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Housing Dynamics

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Abstract

The key stylized facts of the housing market are positive serial correlation of price changes at one year frequencies and mean reversion over longer periods, strong persistence in construction, and highly volatile prices and construction levels within markets over time. We calibrate a dynamic model of housing in the spatial equilibrium tradition of Rosen and Roback to see whether such a model can generate these facts. With reasonable parameter values, this model readily explains the mean reversion of prices over five year periods, but cannot explain the observed positive serial correlation at higher frequencies. The model predicts the positive serial correlation of new construction that we see in the data and the volatility of both prices and quantities in the typical market, and it can account for substantial variation on construction intensity across markets. However, the model cannot explain the most volatile markets in terms of low frequency price changes. More research is needed to determine whether measurement error-related data smoothing or market inefficiency can best account for the persistence of high frequency price changes. With respect to the extremely high house price change volatility in certain coastal markets, more research is needed to ascertain whether shocks to interest rates or better measurement of local income variability can match this moment of data without appealing to some type of animal spirits.

I. Introduction

Housing represents two-thirds of the typical household's portfolio (Tracy, Schneider, and Chan (1999), with Federal Reserve Board data showing that more than $18 trillion of real estate, most of it residential in nature, was owned by the household sector at the end of 2005.[1] Despite the enormous size of this sector, economists' understanding of many aspects of the housing market is incomplete, with the debate over whether the recent boom was a bubble only the latest example of this.[2]

More broadly, four key stylized facts pose serious challenges for any model of housing markets. First, is the predictability of prices that has been well documented by others (e.g., Case and Shiller, 1989; Cutler, Poterba, and Summers, 1991). In our sample of 115 metropolitan areas from 1980 to 2005 for which we have Office of Federal Housing Enterprise Oversight (OFHEO) constant quality house price series, a $1 increase in real house prices in one year is associated with a 71 cent increase the next year. A $1 increase in local market prices over the past five years is associated with a 32 cent decrease over the next five year period. An important question is whether the high frequency momentum and lower frequency mean reversion in price changes can be reconciled with the workings of a rational market.

A second notable feature of housing markets is the high volatility of price changes and construction intensity within markets over time. Over one, three, and five year periods, the standard deviation of house price changes is at least three times the mean price change. The standard deviation of three-year real changes in our sample of metropolitan area average house prices is $26,747 (in 2000 dollars throughout the paper), which is about one-fifth of the median price level. New construction within markets is not as volatile on average as prices, but its standard deviation always exceeds its mean over the same one, three, and five year intervals. Can this volatility be the result of real shocks to housing markets or must it reflect bubbles and animal spirits?

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[1] This figure includes primary homes, second homes, vacant homes for sale, and vacant land. See Table B.100 Balance Sheet of Households and Nonprofit Organizations at http://www.federalreserve.gov/RELEASES/z1/Current/data.htm.
[2] See McCarthy and Peach (2004), Himmelberg, Mayer and Sinai (2005), and Smith and Smith (2006) for recent analyses that conclude there is no large-scale bubble in housing prices. Shiller (2005, 2006) and Baker (2006) argue to the contrary that the bubble is both real and very large.

A third important stylized fact involves the asymmetry of price and quantity changes over longer periods of time. This puzzle is well illustrated in Figures 1 and 2, which document the significantly negative correlation of house price appreciation across the 1980s and 1990s (Figure 1) and the significantly positive correlation of housing unit growth over the same period (Figure 2). Demand-driven housing models predict that prices and quantities should move symmetrically. At a minimum, the mismatch of quantity and price movements at low frequencies seems to suggest that models of housing prices need to more firmly embed supply as well as demand.

A fourth noteworthy fact is that most variation in housing price changes is local, not national. Less than eight percent of the variation in price levels and barely more than one-quarter of the variation in price changes across cities can be accounted for by national year-specific fixed effects.[3] The large amount of local variation and its relationship with macroeconomic variables such as interest rates or national income is another challenge for a consistent economic explanation of housing market dynamics.

In this paper, we present a dynamic, rational expectations model of house price formation to see whether such a framework can explain these salient moments of housing prices and quantities. The model follows the urban tradition of Alonso (1962), Rosen (1979) and Roback (1982) in which housing prices reflect the willingness to pay for one location versus another. In this approach, housing prices are determined endogenously by local wages and amenities, so that local heterogeneity is natural. Our model then extends the Alonso-Rosen-Roback framework by focusing on high frequency price dynamics and by incorporating endogenous housing supply.

In Section II of this paper, the model and four propositions regarding its implications are presented. The predictability of housing price changes is shown to be compatible with a no-arbitrage rational expectations equilibrium. Slow construction responses and mean reverting wage shocks imply that prices will mean revert. And, positive serial correlation of labor demand shocks at high frequencies can generate positive serial correlation of housing prices.

The model can also explain the apparent puzzle of mean reverting prices and persistent quantity changes. Proposition 4 shows that long-term trends to city

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[3] The regression results underlying these claims are provided in the appendix.

2

productivity or local amenities will create persistence in population and housing supply changes, but will have a much smaller impact on prices, since those trends are anticipated and incorporated into initial prices. Price changes are driven by unexpected high frequency shocks, which themselves mean revert, while quantity changes are driven by anticipated low frequency trends that persist.

The model also serves as the basis for the calibrations discussed in Sections III and IV of the paper. Section III presents our estimates of the model's key parameters: the real rate of interest, the degree to which construction responds to higher prices, and the variance and serial correlation of local demand shocks. In Section IV, we compare the moments of the real data with the moments predicted by the model based on the parameter estimates from Section III.

With respect to the serial correlation properties of prices and quantities, the parameter values described in Section III predict that housing prices will mean revert over five year periods at almost exactly the same rate that we see in the data. This mean reversion is due to the mean reversion of economic shocks to local productivity, with lagged responses of new construction also playing a meaningful role in metropolitan areas with relatively elastic supply sides to their housing markets. We fit the modest mean reversion of construction quantities at this relatively low frequency less perfectly, but the patterns in the real data are quite compatible with reasonable parameter values over five year periods.

At higher frequency annual periods, we predict a strong positive serial correlation of new construction, but the true correlation observed in the data is even greater than the level that our model predicts. Price changes also exhibit positive serial correlation at higher frequency one- and three-year intervals, but our model fails to match this pattern, predicting at least modest mean reversion under virtually any reasonable parameterization. There are two possible explanations for this failure. One is that the observed positive serial correlation could be due to the artificial smoothing of the underlying data; the other is less rational factors. Persistence itself is not enough to reject a rational expectations model, but the mismatch between the data and model at annual frequencies indicates that Case and Shiller's (1989) conclusion regarding inefficiency

3

could be right. Future work needs to deal with the data smoothing problem to see whether the actual serial correlation still is far too high relative to the model.

Reasonable parameter values predict variances of new construction and price changes that are quite close to the variances seen in the median metropolitan area in our sample. The model also does a reasonably good job accounting for the extensive heterogeneity in new construction intensity across markets. While a dynamic, rational expectations model can fit the quantity changes observed in housing markets, the same cannot be said for the variation in price changes. The second major shortcoming of the model is that it does not fit the price volatility observed in many coastal markets (California especially), which have huge price changes.

To examine the robustness of the model in this respect, we then consider four additional potential sources of volatility: amenity fluctuation, local taxes, unmeasured income volatility, and volatile interest rates. The one high frequency amenity variable that we have—crime rates—shows little ability to increase predicted demand and price variability. The same conclusion holds for the volatility of state taxes. The role of mismeasured income volatility is more interesting. Our baseline calibrations use average personal income in a market area as determined by the Bureau of Economic Analysis. Data from the Home Mortgage Disclosure Act (HMDA) files document that the volatility of incomes for recent home buyers is much higher than the volatility for average income in a market area. In addition, the variance of income in areas with big price changes also is higher than the variance of incomes for the average market. These two factors combined help explain the high variation of prices in the most volatile markets on the east coast, but still do not allow us to predict the extremely high price change volatility in coastal California markets (where measured income volatility is not especially high).

This leaves interest rate shocks as a remaining rational explanation for observed behavior in those markets. Volatile interest rates will not increase the volatility of prices or construction in markets with prices close to construction costs (or to the national median price in our model), but they can increase the predicted variance in places with permanently high amenities or productivity. Our very preliminary analysis of this issue in Section V suggests that for interest rates to generate the high levels of volatility observed in coastal California markets, the shocks to interest rates must be extremely

large and the areas must be innately extremely attractive. These markets certainly are blessed with high quality amenities, but the required variation in rates exceeds the best estimates of the volatility of interest rate shocks. Hence, the variation of price changes in the most volatile markets remains a puzzle for rational models of housing.

II.   A Dynamic Model of Housing Prices

Our dynamic model of housing prices is based on three equilibrium conditions: (a) a spatial equilibrium that requires equal utility across markets; (b) a housing market equilibrium that requires expected price to equal the cost of construction; and (c) a labor market equilibrium in which wages equal the marginal product of labor to firms in the city.

Following Rosen (1979) and Roback (1982), we require consumers to be indifferent across space at all points in time. This implies that utility U(W, A, R) is equal across space, where W refers to wages, A to amenities, and R to the flow cost of housing. A further simplifying assumption that this spatial equilibrium must hold in all periods is the housing equivalent of assuming no financial transaction costs (as in Hansen and Jagannathan, 1991).

We implement the spatial equilibrium condition by assuming that there is a "reservation locale" that delivers utility of $\underline{U}(t)$ in each period "t" and that the cost of building a home there always equals "C," which reflects the physical costs of construction. Since housing always can be built in the reservation locale at cost C, we assume that the price of a house there always equals C.[4] The reservation locale represents the many metropolitan areas in the American hinterland with steady growth and where prices stay close to the physical costs of construction (Glaeser, Gyourko and Saks, 2005).[5] The annual cost of living in the reservation locale equals the difference

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[4] While it is possible that prices will deviate around this value because of temporary over- or under-building, we simplify and assume that the price of a house always equals C.
[5] Van Neiuwerburgh and Weill (2006) present a similar model in their exploration of long run changes in the distribution of income (also studied by Gyourko, Mayer and Sinai, 2006). Our paper was produced independently of theirs, and our focus on high frequency variation in prices and quantities is quite different from their focus on changes in the long run distribution of housing prices. More generally, the approach taken here differs from most research into housing prices, which employs the user cost approach introduced by Hendershott and Slemrod (1983) and Poterba (1984). That branch of the literature is too voluminous to

between the price of the house at time t and the discounted value of the house at time t+1, or C-C/(1+r) = rC/(1+r), where r is the assumed fixed rate of interest.[6] We abstract from taxes, maintenance costs and allow time-varying interest rates only in Section 5.[7]

Spatial equilibrium requires all cities at all times to deliver to the marginal resident the same utility that always is available in the reservation locale. For simplicity, we focus on the dynamics in a single representative city (which is different from the reservation city). The utility flow for person i living in that city during period t is $W(i,t) + A(i,t)$, or wages plus amenities. We assume that there are a fixed number of firms, each of which has output that is quadratic in labor. This ensures that the marginal product at each firm is linearly decreasing with the number of workers, and that wages in the city are linearly decreasing with the number of workers. These labor demand schedules generated by firm optimization underpin our assumption that wages at the city level include a stochastic time-varying component and a component that is linearly decreasing in total city population.

We assume that the time-specific and individual-specific effects that make up the net utility flow from the city are separable so $W(i,t) + A(i,t) - \underline{U}(t)$ can be written as $D(t) + \theta(i)$. The composite variable D(t) reflects wages and amenities (which in turn reflect exogenous shocks and city size), and $\theta(i)$ is a uniformly distributed taste for living in this particular locale. We let N(t) denote the housing stock in the city and assume that the city's population and labor force equal a constant times the amount of housing.[8] We further assume that D(t) moves linearly with city population to allow for the fact that wages and amenities may fall due to congestion or rise because of agglomeration economies as city size increases. The value of $\theta(i)$ for the marginal resident at time t (denoted $\theta(i*(t))$) is also linearly decreasing in locale size.

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describe in detail. The first three papers referenced in footnote 2 employ a user cost framework to examine the recent housing boom.

[6] This difference would also be the rent that a landlord earning zero profits would charge a tenant.

[7] If maintenance costs are independent of housing values and constant over space, they will not change the analysis. If maintenance costs scale with housing and if there are property taxes, then the cost of owning a house would be higher than the after-tax interest rate. For this reason, we will assume a relatively high real rate in our simulations. See below for more on that.

[8] Glaeser, Gyourko and Saks (2006) provide evidence showing that population is essentially proportional to the size of the housing stock.

The exogenous components of city amenities and wages include a city-specific component (denoted $\overline{D}$), a city-specific time trend (denoted qt) and a mean zero stochastic component (denoted x(t)). Thus, the flow of utility for the city's marginal resident at time t with index i*(t) relative to the reservation locale, $D(t) + \theta(i*(t))$, can be written $\overline{D} + qt + x(t) - \alpha N(t)$, where α captures the assumption that wages, amenities and the taste of the marginal resident for living in the locale can fall linearly with city size. We further assume that x(t) follows an auto regressive moving average (ARMA) (1, 1) process so that $x(t) = \delta x(t-1) + \varepsilon(t) + \theta \varepsilon(t-1)$, where $0 < \delta < 1$, and the $\varepsilon(t)$ shocks are independently and identically distributed with mean zero.

The expected cost of housing in the representative locale equals $H(t)$ minus $E_t(H(t+1))/(1+r)$, where $E_t(.)$ denotes the time t expectations operator. The difference between the cost of housing in the representative city and housing costs in the reservation locale, rC/(1+r), should be understood as the cost of receiving the extra utility flow associated with locating in the city. If extra housing costs in the city equals extra utility delivered by the city then:

(1) $\quad H(t) - \dfrac{E_t(H(t+1))}{1+r} - \dfrac{rC}{1+r} = \overline{D} + qt + x(t) - \alpha N(t)$.

Equation (1) represents a dynamic version of the Rosen-Roback spatial indifference equation, in which differences in housing costs equal differences in wages plus differences in amenities. We assume a transversality condition on housing prices such that $\lim_{j \to \infty} \left(\dfrac{H(t+j)}{(1+r)^j} \right) = 0$.[9]

While our housing market equilibrium assumption that price equals construction costs requires the city to be growing, important insights can be gleaned by briefly considering the case in which housing supply is fixed, so N(t)=N (as might be the situation for a declining city as analyzed in Glaeser and Gyourko, 2005). In that case,

(2) $H(t) = C + \dfrac{(1+r)(\overline{D} - \alpha N + qt)}{r} + \dfrac{(1+r)q}{r^2} + \dfrac{(1+r)x(t) + \theta \varepsilon(t)}{1+r-\delta}$.

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[9] This assumption limits the possible role of housing bubbles. While our focus here is on a purely rational model, we expect that future work will consider dropping this assumption.

Housing prices are a function of exogenous population and exogenous shocks to wages and amenities. The derivative of housing prices with respect to a one dollar permanent increase in wages will be $(1+r)/r$. Note that house price changes are predictable in this framework as long as there are predictable components to changes in urban wages and amenities. The ARMA(1,1) structure of the shocks makes it possible to have the positive correlation of changes at high frequencies and the negative correlation at low frequencies that we see in the data.

In our model, however, the city grows so that $N(t)$ equals $N(t-1)+I(t)$, where I(t) is the amount of construction in time t. [10] The physical, administrative and land costs of producing a house are $C+c_0 t+c_1 I(t)+c_2 N(t-1)$, where $c_1 > c_2$ because current housing production should have a bigger impact on current construction costs than housing production many years ago. [11] Investment decisions for time t are made based on time $t-1$ information, and there is free entry of risk neutral builders. Thus, if there is any building, construction costs will equal the time t expected housing price as described in equation (3):

(3) $E_{t-1}(H(t)) = C+c_0 t+c_1 I(t)+c_2 N(t-1)$. [12] Equations (1) and (3) together describe housing supply and demand.

These equations give us the steady state values of housing prices, investment and

housing stock: $\hat{H}(t) = \dfrac{\begin{pmatrix} c_2^2(q(1+r)+r(rC+\overline{D}(1+r)))+ \\ \alpha(1+r)(C+c_1(q(1+r)-rc_0))+ \\ \alpha c_2(1+r)((c_0+\overline{D}-q)(1+r)+2rC) \end{pmatrix}}{(rc_2+\alpha(1+r))^2} + \dfrac{(1+r)(\alpha c_0+qc_2)}{rc_2+\alpha(1+r)}t$,

$\hat{I}(t)=\hat{I}=\dfrac{q(1+r)-rc_0}{rc_2+\alpha(1+r)}$ and $\hat{N}(t)=\dfrac{\begin{pmatrix} rc_1(rc_0-q(1+r))+\alpha(1+r)(c_0+\overline{D}(1+r)) \\ +c_2(q(1+r)^2+r(\overline{D}(1+r)-rc_0)) \end{pmatrix}}{(rc_2+\alpha(1+r))^2} + \hat{I}t$.

If x(t)=0 for all t, and $\hat{N}(t)=N(t)$ for some initial period, then these quantities would

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[10] For simplicity, we do not allow depreciation which may be reasonable for shorter term housing dynamics, but would not be appropriate for a very long term analysis of city population changes.
[11] We deviate from the investment cost assumptions of Topel and Rosen (1988) by assuming that costs are increasing with the total level of development and not with changes in the level of investment.
[12] The model can be extended to allow for the possibility that, in some states of the world, new construction will be zero. This adds much complication and only a modest amount of insight into our questions.

fully describe this representative city.[13] Secular trends in housing prices can come from trend in housing demand as long as $c_2 > 0$, or the trend in construction costs as long as $\alpha > 0$. If $c_2 = 0$ so that construction costs don't increase with total city size, then trends in wages or amenities will impact city size but not housing prices. If $\alpha = 0$ and city size doesn't decrease wages or amenities, then trends in construction costs will impact city size but not prices.

Proposition 1 then describes housing prices and investment when there are shocks to demand and when $\hat{N}(t) \neq N(t)$. All proofs are in the appendix.

Proposition 1: At time t, housing prices equal
$$H(t) = \hat{H}(t) + \frac{\overline{\phi}}{\overline{\phi} - \delta} x(t) + \frac{\theta}{\overline{\phi} - \delta} \varepsilon(t) - \frac{\alpha(1+r)}{1+r-\phi}\left(N(t) - \hat{N}(t)\right)$$
and investment equals
$$I(t+1) = \hat{I} + \frac{(1+r)}{c_1(\overline{\phi} - \delta)}\left(\delta x(t) + \theta\varepsilon(t)\right) - (1-\phi)\left(N(t) - \hat{N}(t)\right),$$
where $\overline{\phi}$ and ϕ are the two roots of
$c_1 y^2 - ((2+r)c_1 + (1+r)\alpha - c_2)y + (1+r)(c_1 - c_2) = 0$ and $\overline{\phi} \geq 1+r \geq 1 > \phi \geq 0$.

This proposition describes the movement of housing prices and construction around their steady state levels. A temporary shock, ε, will increase housing prices by $\frac{\overline{\phi} + \theta}{\overline{\phi} - \delta}$ and increase construction by $\frac{(1+r)(\delta + \theta)}{c_1(\overline{\phi} - \delta)}$. Higher values of δ (i.e., more permanent shocks) will make both of these effects stronger. Higher values of c_1 mute the construction response to shocks and increase the price response to a temporary shock (by reducing the quantity response). These comparative statics provide the intuition that places which are quantity constrained should have less construction volatility and more price volatility.

The next proposition provides implications about expected housing price changes.

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[13] In this case, the assumption that there is always some construction requires that $q(1+r) > rc_0$.

9

Proposition 2: At time t, the expected home price change between time t and $t+j$ is

$$\hat{H}(t+j) - \hat{H}(t) + \left(\frac{\alpha(1+r)}{1+r-\phi} - \phi^{j-1}\big((1-\phi)c_1 - c_2\big)\right)\big(N(t) - \hat{N}(t)\big)$$

$$- x(t) + \frac{1}{\bar{\phi}-\delta}\left(\frac{1+r}{c_1}\frac{\delta^{j-1}\big((1-\delta)c_1 - c_2\big) - \phi^{j-1}\big((1-\phi)c_1 - c_2\big)}{\phi - \delta} - 1\right)E_t(x(t+1)),$$

the expected change in the city housing stock between time t and $t+j$ is

$$j\hat{I} + \frac{1+r}{c_1(\bar{\phi}-\delta)}\frac{\phi^j - \delta^j}{\phi - \delta}E_t(x(t+1)) - (1-\phi^j)\big(N(t) - \hat{N}(t)\big),$$

and expected time $t+j$ construction is

$$\hat{I} + \frac{1+r}{c_1(\bar{\phi}-\delta)}\left(\frac{\delta^{j-1}(1-\delta) - \phi^{j-1}(1-\phi)}{\phi - \delta}\right)E_t(x(t+1)) - \phi^{j-1}(1-\phi)\big(N(t) - \hat{N}(t)\big).$$

Proposition 2 delivers the implication that a rational expectations model of housing prices is fully compatible with predictability in housing prices. If utility flows in a city are high today and expected to be low in the future, then housing prices will also be expected to decline over time. Any predictability of wages and construction means that predictability in housing price changes will result in our model.

The predictability of construction and prices comes in part from the convergence to steady state values. If $x(t) = \varepsilon(t) = 0$ and initial population is above its steady state level, then prices and investment are expected to converge on their steady state levels from above. If initial population is below its steady state level and $x(t) = \varepsilon(t) = 0$, then price and population are expected to converge on their steady state levels from below.

The rate of convergence is determined by r and the ratios $\dfrac{c_1}{\alpha}$ and $\dfrac{c_2}{\alpha}$. Higher levels of these ratios will cause the rate of convergence to slow by reducing the extent that new construction will respond to changes in demand.

The impact of a shock, x(t), is explored in the next proposition.

Proposition 3: If $N(t) = \hat{N}(t)$, $x(t-1) = \varepsilon(t-1) = 0$, $\theta > 0$, $c_2 = 0$, and $\varepsilon(t) > 0$, then investment and housing prices will initially be higher than steady state levels, but there exists a value j^* such that for all $j > j^*$, time t expected values of time $t+j$ construction and housing prices will lie below steady state levels. The situation is symmetric when $\varepsilon(t) < 0$.

Proposition 3 highlights that this model not only delivers mean reversion, but overshooting. Figure 3 shows the response of population, construction and prices relative to their steady state levels in response to a one time shock. Construction and prices immediately shoot up, but both start to decline from that point. At first, population rises slowly over time, but as the shock wears off, the heightened construction means that the city is too large relative to its steady state level. Eventually, both construction and prices end up below their steady state levels because there is too much housing in the city relative to its wages and amenities. Places with positive shocks will experience mean reversion, with a quick boom in prices and construction, followed by a bust.

Finally, we turn to the puzzling empirical fact that, across the 1980s and 1990s, there was strong mean reversion of prices and strong positive serial correlation in population levels. We address this by looking at the one period covariance of price and population changes. We focus on one-period for simplicity, but we think of this proposition as relating to longer time periods. Since mean reversion dominates over long time periods, we assume $\theta = 0$ to avoid the effects of serial correlation:

Proposition 4: If $N(0) = \hat{N}(0)$, $\theta = 0$, $x(0) = \varepsilon(0)$, cities differ only in their demand trends q and their shock terms $\varepsilon(0)$, $\varepsilon(1)$ and $\varepsilon(2)$, and the demand trends are uncorrelated with the demand shocks, then:
(a) the coefficient estimated when regressing second period population growth on first period population growth will be positive if and only if

$$\frac{Var(q)}{Var(\varepsilon)} > (1 - \delta - \phi)\left(\frac{\delta(rc_2 + \alpha(1+r))}{c_1(\bar{\phi} - \delta)}\right)^2, \text{ and}$$

(b) the coefficient estimated when regressing second period price growth on first period price growth will be negative if and only if $\Omega\left(\dfrac{rc_2 + \alpha(1+r)}{(1+r)c_1c_2(\bar{\phi} - \delta)}\right)^2 > \dfrac{Var(q)}{Var(\varepsilon)}$, where

$$\Omega = \left(\frac{\alpha(1+r)^2\delta}{1+r-\phi} + c_1(1-\delta)\bar{\phi}\right)\left((1-\delta-\phi)\frac{\alpha\delta(1+r)^2}{1+r-\phi} + c_1\left(1-\delta+\delta^2\right)\bar{\phi}\right).$$

Proposition 4 tells us that positive correlation of quantities and negative correlation of prices are quite compatible in the model. The positive correlation of quantities is driven by the heterogeneous trends in demand across urban areas. As long as

the variance of these trends is high enough relative to the variance of temporary shocks, then there will be positive serial correlation in quantities as in Figure 2.

The mean reversion of prices is driven by the shocks, and as long as c_2 is sufficiently low, prices will mean revert. As discussed above, when c_2 is low, trends will have little impact on steady state price growth. The positive trends show up mainly in the level of prices. However, regardless of the value of c_2, unexpected shocks impact prices and, if these shocks mean revert, then so will prices. This suggests two requirements for the observed positive correlation of quantities and negative correlation of prices: city-specific trends must differ significantly and the impact of city size on construction costs must be small. The extensive heterogeneity in city-specific trends is much commented on, with the recent papers by Gyourko, Mayer, and Sinai (2006) and Van Nieuwerburgh and Weill (2006) attempting to explain the phenomenon. The literature on housing investment suggests that the impact of city size on construction costs is small (Topel and Rosen, 1988; Gyourko and Saiz, 2006). Thus, we shouldn't be surprised to see positive serial correlation in quantity changes and negative serial correlation in price changes.

III.   Key Parameter Values for the Calibration Exercises

We now use the model as a calibration tool to see what moments of the data, including its serial correlation properties and variances, can and cannot be explained by our framework. We focus on the movements in prices and construction intensity around steady state levels. The appendix contains the formulae for the predicted values of these moments.[14]

The model's predictions about variances and serial correlations depend on seven parameters: the real interest rate (r), the degree to which demand declines with city population (α), the degree to which construction responds to higher costs (c_1 and c_2), the time series pattern of local economic shocks (δ and θ), and the variation of those

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[14] **Note: this may expand into a section in the text once Charlie is done with his stuff.** We do not use the high frequency correlations of prices with other variables to pin down parameter values. Changes in house prices and changes in income accompany each other at longer horizons (e.g., over the past twenty years, the correlation of the two changes is over 50 percent), but the correlation is much weaker at higher frequencies. Higher frequency correlations are difficult to interpret because the real world information structure may not match that presumed in our model. For example, if income shocks are known a period earlier, this will not matter much for predicted variances and serial correlations, but it will dramatically alter the predicted relationship between income and price changes.

shocks (σ_ϵ^2). Table 1 reports the value of these parameters which are used in the calibration exercise, with the remainder of this section discussing how we estimate or impute them.

The Real Interest Rate (r)

The first row of Table 1 shows that we use a real interest rate (r) of 4 percent in all calibrations. This value is higher than standard estimates of the real rate because it is also meant to reflect other facets of the cost of owning, such as taxes or maintenance expenses, that might scale with housing. The core simulation results are robust to a wide range of alternative values of *r* (e.g., from 2.5-5 percent).

Supply Side Parameters: c_1 and c_2

The housing cost parameters are critical for the model, but we have little guidance from the literature on their values. The parameter c_1 reflects the extent that construction costs—which include land assembly, permitting and physical construction costs--rise with the level of current construction activity. The c_2 parameter measures the sensitivity of costs to the level of overall development, or market size.

Recent work on housing supply emphasizes the heterogeneity across space in both physical construction costs and local land use regulation (Gyourko and Saiz, 2006; Gyourko, Saiz and Summers, 2006). Housing supply is quite different in Las Vegas, which is extremely pro-growth, versus Greater Boston, which has a web of regulations that make construction extremely difficult (Glaeser and Ward, 2006). Hence, we calibrate using a range of construction cost parameters to capture this heterogeneity.

To determine reasonable values for c_1, we begin by examining the relationship between physical construction costs and permitting levels over time for a large number of metropolitan areas. The construction cost data are taken from Gyourko and Saiz (2006) and are based on figures from the R.S. Means Company, a consultant to the homebuilding industry. The R.S. Means Company provides estimates of the costs to construct homes of given qualities. We use annual cost data from 1980-2004 for a 2,000

square foot 'economy' quality home that meets all building code and regulatory requirements in each market.[15]

The baseline specification regresses physical construction costs per square foot of a standard home on annual housing permits and a time trend which controls for the secular decline in costs in most markets (Gyourko and Saiz, 2006). The range of parameter values reported below for c_I is based on estimations that pool across markets within the nine census divisions.[16]

Higher permitting activity is associated with the lowest increase in physical construction costs in the markets in the South Atlantic (FL, GA, NC, SC, VA, WV), Mountain (AZ, CO, ID, MT, NM, NV, UT, WY), and West South Central (AR, LA, OK, TX) divisions. The regression results imply that one thousand additional permits is associated with a $120 increase in the cost of building a standard house in the Mountain division market and a $140 increase in the South Atlantic and West South Central division areas. In these areas, housing supply seems quite elastic.[17]

Higher permitting is associated with the biggest increase in construction costs in the markets in the New England census division (CT, MA, ME, NH, RI, VT). Here, one thousand extra permits implies a $1,900 increase in construction costs for our 2,000 square foot home.[18] In the Pacific census division (CA, OR, WA), we find that 1,000 extra units increases construction costs by $3,680 for the Santa Barbara-Santa Maria metropolitan area and $1,740, for the San Francisco area, but supply seems to be much

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[15] More specifically, the R.S. Means Company assumes this standard home is built according to a common, national specification. It then disaggregates construction of this unit into different tasks that require materials and labor, and surveys local suppliers and builders to determine local prices for the inputs into each construction task. Local physical construction costs are the sum of the materials and labor costs needed to complete each task.

[16] We were able to obtain data on both construction costs and permits over the 1980-2004 period for 161 markets and use   We use information from all those areas in determining the range of parameter values for c_I. If we restrict the analysis to the 115 markets for which OFHEO reports a constant quality price index since 1980, the results are not materially different. We also estimated the relationship for each metropolitan area and comment below on some of those results. As expected, the range of parameter values obtained when pooling to the census division level is smaller than that resulting from the individual metropolitan area estimations, but those differences are not great, as the variation in supply side conditions across census divisions is substantial. All underlying regression results are available upon request.

[17] Naturally, there is variation about that mean estimate when one looks at individual markets. While the results are not precisely estimated for each metropolitan area, the bulk of the results imply that an additional permit is associated with a 10-20 cent increase in physical constructions costs. For example, the estimate for Dallas is 7 cents, that for Phoenix is 12 cents, Atlanta is 18 cents, and Tampa is 26 cents.

[18] The highest individual market estimates are for the metropolitan areas containing the Connecticut and New Jersey suburbs of New York City. They range from $2.43 in New Haven to $3.48 in Trenton.

more elastic in the many interior California markets. We interpret the data as suggesting that something around $2,000 is a reasonable upper bound for the impact of an additional 1,000 permits on physical construction costs.

Given the per unit basis on which the model is calibrated, these estimates suggest that each new unit is associated with increases physical construction costs of between 15¢ and $2. These ordinary least squares estimates are surely biased downwards both because of the endogeneity of new units and because our costs estimates do not include expenses associated with obtaining regulatory approval or land assembly. The importance of these costs probably is quite low in high growth, low regulation markets such as Phoenix and Las Vegas. However, they are likely to be as much as three quarters of costs in some coastal markets (Glaeser and Gyourko, 2003). Hence, we report simulation results for seven values of c_1 ranging from 15¢ to $50 dollars. We think that a value of 50¢ or less is appropriate for the high unit growth markets with very few restrictions on new building activities. For the median market, we believe that a c_1 value of around $2 best captures the reality of the supply side. In high cost areas, the value of c_1 could well be $20 or more, with $50 representing what we believe is an upper bound.[19]

There is even less of a literature to guide our choice of c_2, although there is no doubt that $c_2 < c_1$ because the flow of current construction influences price more than does the flow of past construction. We assume that c_2 scales with c_1, as both reflect general supply conditions in the area. More specifically, we consider a range of ratio of c_2 to c_1 (henceforth denoted ω), that includes 0.0, 0.25, and 0.50.[20]

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[19] This range can be compared to values of c_1 implied by the housing supply elasticites estimated by Topel and Rosen (1988). Those authors used national data and estimated a supply elasticity ranging from 1.4 and 2.2. This supply elasticity is the relationship between the logarithm of investment and the logarithm of price, which in our model equals $H(t)/c_1 I(t)$. Using the mean values of investment and housing prices across our cities and an elasticity of 1.8, this generates a range of c_1 from 1 to 151. The median value is 18, which seems high since it implies that a thousand additional permits (which is not a large number for the typical American metropolitan area) would imply an $18,000 increase in house cost. Hence, we prefer the lower range associated with our estimation.

[20] An alternative method of estimating these parameters suggests a value of 0.25 for ω. That approach to estimating the construction cost parameters follows Rosen and Topel (1988) in inverting the construction cost equation to obtain $I(t) = (1/c_1)(E_{t-1}[H(t) - C] - (c_2/c_1)(N_{t-1})$. In empirically implementing this equation, we used total housing permits in period t to proxy for new construction in period $t+1$, and actual house prices to measure expected values. Obviously, the use of actual prices in lieu of expected prices introduces some bias, but it should be small since the annual time period over which price is measured is relatively short. We also imputed the housing stock ($N(t)$) each year as described above. A simple regression of each market's resulting c_2 value on its c_1 value (with no intercept, as suggested by our assumed functional form)

Increases in Population and the Marginal Valuation of an Area: α

The value of α reflects the impact that an increase in the housing stock will have on the willingness to pay to live in a locale. If population was fixed, equation (2) tells us that the derivative of housing prices with respect to the housing stock equals $-(1+r)\alpha/r$, which can be seen as the slope of the housing demand curve. Typically, housing demand relationships are estimated as elasticities. Consequently, we must transform estimated demand elasticities into a levels estimate by multiplying by r/(1+r).

While many housing demand elasticity estimates are around one (or slightly below, in absolute value), there is a wide range in the literature, so we experiment with a range from 0 to 2. We begin the transformation from an elasticity to a level by multiplying by the ratio of price to population, which produces a range of estimates for $(1+r)\alpha/r$ of from 0 to 3. Multiplying this span by r/(1+r) yields a range from 0 to 0.15. We will use a parameter estimate of 0.1 in our simulations which implies that for every 10,000 extra homes sold, the marginal purchaser likes living in the area $1,000 less per year (see row 5 of Table 1). This estimate seems high to us, but lower estimated values of α do not significantly change the simulations.

Time Series Properties and Variance of Shocks: δ, θ, and σ_ε^2

The model does not separately address wages and amenities, but there is little evidence on high frequency changes in amenities, so we initially assume that the high frequency movement in demand is driven by changes in labor demand, not changes in the valuation of amenities. More specifically, observed wages $W(t)$ are presumed to equal $w_0 - \gamma N(0) + w_1 t + x(t) - \gamma(N(t) - N(0))$, where $w_0 - \gamma N(0)$ is a component of $\overline{D}$, w_1 is a component of q and γ is a component of α. Controlling for a city-specific fixed effect and trend will eliminate the term $w_0 - \gamma N(0) + w_1 t$, and the residual component of wages equals $x(t) - \gamma(N(t) - N(0))$.[21]

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yielded a coefficient of 0.25. The estimated coefficient is 0.21 if we allow for an intercept. The simple correlation between c_1 and c_2 values estimated this way is quite high at 0.92.

[21] Unfortunately, the literature on regional shocks (e.g. Blanchard and Katz, 1992) does not yield the parameter estimates that we need to calibrate the model.

There are two important issues in determining this parameter value. One involves the proper measure of income to use in the analysis; the other requires computing an adjusted income measure that controls for the impact of population changes. The former is by far the more consequential for the results and we now discuss the different income measures used in the calibration exercises.

We would like to measure the volatility of income of potential homebuyers in each market. One such measure is personal income per capita, as reported by the BEA. A considerable benefit of this variable arises from the fact that it is available for each of the metropolitan areas in our sample and is provided in consistent form back to the beginning of our constant quality price series in 1980.[22] One potential cost is that it is a broad measure of income that reflects more than wages and salaries, and includes interest income and dividends, plus a variety of transfer payments (but not capital gains). Our model implies that the appropriate income series is that attributable to living in the local labor market area. At least some dividend and interest payments will not meet this standard.[23]

Other potential drawbacks include that it is a per capita measure and that it covers everyone in the market area, not just potential homebuyers. We need a household level number because that is the unit that demands housing, but readily available information on the average number of persons per household allow us to scale up the per capita figures, albeit with some measurement error. More problematic is the possibility that this measure does not accurately reflect the income volatility of the marginal home purchaser. In particular, we are concerned that using this variable could understate the true variation in the returns from living in a given market for the marginal buyer. For example, if that demander is young, its income might be more volatile. Or, in cities with vibrant economies, homebuyers on the margin might be people in the cities' fastest growing and most volatile industries (e.g., finance in New York and technology in San Francisco).

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[22] Our data series runs from 1980-2004. We do not use the recently released personal income data for 2005 because it was computed in a different way. To speed up the release of this variable, the BEA recently began exptrapolating certain components of personal income by formula when source data were not immediately available. We wish to avoid the inevitable smoothing such procedures will introduce.

[23] The Bureau of Labor Statistics reports disaggregated wage and salary data, but we do not use it here primarily because it is only available for a small set of metropolitan areas (about 40) before the late 1980s. In addition, this source can generate some very small sample sizes for all but the largest markets even in recent years.

To investigate this hypothesis, we turned to the Home Mortgage Disclosure Act (HMDA) files that provide reported income on mortgage applications for those who applied for home loans in all markets across the United States.[24] More specifically, we use the incomes of those mortgage applicants who were approved for loans to purchase a home. State and county identifiers allow us to match observations to the 115 metropolitan areas in our sample, and the HMDA data set provides very large samples even in small population markets.[25] This probably is a better proxy for the income of potential homebuyers than is the per capita income measure because the latter covers all renters and owners in the market. However, the HMDA-based measure still is not perfect because the potential pool of purchasers is larger than those who actually bought with debt. Perhaps a more important downside is that the micro data on incomes is only available back to 1990 at the metropolitan area level.

Any income series used in the analysis must be adjusted to reflect the impact an increase in city size will have on wages. This is captured in the parameter γ in the equation above. This is difficult to estimate precisely, but fortunately, the procedure used has little impact on the estimated properties of x(t). The required adjustment to income is proportional to the impact of labor supply on wage (or the slope of the labor demand function). Customarily, labor demand is estimated as an elasticity,

$$\frac{Labor\ Force}{Wage}\frac{\partial Wage}{\partial Labor\ Force}$$, and most estimates of this elasticity are statistically

indistinguishable from zero (e.g., Card and Butcher, 1991). Borjas (2003) finds a higher estimate of -0.3, although this is at the national level. We use this upper-bound estimate, but note that it has little differential effect on our results compared to assuming an elasticity of zero.

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[24] The Home Mortgage Disclosure Act was enacted to monitor the behavior of Federal Depository Insurance Corporation (FDIC) member banks. The data were purchased from the FDIC. The dataset includes several variables concerning the race, sex, location, and income of the applicants for mortgage loans, as well as information concerning the amount and purpose of the loan (e.g., purchase, home improvement, refinancing). For more detail, see "Home Mortgage Disclosure Act Raw LAR and TS Public Data" (1990-2004). Federal Financial Institutions Examination Council, Board of Governors of the Federal Reserve System . 20th & C Streets, N.W. Mail Stop 502 Washington, D.C. 20551. On the web,see http://www.ffiec.gov/hmda.

[25] For example, in a small market such as Akron, OH, there are about 10,000 observations per year. For larger markets such as Chicago and Los Angeles, the number of observations typically is 10-20 times larger.

Just as with housing demand above, we must convert this elasticity into an estimate of γ. The magnitude of this parameter can be readily seen using the BEA data on personal income per capita in each metropolitan area as the measure of local wages. For our sample of metropolitan areas, the mean of this variable in 1990 (the middle of our sample period) was $26,965 (in $2,000). Mean employment in 1990 across these metropolitan areas was 539,215, so our ratio of wage to the labor force is about 0.05 (~26,965/539,215). Based on these numbers, an elasticity of -0.3 suggests that each worker is associated with 1.5 cents less annual income in the city. In our sample, there are on average 1.26 workers per home, so each extra home is associated with 1.9 cents per year less annual income, which serves as our estimate of γ.

The per capita income series is converted into household income by multiplying by 2.63 (the average ratio of people per household in our sample in 1990). We then adjust this income measure for the size of the local market using our estimate of 0.019 for γ. We use the housing stock to reflect market size. While this has many advantages over other candidate variables such as employment which fluctuate in ways not linked to permanent changes in market size, we must impute the housing stock (the N(t-1) term) because the census provides actual counts of the stock only once each decade. For each metropolitan area, we know the housing stock at the beginning and end of each decade and the permits issued each year in between. Our estimate of the housing stock at time t+j is $N(t) + \dfrac{\sum_{i=0}^{j-1} Permits_{t+i}}{\sum_{i=0}^{9} Permits_{t+i}}(N(t+10) - N(t))$, where N(t) and N(t+10) are the housing stocks measured during the two closest censuses. Thus, the change in housing stock is portioned across years based on the observed permitting activity.

With this corrected income series, we can estimate the time series properties of income shocks at the local level, by fitting an ARMA(1,1) to the wage series that is first demeaned with city and year fixed effects and then corrected for city size changes as discussed above. Using the adjusted-BEA personal income measure, this procedure

yields estimates of δ=0.87, θ=0.17, and σ_ε^2 = \$3.6 million, which are reported in Table 1.[26]

These values are used in our baseline calibrations, but as just discussed, there is good reason to suspect that the volatility of the income shock is a lower bound. Consequently, we also adjusted the HMDA-based income series for market size as described above for the BEA measure.[27] Because the micro data on buyers' incomes in the HMDA files only date back to 1990, we re-estimate the ARMA (1,1) specification on both income series to allow comparison of the volatility of income shocks over a common time period. Doing so finds that the variance of income shocks (σ_ε^2) based on the HMDA income data of those potential buyers whose loan applications were approved is roughly double (\$5.7 million) that found in the mean per capita income series reported by the BEA (\$2.8 million).[28]

Both these estimates of σ_ε^2 are based on all 115 markets in our sample, and thus do not deal with potential heterogeneity in income variability across markets. This could be important for our analysis if, for example, income variability were systematically higher in high price change variance markets. In that case, our model estimates of price volatility would be biased downward in those areas. To investigate this issue, we focused on coastal markets that have particularly volatile house prices. [See below for the documentation of this feature of the data.]

Working first with the BEA series, we re-estimated σ_ε^2 for a sample of 31 markets whose centers are within 50 miles of the Atlantic or Pacific Oceans. While the AR (δ) and MA (θ) components were little changed from those reported for the 115 market national sample, the estimate of σ_ε^2 is almost 50 percent higher in the 31 coastal markets: \$5.3 million versus \$3.6 million. Not surprisingly, income volatility is even higher for

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[26] Largely because γ is so small throughout its relevant range, this adjustment to wages does not have a material impact on our results. If we use a value of 0 for γ, we estimate a value of $\delta = 0.86$, an estimate of $\theta = 0.18$, and an estimate of σ_ε^2=\$3,408,250 In addition, we attempted joint maximum likelihood estimation of δ, θ, and σ_ε^2 for given trend effects and metropolitan area fixed effects, but the program would not converge because the panel was too short relative to the number of markets.

[27] This series uses the median income for each year and MSA of those who were approved for a mortgage for the stated purpose of buying a home.

[28] These results suggest that income volatility was relatively high in the 1980s. Shocks also appear to have been less permanent since then. The estimate of δ is 0.67 in both series (compared to 0.87 for the BEA series since 1980). The moving average component is weaker, too, as θ=0.08 using the HMDA data and θ=0.14 in the BEA data since 1990.

recent buyers in these same markets. The σ_ε^2 estimate from the HMDA series is $7.8 million. This difference in volatility of local wage shocks across different types of markets is large, and to our knowledge, has neither been well-documented nor well-understood. Highly productive coastal areas might specialize in volatile idea-intensive industries, but this is an appropriate topic for future research.

This series of estimates provides a range of local income volatilities that bounds σ_ε^2. The lower end is captured by the $3.6 million figure from the BEA personal income series. In the middle is the $5.7 million value based on the HMDA reported income series. At the upper end is the $7.8 million value derived from the HMDA data for coastal markets. Results using all three values for σ_ε^2 are reported in the tables below, with our baseline simulations employing the $3.6 million estimate. To keep the presentation manageable, we always impose the $\delta=0.87$ and $\theta=0.17$ values from the BEA data. These parameter values do not vary as dramatically depending upon which income series is used, and none of our key results is altered by using other values discussed above.

IV. Calibrating the Model and Matching the Data: Baseline Results

In this section, we calibrate the model using the following parameters values: $r=0.04$, $\alpha=0.1$, $\delta=0.87$, $\theta=0.17$, $\delta=0.87$, and $\sigma_\varepsilon^2=\$3.6$ million. Simulation results using these parameter values are reported for seven different values of c_1 ($0.15, $0.50, $2.00, $5.00, $10.00, $20.00, $50.00) and three values of c_2 (0, 0.25*c_1, and 0.50*c_1). Hence, there are 21 simulation results for each feature of the data we try to match. We first discuss the time series coefficients of prices and construction, and then discuss the volatility of these series. Our "real data" sample is a set of 115 metropolitan areas for which we have continuously defined price data from 1980-2005.

Short-Term Momentum and Longer-Term Mean Reversion in Prices, Rents and Permits

The top row of Table 2 provides evidence on momentum and mean reversion in OFHEO house prices within market over time. We use absolute price changes rather than changes in the logarithm of prices in order to be compatible with the model, but our empirical results are not sensitive to such changes in functional form. Since the OFHEO

index only provides price increases relative to a base year, we convert this into an implied price series by using the median housing value in the metropolitan area in 1980 as a base price in the metropolitan area and then scaling that value by the appreciation in the OFHEO index each year.[29]

The results are estimates from a regression of the current change in prices on the lag change in prices

$$(6) \quad \text{Pr}ice_{t+j} - \text{Pr}ice_t = \alpha_{MSA} + \gamma_{Year} + \beta\left(\text{Pr}ice_t - \text{Pr}ice_{t-j}\right),$$

for j equal to one, three and five years. Because fixed effects estimates such as these which remove market-specific averages can be biased (with spurious mean reversion produced especially when the number of time periods is relatively low), in the first row of Table 2, we report Arellano-Bond estimates which use lagged values of the dependent variable (price changes) as instruments.[30]

Our one year estimate of price change serial correlation is 0.71, implying that a $1 increase in housing prices between time t and t+1 is associated with a 71 cent increase between time t+1 and t+2. This estimate is larger than that reported by the pioneering work of Case and Shiller (1989). It is now well understood that smoothing of the underlying data series can bias one towards finding short-run momentum. Case and Shiller (1989) were able to address this problem by splitting their sample, which consisted of micro data on sales transactions in four markets (Atlanta, Chicago, Dallas, and San Francisco). They report coefficients ranging from 0.2-0.5, although they use the logarithm of prices, not the level, so the results are not exactly comparable. Because we cannot perform any comparable procedure with the OFHEO data, our estimate is surely biased upwards.[31]

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[29] This procedure essentially provides the real price for a constant quality house with the quality being that associated with the median house in 1980. We have experimented with using values from the 1990 and 2000 censuses as the base. All the results reported below are robust to such changes.

[30] See Arellano and Bond (1991) for more detail on this estimation procedure. More specifically, we use the "xtabond" Stata command with year and area fixed effects.

[31] The OFHEO index includes data on repeat sales or refinancings of the same house. The latter typically rely on an appraisal, not a market sale price. Undoubtedly, this results in smoothing of the series and biases upward our estimate of short-run momentum. Even the Case and Shiller (1989) estimates, which rely only on actual sales, could be upward biased. Working with a split sample, bias can result if, randomly, some fraction of homes on which a buyer and seller agree on a price have delayed closings that move their reported sales dates into the next reported period (quarter, year, etc.). Whatever shock there was in period *t* that influenced the agreed upon price, some of its measured impact will spill over into period *t+1*. Obviously, this is potentially more of a problem the shorter the measurement period.

Over three years, there is still momentum. The estimate of 0.27 means that a $1 increase in housing prices between time t and t+3 is associated with a 27 cent increase between time t+3 and t+6. Over five year periods, we estimate a mean reversion coefficient of -0.32, so a $1 increase between times t and t+5 is associated with a 32 cent decline between time t+5 and t+10.[32]

The mean reversion in prices that we estimate over five-year horizons is quite similar in magnitude to that observed for financial assets by Fama and French (1988). Unfortunately, the short time period for which we have constant quality price data at less than decadal frequencies makes it difficult to know whether this mean reversion is a permanent feature of urban life or whether it represents the impact of shocks that are specific to the post-1980 time period. Cutler, Poterba and Summers (1991) also find this pattern of short run momentum and longer- run mean reversion for housing and a number of other asset markets.

Table 3 reports the comparable results for prices based on our simulations using the 21 different c_1 and c_2 combinations discussed above. The first three columns show results for annual serial correlation in prices, the next three columns present the analogous findings over three year periods, with the final three columns being for five year periods.

The one-year predictions document the model's failure to match the positive serial correlation observed in the annual data. In fact, our calibration predicts mean reversion even at such a high frequency. The predicted mean reversion is much higher in low cost, more elastic supply areas than in high cost, inelastic supply areas because more new construction will cause housing prices to fall more rapidly in the first group of markets. The results for three year horizons reported in the middle columns of Table 3 also find a mismatch with the data. Assuming the middle case for omega (ω=0.25,

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[32] These estimates are not an artifact of the Arellano-Bond procedure. The analogous ordinary least squares estimates over 1, 3, and 5 year horizons are 0.74, 0.18 and -0.39, respectively. We also addressed concerns about spurious mean reversion by estimating specifications without metropolitan area fixed effects. If we estimate the following equation, $\mathrm{Pr}ice_{t+5} - \mathrm{Pr}ice_t = \alpha + \gamma_{Year} + \beta(\mathrm{Pr}ice_t - \mathrm{Pr}ice_{t-5})$, the mean reversion coefficient drops to -0.11 and becomes only marginally significant. However, as soon as we include percent of adults with college degrees as a control, the coefficient becomes -0.18 with a t-statistic of three. If we estimate the same change regression using the logarithm of prices instead of the levels, the coefficient is -0.20 (-0.22 with the college graduate control) and has a t-statistic of four.

column 4), we predict mean reversion coefficients from -0.17 to -0.45, not the positive persistence we see in the data as reported in Table 2.

Our model does a much better job of fitting the -0.32 mean reversion seen at five year intervals (columns 6-9). At five year horizons, if c_1 takes on a value from 2 to 5 and ω=0.25, then we come within ten percent of matching the data (see rows 4 and 5, column 8). In fact, almost all of the construction cost parameter values predict levels of mean reversion that are close to those observed in the data. Only in areas with extremely elastic supply do we predict mean reversion that substantially differs from observed levels. Thus, the predictable mean reversion of prices at five year intervals cannot be seen as a challenge for a rational expectations model of housing price movements.

In the model, mean reversion reflects both the tendency of shocks to mean revert and of new construction to cause future declines in prices. New construction will only decrease future housing prices when housing demand is downward sloping, so when demand for housing is perfectly elastic (i.e., α=0), the only force for mean reversion is the mean reversion of shocks. When we restrict α=0, the predicted baseline level of mean reversion is about -0.25, so only a small amount of mean reversion in the average market is due to the effects of new construction. By contrast, if we set δ=1, so that there is no mean reversion in the shocks, then the predicted mean reversion disappears almost entirely, especially in markets with high values of c_1 and c_2. In those markets, the model suggests that the mean reversion of shocks, not new construction activity, drives the mean reversion of prices. However, in markets with lower values of c_1 and c_2, new construction plays a more important role in generating the mean reversion of prices. In these more elastic markets, if α=0, then predicted mean reversion over five year periods falls by a quarter relative to Table 3. In these markets, both new construction and mean reverting shocks play meaningful roles in explaining the mean reversion of prices.

Short run momentum in asset price changes is thought by some to provide evidence of anomalies in the asset markets. If this momentum reflected some asset market quirk, then presumably it should not also appear in rents. In the second row of Table 2, we report the results from rental regressions of the form in equation (7),

$$(7) \quad \mathrm{Re}\,nt_{t+j} - \mathrm{Re}\,nt_t = \alpha_{MSA} + \gamma_{Year} + \beta\left(\mathrm{Re}\,nt_t - \mathrm{Re}\,nt_{t-j}\right).$$

Rental data on apartments is collected by an industry consultant and data provider, REIS Inc. Their data covers only a limited number of metropolitan areas (46 in our sample), and in general, rental units are not similar to owner-occupied housing.[33]

Over one- and three-year horizons, there is strong evidence of persistence, with the Arellano-Bond estimates being 0.27 in both cases. Over five year time horizons, we estimate a mean reversion parameter of -0.64. The presence of high frequency momentum and low frequency mean reversion in rents suggests that these features do not reflect something unique to housing asset markets, but rather something about the changing demand for cities.[34]

Table 4 then reports the predicted values of serial correlation from the simulations of the model. At annual frequencies, we predict serial correlation ranging from -0.26 to 0.09 when $\omega=0.25$ (column 2). Even though there is no predicted mean reversion in higher cost, more inelastic markets, these estimates still are well below the 0.27 estimate observed in the data. Over three year horizons, we consistently predict mean reversion, while there is still a positive serial correlation of rents in the data. For five year intervals, we predict that rent changes should have a mean reversion coefficient of between -0.31 and -0.37 if we assume that c_1 lies between 2 and 5 and $\omega=0.25$ (row 3 and 4, column 8). Higher mean reversion is predicted if c_1 is lower, but our estimates still are only about one-half of the observed mean reversion in that case.

We are again unable to explain the strong positive serial correlations at shorter time horizons. Since there are many reasons to be suspicious about the properties of the rental data, especially because of artificial smoothing, we do not attach much importance to the quantitative mismatch with the data here.[35] However, the short run momentum and long run mean reversion of rents, which are predicted by the model, suggest that these features could reflect something other than irrationality in an asset market.

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[33] Rental units are overwhelmingly in multi-unit buildings, while owner-occupied housing is overwhelmingly single-family detached housing. These differences in housing types and the problem of accurately measuring maintenance costs are two reasons why it is extremely difficult to tell whether housing prices are high or low relative to rents.

[34] The ordinary least squares estimates of these coefficients are 0.28, 0.08 and -0.51 for one, three and five year horizons, respectively.

[35] For example, smoothing is a greater problem in the rental data. The industry consultant that provides the rent data does not survey actual renters, but the landlord owners of apartment buildings. Undoubtedly, averages are being reported.

To examine the dynamics of housing quantities, we turn to housing permit data from the *Census of Construction*. The final set of results reported in Table 2 use housing permits estimated in the following regression: $Permits_t^{t+j} = \alpha_{MSA} + \gamma_{Year} + \beta Permits_{t-j}^t$, where $Permits_{t-j}^t$ refers to the number of permits issued between time t-j and time t. The one-three and five year Arellano-Bond coefficient estimates are 0.84, 0.43, and -0.07, respectively. Thus, construction also displays high frequency momentum, but little or no persistence or mean reversion at longer horizons.[36]

The calibration results for this variable are provided in Table 5. For the case where c_1=5 and ω=0.25, the predicted coefficients are 0.60, 0.28 and 0.05, for one, three, and five year horizons, respectively. These are reasonably close to the actual parameters, and minor changes in the values of one or both of the supply side parameters enable us to fit the data more exactly. While the predictions about the serial correlation of construction are not as accurate as the predictions about the mean reversion in prices, the moments of the real data cannot be said to reject the model.

In sum, the model does a reasonable job at fitting the time series properties of new building and an excellent job at fitting the long term mean reversion of prices. It does a very poor job of fitting the high frequency positive serial correlation of price changes. This failure may be the result of data smoothing causing us to empirically overestimate momentum, or as Case and Shiller (1989) suggest, it could reflect some sort of irrationality in the housing market.

House Price Change and Construction Variances Across Markets

Table 6 reports the variance of price changes and of new construction in our sample.[37] The volatility of both prices and construction varies enormously across cities. The distribution is quite skewed, with the mean variance much higher than the median variance. To address this heterogeneity, we first run a regression for each outcome using all of our markets controlling for year effects, and then compute the variance of the residuals from this regression by metropolitan area. This variance gives us the volatility

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[36] As is the case with the other data, this pattern is not an artifact of our estimation procedure. The analogous ordinary least squares coefficients are 0.82, 0.37, and 0.07, respectively.
[37] Since the rent data are smoothed, we do not believe much, if any, weight should be put on measured variance of rents. Hence, that variable is excluded from this part of the analysis.

of prices and construction, respectively, within a metropolitan area controlling for nationwide effects.

The top panel of Table 6 shows that that the variance of one year price changes equals $14 million in the metropolitan area from the 10th percentile of the distribution and $209 million in the market at the 90th percentile of the distribution. The median market has a one year price change variance of $34 million, which is much smaller that the sample mean of $83 million. This skewness is driven primarily by California markets and Honolulu. The variance of one-year price changes in Honolulu is $763 million, which is the largest in our sample. Five other markets—San Jose, San Francisco, Santa Barbara, Santa Ana and Salinas--had variances that were at least ten times greater than the sample median.

The second and third columns of this top panel of Table 6 report the distribution of variances of three and five year price changes. The distribution of longer horizon price changes is also quite skewed, with the mean price change substantially exceeding the change for the median area. The volatility of price changes is very high at longer horizons. The variance in five-year price changes is $625 million for the median market, and one quarter of the metropolitan areas have variances of at least $1.1 billion.

Table 7 reports predicted price change variances from our simulations with the results arrayed in the same manner as in the serial correlation tables above. At annual frequencies (columns 1-3), we predict a range of price change variances from a low of $16 million to a high of $190 million. Not surprisingly, price change volatility is lower the smaller the values of c_1 and c_2. In those markets, quantities can respond readily to changes in demand. The actual $34 million price variance of the median market does lie within this large range of predicted values, but it is only compatible with a very low value of c_1 : 0.50. For more reasonable values of c_1, we predict much higher variation in one year price changes than we see in the data.

Data smoothing would bias measured price volatility downward over short time periods, and if so, we would expect this problem to be less severe for longer time periods. Our ability to match the volatility of price changes in the median does increase with the horizon over which those changes are measured. If $c_1=2$ (and $\omega=0.25$), the predicted price variance over three-year horizons is $182 million, which is quite close to the $185

variance found in the median market. If c_1=0.5 (and ω=0.25), the predicted variance almost exactly matches that found in the 10th percentile market in the data. However, at this longer horizon, the model fails to match the high price volatility seen in the top quartile of housing markets. Even assuming very little quantity variation (i.e., c_1=$20 or $50), we do not predict a house price change variance much above the $445 million observed in the 75th percentile market.

For the 5-year price change variance predictions listed in the final three columns of Table 7, our range of predictions runs from $29 to $756 million. This captures the lower half of the distribution of actual price change variation reported in the third column of Table 6 (top panel), but our model generally predicts too little price change volatility at this low frequency. For example, if c_1 equals 5 and ω=0.25, then we predict a five year price variance of $421 million, which still is well below the sample median ($625 million). Higher c_1 values, of course, allow us to come much closer, but no reasonable values of c_1 predict the very high price change volatility found in the top quarter of markets. We return to this issue in the next section. Table 7 shows that the model overestimates price volatility at high frequencies, but not at lower frequencies. This pattern could be explained by artificial smoothing of price change data, but it could also reflect a flaw in our model.

The bottom panel of Table 6 reports the variance in units permitted across our 115 metropolitan area sample. As with price changes, there is substantial heterogeneity in the volatility of construction intensity across markets, and this distribution is skewed by a few outliers. For example, the bottom quartile of markets has a new construction variance of about 2 million units per annum, while the top quartile is at least five times more volatile. Moreover, this distribution is skewed by relatively few markets in the right tail that have variances of at least 38 million units (column 1, bottom panel of Table 6). Six markets—Phoenix, Dallas, Riverside-San Bernardino, Atlanta, Los Angeles, and Houston---stand out in this regard, having construction intensity variances that are at least double the next six highest variance markets. There also is great heterogeneity in construction intensity variance over longer horizons, as the second and third columns of this part of Table 6 document.

Table 8 then reports the construction intensity variance estimates from our standard set of simulations. Our model essentially can predict almost any construction variation given the full range of construction costs estimates. At annual periods, the range of actual variances, which run from two million in the 10th percentile market to 38 million at the 90th percentile of the distribution, lies within the range of values predicted when c_1 ranges from 50 cents to ten dollars and $\omega=0.25$. The median market in our sample has a one year standard deviation of 3 million which is in the 2-5 million unit range predicted when c_1 lies between 5 and 10 and $\omega=0.25$.

Over three year horizons, the data also roughly fits the model. For example, the three-year construction variance in the median market is 26 million units (middle column, bottom panel of Table 6). The range of predicted variances is between 11 and 30 million if c_1 lies between 5 and 10 and $\omega=0.25$. The most volatile markets are also compatible with the model, if c_1 is sufficiently low. For example, the variance of 328 million observed for the 90th percentile market, is close to the variance predicted for a market where c_1 equals 0.5 and $\omega=0.25$.

At five year intervals, the actual median market has a variance of 59 million units, which is quite close to the variance of 62 million predicted when c_1 equals 5 and $\omega=0.25$. The 10th-90th percentile range in the data lies between 29-760 million units. When $\omega=0.25$, the predicted range is between 22 and 986 million when c_1 ranges from 0.15 to 10. Overall, the construction variances in the data are well within the range of variances predicted by the data. The values of c_1 and c_2 must be quite low to explain the places with extremely volatile construction levels, but we find that plausible for a number of American housing markets.

That said, it is a fair complaint that the model fails to give tight predictions about construction variation. Hopefully, future work will yield tighter estimates of the construction cost parameters and this will lead to tighter predictions, but the spirit of our calibration exercise is to ask what moments of the data are incompatible with our simple spatial model. This exercise does suggest that supply elasticity can explain a significant amount of the high construction volatility in the Sunbelt, but supply inelasticity cannot explain much of the high price volatility in coastal markets. Hence, we now ask whether

extensions to the model can explain the high volatility of house price changes in those markets.

V. Explaining the High Volatility of Housing Prices

The simplest explanation of highly volatile housing prices are omitted demand shocks, such as changes to local tax rates and amenities, and mismeasurement of income shocks. We examine those hypotheses first and then turn to the potential role of time-varying real interest rates.

Local Tax Rates and Amenities

Changes in local tax rates and amenity flows could increase housing volatility. We use data from the NBER TAXSIM website on the average tax rate on wage income earned in a given state each year to create an after-tax income measure for each market. Our analysis of after-tax income showed that controlling for this factor cannot be responsible for more than a 10 percent increase in local demand variability which would translate into a ten percent increase in price and construction volatility. While Appendix II provides the details, we conclude that changes in state level tax rates cannot be driving the high price volatility in the top quartile of markets.

Unlike taxes, most amenities are relatively permanent characteristics of a place, (e.g., the weather, local architecture). The demand for these amenities may change slowly over time as a society becomes richer or more unequal or as new technologies become available, but it is hard to imagine that their value will fluctuate much at annual frequencies. Crime represents one of the few amenities that does change relatively rapidly and for which there is available data. We collected violent crime rates for the largest cities in each of our metropolitan areas using continuous crime data from 1985-2005.[38] We then created an adjusted income variable that subtracted the negative effect of crime from our BEA real income measure. As detailed in Appendix III, the results showed almost no impact on the variability of local demand from controlling for crime.

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[38] We emphasize that this measure is for the local political jurisdiction, which we then impute to the metropolitan area.

We infer from this that we are unlikely to find an amenity with high frequency variation that can explain much of the observed volatility in prices or construction.

Measurement of Income Shocks

Because the key outstanding price-related puzzle for our model is the high volatility of lower frequency price changes in the top quartile of metropolitan areas, the top panel of Table 9 provides new estimated variances of five-year price changes for markets with high c_1 values of at least 10, assuming more variable local demand shocks.[39] We use the two estimates of σ_ε^2 from the HMDA data discussed above, \$5.6 million and \$7.8 million, to reflect the plausible range of higher income shock volatilities, with the first column reproducing the baseline estimates from column 8 of Table 6 for comparison purposes.

The first row of the second column of Table 9, which assumes a value of \$5.7 million for σ_ε^2, indicates that one still needs an extremely high (c_1, c_2) pair to match the price change volatility of the 75th percentile market. However, if σ_ε^2 is doubled as is assumed in the third column and which seems plausible for the coastal markets using the HMDA data, predicted price change volatility comfortably reaches the \$1.17 billion level observed in the data for the 75th percentile market (see the final column of the top panel in Table 9).

This still leaves the model unable to account for the very high price change variances seen in the top ten percent of markets, which are generally all on the Pacific coast. The 12 most volatile markets in are sample are made up of Honolulu and much of coastal California.[40] There are no east coast markets in this group, with the Nassau-Suffolk and Bethesda-Gaithersburg-Frederick metropolitan areas having the 15th and 16th biggest price change volatilities.

The bottom panel of Table 9 shows that the combination of elastic supply side conditions and higher local income shock variation allow the model to match most of the

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[39] We also assume that $\omega=0.25$ in each simulation.

[40] The top ten percent of the most volatile metropolitan areas in terms of five-year price changes (in ascending order from #104-#115) are as follows: Oakland-Fremont-Hayward, Santa Cruz-Watsonville, San Luis Obispo-Paso Robles, San Diego-Carlsbad-San Marcos, Oxnard-Thousand Oaks-Ventura, Los Angeles-Long Beach-Glendale, San Jose-Sunnyvale-Santa Clara, Salinas, Santa Ana-Anaheim-Irvine, San Francisco-San Mateo-Redwood City, Santa Barbara-Santa Maria, and Honolulu.

highest construction intensity variation markets in the country. We allow demand side volatility to increase only to $5.7 million in these simulations because there are no coastal markets with such low c_1 and c_2 values. We don't need extremely low values for construction costs to account for the construction intensity variation in the most volatile markets if income volatility is more accurately reflected in the HMDA data. Given that we would expect the volatility of the marginal buyer's income to be greater than the average used in the baseline simulations, these results suggest that high construction volatility is not a puzzle.

Time-Varying Interest Rates

We have so far assumed that interest rates are fixed for reasons of tractability, but we do recognize that many authors have claimed that the dramatic rise in house prices, especially in high cost markets, over the past decade is best understood as a response to declining interest rates that make housing in those areas more affordable (e.g., Himmelberg, Mayer and Sinai, 2005). A full treatment of interest rates would require an analysis of long period mortgages and prepayment that lies well beyond the scope of this paper. We can, however, adjust the model modestly to acquire some understanding of the potential impact of time-varying interest rates. In doing so, we assume a spot market for refinancing, as well as linearity in the difference equation describing the shocks to rates.

We begin by decomposing interest rates into permanent and transitory components, $\bar{r}$ and $\rho(t)$, respectively, where $r(t) = \bar{r} + \rho(t)$, and use the approximations $\dfrac{1}{1+r(t)} \approx \dfrac{1}{1+\bar{r}}$ and $r(t)(H(t) - C) = \bar{r}(H(t) - C) + \rho(t)(\overline{H} - C)$, where $\overline{H}$ is meant to reflect the average housing price in the city.[41] If we adjust equation (2) for time-varying interest rates using these approximations, equation (2') results:

$$(2')\ H(t) - \frac{\bar{r}C}{1+\bar{r}} - \frac{E_t(H(t+1))}{1+\bar{r}} = \overline{D} + qt + x(t) - \rho(t)(\overline{H} - C) - \alpha N(t)$$

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[41] The first approximation is minor and would have been unnecessary if we assumed that the utility flow was received at the end of the period rather than the beginning of each period. The second approximation eliminates interactions between transitory changes in value and transitory changes in the interest rate and it may be more consequential.

If we then make the admittedly unrealistic assumption that $\rho(t) = \lambda\rho(t-1) + \eta(t)$ so the difference equation remains linear, the model can be solved in a relatively straightforward fashion.

This results in equation (9)'s description of prices,

(9)
$$H(t) = \hat{H}(t) + \frac{(c_1\phi + \Psi)x(t)}{c_1(\phi - \delta) + \Psi} + \frac{c_1\theta\varepsilon(t)}{c_1(\phi - \delta) + \Psi} - \frac{\alpha(1+r)}{1+r-\phi}\left(N(t) - \hat{N}(t)\right)$$
$$- \frac{(c_1\phi + \Psi)(\overline{H} - C)\rho(t)}{c_1(\phi - \lambda) + \Psi}$$

This differs from the price equation in Proposition 1 because of its last term which multiplies $\dfrac{c_1\phi + \Psi}{c_1(\phi - \delta) + \Psi}$ times the interest rate shock times $\overline{H} - C$ (the gap between average housing prices in the area and construction costs). This term reflects the fact that a decline in interest rates essentially is a positive demand shock for high amenity and productivity places. The shock makes it cheaper to live in such places, pushing up demand and prices.

Since our regressions correct for year effects, this interest rate effect can have no impact on our empirical estimates for the average market which will have prices close to construction costs. The interest rate effect does, however, have the capacity to generate increased variance in both price changes and construction levels for places that are considerably more expensive on average. We consider four different values of $(\overline{H} - C)$: $25,000, $50,000, $100,000 and $200,000, which over the past 25 years captures most of the range of American metropolitan areas.[42]

Initially, we set c_1 and c_2 equal to 3.5 and 0.875, respectively, in order to focus on interest rates effects.[43] We assume that λ=0.90, but experimentation with values as high as 0.95 yield similar results. The variance of interest rates is far more important for the results, and we use a range of standard deviations for $\eta(t)$ from 0.005 to 0.02 which

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[42] For example, in 1980 the highest price metropolitan area had a median house value that was about $170,000 greater than in the median market. The real value of median market's median house price is barely changed between 1980 and 2000. Except for a handful of markets in the upper tail of the metropolitan area price distributions, gaps in excess of $200,000 with the median market do not exist. Finally, we omit runs with a value of zero because they correspond to the simulations from the previous section.

[43] These represent the mid-point between c_1=2 and c_1=5 (assuming ω=0.25), which we believe represent typical supply side conditions.

appears to encompass most assessments of the amount of real rate variation.[44] Simulations suggest that these changes to our baseline model make little difference to the amount of predicted mean reversion. Thus, the results in Table 10 focus on the impact of interest rate volatility on the variance of price changes.

In general, the results from this table show that interest rates shocks can generate significant increases in the variance of price changes if the volatility of rates is at least one percent and if the market has house prices much greater than the average. If H-C<$100,000, the impact on predicted price change variation is minimal. For markets in which H-C=$200,000, the impact typically is quite large. Over high frequency, annual periods, a predicted variance almost equaling the $209 million figure observed for the 90th percentile metropolitan area (see Table 6) results if H-C=$200,000 and if the standard deviation of $\eta(t)$ is one percent. This is roughly double the predicted variance reported in Table 7 for the same supply side conditions (i.e., $c_1=3.5$; $c_2=0.875$). Many of the coastal California markets in question have median house values that are $200,000 more than the national median, and assuming a standard deviation of one percent for the one period rate is not extreme. Moreover, if we run the same simulation, but assume relatively inelastic supply side parameter values (i.e., $c_1=10$; $c_2=2.5$), the predicted price change variance increases to $307.

While this indicates that interest rate shocks may be able to help account for the highest price change volatility observed at high frequencies, the same cannot be said for longer horizons. Recall from Table 6 that the observed price change variance in the 90th percentile market is $1.38 billion for three-year horizons. Table 10's results indicate that to even approach that number not only requires very high house prices, but extremely high short rate volatility. Specifically, if H-C=$200,000 and the standard deviation of $\eta=0.02$, the predicted price change variance is $1.29 billion.[45] A similar pattern is

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[44] Campbell's (2000) review of the asset pricing literature notes that the standard deviation on a one period riskless asset is 1.76 percent, but concludes that "… perhaps half … is due to ex post inflation shocks (p. 1519)." Thus, the lower half of this range may be more plausible. Recent asset pricing papers such as Bansal, Kiku, and Yaron (2006) assume a standard deviation of 1 percent.

[45] Even assuming more inelastic supply with $c_1=10$ and $c_2=2.5$, a more plausible assumption of $\eta=0.01$ yields a predicted price change variance of only $802, which is still only 58 percent of the observed volatility in the 90th percentile market.

observed at five year intervals, except the underprediction of price change volatility is even more extreme.[46]

Table 11 shows that interest rate shocks do not have much influence on predicted variation in construction intensity. There is some increase in predicted construction variance, but it is quite modest. Generally speaking, unless both $\overline{H} - C$ equals \$200,000 and the standard deviation of interest rate shocks is 0.02, the predicted construction volatilities are within the range of values reported in our baseline simulations for markets with c_1 values between 2 and 5 (see Table 8).

In sum, these results are consistent with the claims of Himmelberg, Mayer, and Sinai (2005) and others that time-varying interest rates can play a role in helping explain house price behavior in certain high value markets. However, these findings do not indicate that rate variation does or can explain the extremely high price change volatility observed in certain west coast markets over multiple year periods. Moreover, absent appending a formal model of long-term mortgages with prepayment to the existing theory, these results should be viewed only as a rough heuristic guide on the matter of time-varying rates. They certainly indicate it would be premature simply to assume the excess price change variation is due to irrationality or animal spirits before, but there may be other explanations besides interest rates that could offer a rational explanation of price volatility, as we discuss in the concluding section.

VI.   Conclusion

This paper presents a dynamic rational explanations model of housing markets based on a cross-city spatial equilibrium. The model predicts that housing markets will be largely local, which they are, and that construction persistence is fully compatible with price mean reversion. The model is also consistent with price changes being predictable.

The model has successes and failures at fitting the real data. The model can explain the serial correlation of construction quantities reasonably well and can explain the five year mean reversion of prices almost perfectly. However, the model cannot explain the high frequency positive serial correlation of price changes. The model can

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[46] For example, if we assume a very high interest rate volatility (η=0.02) and inelastic supply (c_1=10; c_2=2.5), the predicted price change variance is \$3.01 billion—still 16 percent below the \$3.58 billion observed in the 90th percentile market.

explain the price and construction volatility of a typical housing market. It does a good job of accounting for the heterogeneity in construction intensity variation across most markets in the country. However, it cannot account for the most volatile markets in terms of low frequency price changes, which are located along California's coast.

This suggests that housing economists begin to focus their attention on high price volatility in coastal markets and on the positive serial correlation of high frequency price changes. The average volatility and longer-term mean reversion of prices should no longer be viewed as puzzles. In addition, the time series properties of new construction and the volatility of changes in building activity are well understood by a dynamic, rational equilibrium model that allows for differences in supply elasticity across markets.

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Table 1: Model Parameters	
r	0.04
δ	0.87
θ	0.17
σ_ε^2	BEA: \$3.6 million; HMDA: \$5.7 million; HMDA (coastal markets): \$7.8 million
α	0.1
c_1	\$0.15, \$0.50, \$2.00, \$5.00, \$10.00, \$20.00, \$50.00 (per housing unit)
c_2	c_2 can be 0%, 25%, or 50% of c_1; hence, there are 21 different combinations of (c_1, c_2) pairs

Table 2: Predicted Moments as a Function of the Information Structure

	CONTEMPORANEOUS KNOWLEDGE	KNOWLEDGE 1 YEAR AHEAD	KNOWLEDGE 5 YEARS AHEAD
Correlation of Price Changes and Income Changes			
1 year	.97 - .99	.095 - .096	.29 - .15
3 year	.96 - .99	.62 - .67	.32 - .18
5 year	.96 - .99	.73 - .77	.31 - .19
Correlation of Price Changes and Construction			
1 year	.70 - .76	-0.051 - 0.032	.31 - .18
3 year	.48 - .59	.29 - .32	.60 - .53
5 year	.52 - .61	.44 - .47	.69 - .65
Correlation of Construction and Income Changes			
1 year	.57 - .64	.81 - .76	.46 - .36
3 year	.46 - .48	.82 - .74	.65 - .56
5 year	.59 - .52	.86 - .80	.80 - .75
Predicted Mean Reversion of Prices			
1 year	-.10 - -.06	.11 - .10	.21 - .14
3 year	-.27 - -.19	-.21 - -.15	.32 - .25
5 year	-.34 - -.28	-.32 - -.25	.04 - .04
Predicted Mean Reversion of Construction			
1 year	.55 - .61	.58 - .63	.85 - .82
3 year	.23 - .31	.23 - .31	.56 - .56
5 year	.00 - .07	.00 - .07	.20 - .23
Predicted Variance Prices, Millions			
1 year	76 - 146	54 - 127	18 - 75
3 year	182 - 382	175 - 413	74 - 285
5 year	245 - 550	249 - 622	150 - 557
Predicted Variance of Construction, Millions			
1 year	17 - 2	54 - 127	18 - 75
3 year	95 - 11	175 - 413	74 - 285
5 year	189 - 22	249 - 622	150 - 557

Note: The range of values shown describes the range of predicted values when c_1 takes on values from 2 to 10 and the ration c_2 over c_1 equals .25

Table 3: Variation in Prices and Quantities Within-Market Over Time
Arellano-Bond Estimates of Coefficients on Lagged Dependent Variable
1, 3, & 5 year horizons

Dependent Variable	1-year changes	3-year changes	5-year changes
House Price Change	0.71 (0.01) N=2,819	0.27 (0.04) N=690	-0.32 (0.07) N=345
Rent Change	0.27 (0.03) N=1,007	0.27 (0.08) N=274	-0.64 (0.17) N=91
New Permits	0.84 (0.01) N=2,645	0.43 (0.04) N=690	-0.07 (0.06) N=460

Notes:
1. Sample for house price, employment, and permit specifications is 115 metropolitan area sample described in text.
2. Sample for rent specification is 46 metropolitan areas tracked by REIS.

Table 4: Predicted Mean Reversion of Prices

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5
$c_1 = .15$	-0.24	-0.27	-0.29	-0.46	-0.45	-0.40	-0.51	-0.47	-0.41
$c_1 = .50$	-0.15	-0.18	-0.18	-0.39	-0.37	-0.30	-0.48	-0.42	-0.34
$c_1 = 2$	-0.08	-0.10	-0.09	-0.28	-0.27	-0.22	-0.39	-0.34	-0.28
$c_1 = 5$	-0.06	-0.07	-0.06	-0.23	-0.22	-0.19	-0.34	-0.30	-0.26
$c_1 = 10$	-0.05	-0.06	-0.05	-0.20	-0.19	-0.18	-0.30	-0.28	-0.25
$c_1 = 20$	-0.04	-0.05	-0.05	-0.18	-0.18	-0.17	-0.28	-0.26	-0.25
$c_1 = 50$	-0.04	-0.05	-0.04	-0.17	-0.17	-0.17	-0.26	-0.25	-0.25

Table 5: Predicted Mean Reversion of Rents

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5
$c_1 = .15$	-0.21	-0.26	-0.32	-0.45	-0.46	-0.47	-0.50	-0.49	-0.48
$c_1 = .50$	-0.08	-0.14	-0.21	-0.36	-0.39	-0.40	-0.46	-0.45	-0.43
$c_1 = 2$	0.03	-0.02	-0.07	-0.24	-0.28	-0.27	-0.36	-0.37	-0.33
$c_1 = 5$	0.06	0.03	0.01	-0.19	-0.22	-0.20	-0.30	-0.31	-0.28
$c_1 = 10$	0.08	0.06	0.05	-0.16	-0.18	-0.17	-0.27	-0.28	-0.26
$c_1 = 20$	0.09	0.08	0.07	-0.15	-0.16	-0.15	-0.25	-0.26	-0.24
$c_1 = 50$	0.09	0.09	0.09	-0.14	-0.14	-0.14	-0.24	-0.24	-0.24

Table 6: Predicted Mean Reversion of Construction

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5	$c_2/c_1 =$ 0	$c_2/c_1 =$ 0.25	$c_2/c_1 =$ 0.5
$c_1 = .15$	0.36	0.28	0.18	0.04	-0.02	-0.08	-0.14	-0.17	-0.20
$c_1 = .50$	0.54	0.43	0.29	0.22	0.10	-0.01	-0.01	-0.10	-0.17
$c_1 = 2$	0.70	0.55	0.36	0.43	0.23	0.04	0.20	-0.00	-0.14
$c_1 = 5$	0.76	0.60	0.39	0.54	0.28	0.06	0.33	0.05	-0.13
$c_1 = 10$	0.80	0.61	0.39	0.60	0.31	0.07	0.42	0.07	-0.12
$c_1 = 20$	0.82	0.62	0.40	0.65	0.32	0.07	0.48	0.08	-0.12
$c_1 = 50$	0.84	0.63	0.40	0.69	0.33	0.07	0.55	0.09	-0.12

	House Price Change Variance (millions of $2000)		
Table 7: Variance in House Price Changes and Construction Intensity 1, 3, and 5 Year Horizons			
	1 year	3 years	5 years
10th percentile market	$14	$69	$183
25th percentile market	$26	$124	$452
50th percentile market	$34	$185	$625
75th percentile market	$70	$445	$1,170
90th percentile market	$209	$1,380	$3,580
Sample mean	$83	$484	$1,310
	Construction Intensity Variance (millions of units)		
	1 year	3 years	5 years
10th percentile market	2	13	29
25th percentile market	2	19	41
50th percentile market	3	26	59
75th percentile market	11	84	212
90th percentile market	38	328	760
Sample mean	21	160	417

Table 8: Predicted Variance of Prices, Millions

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	16	16	18	27	27	28	29	29	32
$c_1 = .50$	28	33	40	59	67	82	70	80	105
$c_1 = 2$	54	76	97	134	182	239	180	245	337
$c_1 = 5$	80	117	140	209	298	365	293	421	528
$c_1 = 10$	102	146	164	272	382	436	391	550	637
$c_1 = 20$	125	167	179	336	446	481	489	650	708
$c_1 = 50$	151	184	190	410	496	513	604	728	756

Table 9: Predicted Variance of Construction, Millions

	One-Year			Three-Year			Five-Year		
	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$	$c_2/c_1 = 0$	$c_2/c_1 = 0.25$	$c_2/c_1 = 0.5$
$c_1 = .15$	117	141	172	538	587	634	946	986	1016
$c_1 = .50$	48	62	80	275	313	337	544	575	569
$c_1 = 2$	13	17	19	87	95	87	191	189	153
$c_1 = 5$	4	5	5	32	30	23	74	62	41
$c_1 = 10$	2	2	1	13	11	7	31	22	13
$c_1 = 20$	1	1	0.4	5	3	2	12	7	4
$c_1 = 50$	0.2	0.1	0.1	1	1	0.3	3	1	1

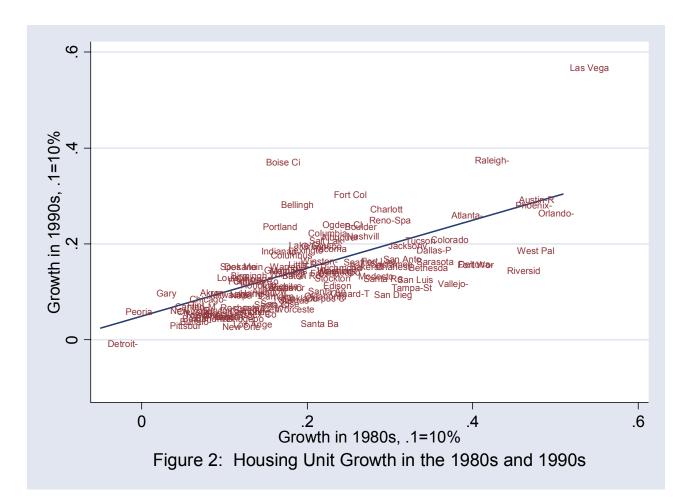| Table 10: The Impact of Greater Local Demand Variability				
	Five-year Price Change Variance ($millions)			
	Baseline σ_ε^2 (=$3.6)	$\sigma_\varepsilon^2 = \$5.7$	$\sigma_\varepsilon^2 = \$7.8$	
$c_1=10$; $c_2=2.5$	550	871	1,192	
$c_1=20$; $c_2=5$	650	1,029	1,408	
$c_1=50$; $c_2=12.5$	728	1,153	1,577	
	Five-Year Quantity Change Variance (millions of units)			
	Baseline σ_ε^2 (=$3.6)	$\sigma_\varepsilon^2 = \$5.7$		
$c_1=0.15$; $c_2=0.0375$	986	1,561		
$c_1=0.50$; $c_2=0.1250$	575	910		
$c_1=2$; $c_2=0.75$	189	299		

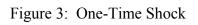| Table 11: Predicted Variance of Price Changes ($millions): Interest Rate Volatility 1, 3, and 5 Year Horizons c_1=3.5, c_2=0.875; all other parameter values as reported in Table 1.									
	One-Year			Three-Year			Five-Year		
$\overline{H}-C$	Stand. Dev. (η) = 0.005	Stand. Dev. (η) = 0.01	Stand. Dev. (η) = 0.02	Stand. Dev. (η) = 0.005	Stand. Dev. (η) = 0.01	Stand. Dev. (η) = 0.02	Stand. Dev. (η) = 0.005	Stand. Dev. (η) = 0.01	Stand. Dev. (η) = 0.02
$25,000	100	102	107	253	256	268	350	355	372
$50,000	102	107	127	256	268	317	355	372	441
$100,000	107	127	206	268	317	511	372	441	714
$200,000	127	206	522	317	511	1288	441	714	1808

Note: All parameter values are as reported in Table 1, expect that c_1=3.5 and c_2=0.875.
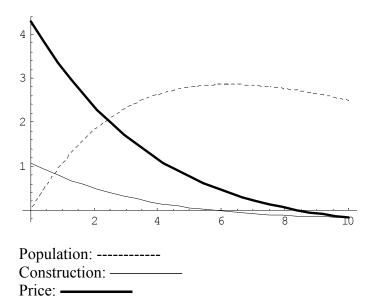
Table 12: Predicted Variance of Construction (millions of units): Interest Rate Volatility 1, 3, and 5 Year Horizons									
	One-Year			Three-Year			Five-Year		
$\overline{H}-C$	Stand. Dev. (η) = 0.005	Stand. Dev. (η) = 0.01	Stand. Dev. (η) = 0.02	Stand. Dev. (η) = 0.005	Stand. Dev. (η) = 0.01	Stand. Dev. (η) = 0.02	Stand. Dev. (η) = 0.005	Stand. Dev. (η) = 0.01	Stand. Dev. (η) = 0.02
$25,000	8	8	9	49	50	53	100	101	107
$50,000	8	9	11	50	53	63	102	107	128
$100,000	9	11	17	53	63	104	107	128	214
$200,000	11	17	44	63	104	270	128	214	558

Note: All parameter values are as reported in Table 1, expect that c_1=3.5 and c_2=0.875.

Figure 1: Real House Price Appreciation in the 1980s and 1990s

Figure 2: Housing Unit Growth in the 1980s and 1990s

Figure 3: One-Time Shock

Population: ------------
Construction: ——————
Price: ━━━━━

$(\alpha = 0.1, c_1 = 3, c_2 = 0.1)$

Appendix 1: Proofs of Propositions

Proof of Proposition 1: We use the change of variables $I(t) = m(t) + \hat{I}(t)$,
$N(t) = n(t) + \hat{N}(t)$, and $H(t) = z(t) + \hat{H}(t)$. Substituting in our definitions of $\hat{I}$, $\hat{N}$, and
$\hat{H}$, we reduce the core pricing equation

$$H(t) = \overline{D} + qt + x(t) - \alpha N(t) + \frac{rC}{1+r} + \frac{E_t(H(t+1))}{1+r} \text{ to}$$

$$z(t) = x(t) - \alpha n(t) + \frac{E_t(z(t+1))}{1+r}, \text{ (*)}$$

the optimality condition for production $C + c_0(t+1) + c_1 I(t+1) + c_2 N(t) = E_t(H(t+1))$
to

$$c_1 m(t+1) + c_2 n(t) = E_t(z(t+1)), \text{ (**)}$$

and the defining equation $I(t+1) = N(t+1) - N(t)$ to

$$m(t+1) = n(t+1) - n(t). \text{ (***)}$$

We seek functions n, z, and m that satisfy the starred equations.

Define $u \equiv \dfrac{\alpha}{c_1}$ and $v \equiv \dfrac{c_2}{c_1}$; $0 \le u$ and $0 \le v < 1$ by the conventions in force. Then

$$\phi = \frac{1}{2}\left(2 + r + (1+r)u - v - \sqrt{r^2 + v^2 + 2(1+r)(2+r)u + (1+r)^2 u^2 + 2v(r - (1+r)u)}\right)$$

and

$$\overline{\phi} = \frac{1}{2}\left(2 + r + (1+r)u - v + \sqrt{r^2 + v^2 + 2(1+r)(2+r)u + (1+r)^2 u^2 + 2v(r - (1+r)u)}\right).$$

Because $0 \le v < 1$, the expression under the radical is positive. Note that
$\phi + \overline{\phi} = 2 + r + (1+r)u - v > (1+r)(1+u) > 0$ and $\phi\overline{\phi} = (1+r)(1-v) > 0$, so $\phi, \overline{\phi} > 0$.
Also, $\sqrt{r^2 + v^2 + 2(1+r)(2+r)u + (1+r)^2 u^2 + 2v(r - (1+r)u)} >$

$$\sqrt{2(1+r)(2+r)u + (1+r)^2 u^2 - 2(1+r)u} > \sqrt{2(1+r)(1+r)u + (1+r)^2 u^2} > 1 + r,$$

so

$$\overline{\phi} > \frac{1}{2}\left(1 + r + (1+r)u + 1 + r\right) \ge 1 + r > 1,$$

which in turn gives

$$0 \le \phi = \frac{(1+r)(1-v)}{\overline{\phi}} < 1 - v \le 1.$$

Now define n by the difference equation

$$n(t) - \phi n(t-1) = \frac{1+r}{c_1(\overline{\phi} - \delta)} E_{t-1}(x(t)). \quad (1)$$

A unique solution n exists because $\overline{\phi} > 1 > \delta$ ensures $\overline{\phi} - \delta \ne 0$ and because $|\phi| < 1$
allows us to solve for n explicitly as

$$n(t) = \frac{1+r}{c_1(\bar{\phi}-\delta)} \sum_{i=0}^{\infty} \phi^i L^i E_{t-1}(x(t)) \quad (2),$$

where L denotes the lag operator. Now that we have defined n, we set

$$z(t) \equiv x(t) + \frac{1}{\bar{\phi}-\delta} E_t(x(t+1)) - \frac{\alpha(1+r)}{1+r-\phi} n(t) \quad (3)$$

and

$$m(t+1) \equiv \frac{1+r}{c_1(\bar{\phi}-\delta)} E_t(x(t+1)) - (1-\phi)n(t). \quad (4)$$

With these choices for z and m, (*) reduces to

$$\frac{\alpha}{1+r-\phi}\big(n(t+1) - \phi n(t)\big) = \frac{\bar{\phi}-1-r}{(\bar{\phi}-\delta)(1+r)} E_t(x(t+1)), \quad (5)$$

which by (1) is equivalent to

$$\frac{u(1+r)}{(1+r-\phi)(\bar{\phi}-\delta)} E_t(x(t+1)) = \frac{\bar{\phi}-1-r}{(\bar{\phi}-\delta)(1+r)} E_t(x(t+1)),$$

which is true, as one sees from cross-multiplying the coefficients and using the previously established formulas for the product and sum of ϕ and $\bar{\phi}$. (**) reduces to

$$\frac{\alpha(1+r)}{1+r-\phi}\left(n(t+1) - \frac{1+r-\phi}{\alpha(1+r)}(c_1(1-\phi)-c_2))n(t) \right) = \frac{\bar{\phi}-1-r}{\bar{\phi}-\delta} E_t(x(t+1)),$$

which is equivalent to (5), and thus true, because

$$\phi = \frac{1+r-\phi}{\alpha(1+r)}(c_1(1-\phi)-c_2)) = \frac{(1+r-\phi)(1-\phi-v)}{u(1+r)},$$

itself evident from cross-multiplying and using the fact that ϕ satisfies the quadratic equation $y^2 - (2+r+(1+r)u-v)y + (1+r)(1-v) = 0$. Finally, (***) reduces to (1). This shows that our choices for n, z, and m solve the starred equations. To recover Proposition 1, we use $I(t) = m(t) + \hat{I}(t)$, $N(t) = n(t) + \hat{N}(t)$, and $H(t) = z(t) + \hat{H}(t)$. [The result then follows from $E_t(x(t+1)) = \delta x(t) + \theta\varepsilon(t)$.]

Proof of Proposition 2: First note that by induction on $i \geq 1$,
$E_t(x(t+i)) = E_t(\delta x(t+i-1) + \theta\varepsilon(t+i-1)) = \delta E_t(x(t+i-1)) = \delta^{i-1} E_t(x(t+1))$, so from (2),

$$E_t(n(t+j)) = \frac{1+r}{c_1(\bar{\phi}-\delta)} E_t \sum_{i=0}^{\infty} \phi^i L^i E_{t+j-1}(x(t+j))$$

$$= \frac{1+r}{c_1(\bar{\phi}-\delta)}\left(\sum_{i=j}^{\infty} \phi^i L^i E_{t+j-1}(x(t+j)) + \sum_{i=0}^{j-1} \phi^i E_t(x(t+j-i)) \right)$$

$$= \frac{1+r}{c_1(\bar{\phi}-\delta)}\left(\phi^j \sum_{i=0}^{\infty} \phi^i L^i E_{t-1}(x(t)) + \sum_{i=0}^{j-1} \phi^i \delta^{j-1-i} E_t(x(t+1)) \right)$$

$$= \phi^j n(t) + \frac{1+r}{c_1(\bar{\phi}-\delta)} \frac{\phi^j - \delta^j}{\phi-\delta} E_t(x(t+1)). \quad (6)$$

Using (4) and (6), we next find that

$$E_t(m(t+j)) = \frac{1+r}{c_1(\bar{\phi}-\delta)} E_t(x(t+j)) - (1-\phi)E_t(n(t+j-1))$$

$$= \frac{1+r}{c_1(\bar{\phi}-\delta)}\left(\delta^{j-1} - (1-\phi)\frac{\phi^{j-1}-\delta^{j-1}}{\phi-\delta}\right)E_t(x(t+1)) - \phi^{j-1}(1-\phi)n(t)$$

$$= \frac{1+r}{c_1(\bar{\phi}-\delta)}\left(\frac{\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)}{\phi-\delta}\right)E_t(x(t+1)) - \phi^{j-1}(1-\phi)n(t). \quad (7)$$

Finally, using (**), (6), and (7), we get

$$E_t(z(t+j)) = c_1 E_t(m(t+j)) + c_2 E_t(n(t+j-1))$$

$$= \frac{1+r}{c_1(\bar{\phi}-\delta)}\left(c_1\frac{\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)}{\phi-\delta} + c_2\frac{\phi^{j-1}-\delta^{j-1}}{\phi-\delta}\right)E_t(x(t+1))$$

$$- \phi^{j-1}(c_1(1-\phi)-c_2)n(t)$$

$$= \frac{1+r}{\bar{\phi}-\delta}\left(\frac{\delta^{j-1}(1-v-\delta)-\phi^{j-1}(1-v-\phi)}{\phi-\delta}\right)E_t(x(t+1)) - c_1\phi^{j-1}(1-v-\phi)n(t). \quad (8)$$

To recover Proposition 2, we use $I(t) = m(t) + \hat{I}(t)$, $N(t) = n(t) + \hat{N}(t)$,
and $H(t) = z(t) + \hat{H}(t)$ with equations (3), (8), (4), (7), and (6).

Proof of Proposition 3: Given the hypotheses, we have
$x(t) = \delta x(t-1) + \theta \varepsilon(t-1) + \varepsilon(t) = \varepsilon(t) > 0$, $E_t x(t+1) = \delta x(t) + \theta \varepsilon(t) = (\delta+\theta)\varepsilon(t) > 0$,
and $n(t) = 0$, so from (3) and (4) we deduce that $z(t) > 0$ and $m(t+1) > 0$: prices and
investment will initially be higher than steady state levels. By assumption, $v = 0$, so by
(7) and (8), each of expected time $t+j$ construction, $E_t(m(t+j))$, and expected time
$t+j$ price, $E_t(z(t+j))$, is negative if and only if

$$\frac{\delta^{j-1}(1-\delta)-\phi^{j-1}(1-\phi)}{\phi-\delta} < 0. \quad (9)$$

If $\phi > \delta$, then (9) holds if and only if

$$\frac{1-\delta}{1-\phi} < \left(\frac{\phi}{\delta}\right)^{j-1},$$

which holds for sufficiently large j because $\phi/\delta > 1$. If $\phi < \delta$, then (9) holds if and
only if

$$\frac{1-\phi}{1-\delta} < \left(\frac{\delta}{\phi}\right)^{j-1},$$

which holds for sufficiently large j because $\delta/\phi > 1$. If $\phi = \delta$, then we reduce (9) to

$$0 < \frac{\phi^j - \delta^j - (\phi^{j-1}-\delta^{j-1})}{\phi-\delta} = \frac{(\phi-\delta)\left(\sum_{i=0}^{j-1}\phi^i\delta^{j-1-i} - \sum_{i=0}^{j-2}\phi^i\delta^{j-2-i}\right)}{\phi-\delta} = \phi^{j-2}(j\phi-j+1),$$

which holds for sufficiently large j because $0 \le \phi < 1$. This shows that there exists j^* such that for all $j > j^*$, time t expected values of time $t+j$ construction and housing prices will lie below steady state levels. When $\varepsilon(t) < 0$, we swap $>$ and $<$ to recover the symmetric case.

Proof of Proposition 4: By assumption, $n(0) = 0$, so from (1), we have

$$n(1) = \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} \varepsilon(0). \quad (10)$$

From (***), $m(1) = n(1)$, and from (4) and (10),

$$m(2) = \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} \big((\delta + \phi - 1)\varepsilon(0) + \varepsilon(1) \big).$$

By definition, $I(t) = \dfrac{q(1+r) - rc_0}{rc_2 + \alpha(1+r)} + m(t)$, so

$$Cov\big(I(2), I(1)\big) = \left(\frac{1+r}{rc_2 + \alpha(1+r)} \right)^2 Var(q) + \left(\frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} \right)^2 (\delta + \phi - 1)Var(\varepsilon),$$

which is positive if and only if

$$\frac{Var(q)}{Var(\varepsilon)} > (1 - \delta - \phi)\left(\frac{\delta(rc_2 + \alpha(1+r))}{c_1(\bar{\phi} - \delta)} \right)^2.$$

From (3) and (10),

$$z(0) = \frac{\bar{\phi}}{\bar{\phi} - \delta} \varepsilon(0) \quad (11)$$

and

$$z(1) = \frac{1}{\bar{\phi} - \delta}\left(\left(\bar{\phi} - \frac{u(1+r)^2}{1+r - \phi} \right)\delta\varepsilon(0) + \bar{\phi}\varepsilon(1) \right). \quad (12)$$

To compute $z(2)$, we use (1) and (10) to get

$$n(2) = \frac{(1+r)\delta}{c_1(\bar{\phi} - \delta)} \big((\phi + \delta)\varepsilon(0) + \varepsilon(1) \big),$$

which yields via (3) that

$$z(2) = \frac{1}{\bar{\phi} - \delta}\left(\left(\bar{\phi}\delta - \frac{u(1+r)^2(\phi + \delta)}{1+r - \phi} \right)\delta\varepsilon(0) + \left(\bar{\phi} - \frac{u(1+r)^2}{1+r - \phi} \right)\delta\varepsilon(1) + \bar{\phi}\varepsilon(2) \right). \quad (13)$$

By definition, $H(t) = \hat{H}(0) + \dfrac{(1+r)(\alpha c_0 + q c_2)}{rc_2 + \alpha(1+r)}t + z(t)$, so from (11), (12), and (13),

$$Cov\big(H(2) - H(1), H(1) - H(0)\big) = \left(\frac{(1+r)c_2}{rc_2 + \alpha(1+r)} \right)^2 Var(q)$$

$$- \frac{\left(\dfrac{u(1+r)^2\delta}{1+r - \phi} + (1 - \delta)\bar{\phi} \right)\left(\dfrac{u(1+r)^2\delta}{1+r - \phi}(1 - \delta - \phi) + (1 - \delta + \delta^2)\bar{\phi} \right)}{(\bar{\phi} - \delta)^2} Var(\varepsilon),$$

52

which is negative if and only if

$$\Omega\left(\frac{rc_2 + \alpha(1+r)}{(1+r)c_1c_2(\bar{\phi} - \delta)}\right)^2 > \frac{Var(q)}{Var(\varepsilon)}.$$

Appendix II: The Contribution of Taxes Local Demand Variance

Data on the average tax rate paid each year in each state was matched to our metropolitan areas using files from the NBER's TaxSim web page. We then multiplied our income numbers by one minus the average tax rate, and calculated new values of δ, θ and σ_ε^2 for this adjusted after-tax income measure.   The new "after-tax" values of the three parameters are very similar to those used in our simulations: δ=0.87, θ=0.18, and σ_ε^2=\$3.3 million. The latter is 92 percent of the \$3.6 million figure obtained without any adjustment for taxes. Hence, correcting for taxes creates an eight percent reduction in the variance and almost no change in the other parameters. Consequently, we conclude that including state level tax rates does not offer any hope of explaining the particularly high volatilities.

Appendix III: The Contribution of Crime to Local Demand Variance

We began by drawing on the hedonic literature on the costs of crime. The range of estimates of the elasticity of property value with respect to the violent crime rate ran from 0.05 to 0.15.[47] To turn these housing price elasticities into estimates of the impact of crime on the flow of utility measured in dollar units, we multiply the elasticity by the average housing price per crime to obtain a relationship between the price of housing and the level of crime. We then followed our model and multiplied this figure by r/(1+r) to generate an estimate of the impact of crime on the flow of utility measured in dollars.

Using this method, our elasticity range from 0.05 to 0.15 implies that the impact of violent crime on the flow of well-being ranges from \$35 to \$105. The upper bound estimate of \$105 dollars implies that, if the violent crime rate in a city increases from 12 violent crimes per 1,000 inhabitants (the national mean) to 24 violent crimes per 1,000 inhabitants, then this is equivalent to an income loss of about \$1,260 dollars, which we believe is a reasonable result.

We then used this upper bound impact to adjust the underlying BEA real income variable and δ, θ and σ_ε^2. As with taxes, crime had little impact on the volatility of the local income shock. Specifically, there is only a 1.4 percent greater shock variance when controlling for crime.[48] While the crime data is far from perfect for our purposes, this exercise leads us to conclude that variation in local amenities will explain little of the high variance price change or construction markets.

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[47] See Thaler (1978) for the lower bound estimate and Schwartz, Susin, and Voicu (2003) for the upper bound number.

[48] We were able to obtain crime data for the major cities of 105 of our 115 metropolitan areas. The ARMA estimates of δ and θ are virtually unchanged depending upon whether income is adjusted for crime in these 105 markets. As noted, the variability of the 'after-crime' income shock is marginally higher.

Appendix IV: Year and Metropolitan Area Fixed Effects Regression Results

Each specification described below was estimated using the data on real house prices (in $2000, created from the OFHEO constant quality price index as described in the text) for the 115 metropolitan areas for which we have continuous annual observations from 1980-2005.

1. Price Levels and Year Fixed Effects (R^2=0.08, nobs=2,990)

$$Price_{i,t} = \alpha + \beta_t*Year_t + \varepsilon_{i,t}$$

where i represents the metropolitan area, t the year, $Year_t$ is a vector of dichotomous year dummies, β_t is the vector of regression coefficients on those year dummies and $\varepsilon_{i,t}$ is the standard error term.

2. Annual Price Changes and Year Fixed Effects (R^2=0.27, nobs=2,875)

$$\Delta Price_{i,t} = \alpha + \beta_t*Year_t + \varepsilon_{i,t}$$

where i represents the metropolitan area, t the year, $Year_t$ is a vector of dichotomous year dummies, β_t is the vector of regression coefficients on those year dummies and $\varepsilon_{i,t}$ is the standard error term.

3. Price Levels and Metropolitan Area Fixed Effects (R^2=0.78, nobs=2,990)

$$Price_{i,t} = \alpha + \gamma_i*MSA_i + \varepsilon_{i,t}$$

where i represents the metropolitan area, t the year, MSA_i is a vector of dichotomous metropolitan area dummies, γi is the vector of regression coefficients on those metropolitan area dummies and $\varepsilon_{i,t}$ is the standard error term.

4. Price Levels with Year and Metropolitan Area Fixed Effects (R^2=0.86, nobs=2,990)

$$Price_{i,t} = \alpha + \beta_t*Year_t + \gamma_i*MSA_i + \varepsilon_{i,t}$$

where i represents the metropolitan area, t the year, $Year_t$ is a vector of dichotomous year dummies, β_t is the vector of regression coefficients on those year dummies, MSA_i is a vector of dichotomous metropolitan area dummies, γi is the vector of regression coefficients on those metropolitan area dummies, and $\varepsilon_{i,t}$ is the standard error term.