# Econ 2450B, Topic 3: Commodity Taxes and Public Goods with Redistributive Concerns

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#### Recap of Topics 1 and 2

Suppose we have a policy that spends more on G targeted towards those earning around \$y of income

#### Need to calculate:

► Individuals WTP out of their own income for additional *G*,  $s(y) = \frac{\frac{\partial u_i}{\partial G}}{\lambda_i} = \frac{u_G}{u_c}$ 

(assume homogenous WTP conditional on income)

- ▶ Total cost to the government inclusive of fiscal externalities
   ▶ 1 + FE<sub>G</sub> = d/dC [q], where q is the aggregate govt budget
- Construct MVPF for each individual with earnings y

$$\textit{MVPF}\left(y\right) = \frac{s\left(y\right)}{1 + \textit{FE}_{\textit{G}}}$$

#### Recap: Aggregation

- Aggregate using either:
- [SWF] Social marginal utilities of income,  $\eta(y) MVPF(y)$
- [Kaldor-Hicks/Kaplow/Mirrlees 1976] Marginal cost of redistributing to those with income y, 1 + FE (y):

$$MVPF^{Tax}(y) = \frac{1}{1 + FE(y)} \leq MVPF(y) = \frac{s(y)}{1 + FE_G}$$

or

$$\underbrace{s\left(y\right)\left(1+\textit{FE}\left(y\right)\right)}_{\text{Taxed Benefits}} \geq^{?} \underbrace{1+\textit{FE}_{G}}_{\text{Net Cost}}$$

# Key Difficulty: Estimating FE...

- Implementing these formulae require estimating two fiscal externalities:
  - Impact of G on tax revenue,  $FE_G$
  - Impact of tax changes to those earning y on tax revenue, FE (y), for all y
- Why are these difficult?
  - Dynamics (impact on tax revenue in 30 years...)
  - Bases (impact of income tax changes on capital taxes, sales taxes, food stamp participation, etc...)
  - And, need rich variation in tax policies to identify FE (y) for all y
    - Made progress in Topic 2 by assuming constant taxable income elasticity/etc.
- This lecture: potentially able to ignore all behavioral responses
  - Literature on optimal commodity taxation and optimal public goods
  - Key (weak?) assumption reduces these empirical requirements: "weak separability"

#### Basic Idea

- Begin with a roadmap of the basic idea
- Many economic models imply a relationship between FE<sub>G</sub> and FE (y)
- ▶ The social benefit of \$1 of spending on *G* is given by:

$$W = \int \left(1 + FE\left(y\right)\right) s\left(y\right) dy$$

• Cost is given by  $1 + FE_G$ 

So, additional spending can increase welfare if and only if

$$\int (1 + FE(y)) s(y) dy \ge 1 + FE_G$$

or

$$\underbrace{\int (s(y) - 1) dy}_{=} \ge \int s(y) FE(y) dy - FE_{G}$$

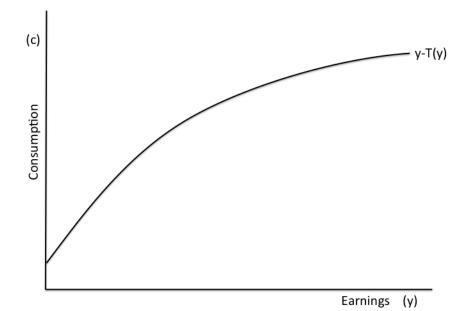
Aggregate Surplus

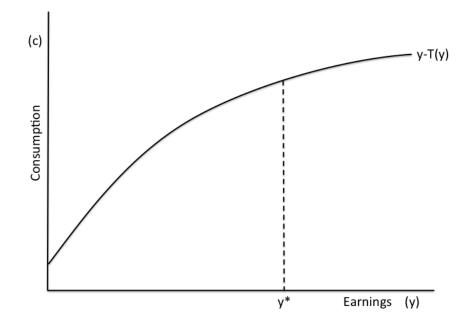
#### Key Insight

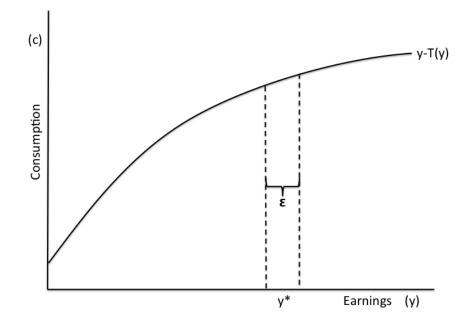
Key insight: In many cases, reasonable to think that

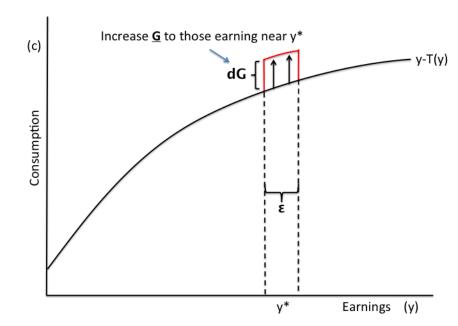
$$\int s(y) FE(y) dy = FE_G$$

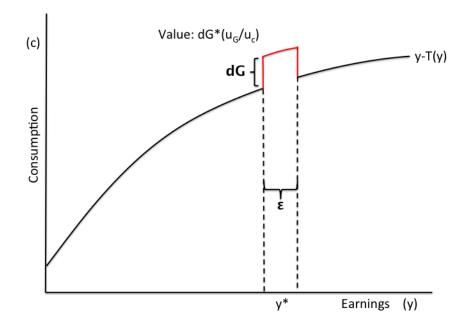


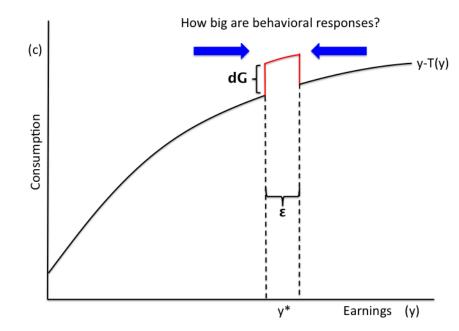


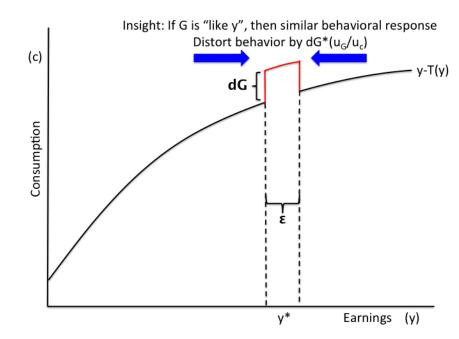


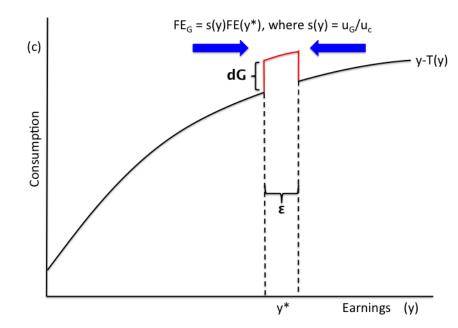












#### Basic Idea

If "G is like y", then ∫ s (y) FE (y) dy = FE<sub>G</sub>, so that additional G can generate a potential Pareto improvement iff aggregate (unweighted!) surplus is positive:

$$\int s\left(y\right) dy > 0$$

Key question: What does it mean for G to be "like y"?

Will formalize as weak separability of utility

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#### This Lecture

 Explore these ideas in two broad context that have been focus of previous literature

- Public goods: Do we subsidize if public good disproportionately helps poor?
  - Follow Kaplow (2006) European Economic Review, "Public Goods and the Distribution of Income"
- Commodities / in-kind subsidies: Do we subsidize if commodity disproportionately consumed by poor?
  - Follow Kaplow (2006) Journal of Public Economics
  - Discuss Ferey, Lockwood, and Taubinsky (2021)
- Discuss other implications/related results
  - Diamond-Mirrlees "Production efficiency" result
  - Zero capital taxation result

Public Goods: Background (Samuelson 1954)

What are Pure public goods?

- Non-rival: My consumption doesn't prevent your consumption
- Non-excludable: Provider can't prevent consumption by those who don't pay

Public Goods benefit several individuals simultaneously
 Lowers effective cost of additional G

- Why might the free market under-provide public goods?
  - Free-riding
  - Public goods create positive externalities, individuals under-provide

#### Optimal Public Goods (Samuelson 1954)

- First Welfare Theorem: Any market equilibrium is Pareto Optimal
  - With public goods, this fails
  - Samuelson (1954) derives condition for a Pareto Optimum
- Consider First Welfare Theorem setup:
  - Individuals indexed by i, two goods, X and G
  - Utility functions U<sup>i</sup>(x<sub>i</sub>, G<sub>i</sub>), standard budget constraint
  - c is the dollar cost of producing G. (Normalize price of x to 1 so  $\frac{p_G}{p_x} = c$ )

Condition for private optimality

$$\bullet \ s_i = \frac{U_G(x_i, G_i)}{U_X(x_i, G_i)} = c \iff s_i = c \ \forall i$$

#### Optimal Public Goods: Failure of FWT

Now, suppose G is a public good

• So each person purchases  $G_i$ , but values  $G = \sum_i G_i$ 

• Utility is 
$$U(x_i, G) = U(x_i, G_i + \sum_{j \neq i} G_j)$$

Condition for private optimality

• Still 
$$\frac{U_G(x_i,G)}{U_x(x_i,G)} = c \iff s_i = c \ \forall i$$

FOC will determine private contribution to public good

But, unweighted social surplus is maximized when

$$\sum_{i} s_{i} = c$$

### Solution: Govt Provision

- Can the government help?
  - Direct provision can avoid the free-rider problem
- ▶ What is the optimal level of public provision of *G*?
  - Samuelson (1954): Pareto efficiency requires maximizing surplus:

$$\sum_i s_i = c$$

- How can we decentralize this?
  - ▶ If  $\sum MRS_i = c$ , then government can find transfers,  $t_i$ , and a change in g to make everyone better off

 $\blacktriangleright \text{ Set } t_i = MRS_i$ 

- But, if we have individual specific lump-sum transfers, what does this say about the social marginal utility of income for rich and poor?
  - Should be equalized!

#### **Optimal Public Goods**

But, we transfer based on observed income
 Implies transfers are distortionary!

What does this mean for optimal public goods? Can still consider taxing back the benefits to each individual i:

$$\int s_i \left(1 + FE\left(y_i\right)\right) di \ge^? 1 + FE_G$$

But, now we need to estimate FE(y) and  $FE_G$ !

Can we do something simpler?

# Kaplow (2006, Euro Econ Review)

- Utility is a function of:
  - A (private) consumption good, *c*
  - The level of government expenditure on a publicly provided good, g (same as "G" in previous lectures)
  - Labor supply /
- Utility satisfies weak separability: there exists a function v (common to all individuals) such that utility is given by

$$u\left(v\left(c,g
ight),I
ight)$$

- Individuals differ in their wage, w
- Consumption given by budget constraint

$$c = wl - T(wl,g)$$

where T(wl,g) is the tax/transfers to individuals with earnings wl

Cannot transfer based on (unobserved) wage, w

Kaplow (2006, Euro Econ Review)

Individuals maximize

$$U\left( \mathsf{g}, T\left( \circ 
ight), \mathsf{w} 
ight) = \mathsf{max}_{\mathsf{l}} \; u\left( \mathsf{v}\left( \mathsf{w}\mathsf{l} - T\left( \mathsf{w}\mathsf{l}, \mathsf{g} 
ight), \mathsf{g} 
ight), \mathsf{l} 
ight)$$

Social welfare given by

$$SW = \int W \left( U \left( g, T \left( \circ \right), w \right) \right) f \left( w \right) dw$$

Government revenue given by

$$R = \int T(wl(w),g)f(w) dw$$

where I(w) is the labor supply choice of type w

Social objective: Choose g to maximize SW subject to R = g

#### Kaplow (2006): Benefit Absorbing Tax

- ▶ What is the optimal level of g?
- Consider a policy that increases g by a small amount
- Define a "benefit-absorbing tax" (analogous to last lecture...)
  - Change T such that utility does not change when both g and T are simultaneously changed
  - Will solve implicitly for what the change in the tax schedule must be
- The total derivative from the policy is given by:

$$\frac{\partial U}{\partial g} = \frac{\partial u}{\partial v} \left[ v_c \frac{-\partial T}{\partial g} + v_g \right]$$

• 
$$v_c = \frac{\partial v}{\partial c}$$
 and  $v_g = \frac{\partial v}{\partial g}$ 

• We assume that the change in g and increase in T is defined such that  $\frac{\partial U}{\partial g} = 0$ 

#### Kaplow (2006): Benefit Absorbing Tax

• What must the tax adjustment look like to set  $\frac{dU}{dg} = 0$ ?

- i.e. how do we change T in response to the increase in g to hold utility constant for everyone?
- For each level of labor earnings, wl, define the marginal change in the tax schedule by

$$\frac{\partial T\left(wl,g\right)}{\partial g} = \frac{v_g}{v_c}$$

Note that this is the individual's WTP for g in units of g.

We "tax back the benefits"

▶ Notice that if we substitute  $\frac{\partial T(wl,g)}{\partial g} = \frac{v_g}{v_c}$  into  $\frac{\partial U}{\partial g} = \frac{\partial U}{\partial v} [v_c c_g + v_g]$  we obtain  $\frac{\partial U}{\partial g} = 0$  for each type w!

#### Kaplow (2006): Benefit Absorbing Tax

Cost depends on the impact on labor supply choices, I

This is where weak separability helps

Define v (I) = v (wI - T (wI, g), g) to be the level of v (c, g) experienced by type w if she chooses I

Labor supply / maximizes

$$I(w) = argmax \ u(v(I), I)$$

Kaplow: Notice that when the policy changes, v (1) is unaffected by the policy change!

$$\frac{dv}{dg}\left(l\right) = v_c \frac{\partial T}{\partial g} + v_g = 0 \quad \forall l$$

- Therefore solution to argmax u (v (I), I) is not affected by the policy change
  - Graphically: Blue arrows for tax adjustment perfectly offset blue arrows from change in g
  - Exercise: Verify this by solving for l(w, g) and showing that  $\frac{\partial l}{\partial g} = 0$  for all w in this policy change.

# Kaplow (2006): Aggregate Surplus

- What is the optimal level of public expenditure on g?
- Dual: Maximize government revenue subject to utility held constant

$$\frac{dR}{dg} = \underbrace{\int \frac{dT(wl,g)}{dg} f(w) dw}_{\text{Denses for Barefs Tarefs}}$$

Revenue from Benefit-Tax

- ▶ But, note that  $\frac{dT(wl,g)}{dg} = \frac{v_g}{v_c} = \frac{u_v}{u_v} \frac{v_g}{v_c} = \frac{\frac{dU}{dg}}{\frac{dU}{dc}} = s(y)$  is each type's willingness to pay (y = wl)
- Re-writing in notation from last class, optimal to increase g whenever aggregate (unweighted!) surplus is positive

$$\int s\left(y\right) dy \geq 1$$

#### Role of Weak Separability

Ensures behavioral response to g is similar to behavioral response for tax cut:

$$FE_{G} = \int s(y) FE(y) dy$$

- Why might weak separability be violated?
- Suppose g is:
  - Job training
  - Medical care
  - Education
  - Food stamps

#### **Commodity Taxation**

- What about commodity taxes? Or taxes on other goods?
  - Subsidize food vs. expensive cars?
- Key papers: Atkinson and Stiglitz (1976) JPubEc and Hylland and Zeckhauser (1979, Scandinavian Journal of Economics)
  - Follow Kaplow (2006, JPubEc) for a nice proof

# Kaplow (2006)

- Setup: individuals indexed by h
- lndividuals choose commodities  $\{c_1, c_2...\}$  and labor effort, I
- Maximize utility function

$$u_h(c_1, c_2, ..., l) = \tilde{u}_h(v(c_1, ...), l)$$

Key assumption: g is the same across people (but ũ<sub>h</sub> can be heterogeneous)

Subject to budget constraint

$$\sum \left( p_{i} + \tau_{i} \right) c_{i} \leq wl - T \left( wl \right)$$

where w is an individual's wage (heterogeneous in population)
wl is earnings and T (wl) is the (nonlinear) tax on earnings

#### Statement

Suppose there is a commodity tax

$$rac{p_i+ au_i}{p_j+ au_j} 
eq rac{p_i}{p_j}$$

for some i and j

Can welfare be improved by re-setting τ<sub>i</sub> = τ<sub>j</sub> = 0 and suitably augmenting the tax schedule T?

Atkinson-Stiglitz/Kaplow: YES.

• Define  $V(\tau, T, wl)$  to be

$$V(\tau, T, wl) = max \ v(c_1, c_2, ...)$$

s.t. 
$$\sum (p_i + \tau_i) c_i \leq wl - T(wl)$$

- V is the value of the consumption argument of the utility function – holds independent of labor effort /!
  - Consumption allocations don't reveal any information about labor supply type w conditional on wl.

# Proof

Define intermediate environment:

- Start with commodity taxes au
- Define new taxes at zero  $\tau_i^* = 0$
- Augment the tax schedule
  - Define T\* to offset the impact on utility so that utility is held constant in this intermediate world
- Specifically, T\* satisfies

$$V(\tau, T, wl) = V(\tau^*, T^*, wl)$$

for all wl

# Proof (Cont'd)

Lemma 1: Every type w chooses the same level of labor effort under τ\*, T\* as under τ, T.

Proof:

Note that

$$U(\tau, T, w, l) = u(V(\tau, T, wl), l)$$
  
=  $u(V(\tau^*, T^*, wl), l)$   
=  $U(\tau^*, T^*, w, l)$ 

- The utility function (as a function of *I*) is the same in both environments
- Therefore, the *I* that maximizes utility in the original world maximizes utility in the intermediate world

# Proof Cont'd

- Lemma 2: The augmented world raises more revenue than the original world
- Proof:
  - Will show that no individual in the intermediate regime can afford the original consumption vector
    - Implies they pay more taxes in intermediate regime
- Suppose type w can afford original vector when there is no commodity tax, τ<sup>\*</sup><sub>i</sub> = 0.
  - Then she strictly prefers a different vector because of change in relative price
    - Utility level hasn't changed, but relative prices have
  - But this would imply intermediate environment is strictly better off
    - Choosing a better bundle than the old bundle would strictly increase utility
  - Contradicts definition of intermediate environment holding utilities constant
    - Therefore, type *w* cannot afford the original bundle

#### Proof Cont'd

- Next: If type w cannot afford original bundle, then aggregate tax revenue must be higher in the intermediate environment
- Because the original bundle is unaffordable, we have:

$$\sum (p_i) c_i > wl - T^* (wl)$$

for all wl (note  $au^* = 0$ )

Budget constraint in initial regime implies

$$\sum_{i} (p_i + \tau_i) c_i = wl - T (wl)$$

so that

$$\sum_{i} p_{i} c_{i} = -\sum_{i} \tau_{i} c_{i} + wl - T(wl)$$

So that

$$-\sum_{i}\tau_{i}c_{i}+wl-T(wl)>wl-T^{*}(wl)$$



 $T^{*}(wl) > \sum \tau_{i}c_{i} + T(wl)$ 

#### Proof Cont'd

- So, the intermediate world generates more tax revenue and holds utility constant
- Why does this mean one can have a Pareto improvement from no commodity tax?
- Generate a third world that gives e benefits to everyone through lowering the tax schedule
  - Implies everyone better off.

# Implications of Atkinson Stiglitz

- Result generally known as the "Atkinson-Stiglitz" theorem
  - Arguably first shown by Hylland and Zeckhauser (1979)
- Incredibly powerful theorem
- Nests many other results:
  - Zero capital taxes in the standard model
  - "Production efficiency" theorem of Diamond and Mirrlees (1971)

# Capital Taxes

- Should we have a tax on capital?
  - Capital owners are rich, doesn't this mean we should tax them if we have redistributive preferences?
- Suppose

$$U(c_{1}, c_{2}, ..., l) = u(c_{1}) - v(l_{1}) + \beta [u(c_{2}) - v(l_{2})] + ...$$

With budget constraint

$$\sum_{i} (p_i + \tau_i) c_i \leq \sum_{i} w_i l_i$$

So

$$g(c_1, c_2, ...) = u(c_1) + \beta u(c_2) + ...$$

- Implies no distortion in relative price of c<sub>1</sub> and c<sub>2</sub>
  - You should prove extension to case with I<sub>i</sub> instead of just I.
- What if more productive types have higher preferences for bequests?

### **Production Efficiency**

- Should we let firms deduct the price of inputs
   E.g. firms don't pay sales tax on their inputs?
- ▶ Diamond and Mirrlees (1971) show a surprising result:
- Suppose C is produced with a bunch of intermediate inputs, x<sub>i</sub>

$$C = f(x_1, ..., x_n)$$

Question: would you ever want to tax these inputs?
Answer: No if C is all people care about

$$u(x, I) = U(C(x), I)$$

- ► The production function for *C* is the same for all people
  - Weak separability holds
  - Implies no taxes on intermediate inputs

When does weak separability fail?

When does this fail?

Is labor supply an "intermediate input"
 No taxes on earnings!?

What if we can't tax profits of an intermediate producer?

# Relation to Mirrlees

- Another way of seeing this: Mirrlees information logic:
  - When commodity choices have desirable information about type conditional on earnings?
    - See Mirrlees (1976, JPubEc)
- What constitutes "desirable information"? (Saez 2002 JPubEc)
  - Information about social welfare weights: Society likes people that consume x<sub>1</sub> more than x<sub>2</sub> conditional on earnings
    - Implement subsidy on good x<sub>1</sub> financed by tax on x<sub>2</sub>
    - First order welfare gain (b/c of difference in social welfare weights)
    - Second order distortionary cost starting at  $\tau = 0$
  - Information about latent productivity: More productive types like x<sub>1</sub> more than x<sub>2</sub> conditional on earnings
    - e.g.  $x_1$  is books;  $x_2$  is surf boards
    - Then, tax the goods rich people like but reduce the marginal tax rate
    - Leads to increase in earnings!
    - Depends on covariance

### Key Lessons

- In general, need to estimate fiscal externalities associated with policy changes
- But, if willing to assume weak separability of utility, can just assume that the FE is the same as an income tax
- Motivates only needing to calculate whether the aggregate surplus is positive
  - Are people WTP for the policy change out of their own income?

Two empirical literatures on Public Goods



Two empirical literatures on public goods:

Measuring willingness to pay 

Measuring private crowd-out of government provision

# Measuring WTP



- Infer based on behavior / prices
- Ask people (Contingent valuation)

# Value of Clean Water

More recently, Keiser and Shapiro (2018, QJE): "Consequences of the Clean Water Act and the Demand for Water Quality"

Cost-benefit analysis of the Clean Water Act

#### Three analyses

- Estimate water pollution from 1962-2001
- Estimate impact of clean water act grants to wastewater treatment plants on pollution
- Estimate WTP for clean water grants from house prices within 25 mi of plants

# Keiser and Shapiro (2017)

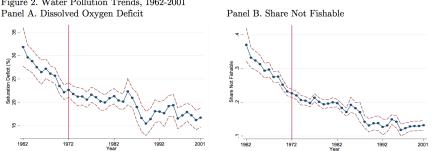


Figure 2. Water Pollution Trends, 1962-2001

Notes: Graphs show year fixed effects plus a constant from regressions which also control for monitoring site fixed effects, a day-of-year cubic polynomial, and an hour-of-day cubic polynomial, corresponding to equation (2) from the text. Connected dots show yearly values, dashed lines show 95% confidence interval, and 1962 is reference category. Standard errors are clustered by watershed.

# Keiser and Shapiro (2018)

$$Q_{pdy} = \sum_{\tau = -10}^{\tau = 25} \gamma_{\tau} \mathbb{1}[G_{p,y+\tau} = 1]d_d + X'_{pdy}\beta + \eta_{pd} + \eta_{py} + \eta_{dwy} + \epsilon_{pdy}$$

#### Event-study design:

- Two observations for each treatment plant: one upstream and one downstream
  - G<sub>p,y+τ</sub> indicator for grant received in year y + τ, where τ indexes years since grant received
  - *d<sub>d</sub>* is an indicator for being downstream from the treatment facility
  - X<sub>pdy</sub> are controls for temperature and precipitation
  - ▶ plant-downstream fixed effects,  $\eta_{pd}$  allow for different mean levels up and down-stream
  - ► plant-year fixed effects,  $\eta_{PY}$ , control for forces like growth of local industry/etc that affect water quality
  - downstream-by-basin-by-year,  $\eta_{dwy}$ , allow upstream and downstream water quality to differ by year in ways common to all plants in a river basin

# Keiser and Shapiro (2018)

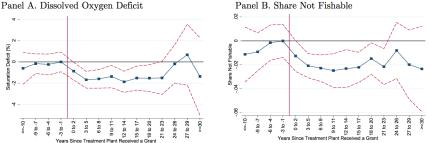


Figure 3. Effects of Clean Water Act Grants on Water Pollution: Event Study Graphs Papel A Dissolved Oxygen Deficit Papel B Share Not Fishable

Notes: Graphs show coefficients on downstream times year-since-grant indicators from regressions which correspond to the specification of Table 3. These regressions are described in equation (5) from the main text. Data cover years 1962-2001. Connected dots show yearly values, dashed lines show 95% confidence interval. Standard errors are clustered by watershed.

# Keiser and Shapiro (2018)

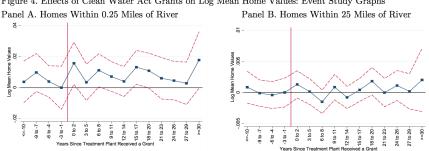


Figure 4. Effects of Clean Water Act Grants on Log Mean Home Values: Event Study Graphs

Notes: Graphs show coefficients on year-since-grant indicators from regressions corresponding to the specification of Table 6, column (3). Connected dots show yearly values, dashed lines show 95% confidence interval. Standard errors are clustered by watershed. Panels A and B show different ranges of values on their y-axes. Data cover decennial census years 1970-2000.

# Value of Clean Water

- Conclusion: Impact on house prices in 25 mile radius is < 1/3 of the costs
- Does this reflect the total WTP for the clean water act?
- What about distributional incidence?

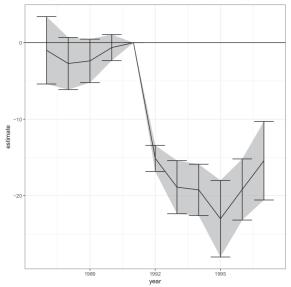
# Value of Clean Air

 Clean Air Act enacted in 1963; 1970 amendment established national ambient air quality standards (NAAQS)

- Specifies minimum level of air quality for six pollutants
- Some counties get affected, others are OK
- Leads to difference in difference design
- Chay and Greenstone (2003) look at impacts on infant mortality
- Isen, Rossin-Slater, and Walker (JPE 2017) use the impact of the Clean Air Act to generate variation in childhood exposure to pollution and study its impact on adult outcomes

### Isen, Rossin-Slater, and Walker (JPE 2017)

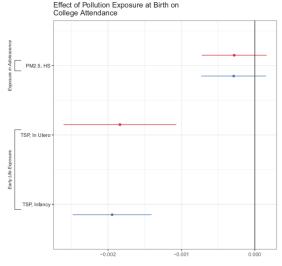
Figure 2: Test of Parallel Trends Assumption, TSP Exposure, Younger Cohort



Source: 2000–2014 IRS 1040, 2005 through 2015 ACS and EPA monitor data Note: This figure shows the year-specific effects of NO<sub>2</sub> Nonattainment designations on TSP and Ozone exposure in utero and during infancy (each panel represents a different model). Confidence intervals that

### Isen, Rossin-Slater, and Walker (JPE 2017)

Figure 4: Graphical Summary of IV Results: College Attendance



Source: 2000-2014 IRS 1040s, 2005-2015 ACS, EPA and ACAG satellite data

Note: This figure shows the effect of pollution exposure at adolescence on college attendance (top row) and the IV estimates of the effect of pollution exposure at birth on college attendance (bottom rows) from Tables 2–13. The point estimates can be interpreted as the effect of a 1 unit  $(\mu_d)m^3$  for TSP and PM2.5

# Value of Clean Air

- How would you measure the WTP for clean air?
- Brookshire et al. (1982)
  - Infer willingness to pay for clean air using effect of pollution on property prices (capitalization)
- Let *P<sub>i</sub>* denote house price of house *i*, regress

 $P_i = \alpha + \beta Pollution_i + \gamma X_i + \epsilon_i$ 

for range of controls,  $X_i$ .

Concerns?

- Chay and Greenstone (2005) look at county-level housing prices using non-attainment status as IV
  - "Improvements in air quality induced by the mid-1970s TSPs nonattainment designation are associated with a \$45 billion aggregate increase in housing values in nonattainment counties between 1970 and 1980."

### APPENDIX: Other things you should know...

### Optimal Commodity Taxation in Ramsey (1927)

- Assume away lump-sum taxation and then try to replicate it with commodity taxes
- Leads to very misleading results because we exogenously restrict the policy space
- "Inverse Elasticity" rule

# Optimal Taxation in Ramsey (1927)

Ramsey (1927): How should commodities be taxed to raise revenue, R > 0.

Modeled by Diamond and Mirrlees (1971)

- Key result: Tax-weighted Hicksian price derivatives are equated across goods
  - "Inverse elasticity rule": tax goods with smaller compensated behavioral responses

# Setup

- Representative Agent (drop *i* subscripts).
- Commodities,  $x_k$ , indexed by k
- Government imposes taxes on commodities,  $\tau_k$ .
- Necessary condition for optimality

$$rac{d\,\hat{V}_P}{d heta}|_{ heta=0}=0$$

for all feasible policy paths P.

- Optimal tax would be lump-sum of size R
  - Assumed to not exist

# Commodity Tax Variation

- Consider policy P (θ) that changes commodity taxes (e.g. lowers tax on good 1 and raises tax on good 2)
- Budget neutral:  $\frac{d\hat{t}}{d\theta} = 0$
- No change in public goods
- So, optimality condition only involves behavioral response:

$$\sum_{k} \hat{\tau}_k \frac{d\hat{x}_k}{d\theta}|_{\theta=0} = 0$$

# Hicksian Elasticity

Diamond and Mirrlees (1971): At the optimum, expand the behavioral response using the Hicksian demands, x<sup>h</sup><sub>k</sub>,

$$\frac{dx_k}{d\theta} = \frac{\partial x_k^h}{\partial \tau_1} \frac{d\tau_1}{d\theta} + \frac{\partial x_k^h}{\partial \tau_2} \frac{d\tau_2}{d\theta}$$

Additional term, 
 <sup>∂x<sup>k</sup><sub>k</sub></sup> <sup>dV<sub>p</sub></sup>/<sub>∂u</sub>, but this vanishes at the optimum.

 Optimality condition is given by

$$\sum_{k} \tau_{k} \frac{\partial x_{k}^{h}}{\partial \tau_{1}} \frac{d\tau_{1}}{d\theta} = \sum_{k} \tau_{k} \frac{\partial x_{k}^{h}}{\partial \tau_{2}} \left( -\frac{d\tau_{2}}{d\theta} \right)$$

- Tax-weighted Hicksian responses are equated across the tax rates
  - Inverse elasticity rule
- What are the needed elasticities?

### Inverse Elasticity Rule

Assume cross elasticities are zero:

$$BC = x_1 \frac{d\tau_1}{d\theta} + \tau_1 \frac{dx_1}{d\theta} + x_2 \frac{d\tau_2}{d\theta} + \tau_2 \frac{dx_2}{d\theta} = 0$$

so

$$x_1\left(1+\frac{\tau_1}{x_1}\frac{\partial x_1^h}{\partial \tau_1}\right)\frac{d\tau_1}{d\theta} = x_2\left(1+\frac{\tau_2}{x_2}\frac{\partial x_2^h}{\partial \tau_2}\right)\left(-\frac{d\tau_2}{d\theta}\right)$$

And optimality implies

$$x_1\left(\frac{\tau_1}{x_1}\frac{\partial x_1^h}{\partial \tau_1}\right)\frac{d\tau_1}{d\theta} = x_2\left(\frac{\tau_2}{x_2}\frac{\partial x_2^h}{\partial \tau_2}\right)\left(-\frac{d\tau_2}{d\theta}\right)$$

### Inverse Elasticity Rule

$$\left(\frac{\tau_1}{x_1}\frac{\partial x_1^h}{\partial \tau_1}\right) = \left(\frac{\tau_2}{x_2}\frac{\partial x_2^h}{\partial \tau_2}\right) = \kappa$$

▶ Translating to price (1+tau) instead of tax (tau) elasticities:

$$\frac{\tau_j}{1+\tau_j}\epsilon^h_{j,(1+\tau_j)} = \kappa$$

Or

So

$$\frac{\tau_j}{1+\tau_j} = \frac{\kappa}{\epsilon^h_{j,(1+\tau_j)}}$$

which is the "inverse elasticity rule".

Key (but Misleading) Result: Inverse Elasticity Rule

- Main result of Ramsey model: Inverse elasticity rule
- Key Assumptions:
  - Representative agent
  - No lump sum taxation

### **Optimal Taxation of Production**

- Diamond and Mirrlees (1971) also consider the issue of production efficiency.
- Commodities, x<sub>k</sub>, indexed by k, transformed into one another (produced) by firms and government
- Producer prices  $p_k$ , Consumer prices  $q_k$

• Tax is wedge  $\tau_k = q_k - p_k$ 

- Consumer *i* solves max  $u_i(\mathbf{x})$  s.t.  $\sum q_k x_k \leq 0$ 
  - Defines consumer (final) demand for each commodity  $x_k^i(\mathbf{q})$
  - and indirect utility  $V_i(\mathbf{q}) = u(\mathbf{x}^i(\mathbf{q}))$
- Note: Consumers are the ones endowed with the initial commodity supply
- Endowments allow them to exchange, consumers are on budget constraint

# Firm side

- Price-taking firms j transform commodities
- Production possibilites represented by input output function f<sup>j</sup>(y) = 0
  - for example,  $y_1 = y_2^{\cdot 3} * y_3^{\cdot 7} \iff y_1 (-y_2^{\cdot 3}) * (-y_3^{\cdot 7}) = 0$
  - Can turn y<sub>2</sub> and y<sub>3</sub> into y<sub>1</sub> (or vice versa, depending of domain)
  - Negative arguments are inputs, positives are outputs

# Firm side: CRS Production

Assumption: constant returns to scale

- Then each firm can produce "as much" or "as little" as desired in fixed proportions
  - Together, many CRS firms define an aggregate production function f(y) = 0
  - No profits for any firm (otherwise infinite production) in equilibrium
  - ▶  $\mathbf{p} \cdot \mathbf{y}^{j} = 0$  must hold in equilibrium, and thus  $\mathbf{p} \cdot \mathbf{y} = \mathbf{p} \cdot (\sum \mathbf{y}_{j}) = 0$
- Under CRS, behavior of many optimizing firms same as one aggregate firm

# Firm side: Firm Objective

Objective: Choose point on frontier to maximize output prices
 input prices

$$\max \mathbf{p} \cdot \mathbf{y} \text{ s.t. } f(y) = 0$$

• Optimality condition: 
$$\frac{\partial f}{\partial y_k} = p_k \iff MRT = \frac{\frac{\partial f}{\partial y_k}}{\frac{\partial f}{\partial y_{k'}}} = \frac{p_k}{p'_k}$$

Why can we ignore lagrange multiplier on f (y) = 0 condition? Because we can normalize the units of f to be in terms of one of the commodities...see Diamond-Mirrlees (1971).

### Govt

- D&M think of Gov't as a planner with a distributive objective but:
  - Can't just pick point on PPF
  - Must deal with consumers through market place using uniform prices
  - Uses:
    - a.) linear commodity taxes to set prices and
    - b.) public production to adjust quantities above and beyond what private sector does given prices
- ▶ Public production follows PPF given by  $g(z) \le 0$

# Objective

- What is the objective here?
  - redistribution-different than Ramsey, since no revenue requirement
- Why would commodity taxes help with no lump sum transfers?
  - differential wealth levels are due to endowment differences
  - Commodity taxes target:
    - Different tastes
    - Value of endowment
  - But commodity taxes cause DWL

Objective

#### Solve

$$\max_{q,p,z}\sum_{i}W(V_i(\mathbf{q})) \text{ s.t. } \sum_{i}x_k^i(\mathbf{q})=y_k(\mathbf{p})+z_k \text{ , } f(\mathbf{y})=\mathbf{0} \text{, and } g(\mathbf{z})$$

Lagrangian

$$\max_{q,p,z} \sum_{i} W(V_{i}(\mathbf{q})) + \sum_{k} \lambda_{k}(y_{k}(\mathbf{p}) + z_{k} - \sum_{i} x_{k}^{i}(\mathbf{q})) + \gamma^{f} f(\mathbf{y}(\mathbf{p})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + z_{k} - \sum_{i} x_{k}^{i}(\mathbf{q})) + \gamma^{f} f(\mathbf{y}(\mathbf{p})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + z_{k} - \sum_{i} x_{k}^{i}(\mathbf{q})) + \gamma^{f} f(\mathbf{y}(\mathbf{p})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + z_{k} - \sum_{i} x_{k}^{i}(\mathbf{q})) + \gamma^{f} f(\mathbf{y}(\mathbf{p})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + z_{k} - \sum_{i} x_{k}^{i}(\mathbf{q})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + z_{k} - \sum_{i} x_{k}^{i}(\mathbf{q})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + y_{k} - \sum_{i} x_{k}^{i}(\mathbf{p})) + \gamma^{g} \lambda_{k}(y_{k}(\mathbf{p}) + \sum_{i} x_{k}^{i$$

# Objective

Production-side and consumer-side variables are additively separable

$$\max_{\boldsymbol{q},\boldsymbol{p},\boldsymbol{z}} \underbrace{\sum_{i} W(V_{i}(\boldsymbol{q})) - \sum_{k} \lambda_{k} \sum_{i} x_{k}^{i}(\boldsymbol{q})}_{consumption} + \underbrace{\sum_{k} \lambda_{k}(y_{k}(\boldsymbol{p}) + z_{k}) + \gamma^{f} f(\boldsymbol{y}(\boldsymbol{p})) + \gamma^{g} g(\boldsymbol{z})}_{production}$$

- Note that FOC for producer prices and government production depend on W only through the shadow value of an endowment unit of k.
- Also, choice of p *directly* implements y, so we can choose y directly

$$\max_{q,y,z} \underbrace{\sum_{i} W(V_{i}(\mathbf{q})) - \sum_{k} \lambda_{k} \sum_{i} x_{k}^{i}(\mathbf{q})}_{consumption} + \underbrace{\sum_{k} \lambda_{k}(y_{k} + z_{k}) + \gamma^{f} f(\mathbf{y}) + \gamma^{g} g(\mathbf{z})}_{production}$$

# Result

$$\blacktriangleright [FOC y_k]\lambda_k = \gamma^f \frac{\partial f}{\partial y_k}$$

$$\blacktriangleright [FOC g_k]\lambda_k = \gamma^g \frac{\partial g}{\partial z_k}$$

Taking ratio, for any social welfare objective, it must be the case that:

$$\frac{\frac{\partial g}{\partial z_k}}{\frac{\partial g}{\partial z_{k'}}} = \frac{\frac{\partial f}{\partial y_k}}{\frac{\partial f}{\partial y_{k'}}} = \frac{p_k}{p_{k'}}$$

The government's decision to intervene in the economy is independent of the objective. MRTs are always equalized, and the only wedge is between consumer and producer prices. Production-side and consumer-side variables are additively separable