Appendices to "Housing Cycles" (Glaeser and Gyourko, April 22, 2006)

Appendix Table: Summary Statistics			
Variable	Mean	Standard Deviation	# of Obs
1-Year House Price Change (in 2000 dollars)	$2,807	$10,947	3,167
3-Year House Price Change (in 2000 dollars)	$7,002	$25,802	929
5-Year House Price Change (in 2000 dollars)	$15,032	$44,035	580
House Price (in 2000 dollars)	$128,501	$67,923	3,306
1-Year Rent Change (in 2000 dollars)	$58	$412	1,167
3-Year Rent Change (in 2000 dollars)	$172	$839	389
5-Year Rent Change (in 2000 dollars)	$427	$1441	194
1-Year Log Change in Employment	0.020	0.030	13,085
3-Year Log Change in Employment	0.050	0.061	2,984
5-Year Log Change in Employment	0.110	0.888	1,865
Log of New Permits, 1 Year	7.343	1.308	10,095
Log of New Permits Over 3 Years	8.437	1.296	2,671
Log of New Permits Over 5 Years	9.000	1.263	1,999
1-Year Personal Income Change (in 2000 dollars)	$315	$682	12,705
3-Year Personal Income Change (in 2000 dollars)	$1,043	$1,212	2,611
5-Year Personal Income Change (in 2000 dollars)	$1,726	$1,689	1,865
1-Year Log Change in Crime	-0.007	0.154	2,701
3-Year Log Change in Crime	-0.029	0.197	774
5-Year Log Change in Crime	-0.046	0.299	479

Appendix: Extending the Model

We now consider two cases, one in which people buy for life and the second in which people buy and then consider reselling. One possible assumption about mobility is that residents buy a house at the start of their lives and transaction costs ensure that they live there in perpetuity. In that case, an individual is willing to pay up to $C+\frac{(1+r)\theta(i)}{r}+E\left(\sum_{j=0}^{\infty} D(t+j)/(1+r)^j\right)$ to live in the city where E(.) is the expectations operator. If there are N homes sold during that time period, then the price of housing will be $C+\frac{(1+r)\theta(N,t)}{r}+\sum_{j=0}^{\infty} D(t+j)/(1+r)^j$. A second plausible mobility assumption is that individuals face no transaction costs and buy with a one-period time horizon. In that case, individuals are willing to pay up to $\theta(i)+D(t)+\frac{rC}{1+r}+\frac{H(t+1)}{1+r}$ for housing in the city. Solving this difference equation for the marginal buyer, assuming that a transversality condition for housing prices holds, implies that housing prices will equal: $C+E\left(\sum_{j=0}^{\infty}\theta(N(t+j),t+j)/(1+r)^j\right)+E\left(\sum_{j=0}^{\infty} D(t+j)/(1+r)^j\right)$.

In the case where people buy permanently, then number of buyers "N" is equal to I(t), the number of new homes brought onto the market, and demand equals $C+\frac{(1+r)\left(\overline{D}+\theta(t)\right)}{r}+\frac{(1+r)\left(D(t)-\overline{D}\right)}{1+r-\delta}-\frac{\alpha(1+r)}{r}I(t)$. The timing of supply means that the relevant supply condition becomes

$$(6)\quad c_1 I(t)+c_2(I(t)-I(t-1))=\frac{(1+r)\left(\overline{D}+\lambda\theta(t-1)\right)}{r}+\frac{\delta(1+r)\left(D(t-1)-\overline{D}\right)}{1+r-\delta}-\frac{\alpha(1+r)}{r}I(t),$$

which has the algebraically attractive attribute that total city population does not influence demand. While this assumption may be somewhat counterfactual, the impact that new housing supply on prices may in fact not have much to do with the impact that these homes have on total city population. At time zero, the steady state assumption implies that $I(0)=\frac{(1+r)\overline{D}}{rc_1+\alpha(1+r)}$. To make things particularly stark, we assume that $\delta=1$, and $\mu(t)=0$ for all t, to avoid any mean reversion that is unrelated to housing supply. Given these assumptions, the following proposition follows:

Proposition 1: If there is a shock to D(t) at time one equal to ε, then the derivative of price growth between time zero and time one with respect to ε will equal $\frac{1+r}{r}$ and the derivative of investment at time 1 with respect to ε will equal $\frac{1+r}{r(c_1+c_2)+\alpha(1+r)}$.

The difference between price at time 1 and expected price in time t will equal

$$-\varepsilon\frac{(1+r)\alpha(1+r)}{r\left(rc_1+\alpha(1+r)\right)}\left(1-\left(\frac{rc_2}{\left(r(c_1+c_2)+\alpha(1+r)\right)}\right)^t\right).$$

The derivative of expected time t investment, as of time 1, with respect to ε will equal $\left(1-\left(\frac{rc_2}{\left(r(c_1+c_2)+\alpha(1+r)\right)}\right)^t\right)\frac{(1+r)}{\left(rc_1+\alpha(1+r)\right)}$ and the derivative of expected price growth between time t and t+1 with respect to price growth between time 0 and 1 will equal $-\frac{\alpha(1+r)}{\left(r(c_1+c_2)+\alpha(1+r)\right)}\left(\frac{rc_2}{\left(r(c_1+c_2)+\alpha(1+r)\right)}\right)^t$.

Proposition 1 makes three simple points. First, mean reversion of prices is perfectly compatible with an extremely rational model with delayed supply responses. In a sense, the delayed supply response ensures that there will be overshooting of prices to changes in demand. Second, mean reversion will be bigger over longer time periods because supply has more of a chance to increase. Third, this simple model predicts strong persistence in housing production, again because of the costs of investment. If $c_2=0$, then investment would immediately shoot up and stay there.

This simple model also yields predictions about the magnitude of the demand and supply response. In this case, a one dollar shock to income yields a massive response in housing prices, despite the fact that prices will eventually mean revert. The people who are buying are correctly forecasting their future income which will, by the random walk assumption, stay high. They are ignoring the fact that future housing production will cause prices to drop because their time horizons are infinite.

We now turn to the case where individuals buy and resell each period. To make the model comparable and simple, we again assume D(t) follows a random walk, so that $E\left(\sum_{j=0}^{\infty} D(t+j)/(1+r)^j\right)$ equals $\frac{(1+r)D(t)}{r}$. Our main simplification is to continue with our assumption that $\theta(N(t+j),t+j)$ equals $-\alpha I(t)$. In most cases, it would be more reasonable to assume that this willingness to pay is a function of the total stock and not of the rate of change of the city. While such a model is certainly plausible, it represents too big a shift from the previous model. In this case, total willingness to pay equals $C+\frac{(1+r)D(t)}{r}-\alpha E\left(\sum_{j=0}^{\infty} I(t+j)/(1+r)^j\right)$. We assume again that the city is at stead state at time zero so that $I(0)=\frac{(1+r)\overline{D}}{rc_1+\alpha(1+r)}$.

Proposition 2: If there is a shock to D(t) at time one equal to ε, then the derivative of price growth between time zero and time one with respect to ε will equal $\frac{(1+r)(1-\phi)(c_1+c_2)\varepsilon}{rc_1+\alpha(1+r)}$ and the derivative of investment at time 1 with respect to ε will equal $\frac{(1-\phi)(1+r)}{rc_1+\alpha(1+r)}$, where

$$\phi=\frac{((1+r)(c_1+\alpha)+(2+r)c_2)-\sqrt{((1+r)(c_1+\alpha)+(2+r)c_2)^2-(1+r)c_2(c_1+c_2)}}{2(c_1+c_2)}.$$

The difference between price at time 1 and expected price in time t will equal $\frac{(\phi c_1-(1-\phi)c_2)(1-\phi^{t-1})(1+r)\varepsilon}{rc_1+\alpha(1+r)}$, where $(1-\phi)c_2>\phi c_1$.

The derivative of expected time t investment, as of time 1, with respect to ε will equal $\frac{(1-\phi^t)(1+r)}{rc_1+\alpha(1+r)}$, and the derivative of expected price growth between time t and t+1 with respect to price growth between time 0 and 1 will equal $\frac{\left(\phi^t(\phi-1)c_1+\phi^{t-1}(1-\phi)^2c_2\right)}{\left(1-\phi c_1+(1-\phi)c_2\right)}$.

As before, this variant of the basic model predicts mean reversion which continue to get larger over longer time horizons. The big difference between the previous model and this one is the quantity response to a shock in the productivity of the city. With mobile residents, the immediate price response to a shock is $\frac{(1+r)(1-\phi)(c_1+c_2)}{r(c_1+\alpha)+\alpha}$ which is approximately equal to $\frac{(1-\phi)(c_1+c_2)}{\alpha}$ when r is small. With lifetime residence, the response is (1+r)/r times that shock. For example if $c_1=c_2=\alpha$ and r=.05, then the impact of a shock with transitory residents is seven percent of the impact of a shock with permanent agents. This gap is surely extreme, but the long time horizons of residents is surely a critical element of wide price fluctuations.

Over a longer time period, unsurprisingly the level of mean reversion is much higher with lifetime residence. In fact, the eventual price response in the two cases is the same, but because there is so much more of an immediate positive price response with lifetime residence, there is so much more mean reversion after that point.

Proof of Proposition 1: The general formula for home prices is $C+\frac{(1+r)(D(t)-\alpha I(t))}{r}$.

At time 0, prices equals $C+\frac{c_1(1+r)\overline{D}}{rc_1+\alpha(1+r)}$, and at time 1, price equals

$C+\frac{(1+r)\varepsilon(1)}{r}+\frac{c_1(1+r)\overline{D}}{rc_1+\alpha(1+r)}$, and the price change equals $\frac{(1+r)\varepsilon}{r}$. Investment solves $I(t)=\frac{(1+r)D(t-1)}{\left(r(c_1+c_2)+\alpha(1+r)\right)}+\frac{rc_2I(t-1)}{\left(r(c_1+c_2)+\alpha(1+r)\right)}$ so expected investment at time t equals

$I(t)=\frac{(1+r)\overline{D}}{(rc_1+\alpha(1+r))}+\left(1-\left(\frac{rc_2}{(r(c_1+c_2)+\alpha(1+r))}\right)^t\right)\frac{(1+r)\varepsilon}{(rc_1+\alpha(1+r))}$ and expected price at time t will equal $C+\frac{(1+r)c_1(\overline{D}+\varepsilon)}{(rc_1+\alpha(1+r))}+\frac{\alpha(1+r)^2\varepsilon}{r(rc_1+\alpha(1+r))}\left(\frac{rc_2}{(r(c_1+c_2)+\alpha(1+r))}\right)^t$. Time one investment equals $\frac{(1+r)\varepsilon}{(r(c_1+c_2)+\alpha(1+r))}$.

The change in price between time t and t+1 will equal $-\frac{\alpha(1+r)^2\varepsilon}{r(r(c_1+c_2)+\alpha(1+r))}\left(\frac{rc_2}{(r(c_1+c_2)+\alpha(1+r))}\right)^t$, and the change in expected price between time 1 and time t will equal

$$-\frac{(1+r)\alpha(1+r)(\varepsilon)}{r(rc_1+\alpha(1+r))}+\frac{\alpha(1+r)^2\varepsilon}{r(rc_1+\alpha(1+r))}\left(\frac{rc_2}{(r(c_1+c_2)+\alpha(1+r))}\right)^t.$$

Proof of Proposition 2: If $D(1)=\overline{D}+\varepsilon$, then expected investment at time t will equal $\frac{(1+r)\overline{D}}{rc_1+\alpha(1+r)}+\left(1-\phi^t\right)\frac{(1+r)\varepsilon}{rc_1+\alpha(1+r)}$ where

$$\phi=\frac{((1+r)(c_1+\alpha)+(2+r)c_2)-\sqrt{((1+r)(c_1+\alpha)+(2+r)c_2)^2-(1+r)c_2(c_1+c_2)}}{2(c_1+c_2)}$$, and

expected price at time t will equal $C+\frac{c_1(1+r)(\overline{D}+\varepsilon)}{rc_1+\alpha(1+r)}-\frac{(\phi^t c_1-\phi^{t-1}(1-\phi)c_2)(1+r)\varepsilon}{rc_1+\alpha(1+r)}$, or $C+\frac{c_1(1+r)(\overline{D}+\varepsilon)}{rc_1+\alpha(1+r)}-\alpha\phi^t\frac{(1+r)^2\varepsilon}{(rc_1+\alpha(1+r))(1+r-\phi)}$. Investment at time 1 will equal $(1-\phi)\frac{(1+r)\varepsilon}{rc_1+\alpha(1+r)}$ and for all subsequent periods, the derivative of investment in period t with respect to investment in period 1 will equal $\frac{1-\phi^t}{1-\phi}$. The price increase between time 0 and time 1 equals $\frac{(1+r)(1-\phi)(c_1+c_2)\varepsilon}{rc_1+\alpha(1+r)}$, and the expected price increase between time t and t+1 equals $\frac{(\phi^t(\phi-1)c_1+\phi^{t-1}(1-\phi)^2c_2)(1+r)\varepsilon}{rc_1+\alpha(1+r)}$, so the derivative of the expected price increase between t and t+1 with respect to the price increase between 0 and 1 will equal $\frac{\left(\phi^{t}(\phi-1)c_1+\phi^{t-1}(1-\phi)^2 c_2\right)}{\left(1-\phi c_1+(1-\phi)c_2\right)}$. The expected price growth between period one and period t will equal $\frac{\left(\phi c_1-(1-\phi)c_2\right)(1-\phi^{t-1})(1+r)\varepsilon}{rc_1+\alpha(1+r)}$, where $\phi c_1>(1-\phi)c_2$ because this holds if and only if

$\left((1+r)(c_1+\alpha)+(2+r)c_2\right)-\sqrt{\left((1+r)(c_1+\alpha)+(2+r)c_2\right)^2-(1+r)c_2(c_1+c_2)}>2c_2$ which holds if and only if

$\left((1+r)(c_1+\alpha)+rc_2\right)>\sqrt{\left((1+r)(c_1+\alpha)+(2+r)c_2\right)^2-(1+r)c_2(c_1+c_2)}$ which holds if and only if $\left((1+r)(c_1+\alpha)+rc_2\right)^2+(1+r)c_2(c_1+c_2)>\left((1+r)(c_1+\alpha)+(2+r)c_2\right)^2$ or

$2r(1+r)(c_1+\alpha)c_2+(1+r)c_2(c_1+c_2)>2(1+r)c_2^2+2(2+r)(1+r)(c_1+\alpha)c_2$ or

$(1+r)c_1c_2>(1+r)c_2^2+4(1+r)(c_1+\alpha)c_2$, which can never hold.

Appendix: Dividing Metropolitan Areas into Different Market Types

Low Growth Markets
(based on being in bottom 1/3 of distribution of population growth between 1950-70)

Allentown-Bethlehem-Easton
Bellingham
Birmingham-Hoover
Boston-Quincy
Buffalo-Niagara Falls
Canton-Massillon
Davenport-Moline-Rock Island
Des Moines
Detroit-Livonia-Dearborn
Essex County
Harrisburg-Carlisle
Newark-Union
New York-White Plains-Wayne
Peoria
Philadelphia
Pittsburgh
Providence-New Bedford-Fall River
Spokane
Toledo
Visalia-Porterville
Wenatchee
Worcester

Coastal Markets
(based on being within 40 miles of the Atlantic or Pacific Oceans; CT metros on coast of Long Island Sound are included in this group)

Bridgeport-Stamford-Norwalk
Cambridge-Newton-Framingham
Charleston-North Charleston
Deltona-Daytona Beach-Ormond Beach
Fort Lauderdale-Pompano Beach-Deerfield Beach
Honolulu
Jacksonville
Los Angeles-Long Beach-Glendale
Nassau-Suffolk
New Haven-Milford
Oakland-Fremont-Hayward
Oxnard-Thousand Oaks-Ventura

San Diego-Carlsbad-San Marcos
San Francisco-San Mateo-Redwood City
San Jose-Sunnyvale-Santa Clara
San Luis Obispo-Paso Robles
Santa Ana-Anaheim-Irvine
Santa Barbara-Santa Maria
Santa Cruz-Watsonville
Santa Rosa-Petaluma
Seattle-Bellevue-Everett
Tacoma
Vallejo-Fairfield
Virginia Beach-Norfolk-Newport News
West Palm Beach-Boca Raton-Boynton Beach

Unconstrained Markets
(interior, growing markets among 139 OFHEO markets)

Akron
Albuquerque
Ann Arbor
Atlanta-Sandy Springs-Marietta
Austin-Round Rock
Bakersfield
Baltimore-Towson
Baton Rouge
Beaumont-Port Arthur
Bethesda-Gaithersburg-Frederick
Boise City-Nampa
Boulder
Camden
Cedar Rapids
Charlotte-Gastonia-Concord
Chicago-Naperville-Joliet
Chico
Cincinnati-Middletown
Cleveland-Elyria-Mentor
Colorado Springs
Columbia
Columbus
Corpus Christi
Dallas-Plano-Irving
Dayton
Denver-Aurora
Durham
Edison

Eugene-Springfield
Flint
Fort Collins-Loveland
Fort Wayne
Fort Worth-Arlington
Fresno
Gary
Grand Rapids-Wyoming
Greensboro-High Point
Hartford-West Hartford-East Hartford
Houston-Sugar Land-Baytown
Indianapolis
Kalamazoo-Portage
Kansas City
Kennewick-Richland-Pasco
Lake County-Kenosha County
Lancaster
Lansing-East Lansing
Las Vegas-Paradise
Lexington-Fayette
Lincoln
Little Rock-North Little Rock
Louisville
Madison
Memphis
Merced
Milwaukee-Waukesha-West Allis
Minneapolis-St. Paul-Bloomington
Modesto
Napa
Nashville-Davidson--Murfreesboro
New Orleans-Metairie-Kenner
Ogden-Clearfield
Oklahoma City
Omaha-Council Bluffs
Orlando-Kissimmee
Phoenix-Mesa-Scottsdale
Portland-Vancouver-Beaverton
Pueblo
Raleigh-Cary
Reno-Sparks
Richmond
Riverside-San Bernardino-Ontario
Rochester
Rockford
Sacramento--Arden-Arcade--Roseville

St. Louis
Salem
Salinas
Salt Lake City
San Antonio
Sarasota-Bradenton-Venice
Stockton
Syracuse
Tampa-St. Petersburg-Clearwater
Topeka
Trenton-Ewing
Tucson
Tulsa
Warren-Farmington Hills-Troy
Washington-Arlington-Alexandria
Wichita
Wilmington
Winston-Salem