

Social Choice with Non Quasi-linear Utilities

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We study social choice mechanisms that aggregate individual preferences and select one among a finite set of alternatives. Our interest is in the existence of strategy-proof mechanisms under non quasi-linear utility functions. In the classical voting problem without money, the seminal Gibbard-Satterthwaite theorem states that if agents' preferences can be any ordering over the alternatives, the only deterministic, onto (i.e. every alternative can be selected) and strategy-proof mechanisms for three or more alternatives are dictatorial. On the other hand, with monetary transfers and quasi-linear utilities, the Vickrey-Clarke-Groves (VCG) mechanism maximizes social welfare in dominant strategies, and can be generalized to implement any affine maximizer of agents' values. By Roberts' theorem, with three or more alternatives, the weighted VCG mechanisms are essentially unique for the quasi-linear domain. However, quasi-linearity is a strong assumption, violated for example in domains with budget constraints, lack of liquidity, wealth effect or risk-aversion.

The main result of this paper is a tight characterization of the maximal utility domain, which we name the *largest parallel domain*, where there exist non-dictatorial mechanisms that are strategy-proof, onto, deterministic, individually rational (IR) and satisfy *no subsidy* (i.e. no positive transfers from the mechanism to the agents). These properties are those of the VCG mechanism in quasi-linear utility domains. As a special case, for utility domains that contain all quasi-linear types but do not reside within the largest parallel domain, the only mechanisms with the above properties are dictatorial. We also provide a negative result for a broader class of mechanisms: by allowing richer utility domains that still differ very slightly from quasi-linearity, we establish the impossibility of non-dictatorial mechanisms even without requiring IR or no subsidy.

A key observation is that the critical property of agents' utilities that enables non-dictatorial mechanisms is not the linear dependency on payments. We say the utility functions of an agent is of *parallel type* if for any two alternatives a and b , within the range of interest, no matter how much the agent is charged for b , to achieve the same utility, the additional amount she is willing to pay for a stays the same. Quasi-linear utility functions have this property, but there can also be non quasi-linear parallel types. Intuitively, a type being parallel requires that regardless of which alternative is selected, the agent's marginal cost for money is the same as long as she has the same utility level— the trade-off with money depends on how happy the agent is, not how much she is paying. A domain where all types are parallel is called a *parallel domain*, and the largest parallel domain is the set of all parallel types. Within the parallel domains, we prove a positive result, that the generalized weighted VCG mechanisms truthfully implement any affine maximizer of agents' willingness to pay. We then generalize Roberts' Theorem to parallel domains with unrestricted willingness to pay, with which we prove our main characterization and impossibility results.

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