12/18/2014

Final exam

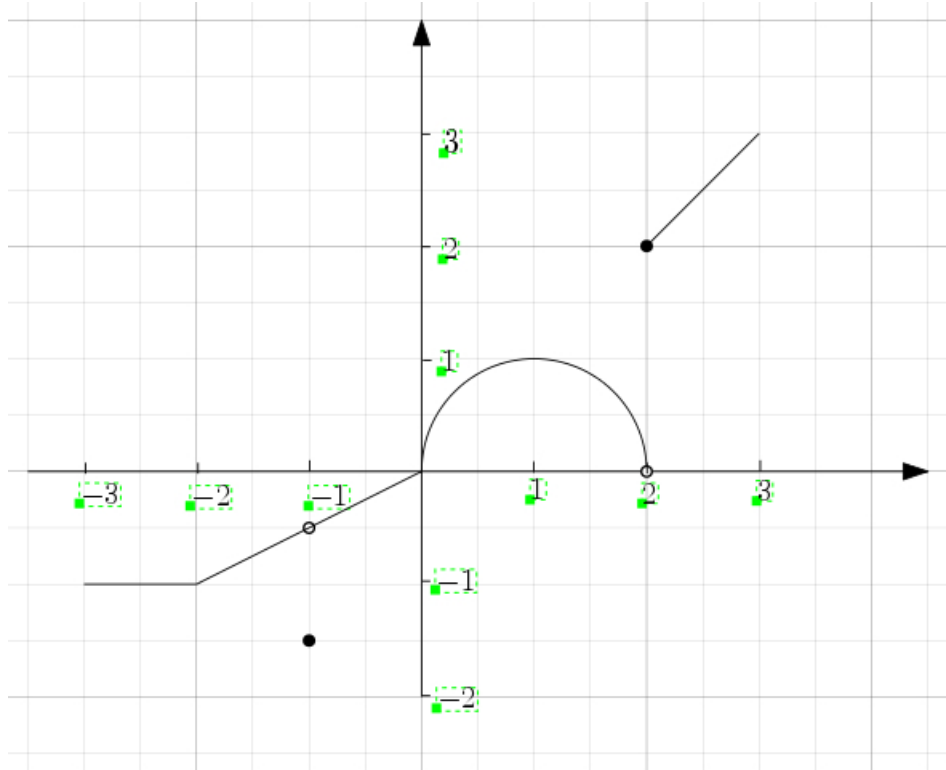Name: [   ]   UNI: [   ]

Instructions.

1. Show all the steps of your work clearly.

2. No textbooks, notes or calculators allowed.

Question	Points	Your Score
1	10	
2	10	
3	8	
4	8	
5	8	
6	12	
7	10	
8	14	
9	10	
10	10	
Bonus	10	
TOTAL	100 + 10	

1) (10 points)

1. At what points is f not differentiable?

2. What is $f'(-\frac{1}{2})$?

3. What is $\int_{-2}^{2} f(x)dx$?

4. What is $\int_{2}^{3} f^{-1}(y)dy$?

5. Is the function $g(x) = \int_{-3}^{x} f(t)dt$ differentiable on $(-3, 2)$? Why?

2) (10 points)

1. (5 points) Use the definition of the derivative to evaluate $f'(x)$ for $f(x) = \sqrt{x-2}$.

2. (5 points) Use linear approximation to estimate $\sqrt{3.99}$.

3) (8 points) A particle is moving along the curve $x^3y^2 = 8$ on the plane. As the particle reaches $(2, 1)$, the y-coordinate is **decreasing** at a rate of 3cm/s. How fast is the x-coordinate of the point changing with respect time at that instant?

4) (8 points) Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{4} + y^2 = 1$. (Write the area of the rectangle as a function of x and y first).
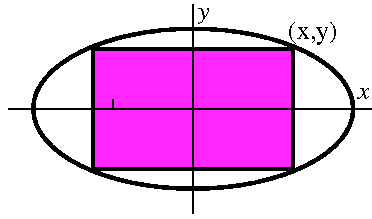
5)(8 points) For what values of a is the following statment true?

$$\lim_{x \to \infty} \left(\frac{x + a}{x - a} \right)^x = e$$

Hint: $f(x)^{g(x)} = e^{\ln f(x)^{g(x)}} = e^{g(x) \ln f(x)}$

6)(12 points) Find the below information for the function

$$f(x) = xe^{-\frac{1}{x}}$$

and use it to sketch its graph on the next page.

1. Domain

2. Horizontal and vertical asymptotes (if any).

3. Intervals on which $f(x)$ is increasing/decreasing

4. Local maxima and minima

5. Intervals on which $f(x)$ is concave up/concave down

6. Points of inflection

7. Graph of $f(x)$

)(**10 points**) Given the definite integral $\int_1^3 \sqrt{\sin(x)}dx$.

1. (8 points) Write it as a limit of left Riemann sums. DO NOT EVALUATE.

2. (2 points) Base on your answer for part (1), which definite integral does the following limit define?

$$\lim_{n\to\infty} \sum_0^{n-1} \frac{2}{n}\sin(1+\frac{2i}{n})$$

8)(14 points)
Let **R** be the finite region bounded by the graphs of $y = x^2$, $y = 2x^2$ and $x = 1$ on the plane. Use definite integrals to find the following quantities.

1. (4 points) Sketch the region and **evaluate the area of R.**

2. (4 points) The volume of the solid obtained when **R** is revolved around the horizontal line $y = -2$. DO NOT EVALUATE.

3. (6 points) The volume of the solid obtained when **R** is revolved around the vertical line $x = 8$. DO NOT EVALUATE.

9) (10 points)

1. (5 points) If $f(x)$ is continuous and $x\sin(\pi x) = \int_0^{x^3} f(t)dt$, then what is $f(1)$?

2. (5 points) Evaluate the following indefinite integral

$$\int \frac{1}{1-\sqrt{x}}dx$$

10)(10 points) Determine if the following statements are true or false. Please provide explanations to your answers. NO partial credits will be given if your explanations are incorrect.

1. You were once exactly 3 feet tall.

2. If $f''(a) = 0$, then $(a, f(a))$ is an inflection point of $f(x)$.

3. If f is continuous on (a, b), then f attains an absolute maximum and absolute minimum at some numbers c and d in the interval (a, b).

4. The following calculations are correct:

$$\int_{-1}^{2} \frac{4}{x^3} dx = -\frac{2}{x^2} \Big]_{-1}^{2} = \frac{3}{2}.$$

5. Given the function $f(x) = \begin{cases} x^2 & \text{if x is an irrational number;} \\ -x^2 & \text{if x is a rational number.} \end{cases}$,
then $\lim\limits_{x \to 0} f(x)$ does NOT exist.

Bonus (+10 points) **Do the bonus question only if have finished all the previous problems. You can choose one of the following problems to do.**

Option 1: Find the two points on the curve $y = x^4 - 2x^2 - x$ that have a common tangent line.

Option 2: The circle of radius 1 shown in the figure touches the curve $y = 2|x|$ twice. Find the area of the region between the two curves.