i}[\bar{n}_2] = (z_2 + \Omega_{2,i})H - (1 + r_1) \int_0^1 d_{1,j} dj \tag{F.10}$$

but the $\int_0^1 d_{1,j}dj$ is correct since I assumed that belief disagreement were common knowledge. Hence the distribution of beliefs about aggregate net worth is uniformly distributed. It thus follows that the average belief about $\bar{n}_2$ is the same with and without heterogeneity, if $\int_0^1 d_{1,j}dj$ is kept constant.

In which direction goes aggregate leverage, $\int_0^1 d_{1,j}dj$? To understand what happens, consider the simplified case where there is no risk, and agents cannot trade their endowment of the risky asset at $t = 1$ (to prevent arbitrage). Then, (perceived) consumption smoothing implies that:

$$e_1 + d_{1,i} = (z_2 + \Omega_{2,i})H - (1 + r_1)d_{1,i} + \phi H q_2(\bar{c}_{2,i}) \tag{F.11}$$

which yields:

$$e_1 + (2 + r_1)d_{1,i} = (z_2 + \Omega_{2,i})H + \phi H q_2(\bar{c}_{2,i}). \tag{F.12}$$

Aggregating over individuals, we get;

$$e_1 + (2 + r_1) \int_0^1 d_{1,i} di = (z_2 + \bar{\Omega}_2)H + \phi H \int_0^1 q_2(\bar{c}_{2,i}) di \tag{F.13}$$

and hence $\int_0^1 d_{1,i}di$ is implicitly defined by this relation since $\bar{c}_{2,i}$ is a function of $\int_0^1 d_{1,j}dj$. This is to be compared with the homogenous relation:

$$e_1 + (2 + r_1)d_{1,i} = (z_2 + \bar{\Omega}_2)H + \phi H q_2(\bar{c}_2). \tag{F.14}$$

Inspecting equations (F.13) and (F.14) shows that the behavior of aggregate leverage is determined by whether $\int_0^1 q_2$ is an increasing or decreasing function with respect to the heterogeneity of beliefs. The concavity of the price function (see Online Appendix Q.10) means that this is a decreasing function, implying that heterogeneity causes *lower* aggregate leverage (the slightly more optimistic financial intermediary takes on less additional leverage than what the pessimistic financial intermediary subtracts). Since the optimal leverage target of the planner is unchanged by the presence of heterogeneity, this heterogeneity reduces the gap between the aggregate decentralized solution and the planner's solution.

G Alternative Measures of Sentiment

I document the covariance between sentiment and financial intermediaries' health using six different measures that are common in the literature:

1. The HY indicator of Greenwood and Hanson (2013);

2. The GZ credit spreads Gilchrist and Zakrajšek (2012);

3. The LTG measure from Bordalo, Gennaioli, La Porta and Shleifer (2020);

4. The PVS indicator of Pflueger, Siriwardane and Sunderam (2020);

5. The BW measure of sentiment of Baker and Wurgler (2007);

6. The $CAPE$ ratio of Campbell and Shiller (1988).

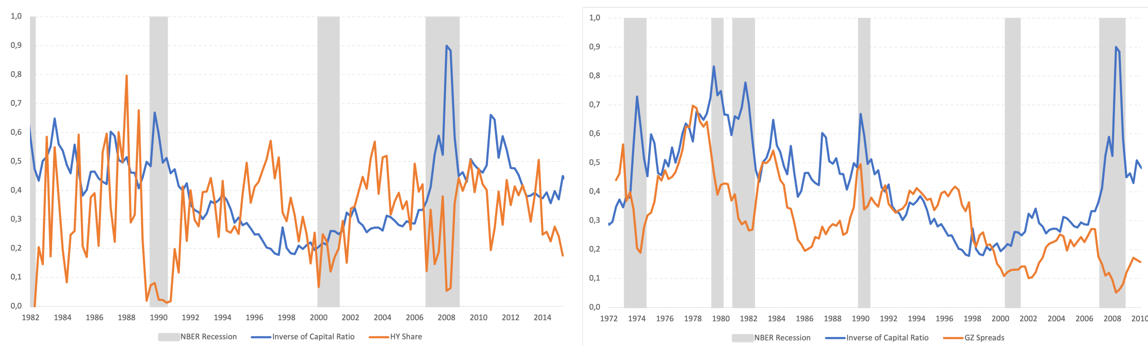

Figure 2: Time-series variation of λ_2 and credit-market proxies for Ω_3. For the financial health of intermediaries λ_2, I rely on He et al. (2017) which computes an intermediary capital ratio. The inverse of this capital ration is proportional to λ_2 when agents have log-utility, as in this model. For Ω_3, I use the High-Yield share of issuance measure of Greenwood and Hanson (2013) on the left panel and invert the credit-spread measure of Gilchrist and Zakrajšek (2012) on the right panel.(credit spreads are high when sentiment is low, and vice-versa).

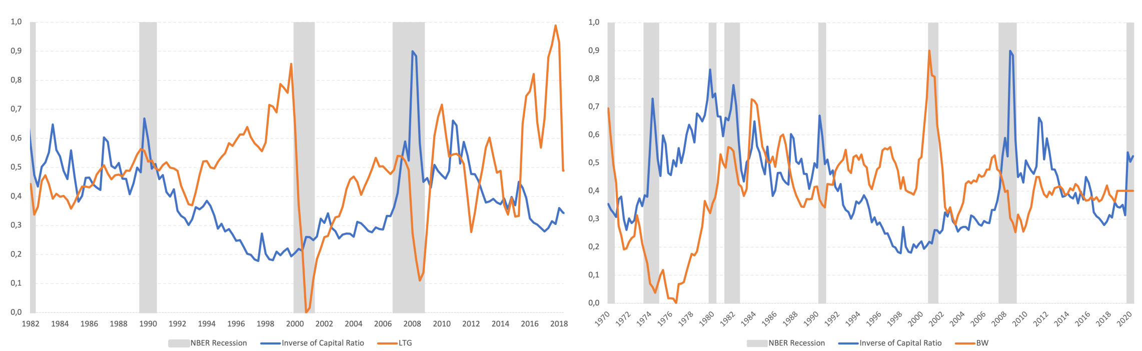

Figure 3: Time-series variation of λ_2 and stock-market sentiment for Ω_3. For the financial health of intermediaries λ_2, I rely on He et al. (2017) which computes an intermediary capital ratio. For Ω_3 on the left panel, I use the Long Term Growth (LTG) measure of Bordalo et al. (2020). This is directly constructed from survey data by aggregating stock market analysts' expectations. For the right panel, I use the Baker-Wurgler index of sentiment of Baker and Wurgler (2007).

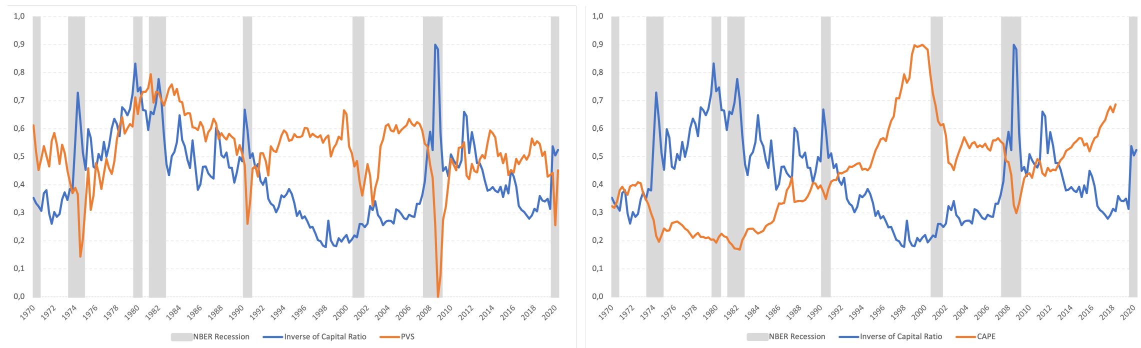

Figure 4: Time-series variation of λ_2 and additional proxies for Ω_3. For the financial health of intermediaries λ_2, I rely on He et al. (2017) which computes an intermediary capital ratio. For the left panel for Ω_3, I use the Price of Volative Stock (*PVS*) measure of Pflueger et al. (2020). For the right panel I use the the CAPE ratio of Campbell and Shiller (1988).

H Additional Results for Ω-Uncertainty

H.1 Ω_3-Uncertainty

This section extends the insights of Section 5 to the case where the uncertainty pertains to Ω_3. I start by studying the realization of only one state of the world, and complete the proof using the linearity of expectations.

I assume that for a given realization of z_2, the planner has a uniform distribution on sentiment during a crisis:

$$w_3 \sim \mathcal{U}\left[\bar{\Omega}_3 - \sigma_\Omega, \bar{\Omega}_3 + \sigma_\Omega\right] \tag{H.1}$$

The integral (denoted by L) used by the social planner to compute the marginal effect on welfare on increasing leverage becomes:

$$L = \frac{1}{2\sigma_\Omega}\int_{-\sigma_\Omega}^{\sigma_\Omega} \frac{\partial \mathcal{W}_2}{\partial n_2}\left(d_1, H; q_2, z_2, \bar{z}_3 - \bar{\Omega}_3 - \omega_3\right) d\omega_3 \tag{H.2}$$

Assume first that for all realisations of ω_3 the resulting equilibrium is a crisis one. This yields:

$$L = \frac{1}{2\sigma_\Omega}\int_{-\sigma_\Omega}^{\sigma_\Omega} \frac{1}{n_2 + \phi H(\bar{z}_3 - \bar{\Omega}_3 - \omega_3)} d\omega_3 \tag{H.3}$$

$$\implies L = -\frac{1}{(2\sigma_\Omega)\phi H}\left[\ln(n_2 + \phi H(\bar{z}_3 - \bar{\Omega}_3 - \omega_3))\right]_{-\sigma_\Omega}^{\sigma_\Omega} \tag{H.4}$$

$$\implies L = \frac{1}{(2\sigma_\Omega)\phi H}\ln\left(\frac{n_2 + \phi H(\bar{z}_3 - \bar{\Omega}_3 + \sigma_\Omega))}{n_2 + \phi H(\bar{z}_3 - \bar{\Omega}_3 - \sigma_\Omega))}\right) \tag{H.5}$$

This is a functions of the type:

$$f(x) = \frac{1}{x} \ln \left(\frac{K+x}{K-x} \right) \tag{H.6}$$

And we can show that this is increasing in x, for $x \in [0, K]$. Indeed, the derivative is given by:

$$f'(x) = \frac{(K^2 - x^2) \ln \left(\frac{K+x}{K-x} \right) + 2Kx}{x^2 (K-x)(K+x)} \tag{H.7}$$

The denominator is clearly positive, but the denominator is indeterminate. Take the derivative of the denominator:

$$\frac{d}{dx} (K^2 - x^2) \ln \left(\frac{K+x}{K-x} \right) + 2Kx = 2x \ln \left(\frac{K+x}{K-x} \right) > 0 \tag{H.8}$$

The denominator is thus increasing and its limit in 0 is 0. Hence, f is increasing on $[0, K]$. Accordingly, L is increasing in σ_Ω.

Left now is the same calculation when for some parts of the uncertainty set, the economy is outside of a crisis. Following the same steps as before, this boils down to the study of, the time:

$$g(x) = \frac{1}{x} \ln \left(\frac{1}{K-x} \right) \tag{H.9}$$

Where the derivative is now:

$$g'(x) = \frac{\frac{x}{a-x} - \ln \left(\frac{1}{K-x} \right)}{x^2} \tag{H.10}$$

And the derivative of the numerator is:

$$\frac{d}{dx} \frac{x}{a-x} - \ln \left(\frac{1}{K-x} \right) = \frac{x}{(a-x)^2} > 0 \tag{H.11}$$

Since $g'(0^+) > 0$, g is increasing. Thus the same result applies. This concludes the proof by linearity of expectations: since this integral is increasing in σ_Ω, all components of the expectations over all future states of the world are increasing, and it then follows that the overall expectation is increasing in σ_Ω.

H.2 Amplification with Price Extrapolation

So far the exercise was done assuming that sentiment was constant state-by-state in period $t = 2$. Do the results change once we extend this to price-dependent biases? The answer lies in the shape of the marginal welfare functions once sentiment moves with prices inside a crisis:

$$\frac{d\mathcal{W}_2}{dn_2} = \lambda_2 + \kappa_2 \phi H \frac{d\Omega_3}{dq_2} \frac{dq_2}{dn_2} \tag{H.12}$$

The question is whether this added part, which is simply the collateral externality, is adding or retrenching convexity. With price extrapolation, we have:

$$\frac{d\Omega_3}{dq_2} = \alpha \tag{H.13}$$

So the only part left is the shape of $\frac{dq_2}{dn_2}$. Fortunately, we showed in Section Q.10 that this is also a convex function: see equations (Q.58) to (Q.63). Hence the marginal welfare function is more convex, amplifying the need for preventive restrictions in the face of uncertainty.

H.3 Ω-Uncertainty and Investment

So far, Proposition 10 was concerned about leverage restrictions. How is uncertainty changing the uninternalized effects of investment in H? Assume that sentiment is exogenous.[7] The first order condition becomes:

$$\lambda_1 c'(H) = \frac{1}{2\sigma_\Omega}\int_0^\infty \left[\int_{-\sigma_\Omega}^{\sigma_\Omega} \lambda_2(z_2 - \bar{\Omega}_2 - \omega_2)(z_2 - \bar{\Omega}_2 - \omega_2 + q_2(z_2 - \bar{\Omega}_2 - \omega_2))d\omega_2\right] f_2(z_2)dz_2. \tag{H.14}$$

Fortunately, it is now straightforward to sign the derivative of this function given the previous proofs. We know that $\lambda_2(z_2 - \bar{\Omega}_2 - \omega_2)$ is convex in ω_2. This is multiplied by a linear and positive function of ω_2 (the dividends), and then by the price realization at $t = 2$.

The price at $t = 2$ is given by:

$$q_2 = \beta(n_2 + \phi M \mathbb{E}_2[z_3])\mathbb{E}_2[z_3] + \phi(1 - n_2 - \phi M \mathbb{E}_2[z_3])\mathbb{E}_2[z_3] \tag{H.15}$$

Which is clearly linear in ω_2 since net worth is linear in ω_2:

$$n_2 = (z_2 - \bar{\Omega}_2 - \omega_2)H - d_1(1 + r_1) \tag{H.16}$$

Hence this function is convex in ω_2, which implies that the right-hand side of the first-order condition is increasing in uncertainty. This time, however, this means that $c'(H)$ in equilibrium needs to be higher than in the decentralised equilibrium. Hence, uncertainty calls for increasing investment (or, in the case with large exuberance, less restrictions on investment). Intuitively, uncertainty increases the SDF that prices the asset, meaning that more consumption should be shifted to the future.

H.4 Ω-Uncertainty and Reversal Externality

How is sentiment uncertainty influencing the optimal conduct of monetary policy? The previous derivations can help us answer that question. The reversal externality that monetary policy explic-

[7] I slightly abuse notations below by not writing Ω_3 for simplicity. This is harmless since we are fixing the first-order condition of private agents and simply study whether the first-order condition of the social planner is increasing or decreasing in σ_Ω.

itly targets is expressed as:

$$\mathcal{R}_q = \mathbb{E}_1 \left[\kappa_2 \phi H \frac{d\Omega_3}{dq_1} \frac{dq_1}{dr_1} \right] \tag{H.17}$$

The only unknown part of this expression, from the perspective of period $t = 1$, is the product $\kappa_2 d\Omega_3/dq_1$. Fortunately, we just showed that κ_2 is a convex object with respect to sentiment uncertainty. It then depends on the shape of $d\Omega_3/dq_1$ with respect to sentiment uncertainty. For instance, with price-extrapolation, $\kappa_2 d\Omega_3/dq_1 = \kappa_2 \alpha$ and so this object is still convex.

Thus, sentiment uncertainty with linear price-extrapolation increases the incentive for the central bank to tighten interest rates when asset prices soar. To conclude, in times of heightened uncertainty about Ω_2 or Ω_3, with price extrapolation, the central planner should:

1. Tighten leverage limits;
2. Relax LTV ratios;
3. Increase the interest rate.

I Infinite-Horizon Model

This section provides a simple infinite-horizon version of the model. It shows how the insights derived in the main paper are not dependent on the 3-period structure assumed.

Financial intermediaries have a utility function given by:

$$U_t = \sum_{i \geq 0}^{+\infty} \beta^{t+i} \ln(c_{t+i}) \tag{I.1}$$

While households have again linear-utility throughout:

$$U_t^h = \sum_{i \geq 0}^{+\infty} \beta^{t+i} c_{t+i}^h \tag{I.2}$$

I assume that the stock of assets H is fixed and given. It can only be held by intermediaries. The budget constraint of financial intermediaries at t are:

$$c_t + d_{t-1}(1 + r_{t-1}) + q_t h \leq d_t + (z_t + q_t)H \tag{I.3}$$

$$d_t \leq \phi h \mathbb{E}_t[z_{t+1} + \Omega_{t+1}] \tag{I.4}$$

Where in equilibrium $h = H$. The first-order conditions, using the same notation for the Lagrange multipliers as in the core of the text, are thus given by:

$$\lambda_t = \frac{1}{c_t} \tag{I.5}$$

$$\lambda_t = \beta(1+r_t)\mathbb{E}_t[\lambda_{t+1}] + \kappa_t \tag{I.6}$$

$$\lambda_t q_t = \beta\mathbb{E}_t[\lambda_{t+1}(z_{t+1} + \Omega_{t+1} + q^r_{t+1})] + \phi\kappa_t\mathbb{E}_t[z_{t+1} + \Omega_{t+1}] \tag{I.7}$$

I assume that the planner can impose a tax on borrowing, or a tax on the holdings of the risky asset. Since H is fixed, this tax only purpose is to change the equilibrium price of the asset. Practically, this policy can be implemented through monetary policy, as explored in Section 6, with spillovers on inflation targeting. I focus on a simple asset tax here for simplicity.

One-Time Policy Intervention Start with the easiest case where the planner intervenes only once and commits to never intervene again afterwards. Thus the equilibrium is the laissez-faire one starting from $t+1$. The planner chooses directly d_t and q_t at t, and takes as given the future values of d_{t+j} and q_{t+j} that will be freely determined in equilibrium.

The social planner maximises:

$$\mathcal{W}_t = \ln(c_t) + \beta\mathbb{E}_t[\mathcal{W}_{t+1}(d_t, q_t)] \tag{I.8}$$

The first-order conditions of the social planner are given by:

$$0 = \lambda_t - \beta\mathbb{E}_t[\lambda_{t+1}] - \sum_{j\geq 1}^{+\infty}\beta^{t+j}\mathbb{E}_t\left[\kappa_{t+j}\phi H\frac{d\Omega_{t+j}}{dq_{t+1}}\frac{dq_{t+1}}{dn_{t+1}}\right] \tag{I.9}$$

$$0 = \sum_{j\geq 0}^{+\infty}\beta^{t+j}\mathbb{E}_t\left[\kappa_{t+j}\phi H\frac{d\Omega_{t+j}}{dq_t}\right] \tag{I.10}$$

The social planner is thus trying to manipulate two things: (i) how future sentiment will be affected by future prices since a change in borrowing today impact prices tomorrow; and (ii) how future sentiment will be affected by current prices.[8]

Discussion of Implementability Constraints The above analysis allowed the Social Planner to directly choose the asset price at t. This simplifies the analysis but at the same time lacks concreteness. It is hard to imagine an infinite-horizon problem where the planner cannot realistically circumvent the market determination of asset prices at each t.

A full analysis of the problem where the social planner chooses short-term debt on behalf of private agents, and asset prices remain market-determined, is outside the scope of this paper. A

[8] The derivates effect $d\Omega_{t+j}/dq_{t+1}$ are assumed to be taking into account the full effects on Ω_{t+j} for conciseness. For example for Ω_{t+2}, it implicitly factors in how prices at $t+1$ directly impact sentiment at $t+2$, but also how the change in Ω_{t+1} changes q_{t+2} and thus Ω_{t+2}. See Section 6.3 for an example on the 4-period model.

few remarks can be made, nevertheless. Bianchi and Mendoza (2018) show that in a setup with a current-price collateral constraint, the optimal policy crucially depends on whether the planner has commitment or not. The intuition goes as follows: during a crisis, the planner would like to promise lower future consumption. This changes the stochastic discount factor, and thus props up asset prices, relaxing the borrowing constraint. This is however time-inconsistent: next period, it will be sub-optimal for the planner to implement this low level of consumption.

Such an effect would also arise here in the case of endogenous sentiment: the planner would like to prop up asset prices at t in order to prop up Ω_{t+1} and relax the collateral constraint (again with belief amplification replacing the traditional role of financial amplification). Note, however, that the problem would be vastly more complicated: the consumption that the planner would promise is not the one expected by private agents, since agents expect future consumption to depend on their biased estimate of future dividends. But to prop up asset prices like suggested by Bianchi and Mendoza (2018), it has to be that agents believe future consumption to be lower than under laissez-faire, since it is private agents' pricing condition that implements asset prices in equilibrium.

This also raises the more general question of policy in models where agents are behavioral. In my baseline setup, agents should be surprised that the planner is intervening: the model does not feature any externality from a rational perspective. There is thus the open question of what agents believe about future policy (an issue I briefly touched upon in Section 6.4), and whether agents should adapt in the face of recurrent intervention. These fascinating issues are left to future research.

J Various Psychological Models of Asset Prices and Ω-Correspondence

J.1 Diagnostic Expectations

Diagnostic expectations are a psychologically founded model of belief formation in light of new data. It builds on the representativeness heuristic of Tversky and Kahneman (1983): agents overweight attributes of a class that are more frequent in that class than in a reference class. Bordalo, Gennaioli and Shleifer (2018) apply this logic to belief formation about aggregate economic condition. Specifically, assume that the state of the world follows an AR(1) process:

$$z_t = bz_{t-1} + \epsilon_t \tag{J.1}$$

with $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. By taking as a reference point the state where there is no news, Bordalo et al. (2018) derive that the diagnostic distribution is also normal, with the same variance, but with mean:

$$\mathbb{E}_t^\theta[z_{t+1}] = \mathbb{E}_t^{SP}[z_{t+1}] + \theta\left(bz_t - b^2 z_{t-1}\right) \tag{J.2}$$

where θ is the parameter governing the representativeness bias. Diagnostic expectations are thus nested as:

$$\Omega_{t+1} = \theta \left(bz_t - b^2 z_{t-1} \right) \tag{J.3}$$

which is close to the reduced-form used in the core of the paper, $\Omega_{t+1} = \alpha \left(z_t - z_{t-1} \right)$. The difference is that, for diagnostic expectations, what matters is not the *per se* movements in z, but the unexpected component of these movements.

J.2 Internal Rationality

Adam and Marcet (2011) present a model where agents are not "externally rational:" they do not know the true stochastic process for payoff relevant variables beyond their control, i.e. prices in my setup. Adam, Marcet and Beutel (2017) apply this idea in an asset pricing framework, giving rise to boom-bust cycles. Here I adapt their idea to my setup with some simplifying assumptions, and show in which circumstances the results change.

Agents are rational regarding the distribution of z_t, but they believe prices evolve according to:

$$q_{t+1} = q_t + \beta_{t+1} + \epsilon_{t+1} \tag{J.4}$$

with ϵ_{t+1} is a transitory shock and β_{t+1} is a persistent component evolving as:

$$\beta_{t+1} = \beta_t + \nu_{t+1}. \tag{J.5}$$

Furthermore, all innovations are jointly normal. Adam et al. (2017) show that under some conditions, and when agents are using a steady-state precision, the filtering problem boils down to expectations evolving as:

$$\tilde{E}_t[q_{t+1}] = (1+g)(q_t - q_{t-1}) + (1-g)\tilde{E}_{t-1}[q_t] \tag{J.6}$$

where g is the equivalent of a Kalman gain, function of the variances of the noise terms. To make progress, I further assume that agents place a low conditional variance on this estimate, such that I can study the limiting case where this point estimate is believed to be certain (i.e. there is no risk for the price next period in agents' mind). I denote by $\tilde{q}_2$ this point estimate, such that agents' optimization yields:

$$q_1 = \beta \mathbb{E}_1 \left[\frac{\lambda_2}{\lambda_1} (z_2 + \tilde{q}_2) \right]. \tag{J.7}$$

Equation (J.7) can be rewritten using the correct price used by the planner q_2:

$$q_1 = \beta \mathbb{E}_1 \left[\frac{\lambda_2}{\lambda_1} (z_2 + q_2 + (\tilde{q}_2 - q_2)) \right]. \tag{J.8}$$

so an equivalent to the Ω_2 used throughout this paper is $\Omega_2^q = \tilde{q}_2 - q_2$: a bias on expected prices that is positive (exuberance) when the forecasted value if above the realized value, and vice-versa.

How does this impact the welfare analysis? It crucially depends on the form of the collateral constraint. If we stay in the benchmark case where the collateral constraint takes the form:

$$d_2 \le \phi H \mathbb{E}_2[z_3] \tag{J.9}$$

then it is clear that since agents are correct about the distribution of fundamentals, they will make no mistake regarding their future net worth or the future borrowing capacity of the economy. Consequently, the only margin that is distorted if the investment margin: agents are too optimistic (pessimistic) regarding the payoffs of their investment, since they are too optimistic (pessimistic) regarding the resale value of the asset they are creating. Thus, only the behavioral wedge for investment is non-zero in this case.

Importantly, there are no externalities anymore. Indeed, decisions during the boom will impact time expectations of prices made at $t = 2$ but these expectations will not affect the tightness of collateral constraints.

This discussion makes clear that for externalities to survive in this case, it is necessary to have a collateral constraint that depends on prices (either current prices, or expected prices), whereas biases on fundamentals impact welfare in a "robust" way. When the collateral constraint takes the form:

$$d_2 \le \phi H q_2 \tag{J.10}$$

then biases impact its tightness: when agents are over-pessimistic regarding future prices at $t = 3$, that impacts the equilibrium value of q_2.[9] In this case externalities survive. But notice that the sign of the key derivative for the reversal externality is clearly ambiguous:

$$\frac{d\Omega_3^q}{dq_1} = \frac{d\tilde{q}_3}{dq_1} = (1-g)\left(\frac{d\tilde{q}_2}{dq_1} - 1\right). \tag{J.11}$$

This is because sentiment is "sticky" with learning. If by reducing asset prices at $t = 1$, the planner makes future agents more pessimistic in a financial crisis, that hurts welfare.

J.3 Overconfidence

In an early behavioral finance survey, De Bondt and Thaler (1995) stated that "perhaps the most robust finding in the psychology of judgment is that people are overconfident." Overconfidence has been most widely used to explain large trading volume, by generating substantial disagreement between investors Odean 1998. Because this paper is about *aggregate* over-optimism or over-pessimism, I will focus in this section on the features of overconfidence that can generate momen-

[9] I am here slightly abusing notation, since strictly speaking there is no price at $t = 3$. But claiming that there is no bias due to internal rationality in the crisis period would only come from the simplifying assumption that the horizon is finite.

tum and reversals.[10] The interested reader can find an exploration of how heterogeneous beliefs among financial intermediaries impact the results in Appendix F.[11]

Financial institutions have a prior over the distribution of dividends in period $t = 2$:

$$z_2 \sim \mathcal{N}(\mu_0, \sigma_0^2) \tag{J.12}$$

and receive a signal $s = z_2 + \epsilon$ with:

$$\epsilon \sim \mathcal{N}(0, \sigma_s^2). \tag{J.13}$$

Overconfident financial intermediaries have a posterior of:

$$z_2 \sim \mathcal{N}\left(\mu_0 + \frac{\sigma_0^2}{\sigma_0^2 + \tilde{\sigma}_s^2}(s - \mu_0), \frac{\sigma_0^2}{1 + \frac{\sigma_0^2}{\tilde{\sigma}_s^2}}\right) \tag{J.14}$$

where $\tilde{\sigma}_s^2 < \sigma_s^2$, which means that overconfident agents believe that the signal has a higher precision than in reality. This directly implies that the bias, relative to the social planner valuation, is given by:

$$\Omega_2 = \frac{\sigma_s^2 - \tilde{\sigma}_s^2}{(\sigma_0^2 + \tilde{\sigma}_s^2)(\sigma_0^2 + \sigma_s^2)}\sigma_0(s - \mu_0) \tag{J.15}$$

so that agents become exuberant after positive news ($s > \mu_0$): $\Omega_2 > 0$.

Notice how the variance of the two distributions are different with overconfidence. As such, the results in Propositions 2 and 5 are not directly applicable. But a higher $\tilde{\sigma}_s^2$ means that agents are using a narrower distribution than the social planner. This is reminiscent of the results presented in Section 5: intuitively, this will create an even larger gap between the two solutions since agents will neglect left-tail and right-tail events. As shown in Proposition 10, this is calling for tighter macroprudential regulation ex-ante.

J.4 Sticky Beliefs

While this paper is mostly concerned with investors that adjust their views too much in response to information, there is also widespread evidence of investors adjusting their beliefs *too little*. A recent example is the work of Bouchaud, Krueger, Landier and Thesmar (2019), where investors form expectations according to:

$$\tilde{\mathbb{E}}_1[z_2] = (1 - \lambda)\mathbb{E}_1^r[z_2] + \lambda\tilde{\mathbb{E}}_0[z_2] \tag{J.16}$$

where $\mathbb{E}_1^r$ is the rational time 1 expectations about the future dividend. When $\lambda = 0$, expectations are fully rational. When $\lambda > 0$, expectations depend on past expectations. In terms of the notation

[10] See e.g. Daniel, Hirshleifer and Subrahmanyam (1998).

[11] Caballero and Simsek (2020a) focus on prudential policies with financial speculation, but in an environment with aggregate demand – rather than pecuniary – externalities.

of my paper, the bias can be expressed as:

$$\tilde{\mathbb{E}}_1[z_2] = \mathbb{E}_1^{SP}[z_2] + \lambda\left(\tilde{\mathbb{E}}_0[z_2] - \mathbb{E}_1^r[z_2]\right) \tag{J.17}$$

so that

$$\Omega_2 = \lambda\left(\tilde{\mathbb{E}}_0[z_2] - \mathbb{E}_1^r[z_2]\right). \tag{J.18}$$

Agents are thus over-optimistic in period t when the objective expected dividend is less than the expectation agents held in period $t-1$. Expanding this expression recursively yields:

$$\Omega_2 = \lambda\left(\mathbb{E}_0^r[z_2] - \mathbb{E}_1^r[z_2]\right) + \lambda\Omega_1. \tag{J.19}$$

which naturally gives rise to a formulation close to the one stipulated in Assumption 7.

Finally, note that this formulation does not necessarily imply pessimism during booms, and so calls for less aggressive macroprudential leverage limits. Indeed, agents are over-optimistic as long as $\tilde{\mathbb{E}}_0[z_2] > \mathbb{E}_1^r[z_2]$. It thus suffices that agents should revise their expectations down to create optimism. The three-period model is not suited to study this kind of dynamics, where a slowdown in growth for example creates irrational exuberance. But the unravelling of sentiment along such a cycle can be understood in the extended framework of Section 6.3. There, I showed that tightening later in the cycle has ambiguous effects since it also makes agents more pessimistic *during* a crisis.

J.5 Inattention

Gabaix (2019) argues that "much of behavioral economics may reflect a form of inattention." He proposes a theory of over- and under-reaction that rests on agents anchoring on a default autocorrelation parameter. Specifically, assume that the dividend process follows and AR(1) as in:

$$z_{t+1} = \rho z_t + (1-\rho)z_0 + \epsilon_{t+1} \tag{J.20}$$

Because agents have to deal with too many such processes, they may not fully perceive each autocorrelation, and instead use ρ_s to make forecasts, with:

$$\rho_s = m\rho + (1-m)\rho_d \tag{J.21}$$

where ρ_d is the average autocorrelation agents encounter. It is then straightforward to show that the bias used in this paper becomes:

$$\Omega_{t+1} = (\rho_s - \rho)(z_t - z_0). \tag{J.22}$$

Agents are thus overreacting when the autocorrelation parameter of the dividend process is less than the anchor value, ρ_d, since $\rho_s - \rho = (1-m)(\rho_d - \rho)$. When this is the case, agents make forecasts thinking that the dividend process is more persistent than in reality, thus putting too much weight on recent data and not enough on the unconditional mean of the process. The opposite happens when $\rho > \rho_d$.

K The Mistakes of Rational Calibration

In Appendix C.2.2, I showed that the collateral externality of the behavioral model differ from the rational counterfactual: sentiment creates two countervailing forces. First, entrenched pessimism makes the asset price less sensitive to changes in net worth, reducing the size of the pecuniary externality. Second, a change in net worth leads to a change in price because of financial amplification, which itself can lead to alleviating pessimism, supporting asset prices. This makes the price more sensitive to changes in net worth.

While it is entirely possible, given the presence of these two countervailing forces, that the introduction of sentiment in this model does not tremendously change the size of the pecuniary externalities, it can still imply large policy differences if the modeler uses the rational expectations hypothesis during a calibration. To understand this, notice that the pecuniary externality is a structural object:

$$\beta \mathbb{E}_1^{SP}\left[\phi \kappa_2 \frac{dq_2}{dn_2}\right] \tag{K.1}$$

and hence is not something that can be measured directly from the data. The pecuniary externality corresponds to a counterfactual exercise, that asks the question "by how much would the price of the collateral asset change if all financial intermediaries were to reduce their leverage exogenously before the crisis happens?" Quantitatively answering this question thus requires a calibration determining the value of each parameter, such as the strength of financial frictions ϕ, that controls the pecuniary externality.

One strategy used in the quantitative macroprudential literature, starting with the seminal work of Bianchi (2011) or more recently by Herreño and Rondón-Moreno (2020), calibrates the financial friction parameter ϕ combining (i) the rational expectation hypothesis, and (ii) a targeted moment on the probability or severity of financial crises.

To illustrate how behavioral forces might hinder this inference, I use a simplified version of the model where the collateral term is ignored in the pricing equation, and without risk.[12] I also assume that the stock of collateral assets H is exogenously fixed to streamline the exposition. Assume that we are aiming at calibrating our model such that a crisis provokes a price drop of X%. We can work through the rational equilibrium conditions to link the targeted moment X to the collateral parameter ϕ as:

$$\frac{1}{X} = 1 + \frac{Hz_3}{2 - \phi H z_3} \tag{K.2}$$

which directly implies that a smaller ϕ (more stringent financial frictions) is needed to match larger

[12]Note that, once again, the Ω-formulation allows me to flexibly work with behavioral biases in a riskless environment. Were one to decide to use a distorted probability measure instead, the task would prove to be more delicate.

asset price crashes. But a smaller ϕ directly implies a weaker sensitivity of the price with respect to net worth in period $t = 2$:

$$\frac{dq_2}{dn_2} = \frac{z_3}{1 - \phi H z_3} \tag{K.3}$$

Intuitively, if financial frictions become extremely stringent, the borrowing capacity of the economy is at zero in period $t = 2$, and a change in net worth does not change this fact. Hence pecuniary externalities disappear when $\phi \to 0$. Calibrating the rational model to match more severe crises therefore automatically reduces the quantitative size of the inefficiencies.

In a behavioral model, however, parts of asset price crashes are attributable to swings in sentiment, and not only to binding collateral constraints.[13] This intuitively allows the calibration to match the same severity of crisis $X\%$ but with a higher value for the parameter ϕ, implicitly giving pecuniary externalities a greater weight.

I graphically illustrate these calibration issues in the case where sentiment is given by $\Omega_{t+1} = \alpha(q_t - q_{t-1})$, and I set q_0 such that there is initially irrational exuberance ($\Omega_2 > 0$). The left panel of Figure 5 presents the calibration step, and should be read from the y-axis to the x-axis. A modeler selects the severity of crisis observed in the data X and infer the value of ϕ. As we intuited earlier, for a given X the value of ϕ is greater in the extrapolative model. The right panel of Figure 5 then constructs the size of pecuniary externalities, by plugging the inferred value of X, read from the x-axis to the y-axis.

The parameters are deliberately chosen to feature small differences in the size of the pecuniary externality for a fixed ϕ. This exercise shows that these slight discrepancies might hide large differences when calibrated to the same moments. As can be seen from Figure 5, calibrating the model to $X = 77\%$ leads the rational model to estimate a pecuniary externality more than three times weaker than in the extrapolative model.

L Multiple Equilibria

The analysis in the main paper as made under the assumption that the equilibrium was unique at $t = 2$ (see footnote 32). When sentiment is exogenous, the uniqueness of the equilibrium is straightforward to prove. It stems from the two equilibrium conditions:

$$q_2 = \beta c_2 \mathbb{E}_2[z_3 + \Omega_3] + \phi(1 - c_2)\mathbb{E}_2[z_3 + \Omega_3] \tag{L.1}$$

$$c_2 = z_2 H - d_1(1 + r_1) + \phi H \mathbb{E}_2[z_3 + \Omega_3] \tag{L.2}$$

[13] Swings in sentiment are also needed to match other moments which are defining features of financial crises: typically the behavior of credit spreads before crashes. Rational models with financial frictions, like Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2019), cannot simultaneously generate elevated probability of crisis with decreasing credit spreads, a robust feature of the data (see Schularick and Taylor 2012 or López-Salido, Stein and Zakrajšek 2017)

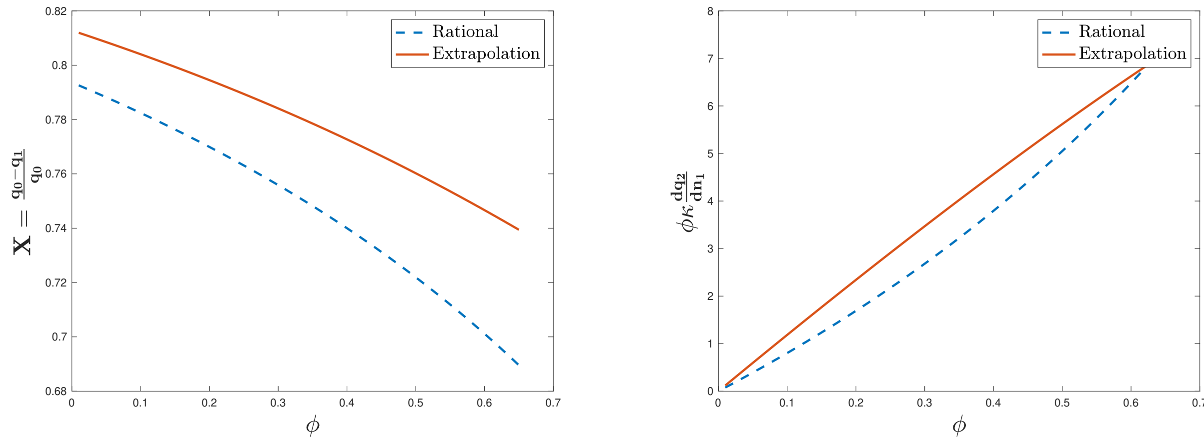

Figure 5: Calibration and Size of Pecuniary Externalities in the Rational and Extrapolative Cases. The behavioral bias is of the price extrapolation form, defined as $\Omega_{t+1} = \alpha(q_t - q_{t-1})$. q_0 is chosen such that $q_0 < q_1$ to feature initial exuberance.

The second condition (coming from the budget constraint) directly pins down the consumption in equilibrium. This in turn directly pins down the asset price, and the equilibrium is unique.

This shows that multiple equilibria can arise only when sentiment depends on asset prices. This creates a feedback effect between prices and consumption, which can be strong enough to generate multiple equilibria. This is reminiscent of the literature on current-price collateral constraints: it is well known that financial amplification can lead to a multiplicity (see Jeanne and Korinek 2019; Schmitt-Grohé and Uribe 2021). Endogenous beliefs reintroduce this two-way feedback effect even in the future-price collateral constraint.[14]

With endogenous biases, the system of equation becomes:

$$q_2 = \beta c_2 \mathbb{E}_2[z_3 + \Omega_3(q_2)] + \phi(1 - c_2)\mathbb{E}_2[z_3 + \Omega_3(q_2)] \tag{L.3}$$

$$c_2 = z_2 H - d_1(1 + r_1) + \phi H \mathbb{E}_2[z_3 + \Omega_3(q_2)] \tag{L.4}$$

which makes it clear that, as long as Ω_3 is strictly increasing in q_2, different equilibrium levels of asset prices result in different equilibrium levels of consumption. The asset price determination is given by:

$$\begin{aligned} q_2 = \beta\left(n_2 + \phi H \mathbb{E}_2[z_3 + \Omega_3(q_2)]\right) \mathbb{E}_2[z_3 + \Omega_3(q_2)] \\ + \phi(1 - (n_2 + \phi H \mathbb{E}_2[z_3 + \Omega_3(q_2)]))\mathbb{E}_2[z_3 + \Omega_3(q_2)] \end{aligned} \tag{L.5}$$

Depending on the shape of $\Omega_3(q_2)$, an arbitrary number of equilibria are possible. I illustrate the problem with a linear function:

$$\Omega_3(q_2) = \alpha q_2 + \chi \tag{L.6}$$

[14] Khorrami and Mendo (2021) explore in general how this two-way feedback creates self-fulfilling fluctuations.

The price condition is now:

$$q_2 = \beta\left(n_2 + \phi H \mathbb{E}_2[z_3 + \alpha q_2 + \chi]\right)\mathbb{E}_2[z_3 + \alpha q_2 + \chi] \\ + \phi(1 - (n_2 + \phi H \mathbb{E}_2[z_3 + \alpha q_2 + \chi]))\mathbb{E}_2[z_3 + \alpha q_2 + \chi] \quad \text{(L.7)}$$

This is a quadratic equation, hence will have at most two solutions. That means, however, that only one of them will be stable: since the consumption equation is linear in q_2, dc_2/dq_2 as computed along the pricing equation is necessarily below the slope of the budget constraint on one of the two equilibria. Figure 6 illustrates this instability. We can thus consider the case of unique equilibrium when sentiment is linear in prices. How more complicated forms of biases interact with frictions to create multiple equilibria is left for future work.

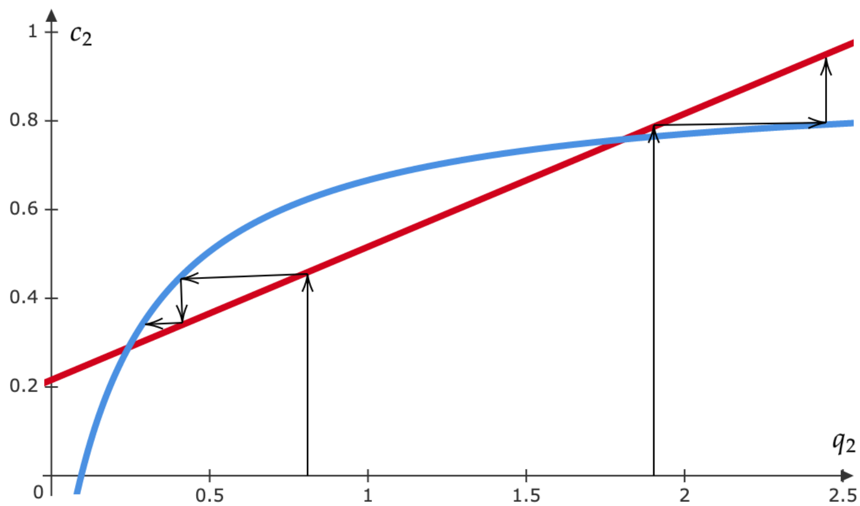

Figure 6: Graphical Illustration of Equilibrium Determination at $t = 2$ with sentiment linear in asset prices. The red line represents the budget constraint, and the blue line represents the pricing condition. The black arrows represent a tâtonnement process that starts at a given price. This price yields a certain level of sentiment and thus of consumption, which then gives rise to a different price, and so on. The right equilibrium is unstable, as the tâtonnement diverges to infinity on the right of the equilibrium, or converges to the stable equilibrium if the starting point is on the left.

M Investment Microfoundations and LTV regulation

M.1 H as Housing

This section provides a concrete and simple example of microfoundations for the investment function, that highlights how LTV regulation impacts the model in practice.

There is a continuum of entrepreneurs, who are looking for funds to finance the construction of houses. Entrepreneurs are denoted by $j \in [0, \infty]$. Entrepreneurs are identical on all dimensions, expect the cost of their project. In particular, all entrepreneurs have the same net worth A, and their project is yielding the same stochastic payoffs Z_t in periods $t = 2$ and $t = 3$. An entrepreneur j

must invest a total of I_j to complete its housing project. Entrepreneur j thus wants to raise $I_j - A$ of outside funds, which financial intermediaries can provide.

At $t = 1$, once they obtained the funds, entrepreneurs can shirk to get private benefits of B next period. When entrepreneurs shirk, the housing project yields no payoff. Entrepreneurs are risk neutral and have no time discounting, and will thus exert effort only when their payoffs z'_t from the project are such that:

$$E_1[z'_2 + z'_3] \geq B \tag{M.1}$$

How the aggregate payoff Z_t is decomposed between z_t and z'_t is irrelevant here: the particular information and contracting frictions will give rise to an equilibrium z_t for the financial intermediaries, which are the payoffs are the risky asset that are used throughout the paper. The important take-away of this microfoundation is that the payoff z_t from project j does *not* depend on the amount $I_j - A$ and so does not depend on j. Payoffs of an individual project are thus fixed irrespective of the aggregate level of H.

Obviously, because of this specific structure, financial intermediaries will start by financing projects with low j since it requires a lower investment amount, but pays the same payoff. The cost of investing into H projects for the financial intermediary is thus:

$$c(H) = \int_0^H (I_j - A)dj \tag{M.2}$$

which is strictly convex in H as long as I_H is strictly increasing in H.

How is LTV regulation entering this problem? The marginal entrepreneur financed by intermediaries is borrowing $I_H - A$, for a total value of investment of I_H. The loan-to-value ratio is thus simply:

$$LTV_H = \frac{I_H - A}{I_H} \tag{M.3}$$

which is strictly increasing in H again, as long as I_H is strictly increasing in H. Therefore, by restricting LTV ratios to be below a certain amount, the regulator will forbid the financing of project by entrepreneurs above a limit $\bar{H}$. Setting an LTV regulation will directly control for the level of H in equilibrium.

Finally, note that I took the example of housing construction to make the model palatable. But a similar interpretation can be given about other types of activities financed by financial intermediaries, such as C&I loans. In this case, the policy instrument would not be LTV ratio regulation but rather "supervisory guidance:" the regulator would nudge intermediaries towards reducing their activities, therefore controlling H exactly like in the housing example.

M.2 H as Mortgage Loans

The collateral assets held by financial intermediaries can be interpreted as mortgage-backed securities, henceforth MBS. Collateralized mortgage obligations (CMO) and MBS still account for roughly 30% of the collateral assets used in repo markets (Securities and Exchange Commission 2021). During the 2007-2008 financial crisis, around 50% of Securities Lenders repo agreements were collateralized by agency securities (Krishnamurthy, Nagel and Orlov 2014). In this section I provide a simple model that microfounds this view, and show how LTV regulations are useful instruments when it comes to regulating the quantity of MBS held by banks, and connect the behavioral bias Ω to behavioral biases directly on house prices.

Setup I make several simplifying assumptions in order to adapt these micro-foundations to the baseline model presented in the paper, which I discuss at the end of this section. I draw on Brueckner (2000) standard model of mortgage default. Mortgage borrowers have a default cost of C, and a repayment of Z in the next period. If a mortgage borrower defaults on its loan, the financial intermediary seizes the house. House prices P next period are distributed according to a density function $F(P)$.

The mortgage borrower optimally defaults when:

$$C < B - P \tag{M.4}$$

since $P - B$ is housing equity. The expected payoff from the mortgage contract is thus:

$$z = \int_0^{B-C} Pf(P)dP + \int_{B-C}^{+\infty} Bf(P)dP. \tag{M.5}$$

The point of MBS is to pool many mortgage. contracts together. Consider for example the case where default costs are heterogenous (and unobserved by banks *ex ante*) and distributed uniformly in $[\underline{C}, \bar{C}]$. For a given price P, assuming that there is enough heterogeneity such that there are defaults and non-defaults for any P in the support of $f(P)$, the average payoff of a mortgage contract is thus:[15]

$$z(P) = \int_{\underline{C}}^{B-P} P\frac{dC}{\bar{C} - \underline{C}} + \int_{B-P}^{\bar{C}} B\frac{dC}{\bar{C} - \underline{C}} \tag{M.6}$$

which is simply equivalent to:

$$z(P) = \frac{P(B - P - \underline{C}) + B(\bar{C} - B + P)}{\bar{C} - \underline{C}} \tag{M.7}$$

$$\Longrightarrow z(P) = \frac{B\bar{C} - P\underline{C} - (B - P)^2}{\bar{C} - \underline{C}}. \tag{M.8}$$

Because a MBS pools many different mortgages, this is the exact payoff of an MBS for given re-

[15] Specifically, for any P such that $f(P) \neq 0$, we have $\underline{C} < B - P < \bar{C}$.

alization of P (by the law of large numbers).[16] Although not immediately obvious, this payoff is unambiguously increasing in house prices:

$$\frac{dz(P)}{dP} = \frac{2(B-P)-\underline{C}}{\bar{C}-\underline{C}} > 0. \tag{M.9}$$

Behavioral Bias Consider now the case where a financial intermediary has a behavioral bias, and believes that house prices will be $P+\omega$ instead of P. Assume, to stay within our assumptions, that the bias is such that there are still defaults as well as non-defaults expected in the pool:

$$\underline{C} < B-P-\omega < \bar{C} \iff B-P-\bar{C} < \omega < B-P-\underline{C} \tag{M.10}$$

The payoff of the MBS for a given price realization P becomes:

$$z(P+\omega) = z(P) + \frac{2\omega(B-P)-\omega^2}{\bar{C}-\underline{C}} \tag{M.11}$$

As can be seen from inspecting this equation, there is no directly relation between ω and Ω. Indeed, the size of the behavioral bias on the payoff depends on P, the underlying stochastic variable. The implicit correspondence, for Ω to be constant, is that ω varies with P and needs to verify:

$$\omega(P) = (B-P) - \sqrt{(B-P)^2 - \Omega(\bar{C}-\underline{C})} \tag{M.12}$$

But note that, to the first-order in the bias:

$$w(P) \approx \Omega\frac{\bar{C}-\underline{C}}{B-P} \tag{M.13}$$

which means that when agents are over-optimistic, there are more optimistic regarding the left-tail of the distribution than over the right-tail.

Discussion This model of mortgage loans was deliberately stylized in order to fit my baseline framework of the core paper. In particular, I kept the payoffs of the loan (and of the housing project in the previous section) constant even when H varies. In general, the risk premium asked by the intermediary, as well as the payments specified in a mortgage contract or when funding entrepreneurs, should depend on the characteristics of the borrower. A more general treatment of these issues, for example following the model of mortgage contracts developed by Campbell and Cocco (2015), is an interesting question left for future research. Second, when collateral assets are loans, like in the MBS case, their payoff profile is generally flat in good times. This implies that behavioral distortions will have different impacts depending on whether they apply to the left-tail

[16] In other words, MBS fully diversify the risk associated with stochastic default costs.

or the right-tail of the distribution.[17]

N General Intertemporal Elasticity of Substitution

The paper made two assumptions on the utility function form of financial intermediaries: (i) log-utility in the first two periods, and (ii) linear utility in the last period. These assumptions were made for tractability, and to avoid over-complicating expressions without bringing any new intuition. In this section, I show that a model with a more general intertemporal elasticity of substitution (henceforth IES) delivers the exact same insights.

The utility function of banks is now given by:

$$U^b = \mathbb{E}_1\left[\frac{c_1^{1-\sigma}}{1-\sigma} + \beta\frac{c_2^{1-\sigma}}{1-\sigma} + \beta^2\frac{c_3^{1-\sigma}}{1-\sigma}\right] \tag{N.1}$$

where σ is the inverse of the IES. The equilibrium is now characterized by the Lagrange multiplier on the collateral constraint, κ, expressed as:

$$\kappa = \lambda_2 - \mathbb{E}_2[\lambda_3] \tag{N.2}$$

where the marginal utility is now given by:

$$\lambda_t = c_t^{-\sigma}. \tag{N.3}$$

The pricing equation at $t = 2$ is thus now slightly more complicated than before:

$$q_2 = \beta\mathbb{E}_2\left[\frac{\lambda_3}{\lambda_2}(z_3 + \Omega_3)\right] + \phi\left(1 - \mathbb{E}_2\left[\frac{\lambda_3}{\lambda_2}\right]\right)\mathbb{E}_2\left[(z_3 + \Omega_3)\right] \tag{N.4}$$

However, it should be clear by now that the uninternalized welfare effects take exactly the same form I presented in Proposition 1 and 4. Why? The welfare of intermediaries at time $t = 2$ during a crisis can be written as:

$$\mathcal{W}_2 = \beta u\left(n_2 + \phi H\mathbb{E}_2[z_3 + \Omega_3(q_2, q_1)]\right) + \beta^2 u\left(\mathbb{E}_2[z_3]H - \phi H\mathbb{E}_2[z_3 + \Omega_3(q_2, q_1)]/\beta\right) \tag{N.5}$$

with u the CRRA utility function and $n_2 = z_2H - d_1(1 + r_1)$, while the Lagrangian corresponding to bankers' problem in period $t = 1$ is given by:

$$\mathcal{L}_{b,1} = \left[u(c_1) + \mathbb{E}_1[\mathcal{W}_2(n_2, H; q_2, z_2)]\right] - \lambda_1\left[c_1 + c(H) - d_1 - e_1\right] \tag{N.6}$$

[17] See Dávila and Walther (2021) for a related exploration of this issue.

the first-order condition on borrowing still gives:

$$\frac{\partial \mathcal{L}_{b,1}}{\partial d_1} = \lambda_1 - \mathbb{E}_1\left[\lambda_2\right] \tag{N.7}$$

where λ_t is the Lagrange multiplier on the budget constraint at time t. The social planner maximizes the same function, but under its own expectations, and by also taking into account how a change in d_1 impacts asset prices in period 2. This leads to the following first-order condition:

$$\frac{\partial \mathcal{L}^{SP}_{b,1}}{\partial d_1} = \lambda_1 + \mathbb{E}^{SP}_1\left[\lambda_2\right] - \beta \mathbb{E}^{SP}_1\left[\kappa_2 \phi H \frac{\partial \Omega_3}{\partial q_2}\frac{\partial q_2}{\partial n_2}\right]\frac{dn_2}{dd_1} \tag{N.8}$$

where the only difference is now that $\kappa_2 = \lambda_2 - \mathbb{E}_2[\lambda_3]$ instead of $\lambda_2 - 1$. Obviously, the same algebra ensures that Proposition 4 is in the same way still valid.

It is less obvious to sign the derivative $\partial q_2/\partial n_2$ in this general case. But inside a financial crisis, this sensitivity is unambiguously positive. Indeed, we have:

$$\frac{dc_2}{dn_2} = 1 + \phi H \frac{d\Omega_3}{dq_2}\frac{dq_2}{dn_2} \implies d\lambda_2 = -\sigma\left(1 + \phi H \frac{d\Omega_3}{dq_2} dq_2\right)\lambda_2^{\frac{\sigma+1}{\sigma}} \tag{N.9}$$

and

$$\frac{dc_3}{dn_2} = -\phi H \frac{d\Omega_3}{dq_2}\frac{dq_2}{dn_2}(1+r_2) \implies d\lambda_3 = \sigma\left(\phi H \frac{d\Omega_3}{dq_2} dq_2 (1+r_2)\right)\lambda_3^{\frac{\sigma+1}{\sigma}} \tag{N.10}$$

which implies that $d\lambda_3$ is of the sign as dq_2. In other words, the IES value (whether it is above or below 1) is irrelevant inside a crisis, because the amount of borrowing is fixed by the collateral constraint. In the case of an exogenous behavioral bias, the price sensitivity can be written:

$$dq_2 = \beta\sigma dc_2 c_2^{\sigma-1}\mathbb{E}_2[\lambda_3(z_3+\Omega_3)] - \phi\sigma dc_2 \mathbb{E}_2[\lambda_3]\mathbb{E}_2[(z_3+\Omega_3)] \tag{N.11}$$

which can be simplified as:

$$dq_2 = \beta\sigma dc_2 c_2^{\sigma-1}\Big((\beta-\phi)\mathbb{E}_2[\lambda_3(z_3+\Omega_3)] + \phi Cov(\lambda_3, z_3)\Big) \tag{N.12}$$

dc_2 is obviously positive when the change is in net worth. Because of Assumption 1, the first term in the parentheses is positive. The second term, however, is negative.[18] While we can entertain the possibility that the covariance is strongly negative, this is not robust to changes in the micro-foundations of the collateral constraint. Indeed, if we assume that agents can default after observing the realization in z_3, the collateral constraint becomes of the form $d_2 \leq \phi H \min z_3$ and in this case the price sensitivity is:

$$dq_2 = \beta\sigma dc_2 c_2^{\sigma-1}\Big(\beta\mathbb{E}_2[\lambda_3(z_3+\Omega_3)] - \phi\mathbb{E}_2[\lambda_3](\min z_3 + \Omega_3)\Big) \tag{N.13}$$

[18] This could not happen in the linear utility at time $t = 3$, since then λ_3 was a constant.

which is unambiguously positive with Assumption 1. in this section, I thus only study the natural case where $dq_2/dn_2 > 0$.[19]

This calculation was made with a fixed Ω_3, but is still valid with an endogenous bias. Indeed, movements in Ω_3 only amplify this price sensitivity:

$$\begin{aligned} dq_2 = \beta\sigma dc_2 c_2^{\sigma-1}\Big((\beta-\phi)\mathbb{E}_2[\lambda_3(z_3+\Omega_3)] + \phi Cov(\lambda_3, z_3)\Big) \\ + \Big(\mathbb{E}_2[\frac{d\lambda_3}{\lambda_2}(z_3+\Omega_3)] - \phi\mathbb{E}_2[\frac{d\lambda_3}{\lambda_2}[\mathbb{E}_2[(z_3+\Omega_3)]]\Big) \\ + d\Omega_3\Big(\beta\mathbb{E}_2[\frac{\lambda_3}{\lambda_2}] + \phi(1-\mathbb{E}_2[\frac{\lambda_3}{\lambda_2}])\Big) \end{aligned} \tag{N.14}$$

where dc_2 also incorporates how the price in q_2 impact Ω_3 and thus the borrowing capacity. There is also a term (the second line) expressing how a change in sentiment brought by a change in asset prices impact future marginal utility, λ_3. Under the same condition as before, this term is also positive (similarly, it is only needed that ϕ is small enough, and this condition disappears under the alternative collateral formulation involving the minimum payoff).

Using the same welfare function:

$$\mathcal{W}_2 = \beta u\left(n_2 + \phi H\mathbb{E}_2[z_3 + \Omega_3(q_2, q_1)]\right) + \beta^2 u\left(\mathbb{E}_2[z_3]H - \phi H\mathbb{E}_2[z_3 + \Omega_3(q_2, q_1)]/\beta\right), \tag{N.15}$$

the general formulation in Proposition 4 is also still valid:

$$\mathcal{W}_H = \left(\beta\mathbb{E}_1^{SP}\left[\lambda_2(z_2+q_2)\right] - \lambda_1 q_1\right) + \beta\mathbb{E}_1^{SP}\left[\kappa_2\phi H\frac{d\Omega_3}{dq_2}\left(\frac{dq_2}{dn_2}z_2 + \frac{dq_2}{dH}\right)\right] \tag{N.16}$$

Here again, however, the sign of dq_2/dH is harder to determine without the linearity of utility in the ultimate period, since movements in H have effects on the future marginal utility. A first thing to notice is that even if for some levels of IES, dq_2/dH becomes negative, that is still unlikely to overturn the result that the collateral externality pushes towards under-investment. Indeed, as I just showed the first term of the collateral externality dq_2/dn_2 is positive. So dq_2/dH needs to be strongly negative to compensate for this effect. In other words, the linearity of utility at $t = 3$ or the log-utility at $t = 2$ are not directly responsible for this result: it is the assumption that $z_2 > 0$ (see Dávila and Korinek 2018 for examples where over-investment arises because dividends are negative in bad states of the world).

But in general, for the same reason dq_2/dn_2 is positive, this derivative will also be positive. Intuitively, dq_2/dH measures how an expansion of the borrowing capacity of financial intermediaries impact the equilibrium asset price. If dq_2/dn_2 is positive, we should expect the same thing for dq_2/dH: an increase in the borrowing capacity is similar to an increase in net worth during a

[19] Dávila and Korinek (2018) also assume that the price of capital assets is increasing in the net worth of the financial sector.

financial crisis. Indeed, consider the case with an exogenous bias for intuition first (remember that these derivatives are keeping the net worth constant):

$$\frac{d\lambda_2}{dH} = -\sigma\phi\mathbb{E}_2[z_3 + \Omega_3]\lambda_2^{\frac{\sigma+1}{\sigma}} > 0 \tag{N.17}$$

$$\frac{d\lambda_3}{dH} = \sigma\phi\mathbb{E}_2[z_3 + \Omega_3](1 + r_2)\lambda_3^{\frac{\sigma+1}{\sigma}} < 0 \tag{N.18}$$

so that it is clear that the stochastic discount factor (λ_3/λ_2) is increasing in H. The price sensitivity can be expressed as always as:

$$dq_2 = \beta\mathbb{E}_2\left[d\frac{\lambda_3}{\lambda_2}(z_3 + \Omega_3)\right] - \phi\mathbb{E}_2\left[d\frac{\lambda_3}{\lambda_2}\right]\mathbb{E}_2\left[(z_3 + \Omega_3)\right] \tag{N.19}$$

which again will be negative only in the case where the covariance is strongly negative:

$$dq_2 = (\beta - \phi)\mathbb{E}_2\left[d\frac{\lambda_3}{\lambda_2}(z_3 + \Omega_3)\right] + \phi Cov(d\frac{\lambda_3}{\lambda_2}, z_3) \tag{N.20}$$

And, once again, this is not robust to alternative collateral constraints like $d_2 \leq \phi H \min[z_3 + \Omega_3]$. Lastly, this goes through with endogenous sentiment (as previously for net worth):

$$dq_2 = (\beta - \phi)\mathbb{E}_2\left[d\frac{\lambda_3}{\lambda_2}(z_3 + \Omega_3)\right] + \phi Cov(d\frac{\lambda_3}{\lambda_2}, z_3) + d\Omega_3\left((\beta - \phi)\mathbb{E}_2\left[\frac{\lambda_3}{\lambda_2}\right] + \phi\right) \tag{N.21}$$

where $d\frac{\lambda_3}{\lambda_2}$ now also incorporates how changes in sentiment affect the SDF. Using $d\Omega_3 = \frac{d\Omega_3}{dq_2}dq_2$, we see that the sign of dq_2 is unchanged, movements in sentiment are simply amplifying the previous sensitivity.

To conclude, the model with a general CRRA utility function across all three periods deliver the same uninternalized welfare effects as in the baseline case. This generality comes at the cost of greater complexity, without bringing anymore intuition. Derivatives are harder to express, and are of the opposite sign as in the baseline case only in extreme situations, that are not robust to small changes in the micro-foundations of the collateral constraint. Importantly, whether the IES is above or below 1 is not the driving force behind the sign of these derivatives inside financial crises. This is because inside a financial crises, there is no ambiguity that additional wealth will be allocated to current consumption rather than future consumption, independent of σ.

O Sophisticated Agents and Optimal Policy

This section provides results in the case where agents are sophisticated, and the social planner is subject to the same biases as private agents. Specifically, private agents now realize that their future selves will have a behavioral bias Ω_3, but are unaware that they are biased today. The planner holds the same beliefs.

Under these conditions, private agents and the planner are effectively maximizing the same welfare function (inside a crisis for brevity):

$$\mathcal{W}_2 = \beta \ln\left((z_2+\Omega_2)H - d_1(1+r_1) + \phi H \mathbb{E}_2[z_3+\Omega_3(q_2,q_1)]\right) \\ + \beta^2\left(\mathbb{E}_2[z_3]H - \phi H \mathbb{E}_2[z_3+\Omega_3(q_2,q_1)]/\beta\right) \quad \text{(O.1)}$$

This expression conceals the intuition for sophisticated agents. First, agents believe that dividends are going to be at a level of $z_2+\Omega_2$, thus are biased. They also take into account that their borrowing capacity is going to be affected by Ω_3, and that this future bias can depend on asset prices $\Omega_3(q_2,q_1)$. But they also know that the payoff of the asset itself is going to be z_3 and not $z_3+\Omega_3$, hence the unbiased expectation in last-period consumption. The amount of debt they need to repay at $t=3$, however, depends on the bias since it corresponds to the maximum amount permitted by the collateral constraint at $t=2$, hence the $z_3+\Omega_3(q_2,q_1)$ in the last position of this expression.

Since private agents and the planner are maximizing the same function, there are no behavioral wedges anymore.[20] Nevertheless, there are still uninternalized welfare effects working through sentiment and prices, exactly like pecuniary externalities:

$$\mathcal{W}_d = -\mathbb{E}_1^{SP}\left[\kappa_2\phi H \boldsymbol{\frac{d\Omega_3}{dq_2}}\frac{dq_2}{dn_2}\right] \quad \text{(O.2)}$$

$$\mathcal{W}_H = \beta\mathbb{E}_1^{SP}\left[\kappa_2\phi H \boldsymbol{\frac{d\Omega_3}{dq_2}}\left(\frac{dq_2}{dn_2}z_2+\frac{dq_2}{dH}\right)\right] \quad \text{(O.3)}$$

$$\mathcal{W}_q = \beta\mathbb{E}_1^{SP}\left[\kappa_2\phi H \boldsymbol{\frac{d\Omega_3}{dq_1}}\right] \quad \text{(O.4)}$$

While agents are aware that their leverage today will have a negative impact on asset prices which itself can aggravate pessimism, they cannot take it into account in their maximization program since private agents are infinitesimal. The same applies to their level of investment and the equilibrium price at $t=1$.

This specific case exemplifies the robustness of my results. Even in a framework that does not feature any externality in a rational benchmark, even by having sophisticated agents, and even by having the planner sharing the same beliefs, these three externalities survive. Thus, even in this extreme case should the planner have an additional instrument in order to tame asset prices. Note that in this case, not only are equilibrium prices *unbiased*, they can even be lower than in a rational counterfactual. Indeed, agents factor in their expectation that they will be over-pessimistic in a crisis, which reduces q_1 since it worsens the severity of future crises. Finally note that, since there is no behavioral wedge, this case unambiguously calls for investment *subsidies*, not restrictions.

[20] Recall that behavioral wedges were quantifying the difference in expectations in Propositions 1 and 4, which are now 0.

P State-Space Representation of Belief Distortions

The paper works with an analytically convenient formulation for belief distortions, where behavioral biases are represented as a shifter in the pricing equation:

$$q_1 = \mathbb{E}_1 \left[\frac{\lambda_2}{\lambda_1} \left(z_2 + \Omega_2 + q_2^r \right) \right] \tag{P.1}$$

where Ω_2 is a constant. This definition might seem unconventional, since a large part of the behavioral literature instead works with distortions in *probability density*: agents use a density $\tilde{\pi}$ instead of the objective density π.[21] This section presents the formal correspondence between the two concepts. To this end, the relationship between endogenous objects and the state of world z_2 is made explicit. In particular, the marginal utility of the financial sector at $t = 2$ is expressed as $\lambda_2(z; \Omega(z))$ to highlight how its equilibrium value depends on the realization of the state of the world, and of the future behavioral bias at $t = 2$, which itself depends on the state of the world.

P.1 Correspondence at $t = 2$

From $\tilde{\pi}$ to Ω: Behavioral agents set their expectations using a distorted probability density function $\tilde{\pi}$:

$$\frac{1}{\lambda_2} \int z_3 \tilde{\pi}_3(z_3) dz_3 \tag{P.2}$$

which implies that we can simply set Ω_3 such that:

$$\Omega_3 = \int (\tilde{\pi}(z_3) - \pi(z_3)) z_3 dz_3 \tag{P.3}$$

This does not mean, however, that all results are the same whether we are using an Ω or a distorted density. With a distorted density, the expressions for the first-order approximations for example do not hold, and similarly the expressions for the collateral and reversal externalities. This is the reason why working with this Ω-formulation is convenient for welfare analysis.

From Ω to $\tilde{\pi}$: Since we are summarizing an entire function with a single scalar, there are an infinite number of ways to proceed. A convenient approach for exposition is to define z_3^m as the median of the stochastic process, and distort the density with a constant factor above the median:

$$\tilde{\pi}_3(z_3) = \begin{cases} \tilde{X}_1 \pi_3(z_3) & \text{if } z_3 \geq z_3^m \\ (2 - \tilde{X}_1) \pi_3(z_3) & \text{if } z_3 < z_3^m \end{cases} \tag{P.4}$$

[21] As in Bordalo et al. (2018), Caballero and Simsek (2020b) or Dávila and Walther (2021) among others.

where $\tilde{X}_1$ is defined as:

$$\tilde{X}_1 = \frac{\int_{z_3^m}^{\infty} \pi_3(z_3) z_3 dz_3 - \int_0^{z_3^m} \pi_3(z_3) z_3 dz_3 + \Omega_3}{\int_{z_3^m}^{\infty} \pi_3(z_3) z_3 dz_3 - \int_0^{z_3^m} \pi_3(z_3) z_3 dz_3} \tag{P.5}$$

which is greater than 1 as long as $\Omega_3 > 0$, which means that agents use a distorted density that exaggerates the probability of being above the median z_3^m.

P.2 Correspondence at $t = 1$

We now have to take into account the influence of the dependence of the SDF with respect to the state of the world.

From $\tilde{\pi}$ to Ω: A rational agent, recognizing the future biases of agents, would set its expectation of the discounted payoff of the asset as:

$$\int \left(\lambda_2(z_2; \Omega_3(z_2)) z_2 + z_3 + \Omega_3(z_2)\right) \pi_2(z_2) dz_2 \tag{P.6}$$

where Ω_3 can possibly be defined with distorted probabilities as shown above. As such, if behavioral agents discount the exact same payoffs state-by-state by use a distorted density $\tilde{\pi}_2$, we can implicitly define Ω_2 as[22]:

$$\int \left(\lambda_2(z_2 + \Omega_2; 0)(z_2 + \Omega_2) + z_3\right) \pi_2(z_2) dz_2 = \int \left(\lambda_2(z_2; \Omega_3(z_2)) z_2 + z_3 + \Omega_3(z_2)\right) \tilde{\pi}_2(z_2) dz_2 \tag{P.7}$$

From Ω to $\tilde{\pi}$: Given Ω_2, behavioral agents set their expectations according to:

$$\int \left(\lambda_2(z_2 + \Omega_2; 0)(z_2 + \Omega_2) + z_3\right) \pi_2(z_2) dz_2 \tag{P.8}$$

where π_2 is the objective probability density function of dividends at $t = 2$. This expression makes clear that agents do not take into account that their future selves might be subject to behavioral biases, represented by the 0 in $\lambda_2(z_2 + \Omega_2; 0)$. I similarly use the median z_2^m to construct the distorted probability measure correspondence:

$$\tilde{\pi}_3(z_2) = \begin{cases} \tilde{X}_2 \pi_3(z_2) & \text{if } z_2 \geq z_2^m \\ (2 - \tilde{X}_2) \pi_3(z_2) & \text{if } z_2 < z_2^m \end{cases} \tag{P.9}$$

[22] Ω_2 is uniquely defined as long as the discounted payoff of the asset is increasing in optimism, a natural condition we shall assume.

where $\tilde{X}_2$ is defined as:

$$\tilde{X}_2 = \frac{\int_0^{\infty}(\lambda_2(z_2+\Omega_2;0)(z_2+\Omega_2)+z_3)\pi_2(z_2)dz_2 - 2\int_0^{z_2^m}(\lambda_2(z_2;\Omega_3(z_2))z_2+z_3+\Omega_3(z_2))\pi_2(z_2)dz_2}{\int_{z_2^m}^{\infty}(\lambda_2(z_2;\Omega_3(z_2))z_2+z_3+\Omega_3(z_2))\pi_2(z_2)dz_2 - \int_0^{z_2^m}(\lambda_2(z_2;\Omega_3(z_2))z_2+z_3+\Omega_3(z_2))\pi_2(z_2)dz_2} \tag{P.10}$$

This exercise highlights two advantages of the Ω notation. First, it makes the handling of endogenous variables (and their dependence, in expectations, to sentiment) more convenient. Second, it can span a larger set of behavioral biases than the distorted density: while the range of values attainable by the distorted density is restricted by the support of the exogenous process, there is no such constraint when adding directly a location shifter to the dividend distribution.

Q Proofs and Derivations for the Current-Price Collateral Constraint

Q.1 Full expressions for Appendix C.1

This part provides the expressions of Section C.1 once the collateral pricing part is taken into account.

Fundamental Extrapolation: When $\Omega_3 = \alpha(z_2 - z_1)$ the price is determined by the quadratic equation:

$$q_2 = \beta(n_2 + \phi H q_2)\big(\mathbb{E}_2[z_3] + \alpha(z_2 - z_1)\big) + \phi q_2(1 - n_2 - \phi H q_2) \tag{Q.1}$$

leading to the full expression:

$$q_2 = \frac{-(1-\beta\phi(\mathbb{E}_2[z_3]+\alpha(z_2-z_1)) - \phi(1-n_2))}{2H\phi^2} + \frac{\sqrt{(1-\beta\phi(\mathbb{E}_2[z_3]+\alpha(z_2-z_1)) - \phi(1-n_2))^2 + 4H\phi^2\beta n_2}}{2H\phi^2} \tag{Q.2}$$

the sensitivity of the price with respect to changes in net worth is given by:

$$\frac{dq_2}{dn_2} = \frac{\beta(\mathbb{E}_2[z_3]+\alpha(z_2-z_1)) - \phi q_2}{1 - \beta\phi H(\mathbb{E}_2[z_3]+\alpha(z_2-z_1)) - \phi(1-n_2-\phi^2 H q_2)} \tag{Q.3}$$

□

Price/Return Extrapolation: When $\Omega_3 = \alpha(q_2 - q_1)$ the quadratic equation becomes:

$$q_2 = \beta(n_2 + \phi H q_2)\big(\mathbb{E}_2[z_3] + \alpha(q_2 - q_1)\big) + \phi q_2(1 - n_2 - \phi H q_2) \tag{Q.4}$$

leading to the full expression:

$$q_2 = -\frac{(1-\beta\phi(\mathbb{E}_2[z_3]-\alpha q_1)-\phi(1-n_2))}{2(H\phi^2-\beta H\phi\alpha)}$$

$$+\frac{\sqrt{(1-\beta\phi(\mathbb{E}_2[z_3]-\alpha q_1)-\phi(1-n_2))^2+4\beta n_2 H(\mathbb{E}_2[z_3]-\alpha q_1)(\phi^2 H\phi\alpha)}}{2(H\phi^2-\beta H\phi\alpha)} \tag{Q.5}$$

and the price sensitivity:

$$\frac{dq_2}{dn_2} = \frac{\beta(\mathbb{E}_2[z_3]-\alpha q_1)-\phi q_2}{1-\beta\phi H(\mathbb{E}_2[z_3]-\alpha q_1)-\phi(1-n_2-\phi^2 H q_2)-\beta(n_2+\phi H q_2)\alpha} \tag{Q.6}$$

□

Q.2 Proof of Proposition 15

At time $t = 2$, the welfare of borrowers can be written as:

$$\mathcal{W}_2 = \begin{cases} \beta \ln\left(z_2 H - d_1(1+r_1) + \phi H q_2\right) + \beta^2\left(\mathbb{E}[z_3]H - \phi H q_2/\beta\right) & \text{if } z_2 \geq z^* \\ \beta\left(\beta\mathbb{E}[z_3]H + z_2 H - d_1(1+r_1)\right) & \text{otherwise} \end{cases} \tag{Q.7}$$

while the Lagrangian corresponding to bankers' problem in period $t = 1$ is given by:

$$\mathcal{L}_{b,1} = \left[u(c_1) + \mathbb{E}_1[\mathcal{W}_2(n_2, H; q_2, z_2)]\right] - \lambda_1\left[c_1 + c(H) - d_1 - e_1\right] \tag{Q.8}$$

the first-order condition on borrowing gives:

$$\frac{\partial\mathcal{L}_{b,1}}{\partial d_1} = \lambda_1 - \mathbb{E}_1\left[\lambda_2\right] \tag{Q.9}$$

The social planner maximizes the same function, but under its own expectations, and by also taking into account how a change in d_1 impacts asset prices in period 2. This leads to the following first-order condition:

$$\frac{\partial\mathcal{L}^{SP}_{b,1}}{\partial d_1} = \lambda_1 - \mathbb{E}^{SP}_1\left[\lambda_2\right] - \mathbb{E}^{SP}_1\left[\phi H\kappa_2\frac{\partial q_2}{\partial n_2}\right] \tag{Q.10}$$

Hence simply by incorporating $\mathbb{E}_1[\lambda_2]$ we can express the total change in welfare as internalized plus uninternalized effects:

$$\frac{\partial\mathcal{L}^{SP}_{b,1}}{\partial d_1} = \underbrace{\lambda_1 - \mathbb{E}_1\left[\lambda_2\right]}_{\text{Internalized}} + \underbrace{\mathbb{E}_1\left[\lambda_2\right] - \mathbb{E}^{SP}_1\left[\lambda_2\right] - \mathbb{E}^{SP}_1\left[\phi H\kappa_2\frac{\partial q_2}{\partial n_2}\right]}_{\text{Uninternalized}} \tag{Q.11}$$

which proves Proposition 15. □

Q.3 Proof of Proposition 16

I compute the difference between λ_2 expected by private agents and λ_2 expected by the Planner state by state z_2. When both expect a realization z_2 not to produce a financial crisis, marginal utilities are equalized to 1, so the difference disappears. For the rest there are two cases: either both marginal utilities correspond to binding collateral constraints, either one agent expect the friction to bind and the other not. The first case yields:

$$\frac{1}{c_2(z_2+\Omega_2,0)} - \frac{1}{c_2(z_2,\Omega_3)} = \\ \frac{1}{(z_2+\Omega_2)H - d_1(1+r_1) + \phi H q_2(z_2+\Omega_2;0)} - \frac{1}{z_2 H - d_1(1+r_1) + \phi H q_2(z_2;\Omega_3)} \tag{Q.12}$$

I take the first-order approximation around the REE $\lambda_2 = 1/(z_2 H - d_1(1+r_1) + \phi H q_2(z_2;0)) = 1/c_2(z_2,0)$. It gives:

$$\frac{1}{(z_2+\Omega_2)H - d_1(1+r_1) + \phi H q_2(z_2+\Omega_2;0)} = \frac{1}{c_2(z_2,0)} \frac{1}{1+\frac{\Omega_2 H + \phi\Omega_2 \frac{dq_2}{n_2}}{c_2(z_2,0)}} \\ = \lambda_2 \left(1 - \frac{\Omega_2 H + \phi\Omega_2 \frac{dq_2}{n_2}}{c_2(z_2,0)}\right) \tag{Q.13}$$

While the same algebra for the second part of equation (Q.12) yields similarly:

$$\frac{1}{z_2 H - d_1(1+r_1) + \phi H q_2(z_2;\Omega_3)} = \frac{1}{c_2(z_2,0)} \frac{1}{1+\frac{\phi\Omega_3 H \frac{dq_2}{z_3}}{c_2(z_2,0)}} = \lambda_2 \left(1 + \frac{\phi H \Omega_3 \frac{dq_2}{z_3}}{c_2(z_2,0)}\right) \tag{Q.14}$$

Taking the difference gives:

$$\frac{1}{c_2(z_2+\Omega_2,0)} - \frac{1}{c_2(z_2,\Omega_3)} = \lambda_2^2 \left(H\Omega_2 + \phi \frac{dq_2}{dn_2}\Omega_2 - \phi H \frac{dq_2}{dz_3}\Omega_3\right) \tag{Q.15}$$

Lastly we need to consider the cases where the social planner and private agents disagree about the occurrence of a crisis for a given z_2. Without loss of generality, I assume that private agents are over-optimistic so for some range of states, $[z^* - dz, z^*]$ they expect to be at $c_2 = 1$, while the Planner expects the collateral constraint to be binding (where z^* is the crisis cutoff in the RE case). The size of the band is infinitesimal since, as can be seen in equations (32) and (33), the cutoff is only moving because of Ω_2 and Ω_3 which are small.

The difference, integrated on the band, can be expressed through a triangle approximation:

$$\int_{z^*-dz}^{z^*} \left(1 - \frac{1}{c_2(z_2, \Omega_3)}\right) \pi(z_2) dz_2 = \frac{dz\pi(z^*)}{2} \left(1 - \frac{1}{c_2(z^* - dz, \Omega_3)}\right) \tag{Q.16}$$

Because the difference between $t = 1$ and $1/c_2(z^* - dz, \Omega_3^*)$, where Ω_3^* is the bias at the cutoff, is also infinitesimal, this term is negligible compared to the previous one.[23] It thus follows that, to the first order:

$$\mathcal{B}_d = \Omega_2 \mathbb{E}_1^{SP} \left[\lambda_2^2 \left(H\Omega_2 + \phi \frac{dq_2}{dn_2}\right)\right] - \phi H \mathbb{E}_1^{SP} \left[\Omega_3 \lambda_2^2 \frac{dq_2}{dz_3}\right] \tag{Q.17}$$

□

Q.4 Proof of Proposition 17

The asset price is determined in a crisis equilibrium by:

$$q_2 = \beta \left(n_2 + \phi H q_2\right) \mathbb{E}[z_3 + \Omega_3] + \phi q_2 (1 - n_2 - \phi H q_2) \tag{Q.18}$$

A total differential yields:

$$\begin{aligned} dq_2 = \beta dn_2 \mathbb{E}[z_3 + \Omega_3] + \beta\phi H dq_2 \mathbb{E}[z_3 + \Omega_3] + \beta \left(n_2 + \phi H q_2\right) d\Omega_3 \\ + \phi dq_2 (1 - n_2 - \phi H q_2) + \phi q_2 (-dn_2 - \phi H dq_2) \end{aligned} \tag{Q.19}$$

Because the variation in Ω_3 can only come from q_2 by assumption, rearranging gives:

$$\frac{dq_2}{dn_2} = \frac{\beta \mathbb{E}[z_3 + \Omega_3] - \phi q_2}{1 - \beta\phi H \mathbb{E}[z_3 + \Omega_3] + 2\phi^2 H q_2 - \beta c_2 \frac{d\Omega_3}{dq_2}} \tag{Q.20}$$

□

Q.5 Collateral Externality Perturbation

We can perform a perturbation analysis around the REE equilibrium to gain intuition about how the collateral externality is changed by sentiment. Let us elaborate on the difference with a social planner that would entirely respect the beliefs of private agents. I develop the Taylor expansion of

[23] For completeness, its value can be approximated as:

$$\int_{z^*-dz}^{z^*} \left(1 - \frac{1}{c_2(z_2, \Omega_3)}\right) \pi(z_2) dz_2 \approx -(\Omega_2 - \phi\Omega_3(z^*)) \frac{(\Omega_2 - \phi\Omega_3(z^*)) \left(1 + \phi \frac{dq_2}{dn_2}\right) - \phi\Omega_3(z^*)}{2} \pi(z^*)$$

Ω_2 enters this equation because it parametrizes the value of dz, i.e. the size of the band where agents do not expect a financial crisis but the planner does.

the difference between the two expectations, where $(dq_1/dn_2)^e$ is the price sensitivity in the rational world, and similarly κ_2^e for the Lagrange multiplier (both are defined state-by-state).

$$\mathbb{E}_1^{SP}\left[\phi\kappa_2\frac{dq_2}{dn_2}\right] - \mathbb{E}_1\left[\phi\kappa_2\frac{dq_2}{dn_2}\right] = -\mathbb{E}_1\left[\left(\frac{dq_1}{dn_2}\right)^e \mathcal{B}_d\right]$$
$$+\mathbb{E}_1\left[\kappa_2^e\frac{\mathbf{\Omega_3}}{1-\phi H(\mathbb{E}_2[z_3])+\phi^2 Hq_2 - c_2\frac{d\mathbf{\Omega_3}}{dq_2}}\left(1+\frac{z_3-\phi q_2}{1-\phi H(\mathbb{E}_2[z_3])+\phi^2 Hq_2 - c_2\frac{d\mathbf{\Omega_3}}{dq_2}}\right)\right] \quad \text{(Q.21)}$$

This expression shows that the difference is increasing when the behavioral wedge $\mathcal{B}_d$ becomes more negative. Future pessimism reduces this difference, while a higher sensitivity of future sentiment with respect to asset prices increases it. Finally, note that Ω_2 is not part of the second term of this expansion. This is because, in my specific modeling framework with log utility, the price sensitivity in the rational benchmark is constant with respect to net worth.

□

Q.6 Proof of Proposition 18

At time $t = 2$, the welfare of borrowers can be written as:

$$\mathcal{W}_2 = \begin{cases} \beta\ln\left(z_2 H - d_1(1+r_1) + \phi H q_2\right) + \beta^2\left(\mathbb{E}[z_3]H - \phi H q_2/\beta\right) & \text{if } z_2 \geq z^* \\ \beta\left(\beta\mathbb{E}[z_3]H + z_2 H - d_1(1+r_1)\right) & \text{otherwise} \end{cases} \quad \text{(Q.22)}$$

while the Lagrangian corresponding to bankers' problem in period $t = 1$ is given by:

$$\mathcal{L}_{b,1} = \left[u(c_1) + \mathbb{E}_1[\mathcal{W}_2(n_2, H; q_2, z_2)]\right] - \lambda_1\left[c_1 + c(H) - d_1 - e_1\right] \quad \text{(Q.23)}$$

the first-order condition on investment yields:

$$\frac{\partial\mathcal{L}_{b,1}}{\partial H} = \lambda_1 c'(H) - \mathbb{E}_1\left[\lambda_2(z_2+\Omega_2+q_2(z_2+\Omega_2))\right] \quad \text{(Q.24)}$$

The social planner maximizes the same function, but under its own expectations, and by also taking into account how a change in d_1 impacts asset prices in period 2 (recall that $q1$ is fixed by assumption). This leads to the following first-order condition:

$$\frac{\partial\mathcal{L}_{b,1}^{SP}}{\partial H} = \lambda_1 c'(H) - \beta\mathbb{E}_1^{SP}\left[\lambda_2(z_2+q_2)\right] - \beta\mathbb{E}_1^{SP}\left[\kappa_2\phi H\left(\frac{\partial q_2}{\partial n_2}z_2 + \frac{\partial q_2}{\partial H}\right)\right] - \beta\mathbb{E}_1^{SP}\left[\kappa_2\phi H\frac{dq_2}{dq_1}\frac{dq_1}{dH}\right] \quad \text{(Q.25)}$$

where the last part quantifies how a change in price today impacts the aggregate borrowing capacity of the financial sector. In most models, this term is zero since $dq_2/dq_2 = 0$: there is no reason a change in price today should directly change the price tomorrow. But in the case where sentiment

Ω_3, which enters the determination of prices at period 2, depends on past prices, this derivative is not zero anymore.

Proposition 18 is then proved once we notice that $q_1 = c'(H)$ in equilibrium, so that $dq_1/dH = c''(H)$, while:

$$\frac{dq_2}{dq_1} = \frac{dq_2}{d\Omega_3}\frac{d\Omega_3}{dq_1} \tag{Q.26}$$

which yields the final formula for the uninternalized effects of marginally increasing investment:

$$\begin{aligned}\frac{\partial \mathcal{L}^{SP}_{b,1}}{\partial H} \quad = \quad & \underbrace{\lambda_1 q_1 - \beta \mathbb{E}_1\left[\lambda_2(z_2+q_2)\right]}_{\text{Internalized}} \quad + \\ & \underbrace{\beta \mathbb{E}_1\left[\lambda_2(z_2+q_2)\right] - \beta \mathbb{E}^{SP}_1\left[\lambda_2(z_2+q_2)\right] - \beta \mathbb{E}^{SP}_1\left[\kappa_2 \phi H\left(\frac{\partial q_2}{\partial n_2} z_2 + \frac{dq_2}{dH}\right)\right] - \beta \mathbb{E}^{SP}_1\left[\kappa_2 \phi H \frac{\partial q_2}{\partial \Omega_3}\frac{\partial \Omega_3}{\partial q_1} c''(H)\right]}_{\text{Uninternalized}}.\end{aligned} \tag{Q.27}$$

□

Q.7 Proof of Proposition 19

I use the same notation as for the proof of Proposition 16, presented in Online Appendix Q.3. The behavioral wedge for investment can consequently be expressed state-by-state as:

$$\mathcal{B}_H(z_2) = \left[\lambda_2(0;\Omega_3)(z_2+q_2(0;\Omega_3))\right] - \left[\lambda_2(\Omega_2;0)(z_2+\Omega_2+q_2(\Omega_2;0)\right] \tag{Q.28}$$

As for leverage, it is sufficient to only look at states where the borrowing constraint binds both in the expectation of the social planner and of private agents. To the first-order, we can write:

$$\mathcal{B}_H(z_2) = (\lambda_2(0;\Omega_3) - \lambda_2(\Omega_2;0)(z_2+q_2^r)) + \lambda_2^r\left(\Omega_3\frac{dq_2}{dz_3} - \Omega_2\left(1+\frac{dq_2}{dz_2}\right)\right) \tag{Q.29}$$

The part $\lambda_2(0;\Omega_3) - \lambda_2(\Omega_2;0)$ exactly corresponds to the behavioral wedge for leverage state-by-state, that we will denote by $\mathcal{B}_d(z_2)$ for conciseness. The behavioral wedge for investment can thus be expressed as:

$$\mathcal{B}_H(z_2) = \mathbb{E}^{SP}_1\left[\mathcal{B}_d(z_2)(z_2+q_2^r)\right] - \Omega_2\mathbb{E}^{SP}_1\left[\lambda_2^r\left(1+\frac{dq_2}{dz_2}\right)\right] + \mathbb{E}^{SP}_1\left[\lambda_2^r\Omega_3\frac{dq_2}{dz_3}\right] \tag{Q.30}$$

where

$$\mathcal{B}_d(z_2) = \Omega_2\lambda_2^2\left(H\Omega_2+\phi\frac{dq_2}{dn_2}\right) - \phi H\Omega_3\lambda_2^2\frac{dq_2}{dz_3}. \tag{Q.31}$$

□

Q.8 Derivation of Equation C.20

I proceed as for the derivation of the price sensitivity to swings in sentiment, Proposition 17, as in Online Appendix Q.4. I start from the equilibrium condition that links the asset price at time $t = 2$ to consumption through the collateral constraint:

$$q_2 = \beta\,(n_2 + \phi H q_2)\,\mathbb{E}[z_3 + \Omega_3] + \phi q_2(1 - n_2 - \phi H q_2) \tag{Q.32}$$

I then differentiate with respect to H, acknowledging that q_2 and Ω_3 will be modified as a result:

$$\begin{aligned} dq_2 = \beta\phi q_2 dH \mathbb{E}[z_3 + \Omega_3] + \beta\phi H dq_2 \mathbb{E}[z_3 + \Omega_3] + \beta\,(n_2 + \phi H q_2)\,d\Omega_3 \\ + \phi dq_2(1 - n_2 - \phi H q_2) + \phi q_2(-\phi q_2 dH - \phi H dq_2) \end{aligned} \tag{Q.33}$$

Rearranging gives the desired result:

$$\frac{dq_2}{dH} = \frac{\beta\phi q_2 \mathbb{E}_2\left[z_3 + \boldsymbol{\Omega_3}\right] - \phi^2 q_2^2}{1 - \beta\phi H(\mathbb{E}_2\left[z_3 + \boldsymbol{\Omega_3}\right]) + 2\phi^2 H q_2 - \beta c_2 \frac{d\boldsymbol{\Omega_3}}{dq_2}}. \tag{Q.34}$$

□

Q.9 Proof of Proposition 20

Using equation (Q.22), the derivative of total welfare with respect to changing asset prices at $t = 1$ is:

$$\frac{\partial \mathcal{W}_1}{\partial q_1} = \beta \mathbb{E}_1^{SP}\left[\kappa_2 \phi H \frac{dq_2}{dq_1}\right] \tag{Q.35}$$

In most models, this term is zero since $dq_2/dq_1 = 0$: there is no reason a change in price today should directly change the price tomorrow. But in the case where sentiment Ω_3, which enters the determination of prices at period 2, depends on past prices, this derivative is not zero anymore.

Proposition 20 is then proved once we notice that

$$\frac{dq_2}{dq_1} = \frac{dq_2}{d\Omega_3}\frac{d\Omega_3}{dq_1} \tag{Q.36}$$

which yields the final formula for the welfare effects of marginally changing asset prices:

$$\mathcal{W}_q = \beta \mathbb{E}_1^{SP}\left[\kappa_2 \phi H \frac{\partial q_2}{\partial \Omega_3}\frac{\partial \Omega_3}{\partial q_1}\right] \tag{Q.37}$$

□

Q.10 Proof of Proposition 10 for the Current-Price Collateral Constraint

As explained in the main text, the social planner's optimality condition under the premises of Proposition 10 can be expressed as:

$$u'(c_1) = \frac{1}{2\sigma_\Omega}\int_0^\infty \left[\int_{-\sigma_\Omega}^{\sigma_\Omega} \frac{\partial \mathcal{W}_2}{\partial n_2}\left(d_1, H; q_2, z_2 - \bar{\Omega}_2 - \omega_2\right) d\omega_2\right] f_2(z_2) dz_2. \tag{Q.38}$$

Key to this proposition is the shape of $\partial\mathcal{W}_2/\partial n_2$ with respect to z_2. First recall that:

$$\mathcal{W}_2 = \begin{cases} \beta \ln\left(z_2 H - d_1(1+r_1) + \phi H q_2\right) + \beta^2\left(\mathbb{E}[z_3]H - \phi H q_2/\beta\right) & \text{if } z_2 \geq z^* \\ \beta\left(\beta\mathbb{E}[z_3]H + z_2 H - d_1(1+r_1)\right) & \text{otherwise} \end{cases} \tag{Q.39}$$

so that the first derivative is equal to:

$$\frac{\partial \mathcal{W}_2}{\partial n_2} = \begin{cases} \beta\left(1 + \phi H \frac{dq_2}{dn_2}\right)\lambda_2 - \beta\phi H \frac{dq_2}{dn_2} & \text{if } z_2 \geq z^* \\ \beta & \text{otherwise} \end{cases} \tag{Q.40}$$

which is constant outside of a crisis, as expected. More important is the behavior of this derivative inside of crises, which can be rewritten:

$$\frac{\partial \mathcal{W}_2}{\partial n_2} = \frac{\beta\left(1 + \phi H \frac{dq_2}{dn_2}\right)}{z_2 H - d_1(1+r_1) + \phi H q_2} - \beta\phi H \frac{dq_2}{dn_2} \quad \text{if } z_2 \geq z^* \tag{Q.41}$$

or for convenience:

$$\frac{\partial \mathcal{W}_2}{\partial n_2} = \lambda_2 + \phi H(\lambda_2 - 1)\frac{dq_2}{dn_2} \quad \text{if } z_2 \geq z^*. \tag{Q.42}$$

I use the following notation to simplify the exposition of the proof. First, the expectation over z_2 for a given w_2 is denoted by:

$$g(w_2) = \int_0^{+\infty} \frac{\partial \mathcal{W}_2}{\partial n_2}(d_1, H; q_2, z_2 - \bar{\Omega}_2 - \omega_2) f_2(z_2) dz_2 \tag{Q.43}$$

while the integral taken over the uncertainty band is:

$$G(\sigma_\Omega) = \int_{-\sigma_\Omega}^{+\sigma_\Omega} \frac{g(w_2)}{2\sigma_\Omega} dw_2. \tag{Q.44}$$

Given the continuity of $\partial\mathcal{W}_2/\partial n_2$ (see equation Q.39) we can differentiate with respect to σ_Ω:

$$\begin{aligned} G'(\sigma_\Omega) = &-\frac{1}{2\sigma_\Omega^2}\int_{-\sigma_\Omega}^{+\sigma_\Omega}\int_0^{+\infty} \frac{\partial \mathcal{W}_2}{\partial n_2}(d_1, H; q_2, z_2 - \bar{\Omega}_2 - \omega_2) f_2(z_2) dz_2 dw_2 + \\ &\int_0^{+\infty} \frac{\partial \mathcal{W}_2}{\partial n_2}(d_1, H; q_2, z_2 - \bar{\Omega}_2 - \sigma_\Omega) f_2(z_2) dz_2 - \int_0^{+\infty} \frac{\partial \mathcal{W}_2}{\partial n_2}(d_1, H; q_2, z_2 - \bar{\Omega}_2 + \sigma_\Omega) f_2(z_2) dz_2 \end{aligned} \tag{Q.45}$$

which can be expressed in terms of the notation just defined above as:

$$G'(\sigma_\Omega) = -\frac{G(\sigma_\Omega)}{\sigma_\Omega} + \frac{1}{2\sigma_\Omega}(g(\sigma_\Omega) - g(-\sigma_\Omega)) \tag{Q.46}$$

Before proceeding further, remember that the social planner optimally sets leverage such that:

$$u'(c_1) = G(\sigma_\Omega) \tag{Q.47}$$

while the decentralized equilibrium is independent of σ_Ω. Thus, leverage restrictions will be increasing in σ_Ω if and only if G is increasing in σ_Ω. This condition is then equivalent, using the derivative just computed, to:

$$\frac{g(\sigma_\Omega) - g(-\sigma_\Omega)}{2} > \int_{-\sigma_\Omega}^{+\sigma_\Omega} \frac{g(w_2)}{2\sigma_\Omega} dw_2. \tag{Q.48}$$

Since $\partial\mathcal{W}_2/\partial n_2$ is continuous in z and in ω_2, and since ω_2 is defined in the compact set $[-\sigma_\Omega, \sigma_\Omega]$, g is continuous (by continuity of parametric integrals) and Fubini's theorem implies that a sufficient condition for $G'(\sigma_\Omega) > 0$ is that[24]:

$$\frac{1}{2}\left(\frac{\partial\mathcal{W}_2}{\partial n_2}(z_2 + \sigma_\Omega) - \frac{\partial\mathcal{W}_2}{\partial n_2}(z_2 - \sigma_\Omega)\right) > \int_{-\sigma_\Omega}^{+\sigma_\Omega} \frac{\partial\mathcal{W}_2}{\partial n_2}(z_2 + \omega_2)\frac{d\omega_2}{2\sigma_\Omega} \quad \forall z_2 \in supp(f_2). \tag{Q.49}$$

In other words, this condition requires that the average taken over a segment is below the average of the two extreme points of this same segment.

Next, notice that any convex function satisfies this requirement. For a convex function φ, Jensen's inequality yields:

$$\varphi(t\sigma_\Omega - (1-t)\sigma_\Omega) \le t\varphi(\sigma_\Omega) + (1-t)\varphi(-\sigma_\Omega) \quad \forall t \in [0,1]. \tag{Q.50}$$

Now integrate this inequality over t to get:

$$\int_0^1 \varphi(t\sigma_\Omega - (1-t)\sigma_\Omega)dt \le \int_0^1 t\varphi(\sigma_\Omega)dt + \int_0^1 (1-t)\varphi(-\sigma_\Omega)dt. \tag{Q.51}$$

A change of variable $t \to (x - \sigma_\Omega)/(2\sigma_\Omega)$ in the left-hand side thus yields:

$$\int_{-\sigma_\Omega}^{+\sigma_\Omega} \frac{\varphi(x)}{2\sigma_\Omega} dx \le \frac{\varphi(\sigma_\Omega) - \varphi(-\sigma_\Omega)}{2} \tag{Q.52}$$

which is exactly the relationship in equation (Q.49).

We now have to prove that $\partial\mathcal{W}_2/\partial n_2$ is convex to end the proof of Proposition 10. Going back to

[24] $\bar{\Omega}_2$ does not need to appear in this condition since this inequality is required to hold for all z_2 in the support of the definition, so equivalently for all $z_2 - \bar{\Omega}_2$ also in the support.

equation (Q.39), denote $\partial\mathcal{W}_2/\partial n_2$ by $\mathcal{W}_{2,n}$. Start with the derivative of marginal utility. We have:

$$\frac{d\lambda_2}{dz_2} = -\frac{H + \phi H \frac{dq_2}{dz_2}}{c_2^2} \tag{Q.53}$$

and so:

$$\frac{d^2\lambda_2}{dz_2^2} = -\frac{2}{c_2^3}\left(H + \phi H \frac{dq_2}{dz_2}\right) \tag{Q.54}$$

and notice that we will have the following for the derivative of consumption given the log-utility assumption:

$$\frac{dc_2}{dz_2} = H + \phi H \frac{dq_2}{dz_2} = -\frac{d\lambda_2}{dz_2} c_2^2 \tag{Q.55}$$

Turning now to the whole marginal welfare derivative:[25]

$$\frac{d\mathcal{W}_{2,n}}{dz_2} = \frac{d\lambda_2}{dz_2} - \phi H \frac{dc_2}{dz_2}\frac{dq_2}{dn_2} + \phi H(1-c_2)\frac{d^2 q_2}{dz_2 dn_2}. \tag{Q.56}$$

Finally we can inspect the second derivative of $\mathcal{W}_{2,n}$ to sign it. Basic algebra yields:

$$\begin{aligned}\frac{d^2\mathcal{W}_{2,n}}{dz_2^2} = -\left[-\frac{2}{c_2^3}\left(H + \phi H\frac{dq_2}{dz_2}\right) + \phi H \frac{d^2 q_2}{dn_2 dz_2}\right]\frac{dc_2}{dz_2} \\ - \left[\frac{1}{c_2^2} + \phi H \frac{dq_2}{dn_2}\right]\phi H \frac{d^2 q_2}{dn_2 dz_2} - \left[H + \phi H \frac{dq_2}{dz_n}\right]\phi H \frac{d^2 q_2}{dn_2 dz_2} \\ + \phi H (1-c_2)\frac{d^3 q_2}{dn_2 d^2 z_2}\end{aligned} \tag{Q.57}$$

Inspecting the signs of the different terms, one can notice that $c_2 > 0$, $dq_2/dn_2 > 0$, $dc_2/dz_2 > 0$, and $1 - c_2 > 0$ since we are in a financial crisis (or equivalently, $\kappa_2 > 0$). This directly implies that a sufficient condition (but far form necessary) for this second derivative to be positive (and hence the function of interest convex) is that both $d^2q_2/(dn_2 dz_2) < 0$ and $d^3 q_2/(dn_2 d^2 z_2) > 0$. We can now conclude this proof by computing these two objects. First the second derivative of the price in a financial crisis:[26]

$$\frac{dq_2^2}{dn_2 dz_2} \propto -\phi \frac{dq_2}{dn_2}\left((1 - \beta\phi H \mathbb{E}_1[z_3] + 2\phi^2 H q_2) + 2\phi H(\beta \mathbb{E}_1[z_3] - \phi q_2)\right). \tag{Q.58}$$

and both terms inside the large parentheses are positive: they correspond to the denominator and numerator of the sensitivity dq_2/dn_2. The coefficient of proportionality is positive since it corre-

[25] The behavior of λ_2 would be enough to characterize how leverage should move with sentiment uncertainty if we were to look at an infinitesimal agent. But the social planner takes pecuniary externalities into account, so we need to also compute the derivatives of the pecuniary externalities.

[26] For the sake of brevity, Ω_3 is left ou of the expression as, by assumption, it is a constant. It thus only shifts the value of $\mathbb{E}_1[z_3]$ and that has no impact on the sign of these derivaties as long as $\mathbb{E}_1[z_3] + \Omega_3 > 0$, which we always assume to be the case.

sponds to the square of the denominator of this exact same sensitivity. Hence we unambiguously have:

$$\frac{dq_2^2}{dn_2 dz_2} < 0 \tag{Q.59}$$

Now we are left with the the third derivative of the price function inside a financial crisis. Rather than directly compute its expression – which is extremely involved – a possible short-cut is instead to write the price sensitivity of the form:

$$\frac{dq_2}{dn_2} = b(z_2) = \frac{\beta z_3 - \phi q_2}{D(z_2)} > 0. \tag{Q.60}$$

This short notation implies that deriving this expression yields, using the computation just above:

$$b'(z_2) = -\phi \frac{dq_2}{dz_2} \frac{D(z_2) + 2\phi H(\beta z_3 - \phi q_2)}{D^2(z_2)} < 0 \tag{Q.61}$$

which can be rewritten for convenience as:

$$b'(z_2) = -\frac{\phi \frac{dq_2}{dz_2}}{D} - \phi \frac{dq_2}{dz_2} \frac{2\phi H b(z_2))}{D(z_2)} < 0 \tag{Q.62}$$

since $(\beta z_3 - \phi q_2)/D^2 = b/D$. This eases the computation of the next derivative, yielding:

$$\begin{aligned} b''(z_2) = -\frac{\phi \frac{d^2 q_2}{dz_2^2}}{D(z_2)} + \phi \frac{\frac{dq_2}{dz_2} D'(z_2)}{D^2(z_2)} - \phi \frac{\phi \frac{d^2 q_2}{dz_2^2}}{D(z_2)} 2\phi H b(z_2) + \\ \phi \frac{\phi \frac{dq_2}{dz_2}}{D(z_2)} 2\phi H b(z_2) D'(z_2) - \phi \frac{\phi \frac{dq_2}{dz_2}}{D(z_2)} 2\phi H b'(z_2) \end{aligned} \tag{Q.63}$$

and all terms are positive, since $dq_2/dz_2 > 0$, $d^2 q_2/dz_2^2 < 0$, $D'(z_2) > 0$ and $b'(z_2) < 0$.[27]

Summing up, $d^2 q_2/(dn_2 dz_2) < 0$ and $d^3 q_2/(dn_2 d^2 z_2) > 0$. This implies that $d^2 \mathcal{W}_{2,n}/dz_2^2$ is a convex function. This leads to G' being positive, thus the right-hand side of equation (Q.38) to be increasing in σ_Ω. Since the optimality condition of private agents is independent of σ_Ω, this equivalently means that optimal leverage restrictions are increasing in σ_Ω. This concludes the proof. □

[27] Indeed, $D'(z_2) \propto dq/dn > 0$.

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