ERRATUM

Songzi Du pointed out to us that the proof of Lemma 5.5 in "The Folk Theorem with Imperfect Public Information" (Fudenberg, Levine, and Maskin, *Econometrica* (1994)) is incorrect. The lemma is valid as stated. Here is a correct proof; the notation and all cited lemmas are taken from the original paper.

Lemma 5.5: If a pure-action profile is enforceable, and pairwise identifiable for all pairs of players, it is enforceable with respect to all regular hyperplanes.

Proof: Suppose pure action profile a^* is enforceable, and pairwise identifiable for all pairs *i,j;* normalize payoffs by setting $g(a^*) = 0$. By lemma 4.3, enforceability on hyperplanes is independent of the discount factor and the overall payoff to be enforced, so we can set the discount factor to ½ and the overall payoff to be 0. Thus for each player i there is a continuation payoff function w_i such that

(1) $$\sum_y \pi(y \mid (a_i, a_{-i}^*) w_i(y) \leq -g_i(a_i, a_{-i}^*) \text{ with equality if } a_i = a_i^* .$$

By lemma 5.3, it is sufficient to consider pairwise hyperplanes, so we want to show that for any pair j,k and non-zero coefficients β_j, β_k there are continuation payoff functions w_j', w_k' that satisfy (1) and also satisfy $\beta_j w_j'(y) + \beta_k w_k'(y) = 0 \, \forall y$. Substituting $w_k'(y) = -(\beta_j / \beta_k) w_j'(y)$ our goal is to find w_j' such that

(2) $$\begin{aligned} &\sum_y \pi(y \mid (a_j, a_{-j}^*) w_j'(y) \leq -g_j(a_j, a_{-j}^*) \text{ with equality if } a_j = a_j^* \\ &\sum_y \pi(y \mid (a_k, a_{-k}^*) (-\beta_j / \beta_k) w_j'(y) \leq -g_k(a_k, a_{-k}{}^*) \text{ with equality if } a_k = a_k^* . \end{aligned}$$

Now consider the linear programming problem of maximizing the constant objective function 0 subject to the constraints (2); this problem has a solution if (and only if) the dual is bounded.

The dual of this LP is

(3) $\max_{\mu_j,\mu_k} \sum_{a_j \in A_j} \mu_j(a_j) g_j(a_j, a_{-j}^*) + \sum_{a_k \in A_k} \mu_k(a_k) g_k(a_k, a_{-k}^*)$

s.t.

(4) $\sum_{a_j \in A_j} \mu_j(a_j)\pi(y \mid (a_j, a_{-j}^*)) + \sum_{a_k \in A_k} \mu_k(a_k)(-\beta_j / \beta_k)\pi(y \mid (a_k, a_{-k}^*)) = 0 \ \forall y$

(5) $\mu_j(a_j) \geq 0 \ \forall a_j \neq a_j^*$, $\mu_k(a_k) \geq 0 \ \forall a_k \neq a_k^*$.

Pairwise identifiability says that

$$\operatorname{rank} \Pi_{jk}(a^*) = \operatorname{rank} \Pi_j(a_{-j}^*) + \operatorname{rank} \Pi_k(a_{-k}^*) - 1.$$

Let e_j, e_k be the basis vectors placing weight one on a_j^*, a_k^* respectively. Since $(e_j, -e_k)\Pi_{jk}(a^*) = 0$, the null-space of $\Pi_{jk}(a^*)$ is the direct sum of the subspace spanned by (e_j, e_k) and the subspace $(\tilde{\mu}_j, \tilde{\mu}_k)$ defined by

$$\sum_{a_j \in A_j} \tilde{\mu}_j(a_j)\pi(y \mid (a_j, a_{-j}^*)) = 0 \, \forall y \text{ and}$$

$$\sum_{a_k \in A_k} \tilde{\mu}_k(a_k)\pi(y \mid (a_k, a_{-k}^*)) = 0 \, \forall y.$$

Plugging into the objective function, since $g_i(a^*) = 0$,

$$\sum_{a_j \in A_j} \mu_j(a_j) g_j(a_j, a_{-j}^*) + \sum_{a_k \in A_k} \mu_k(a_k) g_k(a_k, a_{-k}^*) =$$
$$\sum_{a_j \in A_j} \tilde{\mu}_j(a_j) g_j(a_j, a_{-j}^*) + \sum_{a_k \in A_k} \tilde{\mu}_k(a_k) g_k(a_k, a_{-k}^*).$$

Thus, to show that the solution is bounded, it suffices to consider for $i = j, k$ the simpler problem

(6) $\max_{\mu_i} \sum_{a_i \in A_i} \mu_i(a_i) g_i(a_i, a_{-i}^*)$

s.t.

(7) $\sum_{a_i \in A_i} \mu_i(a_i)\pi(y \mid (a_i, a_{-i}^*)) = 0 \, \forall y$

(8) $\mu_i(a_i) \geq 0 \ \forall a_i \neq a_i^*$.

This is the dual of the linear program corresponding to (individual) enforceability, which is to maximize 0 subject to the incentive constraints for player *j*. Because profile a^* is enforceable, the primal has a solution, so this dual has a solution as well , and in particular is bounded.

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