What can symplectic geometry tell us about Hamiltonian dynamics?

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Thank you!

Thank you for inviting me to give this talk!

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1 Preliminaries

2 Weinstein's conjecture

3 Refinements of the Weinstein conjecture

4 The restricted three-body problem

5 Non-autonomous Hamiltonians

6 Future directions

Phase space

We will be primarily talking about $\mathbb{R}^{2n}$, with *position* coordinates $x_1, \ldots, x_n$ and *momentum* coordinates $y_1, \ldots, y_n$.

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We call any function $H : \mathbb{R}^{2n} \longrightarrow \mathbb{R}$ a (autonomous) *Hamiltonian*. Our Hamiltonians will generally be smooth.

Hamilton's ODEs

The basic object of study in this talk will be trajectories $(x(t), y(t))$ such that

$$\frac{dx_i}{dt} = \frac{\partial H}{\partial y_i}, \qquad \frac{dy_i}{dt} = -\frac{\partial H}{\partial x_i}.$$

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Note that the x_i and the y_i in Hamilton's equations of motion are "intertwined". Symplectic (which means intertwined) geometry is a way of capturing this.

Periodic trajectories and conservation of energy

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A basic fact about Hamilton's equations are that they preserve H. Specifically, if $(x(t), y(t))$ solves Hamilton's equations, then $H(x(t), y(t))$ is always constant. Hence, Hamiltonian trajectories always travel along level sets of H.

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Theorem 1

(Hofer-Zehnder, Steuwe, 1990). Let $H : \mathbb{R}^{2n} \longrightarrow \mathbb{R}$ *be a proper smooth function. Then there is a closed periodic Hamiltonian trajectory along* $H^{-1}(E)$ *for almost every* $E \in \mathbb{R}$ *such that* $H^{-1}(E) \neq \emptyset$.

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While the proof of this theorem uses some ideas from symplectic geometry, it will not be the focus of this talk.

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Specifically, Ginzburg-Gurel (2003) found a proper C^2 Hamiltonian H on $\mathbb{R}^4$ with a regular level set with no closed Hamiltonian orbits. C^∞ counter examples are also known in $\mathbb{R}^{2n}$ for $n > 2$.

When does a Hamiltonian have a closed orbit along a level set?

Basic calculation: if Y is a hypersurface in $\mathbb{R}^{2n}$ that is a reguar level set of two different Hamiltonians H and K, then the existence of a closed Hamiltonian trajectory depends *only on* Y and not on H and K.

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However, as I will explain very shortly, the existence of a closed orbit is a "symplectic" condition, while convexity *is not*.

Symplectic geometry

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There is a bilinear product b on $\mathbb{R}^{2n}$ that captures this intertwinedness. It is given for $n = 2$ by

$$b((x_1, x_2, y_1, y_2), (x_1', x_2', y_1', y_2')) = x_1 y_1' - x_1' y_1 + x_2 y_2' - x_2' y_2,$$

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and extended for any n by this pattern. Unlike the dot product, this product is *anti*-symmetric, hence not *positive-definite*.

Symplectic geometry (cont.)

A *symplectic transformation*

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Many interesting symplectic transformations. Example: product of two area preserving maps is a symplectic transformation of $\mathbb{R}^4$. Symplectic geometry is essentially the geometry of symplectic transformations.

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The point: by a simple calculation, if Y is a hypersurface carrying a closed Hamiltonian orbit, and T is a symplectic transformation, then $T(Y)$ also has a closed orbit. Moreover, it is easy to construct examples (e.g. $n = 1$) where Y is convex but $T(Y)$ is not.

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Weinstein therefore sought a condition for Y to carry a closed orbit that is invariant under symplectic transformations. He conjectured that Y should be of "contact type".

Weinstein's conjecture

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Conjecture 2

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This was proved by Viterbo in 1987, but there are many important phases spaces, called *symplectic manifolds*, that are not $\mathbb{R}^{2n}$. The analogue of Weinstein's conjecture for symplectic manifolds remains open, except for dimensions 2 and 4.

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Global surfaces of section

For the remainder of the talk, I want to discuss situations where one can find much more structure than one periodic orbit. In dimension 4, there is a beautiful body of work by Hofer, Wysocki, and Zehnder on finding *global surfaces of section*.

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Let Y be a compact hypersurface in $\mathbb{R}^4$ that is the level set of some Hamiltonian H. A *global surface of section* for Y is an embedded compact surface $\Sigma \subset Y$ such that

- The boundary components of Σ are periodic Hamiltonian trajectories
- Every trajectory is transverse to the interior Σ^o and intersects the interior both forwards and backwards in time (other than the boundary components).

Poincare return map

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It is defined by following a point $p \in \Sigma^o$ along its trajectory until the first time it hits Σ^o again. We can use the Poincare return map to reduce the study of our four-dimensional Hamiltonian system to studying an area preserving map of Σ^o and its iterates.

HWZ's theorem

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Theorem 3

(Hofer, Wysocki, Zehnder 1998) Any Hamiltonian on $\mathbb{R}^4$ *possesses a global surface of section along any strictly convex energy hypersurface.*

In fact, they show that one can always take this surface of section to be a disc.

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Similarity with Weinstein conjecture: strictly convex condition not a symplectic condition. HWZ find a symplectic condition, called "dynamical convexity", which yields the same results.

Pseudoholomorphic curves

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This is beyond the scope of this talk, but these are basically surfaces in $\mathbb{R}^4$ that are quite similar to images of holomorphic functions from

$$\mathbb{C} \longrightarrow \mathbb{C}^2,$$

but are more flexible. They are central to modern symplectic geometry.

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$$H(q,p) = \frac{1}{2}|p|^2 + \langle p, iq \rangle - \langle p, i\mu \rangle - \frac{1-\mu}{|q|} - \frac{\mu}{|q-1|},$$

where $\mu \in [0,1]$ is the mass ratio $\frac{m_S}{m_E+m_S}$. This is $\approx .999997$ for the actual sun/earth.

Lagrange points

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If energy $c < H(L_1)$, then $H^{-1}(c)$ has three connected components: one near earth, one near sun, and one near ∞. Components *not* compact (because of collisions), but can be "regularized", i.e. noncompactness can be removed.

Convexity?

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They also show that as c approaches the first Lagrange point from below, the component of $H^{-1}(c)$ *fails to be strictly convex*. They conjecture, however, that $H^{-1}(c)$ is *dynamically convex*, which would still imply the existence of a global surface of section.

What about above the first Lagrange point?

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This has the effect that the level sets for c just above the first Lagrange point are a "connect sum" of two $\mathbb{RP}^3$s. For topological reasons, these can not carry a global surface of section. However, Fish and Siefring conjecture that they should carry a "finite energy foliation", which is a closely related idea.

Back to hypersurfaces of contact type

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Theorem 5

(CG., Hutchings) Any contact type hypersurface in a symplectic 4-manifold must carry at least 2 closed orbits for any Hamiltonian.

Implications for the restricted three-body problem

Albers, Frauenfelder, Van Koert, and Paternain: for (circular) planar restricted three-body problem, $H^{-1}(c)$ is always a hypersurface of contact type for c below $H(L_1)$ and also for c just slightly above $H(L_1)$ (they also conjecture that this should hold for all energy levels).

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My result with Hutchings therefore applies to show that these hypersurfaces carry at least two closed orbits. Actually I believe that the connect sum of two $\mathbb{RP}^3$s should always carry infinitely many closed orbits for any Hamiltonian for which it is a contact-type hypersurface.

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The Conley conjecture

Symplectic geometry can also be used to study Hamilton's ODEs for *non-autonomous* Hamiltonians, i.e.

$$H : \mathbb{R}^{2n} \times \mathbb{R}/\mathbb{Z} \longrightarrow \mathbb{R}.$$

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The Arnold conjecture

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1. Does every compact contact type hypersurface in a 4-dimensional symplectic manifold carry a "short" Hamiltonian trajectory?
2. What do the local dynamics look like around the periodic orbits that do appear?
3. Do all "topologically complicated" contact type hypersurfaces carry infinitely many periodic orbits?