Interviewing in Matching Markets[*]

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Abstract

Interviewing decisions shape labor market outcomes. This paper proposes a framework for interviewing in a many-to-one matching market where firms and graduating students have capacity constraints on the number of interviews. While intuition suggests that relaxing interviewing constraints should improve the market's operation, this is not always the case. As participants are strategic in their interviewing decisions, interviewing constraints carry subtle implications for aggregate surplus, employment, and the distribution of welfare. We relate the insights obtained to various features of matching markets. First, firms frequently pass over even stellar candidates at the market's interviewing stage and as a result, some highly-skilled students may "fall through the cracks." Second, relaxing students' interviewing constraints benefits all firms and only the best students, but it adversely impacts the lower-ranked students. Third, this increase in students' capacities improves the social surplus but may decrease the number of matched agents. This may be undesirable if a social planner cares about the number of matched agents along with or as compared to, the social surplus. Fourth, in some cases a higher-ranked student may be worse off than a lower-ranked student due to firms' interviewing constraints. We show how credible signaling can ameliorate such inefficiencies. Lastly, interviewing in the presence of a low capacity acts as a sorting mechanism and an increase in the students' interviewing capacity may even lead to a decrease in social welfare due to reduced sorting.

Keywords: Two-sided Matching, Interviewing, Stability, Market Design, Uncertainty, Labor markets

JEL: C78, D47

Matching markets are characterized by agents who have preferences over whom they interact with, unlike the commodity markets or stock markets. The major focus of studies in matching markets has been on what happens *after* the preferences are formed.[1] This paper presents a theoretical model of interviewing, a process *before* preferences are finalized. It answers positive and normative questions related to the interviewing process and the implications of altering interviewing constraints at the margin. It sheds light on the existing design of certain markets and provides a framework to better design some others.

In both centralized and decentralized labor markets, agents' preferences are formed over a long process of multiple interactions between the two sides. As an example consider the annual matching process of doctors to residency positions organized by the National Resident Matching Program (NRMP). Like any other job matching process, the doctor-residency matching process has multiple stages—application, screening of applications, interviewing, and a final matching or market clearing. The 'preference formation' stages before the final matching process, impose constraints due to financial and time costs. For instance, medical fellowship candidates are expected to use their vacation days for interviewing and that imposes a clear constraint on the number of interviews a candidate can take up.[2]

A compelling intuition suggests that if the interviewing constraints were completely relaxed, the first-best outcome would obtain. In practice, however, this is hardly a feasible or realistic policy prescription. Instead, the practical market design quandary is about the consequences of increasing interviewing (or application) capacities at the margin.[3] Will increasing interviewing capacity move the economy towards greater surplus? What are the distributional welfare consequences? This paper provides answers to these and related questions. One may expect that a rising capacity (of interviewing) will lift all boats. However, in many situations, jobs offered to some candidates may

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[1]As a result, we know about various aspects of these markets, once these final preferences shape up. For instance, among other things we know about the existence of a stable matching [Gale and Shapley, 1962], the incentives to participate in such markets [Roth, 1982], and the dynamics when the relationships last over multiple periods [Kadam and Kotowski, 2015a,b]. Also see Roth [1984], Roth [1990], Roth and Peranson [1999], Abdulkadiroglu and Sönmez [2003], Roth et al. [2004], and Roth [2008] along with the references therein. Important exceptions which study application or interviewing processes are Lee and Schwarz [2012], Chade et al. [2014] and Che and Koh [2015] respectively. We discuss the related literature in Section 2.

[2]This is based on personal communication with Dr. Niesen. Niesen et al. [2015] find that the median fellowship applicant accepted 10 interviews and canceled 1 interview citing costs associated with interviewing in their survey results.

[3]Interviewing constraints matter in many markets. Although interviewing data is hard to get, we report some aggregate numbers from the summary reports of NRMP's proprietary survey data. This data shows that doctors reject interview invitations, sometimes regrettably as they are left unmatched through the main rounds. The survey indicates that there are some positions that remain unmatched after the main rounds but get filled up in the secondary rounds. We take this coupled with another survey data from Niesen et al. [2015] as directional evidence for the relevance of interviewing constraints. We present the detailed evaluations in appendix Section A.2.

come at a cost of not being able to offer them to some other candidates. Hence the above welfare evaluation may be, and as we show indeed is, imprecise.

We propose a two-sided matching model with non-transferable utility[4] to study the impact of changing interviewing capacities. There is a finite set of firms and a continuum mass of students. All the students and firms have *almost* identical preferences over each other which are common knowledge.[5] Preferences, although almost aligned, are crucially dependent on what we call a 'fitness factor.'[6] This parameter takes one of the two values—'fit' and 'misfit'—for each firm–student pair. A firm learns the fitness factors only through interviewing the students. Moreover, a firm learns this information only for those students whom they interview, specific to itself. Particularly, a firm does not see the fitness values for any student with any other firm. For any firm, the misfit and uninterviewed applicants remain unacceptable. Among those students who are interviewed and found fit, a firm prefers hiring students with higher ability and all firms evaluate the students' abilities identically. This keeps the interviewing process relevant while keeping the problem tractable. Firms and students choose their interviews optimally subject to their constraints and knowing that the fitness factor will be discovered during the interviewing process. Based on the actual interviews that take place, firms and students form preferences and match. We focus on a stable matching outcome as it has been recognized as paramount for a market's successful operation [Roth, 2002].

We nest the above environment into a multi-stage game. In the game's first stage, the applicants apply to (all) firms (as applications are assumed to be costless). In the next stage, the firms strategically extend interview invitations to some students. The students accept some of the interview offers given their capacity. All interviews take place and then the final matching is realized.[7] Note that both firms and students are limited in the number of interviews they can conduct.

In our model, since fitness is the only information discovered through the interviewing process, a firm extends a final job offer to all students who are found fit through the interviewing process.[8]

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[4]Specifically, we do not consider the models where the salary is a part of the negotiation. This is a good approximation for many entry-level labor markets where many contracts are relatively standardized [Agarwal, 2015, Avery et al., 2007, Roth, 1984].

[5]For instance, consider the matching market of entry-level hiring for college graduates. The students are ranked by firms as per their credentials (almost) identically by all firms. There is also a ranking over various firms (within an industry) from the students' perspectives, which is roughly identical. Students may consider the ranking over various consulting or law firms published by some companies [Vault Rankings & Reviews, 2015] to guide their preferences.

[6]In the real world, the preference of firms is determined by how well the candidates performs on the interviews and is perceived to fit with a firm's culture in various rounds of interviewing. A recent opinion piece discussed this issue about the potential candidate's cultural fit [Rivera, 2015]. See also Chatman [1991] and Rivera [2012].

[7]Specifically, we rule out the issues related to timing in these markets. We assume that it is prohibitively expensive for a firm to wait for the firms at the top to finish their hiring because hiring is a time-consuming process. We talk briefly about the timing of markets in Appendix Section A.3.

[8]If a firm were to not extend a final job offer to a student despite finding her fit, the firm could choose to not extend

Moreover, a misfit student is never extended a job offer as the firm strictly prefers to leave the position empty or offer it to some other fit student. If this is the case, a student only needs to choose the best interview offers when the fitness factors across firms is identically and independently drawn. The firms are, however, more strategic about their choices and have a mix of (some good, some average and some safe) candidates to whom they extend their interview offers (Proposition 1). This suggests that the firms expend their scarce interviewing resource by evaluating a student's ability and also the probability of being able to successfully hire her. We establish the existence of an (essentially) unique equilibrium of the application, interviewing, and matching game (Theorem 1). We compare the different equilibrium outcomes of these games as the students' interviewing capacity changes. When the student interviewing capacities increase, the total surplus generated from the matching also increases (Proposition 2). However, this is not a Pareto improvement and some students at the bottom are worse off (Proposition 3).

In the setting analyzed so far if a social planner has to choose the levels for interviewing capacities, increasing them as much as possible seems optimal. However, in many real-world settings the interviewing capacity for students is restricted and, sometimes, chosen to be so.[9] A case in point is that of the job placement process for graduates of Indian management and engineering institutes. Students at these institutions choose the list of firms they want to interview with, as many firms are concurrently invited to interview by the institutions. Although there is no explicit capacity on interviewing for the students, implicitly there is a constraint as is the case in many other settings. The placement processes at these institutes are crafted to ensure that the students get good job offers and a maximum number of students are placed.[10] Although an increase in interviewing capacity increases utilitarian welfare, it may decrease the number of students who get matched (Proposition 4). This suggests that the capacity constraints on students may be aligned with the placement process objective.

The intuition for the above results can be easily obtained by considering a simple setting with two firms, A and B and two students i and j. It is common knowledge that firm A is better than firm B and student i is better than student j from an ex-ante perspective. However, there are fitness factors specific to every firm-student pair which can be discovered only through interviewing. Suppose the fitness factors are independent and each firm-student pair is equally likely to be a fit

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an interview invitation at the first place. This plays out at equilibrium of the game and discussed in detail in the main model.

[9]This is particularly true in cases where there is *congestion* at the interviewing stage [Roth, 2008, Avery et al., 2001].

[10]This is based on personal communication with placement directors, alumni and current students. In addition to the number of students placed, the time it takes for each institute to complete the placement process also gets media attention [Economic Times Bureau, 2015, Financia Express Bureau, 2015].

| | Firms | |
Students	A	B
i	200 or -250	120 or -250
j	100 or -250	30 or -250

Table 1: An example of two firms A and B and two students i and j. The positive numbers represent the match surplus accrued to the firm when the student is found fit in the interviewing process by the firm and the negative number is the firm's share of the surplus if a firm hires a misfit candidate.

or a misfit. The values of surplus accrued to the firm in various matches are shown in Table 1. The (negative) positive value is accrued to the firm if the student is a (mis)fit for the firm. The student also gets the exact same values as the firm from a given match.

Under the absence of interviewing constraints, both firms and students interview with each other. The expected sum of surplus generated from the first-best outcome is $2 \times 164.375 = 328.75$ and the expected number of matches is 1.325.[11] Now suppose that interviewing is so costly that a firm and a student can only sign up for a maximum of one interview each. With the interviewing constraints in place, firm A and student i interview with each other. Note that firm B would have preferred to interview student i although it knows that it stands a chance to hire student i only if she is found misfit for firm A and a fit for itself. This is the case as the value generated by student i (120) and the value generated by student j (30) are such that $0.25 \times 120 > 0.5 \times 30$. However, given the interviewing constraints, firm B chooses to interview student j. The expected surplus from these interviewing choices under constraints falls to $0.5 \times 200 + 0.5 \times 30 = 115$. The expected number of matches is now $0.5 + 0.5 = 1$.

Consider the case where the constraints on the students are relaxed but those on the firms are still in place. Firm B prefers to interview student i over spending the only interview spot on student j. The expected social surplus is $0.5 \times 200 + 0.25 \times 120 = 130$ and the expected number

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[11]This considers all the different possibilities of fitness factors for the four pairs between the firms (A and B) and students (i and j). The following table summarizes these outcomes where 1 stands for the firm-student pair being fit, $\times$ represents a misfit, and $\cdot$ stands for any realization of the fitness value for the pair.

Ai	Bj	Aj	Bi	Probability	Firms' share of surplus	No. of matches
1	1	$\cdot$	$\cdot$	$1/4$	230	2
1	$\times$	$\cdot$	$\cdot$	$1/4$	200	1
$\times$	$\cdot$	1	1	$1/8$	220	2
$\times$	$\cdot$	1	$\times$	$1/8$	100	1
$\times$	$\cdot$	$\times$	1	$1/8$	120	1
$\times$	1	$\times$	$\times$	$1/16$	30	1
$\times$	$\times$	$\times$	$\times$	$1/16$	0	0

of matches is $0.5 + 0.25 = 0.75$ with a higher interviewing capacity for the students. The expected social surplus has increased in this case. However, the expected number of matches has gone down from 1 to 0.75. The welfare, in terms of the expected utility and probability of getting a job, for student i has increased at the cost of welfare for student j. The expected welfare for firm 2 strictly increases.

It is not surprising that the system approaches efficiency and the surplus from matching increases as we relax the interviewing constraints. Interviewing capacities act like frictions and prevent the attainment of the first-best. Roughly speaking, it is expected that when the frictions are reduced, more surplus will be generated.[12] The surprising feature is that the increase in surplus comes with a reduced number of matched agents. From a social planner's perspective, if the goal is not only to raise the social surplus but also to ensure that a maximum number of positions get filled up then the two objectives can come directly in conflict with each other. Moreover, if students' preferences exhibit risk aversion where being unemployed leaves the students with a significantly lower utility (e.g a large negative value rather than 0 in the context of the above example), this could also reduce the utilitarian surplus.[13] We however, assume risk neutrality of all the agents for most of the discussion.

The baseline model expands upon the preceding example's conclusions. After evaluating the impact of interviewing capacity change on the overall outcomes, we investigate the effects on individual welfare for different agents in the economy. We show that increasing capacity for the students increases the welfare for all firms and the students at the top. However, the students in the lower ability range are worse off due to the capacity increase. It is clear that since the number of matched agents could be lower, it does not lead to a Pareto improvement.[14] We can precisely identify the agents who benefit and those who are hurt from a capacity increase.

To check the robustness of our results, we extend our model in various directions. Specifically, we introduce correlation in the information discovered during the interviewing process, fitness, and also different ex-ante preferences. We also show that although basic intuition suggests that interviewing constraints are wasteful and their impact should be minimized, they retain a key benefit. They facilitate sorting of agents among different firms if there are information asymmetries. When interviewing capacities increase, the informativeness of a student choosing to interview with a firm

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[12]This holds if there is enough agreement about preferences before the interviewing stage. We present an example in the next section, which actually highlights that this intuition can be misleading when there is enough heterogeneity.

[13]This result relies on the assumption that being unemployed or leaving a job unfilled has different implications for the students and firms respectively. This asymmetry drives the wedge between socially optimal first-best and the second-best outcome.

[14]This channel is different from the endogenous channel that leads to different kind of jobs being offered by an employer when search frictions reduce as discussed by Acemoglu [1999] and Autor [2001].

decreases and this may lead to a reduced social surplus.

However, we do not mean to suggest that reducing interviewing constraints is bad for the overall economy but just that a social planner should be aware of the potential winners and losers from such a reduction to evaluate the correct impact. We also relate various insights from our framework to some anecdotal phenomena (e.g. strategic choices by firms, students falling through the cracks, etc.) and some institutional settings (e.g. the hiring processes at the Indian Institutes of Management (IIMs)). We hope the tractable framework we provide will be further used for the study of interviewing processes.

The remainder of the paper is organized as follows. Section 1 presents some examples with 2 firms and a continuum of students to describe our general model's main results in simple settings. Section 2 describes the connection of this paper with the existing literature. In Section 3, we present the general model. Sections 4 and 5 establish the main results. Section 6 discusses applications and extensions of the main model where we introduce correlation in the fitness factor, and ex-ante preference heterogeneity amongst students and firms. Section 7 concludes.

1 Numerical examples with 2 firms

In this section, we present three numerical examples to describe some of the key insights from this paper. The first example presents the main results about strategic choices by the firms and the impact of an increase in interviewing capacity on the overall welfare, the number of matched agents, and changes in the distribution of welfare. This illustrates our findings in the baseline model and the extensions. Thereafter, we consider two variants where not all of the results in the main model hold. These variants highlight the crucial assumptions which drive the main results and show some surprising effects of interviewing constraints. In the first variant we introduce heterogeneity on the fitness factors with respect to the firms and show that some higher-ranked students could be worse off ex-ante than lower-ranked students. In the second variant, we consider heterogeneous student preferences and show that an increase in interviewing capacity can lead to a lower welfare.

We start by describing the general setting which is common to all the examples. There are 2 firms and a continuum of students of mass 1. The two firms are labeled 1 and 2. A student's type is $\theta = (\succ^\theta, e^\theta, f^\theta)$ drawn from a distribution G with support $\Theta = \{1 \succ 2, 2 \succ 1\} \times [0,1] \times \{-1,1\}^2$. The first component is the student's preference over firms. The next component e^θ is the students' 'ability' and the last component is a 2-dimensional vector which has the student's firm-specific 'fitness factors.' We assume that the distribution of the students over Θ is such that e^θ
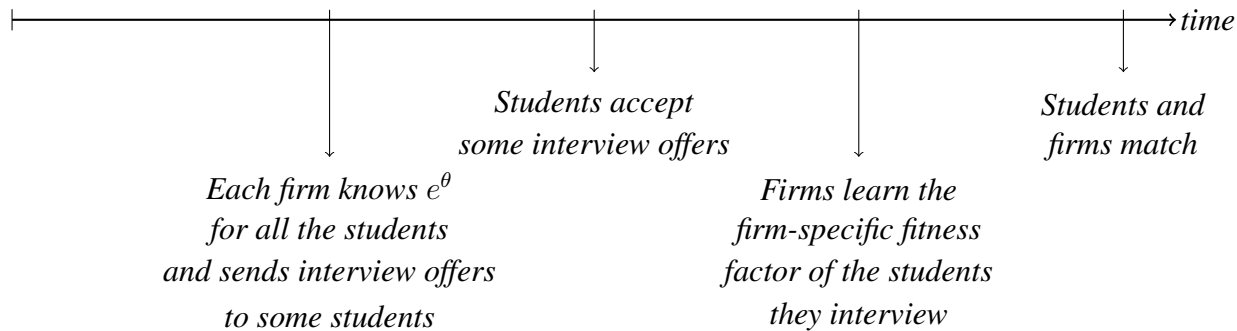
Figure 1: The timing of interactions between the firms and students

is uniformly distributed over $[0, 1]$. Moreover, the fitness factor with firm k is independent of $\succ^\theta$, e^θ and other fitness factors, and takes value 1 (which indicates that the student is a fit for the firm) with probability p_k and -1 (indicating a misfit) with probability $1 - p_k$.

We outline in Figure 1 the market's timeline. We assume that the ability parameter is common knowledge and based on that information firms decide their interview offers. A student accepts some interview offers from the ones she receives. The interviews take place and the firms learn the fitness factor perfectly for the students they interview. The students and firms match based on the preferences formed at the end of the interviewing process.[15]

1.1 Students' preferences agree about firms' rankings

In the first example, all students prefer firm 1 over firm 2. The fitness factor with firm k is 1 with probability p_k. Here we assume that $p_1 = p_2 = 0.2$. Each firm has a hiring quota of 0.07 mass of students. There is a maximum number of interviews that the students (firms) can take up (conduct) which represents the capacity constraint. Each firm can interview up to 0.39 mass of students.[16] Similarly, the students have a capacity on the total number of interviews they can take up. In the low-capacity regime, students can interview with up to $k_{LC} = 1$ firm and in the high capacity regime they can interview with up to $k_{HC} = 2$ firms. We keep the firm interviewing capacity fixed at 0.39 in both the regimes for ease of comparison. We also make an assumption that the firms have a slight distaste for interviewing. This assumption will ensure that a firm will not interview more students than it needs to to fill up its hiring quota. A firm will never interview students just

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[15]We specifically assume that firms do not extend offers immediately after they discover a student's fitness factor. The strategic choice about the timing of the interviews and/or job offers is an important issue, which we abstract away from, in the scope of this discussion. We briefly discuss this issue in appendix Section A.3.

[16]The results are not knife-edge and do not depend on the exact choice of these numbers. We use these numbers for ease of calculations.

to fill up its interviewing capacity.

The first component e^θ summarizes the relative desirability of the student, if she is found fit. More precisely, the surplus generated by firm k and student θ is given by $2U(k, e^\theta)$ and is assumed to be split equally;[17] where

$$U(k, e^\theta) = \begin{cases} \frac{e^\theta}{k} & \text{if the student is a fit for firm } k, \\ -\infty & \text{if the student is a misfit for firm } k. \end{cases}$$

We describe the strategic decisions of firms and students. As the interactions take place over time, we can start evaluating the decisions from the end of the timeline. Once the preferences are formed based on the interviewing process outcomes, there is a unique stable matching[18] between the firms and students as all students' rank ordering over (acceptable) firms is identical. We have assumed that the firms dislike interviewing more students than 'required' and hence they will extend a job offer to all the students who are found fit. Since fitness is the only information revealed in the interviewing process if a firm were to not extend an offer to a student found fit, it will not interview them at the first place or its interview offers will be rejected by the students accordingly.[19]

We first investigate the firm interviewing strategies in the low-capacity regime. A student always accepts an interview offer from the best firm. The best firm knows that if it interviews some mass of students, it would find 20% of them to be fit or put differently, a fifth of that mass of students will be employable by firm 1. Since firm 1 has a hiring quota of 0.07, it decides to interview the best 0.35 mass of students. Thus it chooses to extend interview offers to all students with ability $e^\theta \in [0.65, 1]$. In the low-capacity regime, since each student can only accept a single interview offer all the students who would be interviewed by firm 1 are effectively not available to firm 2 for interviewing. The fitness factor with firm 2 is 1 with probability 0.2. Firm 2 will also find 20% or a fifth of the mass employable from any set of students it chooses to interview. Firm 2 will interview the best students not interviewed by firm 1 as each student can only interview with one firm. Hence it will extend interview offers to all students with $e^\theta \in [0.3, 0.65)$. These interviewing choices are summarized in Figure 2.

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[17]Note that the match utility is such that there will be a positive assortative matching absent any constraints, due to supermodularity.

[18]The definition of stability is adapted from Azevedo and Leshno [2015]'s definition of stability to account for multiplicities introduced by the continuum setting. This will be discussed in detail in the next section.

[19]Note that due to the continuum assumption and continuity of firm utility on the ability dimension, a firm will never have to randomize in extending job offers after the student is found fit. For example, firm 1 can choose to interview the top $5x$ mass if it is looking to hire x students since the probability of finding a student fit is $p = 0.2$.

Students' ability   0 ├──┤ 1

Firm 1's
interview offers   ├──────────────┤
                                            0.65

Firm 2's
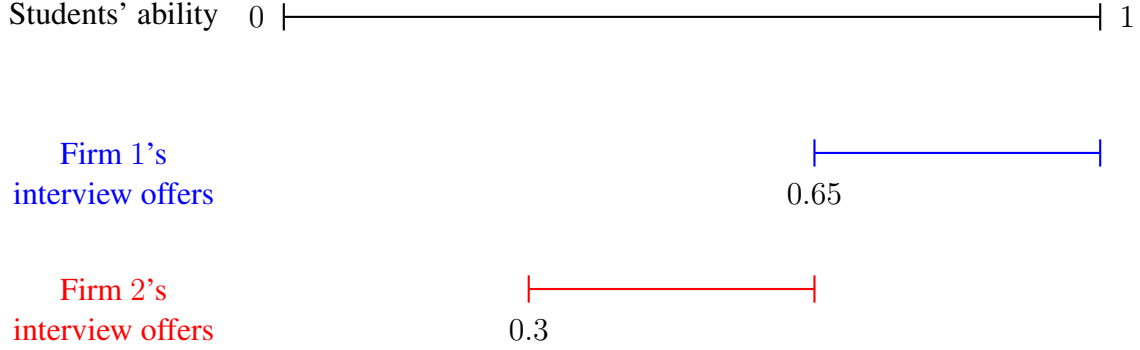interview offers   ├──────────────┤
                       0.3

Figure 2: Interview offers by firms 1 and 2 when students can accept only one interview offer. Firm 1 interviews the best students available. Firm 2 interviews the best students who can accept its interview offer.

We now continue with the analysis and look at the firms' interviewing strategies in the high-capacity regime. In this setting, a student can interview with up to 2 firms. The choice for the first firm is unaffected. Firm 1 continues to extend interview offers to all students with ability $e^\theta \in [0.65, 1]$.

In the high-capacity regime, firm 2 can extend interview offers to any of the students and they will accept the offers. Let us assume that the firm's optimal strategy is to continue with its old interviewing strategy of extending interview offers to students with ability $e^\theta \in [0.3, 0.65)$. We can check if the firm has a profitable deviation from this conjectured optimal. The lowest ability student is found fit with probability 0.2 and hence generate a utility of 0.06. Note that the students at the top with ability close to 1 are being interviewed by firm 1. These students are found misfit, and hence available for firm 2, with probability 0.8. Firm 2 is only interested in hiring those candidates who are fit for it. Thus, firm 2 will find the student at the top employable with probability of $0.8 \times 0.2 = 0.16$ and expected utility of $0.16 \times e^\theta = 0.16 \times 1 = 0.16$. This shows that the firm will be better off to interview the students with ability close to 1 rather than interviewing students with ability around 0.3. In fact firm 2 continues to move its interview offers to the students who are also being interviewed by firm 1 till it no longer remains optimal. In this case, we observe that $0.2U(2, 0.56) = \frac{0.2 \times 0.56}{2} = 0.16U(2, 0.7)$. Incidentally, this does not violate the firm's hiring quota of 0.07 mass of students and its interviewing capacity of 0.39. The interviewing strategies for both firms are summarized in Figure 3.

In the low-capacity regime, firm 2 interviewed only 0.35 mass of students and successfully filled up its hiring quota. However, in the high-capacity regime, although firm 2 fills up its inter-
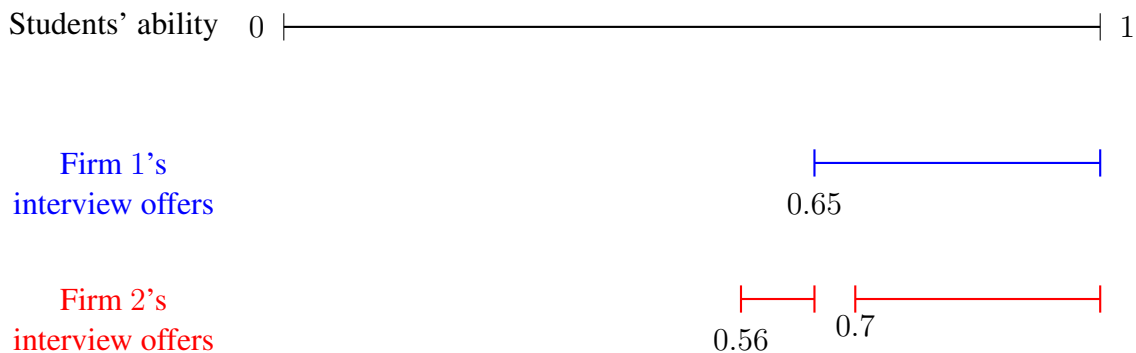
Figure 3: Interview offers by firms 1 and 2 when students can accept up to two interview offers. Firm 1 interviews the best students available. Firm 2, however, chooses not to and strategically extends interview offers with a gap in the middle.

viewing capacity of 0.39, it now has vacant positions following the match outcome.[20]

We pause here to highlight the important characteristics of the interview offers and the resulting outcomes across the two regimes.

1. In the low-capacity regime, firm 2 would have preferred to interview the students at the very top but can not as the students' interview capacity binds. The effective value of expending an interview slot on the students with $e^\theta = 1$ is greater than that on the students with $e^\theta = 0.3$.

2. All students in the ability region $[0.65, 0.7)$, who are not hired by firm 1 but would have been found fit for firm 2 (if interviewed) i.e. $f_2^\theta = 1$, fall through the cracks.

3. Firm 2 does strictly better under the high-capacity regime as it still has the option to extend the same interview offers as in the low-capacity regime but chooses not to.

4. Firm 2 hires a smaller mass of students in the high-capacity regime in spite of interviewing more students (and exhausting its interviewing capacity).

The last two points above highlight the increase in surplus that may be accompanied by a decrease in the number of matched agents. The welfare for the second firm is strictly higher. The expected utility for students above the ability threshold of $e^\theta = 0.7$ increases from $0.2U(1, e^\theta)$ to $0.2U(1, e^\theta) + 0.16U(2, e^\theta)$. The probability of getting a job for these students increases from 0.2 to 0.36. However, the expected utility and probability of getting a job for students with ability

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[20]Firm 2 hires 0.066 mass of students or 5.7% fewer students. It gets $0.2 \times 0.09 = 0.018$ mass from interviewing the students without an interview from the best firm and $0.2(0.8) \times 0.3 = 0.048$ mass from interviewing those with such an offer.

$e^\theta \in [0.3, 0.56)$ decrease from positive values to 0. These results are not particular to this example and we prove through the main results that they hold in a more general setting as well.

1.2   A better student does worse than a worse student

We consider a slightly different setting to highlight the possibility that a student with a better ex-ante score or evaluation can be worse off. Consider the setting where the students prefer a job from firm 1 over that from firm 2. The probabilities that a student is fit are not the same across firms. Specifically, for firm 1, this value p_1 is 0.2 and that for firm 2, p_2 is 0.5. The two firms want to hire 0.07 and 0.18 mass of students respectively. They both have an interviewing capacity of 0.39. The students can interview with up to 2 firms. The surplus generated by firm k and student θ when the student is a fit for the firm is given by $2U(k, e^\theta)$;[21] where

$$U(k, e^\theta) = \begin{cases} \frac{e^\theta}{k} & \text{if the student is a fit for firm } k, \\ -\infty & \text{if the student is a misfit for firm } k. \end{cases}$$

As in the example above, the firms make the strategic decisions about their interview offers. Again, it is easy to see that the first firm interviews the top 0.35 mass of students and the choice for firm 2 is exactly the same as described in Figure 3.[22] We observe that the students with ability $e^\theta \in [0.65, 0.7)$ fall through the cracks if they do not get a job offer from firm 1. However in this case, these students are worse off even in an ex-ante sense as compared to students with ability $e^\theta \in [0.56, 0.65)$. Consider the ex-ante expected utility of a student with ability e^θ who gets an interview offer from firm 1. It is $p_1 \times U(1, e^\theta) = 0.2e^\theta$. This is smaller than the ex-ante expected utility for this student, if she got an interview offer from firm 2 instead. This expected utility is $p_2 \times U(2, e^\theta) = 0.5 \times \frac{e^\theta}{2} = 0.25e^\theta$. Counter-intuitively, the students with ability in $[0.65, 0.7)$ regret having a greater score on the ability dimension. Note that it is not an equilibrium for firm 2 to extend an interview offer to students in this range.

1.3   Students' preferences differ about firms' rankings

Now suppose there is uncertainty about students' preferences. They are equally likely to be $1 \succ 2$ or $2 \succ 1$. This is perfectly and privately known to the students even before the interviewing stage.

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[21] Note that the match utility is such that there will be a positive assortative matching absent any constraints.

[22] Note that $p_1 = 0.2$ and $p_2 = 0.5$. Firm 2 wants to hire 0.18 mass of students and its interviewing capacity is 0.39. It chooses to interview the students above ability 0.7 and those in the ability range of $[0.56, 0.65)$ as $p_2 U(2, 0.56) = 0.5 \times \frac{0.56}{2} = p_2(1 - p_1)U(2, 0.7)$.

Students' ability 0 ├────────────────────────────────┤ 1

Firm 1's
interview offers
├ - - - - - - - - - - ┤
0.64

Firm 2's
interview offers
|- - - - - - - - - - - |
0.64

Figure 4: Interview offers by firms 1 and 2 when students can accept only one interview offer and students' preference over firms vary.

The student is a fit for a firm with an equal and independent probability of 0.5, i.e. $p_1 = p_2 = 0.5$. Each firm has a hiring need of 0.09 mass of students and an interviewing capacity of 0.18. The students can interview with up to 1 firm in the low-capacity regime and with up to 2 firms in the high-capacity regime. A firm k and a student with ability e^θ generate an expected surplus of $2U(k, e^\theta)$. The firm always gets $U(k, e^\theta)$ given by the following.

$$U(k, e^\theta) = \begin{cases} e^\theta & \text{if the student is a fit for firm } k, \\ -\infty & \text{if the student is a misfit for firm } k. \end{cases}$$

A student, however, gets an ϵ higher utility if the firm is her (ex-ante) first choice firm and an ϵ lower payoff if it is not. Thus a student of ability e^θ with a preference $1 \succ 2$ gets a utility of $U(1, e^\theta) + \epsilon = e^\theta + \epsilon$ if she gets a job offer from firm 1 and if she is a fit for firm 1. This student gets a utility of $e^\theta - \epsilon$ with a job offer from firm 2 if she is a fit for the firm.

In the low-capacity regime, a student simply chooses to interview with her top choice firm if faced with two interview offers. A firm chooses to extend interview offers to the best students knowing that only half of them will accept its interview offers. Thus the firms extend interview offers to the top 0.36 mass of students and will be sure that only 0.18 mass of students will accept each firm's interview offers. Thus interviewing acts as a sorting mechanism in this case. The interview offers for both firms are summarized in Figure 4. The dashed lines indicate that the firms do not interview all students in that ability range but only those who accept their interview offers.

In the high-capacity regime, a student can accept offers from both firms. The firms can no longer extend interview offers and expect the students to sort themselves according to their preferences. Thus, the sorting benefit is lost with increased interviewing capacity. The strategic choices

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by the firms are shown in Figure 5. The solid lines indicate that firms extend interview offers to all students in that ability range. The dotted line indicates that only one of the firms extends an interview offer to the student with that ability. However, how the firms decide sharing of these students is not uniquely defined.[23]

To evaluate the optimality of these, we need to find the probability that a student is available for a given firm when the student is interviewed by both firms. Consider a high ability student who gets an interview offer from both firms. Recall that p_1 is the probability that a student is fit for firm 1 and p_2 is the same for firm 2. The probability that firm 1 is able to hire this candidate = Probability (student's preference is $1 \succ 2$) $\times p_1$ + Probability (student's preference is $2 \succ 1$) $\times (1 - p_2) \times p_1 = 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5 = 0.375$. A low-ability student, who gets an offer from only one of the two firms, is found fit with probability 0.5. The firms' strategies are optimal if the expected utility from interviewing the lowest-ability students in the two regions, the one where both compete and the one where they divide up the set of students, are equal. Both firms choose to extend interview offers to the students at the top in the ability range $[0.94, 1]$ and the students in the ability range $[0.7, 0.94)$ get an interview offer from only one of the two firms.[24]

In this case, it is easier to see that each firm hires a smaller mass of students than when there was no overlap in the interview regions. It is less obvious that the social surplus decreases. However, the sum of utilities for the firms decreases by 4% along with a 9% reduction in the number of matched agents when the interviewing costs reduce.[25] The sum of utilities for the students should account for the idiosyncratic components that the students get. It is the case that even the sum of

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[23]There are multiple strategies for the two firms which are effectively similar. Firm 1 can choose to extend interview offers to students such that $e^\theta \in [0.7, 0.79) \cup [0.85, 0.88) \cup [0.94, 1]$ and firm 2 can choose to extend interview offers to those with ability in $[0.79, 0.85) \cup [0.88, 0.94) \cup [0.94, 1]$. Clearly this is not the only distribution of students which is optimal. The other choices differ by which firm extends an interview offer to a student of a particular ability but they are essentially equivalent as only one of them extends an offer to the students in the ability range of $[0.7, 0.94)$.

[24]We can start with a conjecture for the optimal strategy as interview offers similar to the one when the students faced a capacity constraint. Suppose that the firms 'share' the top 0.36 mass of students in 'some' way to meet their hiring needs while not violating the interviewing constraint and not competing with each other. Consider the firm which interviews students with ability 0.64 and gets an expected utility of 0.32. However, it may choose to extend an interview offer to a student with ability greater than 0.8534 and it will result in an expected utility greater than 0.32. Consider y as the mass of interview offers that are shifted away to the students at the top. This move is optimal if $(0.64 + y) \times 0.5 = (1 - y) \times 0.375$. This leads to $1 - y = \frac{164}{175} = 0.9371$ which we represent as 0.94 above for simplicity. The exactly optimal interview offers are such that both firms interview students with ability $[\frac{164}{175}, 1]$ and 'share' the students in the ability range $[\frac{123}{175}, \frac{164}{175})$.

[25]The sum of firms' utilities in the low-capacity regime is given by the following.

$$\int_{0.64}^{1} 0.5t dt = \frac{1 - (0.64)^2}{4} = 0.1476$$

In the high-capacity regime, both firms extend interview offers to students in the ability range $[1 - y, 1]$ where $y = \frac{11}{175}$

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Figure 5: Interview offers by firms 1 and 2 when students can accept both interview offers and the students' preferences over firms vary.

students' utilities is lower in the high-capacity regime.[26]

2   Related Literature

In this section, we review various strands of literature that are related to this work. We start with papers that have some stages of costly interviewing in the context of matching models. After interviewing models, we review the work related to application stages which are also one of the preference formation processes. We will then connect our work to the matching theory and search theory literature.

In the domain of interviewing, the closest paper is Lien [2013]. He provides a model of interviewing with a finite set of firms and a finite number of students who are each looking for one position. In his setup, the firms can interview up to 2 candidates. He focuses on the non-assortative nature of interview offers and the final match in this setting. For certain parameter specifications, some students fall through the cracks due to the non-assortative nature of matching at the interviewing stage. We focus on a general many-to-one matching setting where both sides of the market face interviewing capacities and more importantly, analyze the effect of changing these capacities on welfare, the number of matched agents, and the distributional consequences.

Lee and Schwarz [2012] focus on the network aspect of an interview schedule for multiple

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and share the students in the ability range $[0.64 + y, 1 - y]$. The sum of firms' utilities is the following.

$$2\Big[\int_{1-y}^{1} 0.375 t dt \Big] + \int_{0.64+y}^{1-y} 0.5 t dt \quad = \quad 0.14172$$

[26]We refer an interested reader to Appendix Section A.4 for the calculations.

firms and multiple agents in a one-to-one matching market. They find that interviewing schedules with maximum overlap are welfare improving as compared to the ones with less overlap. Ely and Siegel [2013] analyze the implications of revelation of intermediate interviewing decisions by firms in a common-value labor market. Their focus is on a setting where firms compete for a single worker or multiple workers who are not substitutes. They show that severe adverse selections shuts off all the firms except the top firm(s) from participation in recruiting.

Josephson and Shapiro [2013] look at information-based unemployment resulting from a schedule of interviews presented to the participants (by a central coordinating organization). Rastegari et al. [2013] solve the problem of centralized interview schedule for partially informed agents with the objective of stability and a minimum number of interviews. Using a one-to-one model they establish a computationally-efficient interview-minimizing policy. Das and Li [2014] study the impact of greater commonality about the candidates' evaluations available before interviewing using simulations. They find that more commonality in the ex-ante quality signal can cause firms to focus on the same candidates and reduce the match probabilities.

The process of interviewing attracts attention from applied sociologists and psychologists. Our study is the closest in its premise regarding fitness factor to findings reported by Rivera [2012]. She studies the hiring decisions of elite professional service firms and provides evidence that '··· [Hiring] is also a process of *cultural matching* between candidates, evaluators, and firms.'[27] She further suggests that, 'Concerns about shared culture were highly salient to employers and often outweighed concerns about absolute productivity.' Chatman [1991] finds a positive impact of person-organization fit on hiring, acclimatization, satisfaction and tenure of individuals in different organizations.

In the college-applications literature, the recent work by Che and Koh [2015] analyzes the strategic choices by colleges in extending admissions decisions when there is aggregate uncertainty about student preferences and students face costless applications. They show that there exist equilibria where colleges give more weight to the idiosyncratic elements of a student's application to minimize head-on competition and aggregate uncertainty. We focus on the presence of interviewing constraints on both sides of the market and analyze the impact of interviewing capacity changes on the utilitarian welfare as well as welfare of different agents, i.e. top-ranked and medium-ranked students and various firms.

Avery and Levin [2010] study the college admissions problem and focus on the presence of early admissions. They show that early admissions applications being credibly limited to only one college, are a way for a student to express enthusiasm about attending a college especially when the

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[27]Emphasis is present in the original text.

colleges care about it. Avery et al. [2014] study the timing game for admission decisions where the colleges are the strategic players. They create a model where students have different information about their chances of admissions under different regimes. When students have more information when applying, a second ranked college prefers allowing more applications per students to less, or in terms of their model select a different examination date than the best ranked college and thus effectively allows students to apply to multiple colleges. Yenmez [2015] addresses the college admissions offers as a many-to-many matching problem with contracts where the contractual terms represent an early admission or regular decision among other things.

Chade and Smith [2006] focus on the problem of portfolio choice for applications for a single student. They present a greedy algorithm which solves the combinatorial optimization problem. Chade et al. [2014] talk about the equilibrium model of college admissions in a setup with two ranked colleges. With incomplete information about the student quality as seen by the colleges and incomplete information about the portfolio of students by the colleges, they generate interesting results about 'stretch' and 'safety' application portfolios. Hafalir et al. [2014] undertake a theoretical and experimental investigation about the efforts put by students when they can apply to all, i.e. in their setting both, colleges versus that exerted by the students in a decentralized setting where they can only apply to one college.

On the technical front, this paper borrows from and expands upon the setup of a finite set of firms and a continuum mass of students in Azevedo and Leshno [2015]. We adapt and use their definition of stability. We make two significant changes to their setup for the problem we study. First, the preferences of both sides of the market are not completely known to the market participants. The preference of firms over students is dependent on two parameters–student's 'ability' and mutual 'fitness.' The second departure from Azevedo and Leshno [2015] setup is that we focus on a specific case of complete agreement on the firm ranking by the students and also a complete agreement on the 'ability' parameter of the students by firms for most of our discussion. This keeps the model tractable and still keeps the possibility of idiosyncratic preferences of firm and students open through the fitness factor.

Kadam and Kotowski [2015a] have a section dealing with preference discovery over a multi-period horizon of the agents. Their analysis is restrictive to the cases where the first period can be viewed as interviewing but each side is limited to only one interview. They take the preferences of the agents as given and do not consider the strategic choices that we focus on in this paper. Other papers with dynamic matching models [Akbarpour et al., 2015, Baccara et al., 2015] focus on the uncertainty of the number and nature of agents present in an economy and evaluate the puzzle of when to match an accumulated pool of agents. Arnosti et al. [2014] discuss the impact

of congestion as application costs are reduced in a dynamic matching market.

Chakraborty et al. [2010] generalize the study of stable matching markets to include incomplete and interdependent valuations for one side of the market. In their setting, they focus on the ex-post robustness of the match outcomes. However, the approach we take here is that the preferences are revealed before the final match through the preference formation processes. Arnosti [2015] studies the impact of short lists and various types of preferences between the participants in a centralized clearinghouse for a one-to-one matching market. He studies and highlights the preferences of participants that lead to a large number of matching and a greater quality of matching. In his setting, all agents are ex-ante homogenous and hence resolve the strategic decisions purely randomly.

Search-theoretic matching models in the tradition of DMP models [Diamond, 1971, 1982, Mortensen, 1970, 1982, Pissarides, 1979, 1985], have been successfully applied to explain many labor market phenomena. Rogerson et al. [2005] provide a comprehensive survey of the work in this field. The search-theoretic literature seeks an explanation for the equilibrium phenomenon about employment-unemployment spells, layoffs, job switches, wage dispersion, etc. Albrecht et al. [2006] study the impact of multiple applications in a directed search model and find that more than one applications results in inefficient equilibrium with wage dispersion. In this literature, most of the interactions are sequential and the market participants have to decide instantaneously whether or not to exit the market with the current match. However, in many settings especially in the entry-level markets, interviews take place over time and the uncertainty is due to strategic decisions by the agents involved. Through the current paper, we have explored the later phenomenon. There is some recent work in labor economics which use the settings of modern online labor markets which provide a greater transparency about contacts made between employers and potential employees. They analyze experimental and quasi-experimental settings to analyze the impact of reduced search costs on the aggregate outcomes of the market. In an online experiment, Horton [2015] shows that reducing employer search costs increases the number of filled vacancies by 20%.

3 General Model

We now describe the general model and also define the metrics—social surplus and the number of matched agents—we use to compare different interviewing capacity regimes. We describe the elements of the general model in the following order: the market participants, their preferences, and the utilities from a match.

We consider a many-to-one matching market of a finite set of F firms and a continuum of students S of mass 1. We denote the set of firms with a minor abuse of notation as $F = \{1, 2, \cdots, F\}$.

Each firm i wants to match to a continuum of students and hire only up to q_i mass of students. These hiring capacities for all F firms can be summarized as an F-dimensional vector $q \in [0, 1]^F$. All students agree on the ranking of the firms to be $\succ_S$: $1 \succ 2 \succ \cdots \succ F$. Students are of different types. The type $\theta = (\succ^\theta, e^\theta, f^\theta)$ is drawn from a continuous distribution G over $\Theta = \{\succ_S\} \times \Theta_e \times \{-1, 1\}^F$ where $\Theta_e = [0, 1]$.[28] The F-dimensional vector, f^θ summarizes the idiosyncratic component of the firm's preferences over the students. We assume that firm i wants to hire only those students who are known with certainty to be 'fit' for the firm as summarized by the ith component of f^θ, i.e. $f_i^\theta = 1$. The fitness factor for any firm is assumed to be independent of the ability scalar e^θ, independent across the fitness factors for other firms, and is 1 with probability p. The distribution G over Θ is such that it satisfies the above conditions and that the marginal distribution over $[0, 1]$ is uniform. Let $\eta(S')$ be the mass of a measurable set $S' \subseteq S$. We also assume that the firms are on the short side of the market, precisely $\Sigma_{i \in F} \frac{q_i}{p} \leq 1$. Intuitively, this assumption says that there are enough candidates to guarantee that all firms will be able to meet their hiring needs.

A firm and a fit student match to generate a surplus of $2U(i, e^\theta)$ which is split equally between them.[29] The function $U : F \times [0, 1] \to \mathbb{R}^+$ monotonically increases with the student's ability (e^θ) and the desirability of the firm. We further assume that

$$U(i, e^\theta) = h(i)V(e^\theta)$$

such that $h : F \to \mathbb{R}_+$ and $V : [0, 1] \to \mathbb{R}_+$ are the parts of surplus due to the firm and the student respectively. For all $i, j \in F$, $h(i) > h(j)$ if $i \succ j$ and $V(\cdot)$ is an increasing function of the student ability. Essentially, we assume that the total surplus generated will be of increasing differences and separable in the firm and student identities.

We define the economy as $E = [G, q, U]$. In addition to these parameters, the constraints on interviewing determine the market outcome. Each firm M can interview with up to $q_M k_M$ students and each student can interview with up to k_S firms. We can summarize the interviewing constraints for the firms and students as a vector k. In our discussion we will focus on two interviewing capacity regimes, a low-capacity regime (LCr) and a high-capacity regime (HCr). The student interviewing capacity is smaller in the LCr than in the HCr while firm capacities are assumed to be the same.

The timing of the model can be summarized as in Figure 6, which is similar to that in the

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[28]We include the preference $\succ^\theta$ in a student's type although all students' preferences agree. We later relax this requirement in Section 6.2.

[29]Equal sharing of surplus is not required for any of the results. Any fixed surplus splitting arrangement between the firms and students is sufficient.
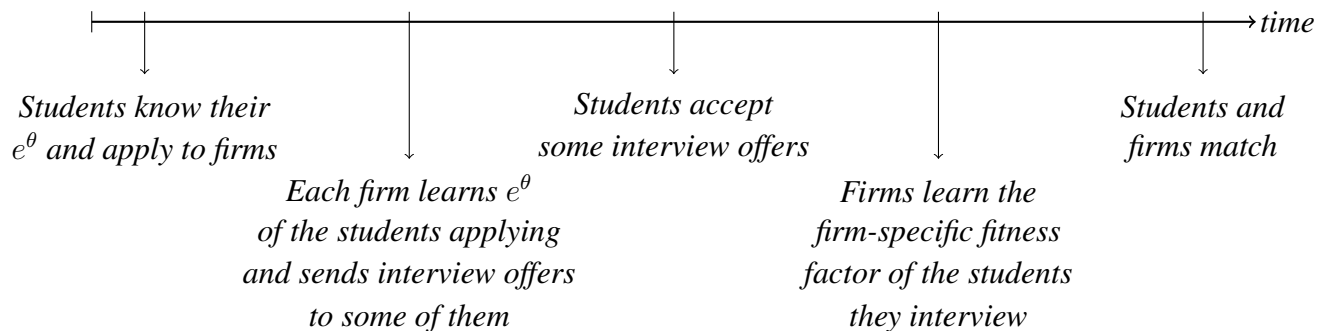
Figure 6: The timing of the model.

example above in Section 1.1. For the purpose of the current discussion, we assume applications are costless and hence all students apply to all firms. Equivalently, we can assume that the firms know e^θ for all the students. Nevertheless, we place the "application phase" in to the timeline. Our purpose of doing so is twofold. First, it relates a firm's discovery of the e^θ values with the realistic phenomenon of students sending their applications including their resumes and letters to the firm. Second, it leaves the model general enough to include application costs, as we plan on doing, in an extension.

3.1 Stable Matching and Equilibrium

We start by defining a nondegenerate set of students to remove among other things, those sets that have zero measure holes.

Definition 1. A *nondegenerate set of students* is defined as a set $X \subseteq \Theta_e$ such that the following holds.[30]

1. If there is a decreasing sequence $e^{\theta^j} \in X$ that converges to e^θ, then $e^\theta \in X$. Also if there is an increasing sequence $e^{\theta^j} \in X$ and e^{θ^j} converges to 1, then $1 \in X$.

2. There does not exist a student with $e^\theta \in X$ such that for every decreasing sequences e^{θ^j} that converge to e^θ, we can find a corresponding J such that for all $j' \geq J$, we have $e^{\theta^{j'}} \notin X$.

Condition (1) implies that for every sequence of students, who are of progressively lower ability and have an interview offer from a firm, the limit ability student is also included in the interview offer by the firm. The next part looks at a similar increasing ability sequence converging to ability

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[30]Technically, we want to restrict attention to finite unions of intervals of the form $[a, b)$ if $b < 1$ and $[a, 1]$ for $a \neq 1$.

of 1. Condition (2) implies that for every student who is an element of the set, we can always find a neighborhood on the right, i.e. students with higher ability, who are also included in the set.

We will work with interview offers and match sets which respect the firm interviewing capacities and firm hiring capacities respectively. In a continuum setting, it is always possible to include or exclude countable massless points and change the interview offers (or matches) while still maintaining the capacity constraint. The nondegeneracy requirement rules out such multiplicities at the interviewing and matching stage.

Definition 2. A set of *nondegenerate interview offers* from firm i is defined as $\sigma_i : 2^{\Theta_e} \to 2^{\Theta_e}$ such that the following holds.

1. $\sigma_i(X) \subseteq X$ for all $X \subseteq \Theta_e$.

2. $\sigma_i(X)$ is a nondegenerate set of students.

Condition (1) in the above definition is just a translation of what an interview offer would mean (even in a discrete setting). It ensures that the function maps a set of students for each firm to a subset of students to reflect the decision of a firm to extend interview offers to a subset of students who applied.

A firm does not want to hire those students whom it has not interviewed. Hence, the preferences for all firms, which we denote as P_F, are defined with respect to a specific interview assignment that results from the interview offers and acceptances from the students. We now define stability for a given set of preferences and denote $\varnothing$ as the outside option for a firm which is preferred over a misfit partner or a partner with unknown fitness.

Definition 3. A *stable matching* with respect to preferences P_F and P_Θ for all the firms and students is a function $\mu : F \cup \Theta \to F \cup \Theta \cup 2^\Theta$ such that

1. $\mu(\theta) \in F \cup \{\theta\}$ for all $\theta \in \Theta$.

2. $\mu(i) \subseteq \Theta$ for all $i \in F$ such that $\eta(\mu(i)) \leq q_i$ and for all $\theta \in \mu(i), \theta \succ_{P_i} \varnothing$.

3. $i = \mu(\theta)$ if and only if $\theta \in \mu(i)$.

4. $\nexists$ a firm i and a student θ such that $\theta \succ_{P_i} \varnothing$, $i \succ_{P_\theta} \mu(\theta)$, and either $\eta(\mu(i)) < q_i$ or $e^\theta > e^{\theta'}$ for some $\theta' \in \mu(f)$.

Our definition is identical to the standard notion of stability. The first three conditions focus on an individually rational many-to-one matching between the firms and the students. Specifically, the

first condition ensures that a student is matched to a firm or itself. A firm is matched to a subset of students such that its hiring quota is not violated and it prefers to match with all the students rather than not matching with some and leaving some positions unfilled. The third condition ensures that a firm is matched to only those students who are matched with it. The fourth condition ensures that there is no blocking set consisting of a firm and a student.[31] We now define a nondegenerate stable matching to avoid multiplicities of the kind that we ruled out when we defined nondegenerate interviewing offers.

Definition 4. A *nondegenerate stable matching* with respect to preferences P_F and P_Θ for all the firms and students is a matching μ such that it is stable and the set of students matched to each firm is a nondegenerate set.

We use nondegenerate stable matching using Azevedo and Leshno [2015]'s insight to avoid inconsequential zero-mass multiplicites which could be artificially created in a continuum setting.

Note that if the preferences for firms and students are formed after nondegenerate interview offers, the stable matching can still leave out a countable number of acceptable students.[32] We rule out such possibilities using the above definition. We implicitly assume that the final matching is generated using a student-proposing deferred acceptance algorithm,[33] but will shortly prove that the choice of assignment mechanism (or the implicit presence of a centralized matching mechanism), is inconsequential to the final outcome. Now we are ready to define the equilibrium of the application, interviewing, and matching game.

Definition 5. An *equilibrium* of the application, interviewing, and matching game is

1. a strategy of applications for each student, $\sigma_S : \Theta_e \to 2^F$,

2. a strategy for each firm to extend interview offers, $\sigma_i : 2^{\Theta_e} \to 2^{\Theta_e}$ for all $i \in F$,

3. a strategy of interview acceptances for each student $\sigma_{\theta_e} : 2^F \to 2^F$ for all $\theta_e \in \Theta_e$, and

4. a set of preference reports P_θ for all $\theta \in \Theta$ and P_M for all $M \in F$

such that each firm and student find its/her strategies optimal given those of the other firms and students and a nondegenerate stable matching results from the use of student-proposing deferred acceptance algorithm on the reported preferences.

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[31]Under responsive preferences, pairwise stability is sufficient for a more general stability concept defined with a blocking coalition of a firm and a set of students [Roth, 1985].

[32]This is true because the absence of blocking pair requires that mass of students matched to the firm be strictly less than its hiring quota.

[33]See Gale and Shapley [1962] or Roth [1990] for the description of the deferred acceptance algorithm.

An equilibrium comprises of strategies for all the firms and students at each stage of the game–application, interview offers, acceptance of the interviews and the final matching. We ruled out strategies that may differ in zero measure sets using nondegenerate interview offers and matching. There is still some possibility of artificially different looking equilibria in the following sense. Consider an equilibrium where firm M extends interview offers to all students with ability in $[e_1^\theta, e_2^\theta)$ and all those students reject firm M's interview offers. Consider a second equilibrium where firm M does not extend interview offers to these students in the first place. This following condition identifies these equilibria as essentially identical. Two equilibria are *essentially identical* if 1) the interview stage matching is identical, and 2) the final matching is identical.

Definition 6. An equilibrium is *essentially unique* if all equilibria that may exist are essentially identical to each other.

We use the standard measure of welfare, the social surplus generated from a matching. We also keep track of the number of matched agents to measure the aggregate employment.

Definition 7. The *social surplus from a matching* is defined as the sum of the total utilities from a match outcome.

$$\text{Surplus of a match } \mu = \sum_{i \in F} \int_{\mu(i)} 2U(i, e^\theta) d\eta(\theta)$$

Definition 8. The *number of matched agents* is defined as the total number of positions which are filled in the match outcome.

$$\text{Number of matched agents in a match } \mu = \sum_{i \in F} \eta(\mu(i))$$

The surplus of a matching measures the overall efficiency of the match and is a useful benchmark to evaluate the impact of the interviewing capacities. The number of matched agents measures the number of positions that get filled up. We also use surplus and the number of matched agents in the context of a single firm to mean the surplus and the number of positions filled for the firm much like the above definitions.

4   Equilibrium existence

We know that the firms and students face capacity constraints on the number of interviews they can conduct or participate in. We start by evaluating the strategies from the end of the timeline.

We describe the choices made at the preference reporting stage, the interviewing stage, and then combine them in to an equilibrium existence result.

4.1 Final match

The final match is generated by a student-proposing deferred acceptance algorithm. The following lemmas, which are proved in Appendix Section B.1, establish the final outcome and the strategic choices. All students agree over the ranking of the firms and find truth-telling (trivially) optimal.

Lemma 1. *All students report their true preference at the preference reporting stage.*

For a many-to-one matching, we know that truth-telling need not be optimal for the firms (Roth [1990] Theorem 5.10 and 5.14). The choice for the firms is less obvious but it turns out that the firms also tell the truth.

Lemma 2. *Firms' optimal strategy includes truth-telling, i.e. each firm lists all the acceptable students in the correct order.*

We know that a stable matching exists for any set of reported preferences from Gale and Shapley [1962]. The uniqueness is related to the uniqueness in Azevedo and Leshno [2015] but we give an independent proof which is applicable in our setting.

Lemma 3. *There exists a stable matching and it is unique for the reported preferences by firms and students.*

The uniqueness of the stable matching proves that the choice of the stable matching algorithm or even the existence of such a centralized procedure to obtain a matching is inconsequential as long as we focus on the final outcomes which are stable. We nonetheless continue to use the lens of student-proposing deferred acceptance algorithm as it splits the decision for the students and firms in to interviewing decisions and preference reporting decisions, making the analysis more manageable and in line with the existing literature. We could have completely abstracted away from a centralized mechanism and suggested that one of the possibly many stable matchings are randomly chosen after the preferences are formed.

4.2 Interviewing stage

With a unique stable matching and no strategic choices for the firms or students at the preference reporting stage, we focus on the strategic choices at the interviewing stage. A student faced with

more interview offers than her capacity k_S, has to decide which ones to accept. We make the following assumption which is motivated from a distaste for interviewing to simplify students' strategic choices significantly.[34]

Definition 9. At equilibrium, a firm M is *indifferent between interviewing* two sets of students S_1 and S_2, if the final set of students matched is exactly identical across the two choices, all else equal.

Assumption 1. *Distaste for interviewing If a firm is indifferent between interviewing a set S_1 and set S_2 of students such that $S_1 \subset S_2$ and $\mu(S_1) < \mu(S_2)$, then the firm selects the smaller set S_1.*

This assumption gives us the following lemma and proves the unique strategies used by the students and firms.

Lemma 4. *The nondegenerate interview offers by firms are uniquely solvable by iterated elimination of dominated strategies when all firms have a distaste for interviewing. Moreover, the students accept the best interview offers up to their capacities.*

Thus, we note that the students always select the best interview offers they receive but the choices for the firms are not as obvious.

Proposition 1. *All firms other than the best firm, need not always interview the best students when filling up its interview capacity.*

Firm 2's strategic choice in the high-capacity regime in Section 1.1 confirms this conclusion and we skip the proof.

4.3 Essentially unique equilibrium

We prove the the existence of an essentially unique equilibrium outcome although there may be many other equilibria which differ only in zero measure sets.

Theorem 1. *For the economy $E = [G, q, U]$ where agents face interviewing constraints described by k, there exists an equilibrium and it is essentially unique.*

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[34]If we do not assume this, we just shift the burden of strategic choice on to the students. For instance, consider the interview offers from firm 1 in the example in Section 1.1. Firm 1 could have extended interview offers to students with ability $e^\theta \in [0.61, 1]$. However all students in the ability range $[0.61, 0.65)$ would choose to reject the interview offers in the low-capacity regimes as they know that inspite of getting interview invitations they will never get the final job offer. Firm 1 will exhaust its needs from interviewing the top 0.35 mass. We abstract away from all these strategic choices by making the above simplifying assumption.

We provide a brief sketch of the proof here and relegate the details to Appendix Section B.1. The assumption about firms' interview aversion pins down the students' strategic choices and in turn those of the firms following the interviews. The interviewing decision of the best firm is not dependent on that of any other firm or the students as per our discussion above and Lemma 4. The best firm, i.e. firm 1, interviews the required number of students at the top to fill up its quota q_1. It knows that the students are found fit with probability p and hence the top q_1/p mass will be interviewed if the interviewing capacity allows (i.e. $q_1/p < k_1 q_1$). The second firm's choice will account for the fact that some of the top students have an interview offer from the first firm and will be available to it only when they are found fit for itself but are found misfit with firm 1. This can be iteratively continued. Thus, the best response can be found by a firm to the strategies by other better firms. Thus, an equilibrium exists where the firm's interview offers can be found and the students accept the best interview offers. Moreover, the continuity along the ability dimension and the exact probabilities give us a unique strategy for each firm which will be optimal up to the redundancies of zero mass of students. Thus, the equilibrium is essentially unique.

4.4 Equilibrium characterization

We established a firm's optimal strategy to interview a discontinuous set of students along the ability dimension in Proposition 1. We define such firm strategies as interview offer strategies with 'gaps.' More formally, we mean the following.

Definition 10. Firm i has *gaps in its interview offers* if there exists a student θ_1 with ability e^{θ_1} such that $e^{\theta_1} \notin \sigma_i(\Theta_e)$ and there are at least two students $\hat{\theta}, \tilde{\theta}$ such that $e^{\tilde{\theta}}, e^{\hat{\theta}} \in \sigma(i, \Theta_e)$ such that $e^{\theta_1} \in (e^{\hat{\theta}}, e^{\tilde{\theta}})$.

Some firms may extend their interview offers with lots of gaps and we define a concept which will be helpful for our analysis.

Definition 11. Firm i has a *sufficiently large number of gaps* in its interview offers if for every pair of students $\hat{\theta}, \tilde{\theta}$ with $e^{\tilde{\theta}}, e^{\hat{\theta}} \in \sigma_i(\Theta_e)$ such that the two students have different number of interviews from firms better than i, there exists a student θ_1 with ability $e^{\theta_1} \in (e^{\hat{\theta}}, e^{\tilde{\theta}})$ such that $e^{\theta_1} \notin \sigma_i(\Theta_e)$.

Note that firm 2's interview offers in the example in Section 1.1 had sufficiently large number of gaps. We present the above conditions to present some general sufficient conditions under which we get a decrease in the number of matches.[35]

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[35]Loosely speaking, the firms have a sufficiently large number of gaps in their interview offers when the utility for students is not identical and the firms do not have large interviewing capacities.

5 Main Results

We will compare the equilibrium outcomes under two regimes—a low-capacity regime and a high-capacity regime. Consider the students' interviewing constraint in the low-capacity regime to be $k_S = k_{LC}$ and that in the high-capacity regime to be $k_S = k_{HC}$. The firm interviewing capacities are assumed to remain the same across the two regimes. We also say that all firms do not have excess interview capacity under the low-capacity regime if at equilibrium for each firm the mass of students interviewed is exactly equal to its interviewing capacity. Mathematically, we mean for all $M \in F$, we have $\eta(\sigma_M(\Theta_e) \cap \{\theta | M \in \sigma_{\theta_e}(\cdot)\}) = q_M k_M$. We have the following two propositions about the impact of an increase in the students' interviewing capacity.

Proposition 2. *When the interviewing regime moves from a low-capacity regime to a high-capacity regime, the utilitarian surplus of the match weakly increases. If firms do not have any excess interview capacity and the surplus strictly increases, the number of matched agents strictly decreases for at least one firm.*

Proposition 3. *When the interviewing regime shifts from a low-capacity regime to a high-capacity regime and the interviewing offers are different, there exist two threshold abilities $e^{\theta_1}(< 1)$ and $e^{\theta_2}(> 0)$ such that:*

1. *All students with ability at or above e^{θ_1} are weakly better off and there exists a non-zero mass of students who are strictly better off, in terms of the expected utility from a match as well as the ex-ante probability of finding a match.*

2. *All students with ability strictly below e^{θ_2} are weakly worse off and there exists a non-zero mass of students who are strictly worse off, in terms of the expected utility from a match as well as the ex-ante probability of finding a match.*

Moreover, all firms are weakly better off and there exists a non-empty set of firms such that all firms in that set are strictly better off.

The broad idea of the proofs of these propositions can be drawn from our discussion in the example in Section 1.1 above. When students' interviewing capacity increases, medium-tier firms see a change in the set of students they can consider. A medium-tier firm can choose to extend interview offers to some students at the top for whom the added interviewing capacity has relaxed their constraints on the number of interviews they can accept. If such a firm has a binding interviewing capacity, it may choose to get the better students, who are available with a lower probability, than the average students, who are available with a higher probability. Moreover, the

first such firm to make such a switch has a positive externality on the subsequent firms as they are left with better quality student pools. With the better quality student pools the next firm, too, chooses to interview more in the region of high 'ability' students and less in the region of average students. Thus, the surplus for each firm and hence the utilitarian surplus, weakly increases.[36] When the firm interview offers are different, there will be some students who receive incremental interview offers from firms in the high-capacity regime. These students will be better off. As the firms' interviewing capacities bind, new offers to the above set of students come at the cost of not extending them to some lower ranked students and these students are worse off.

We now identify some sufficient conditions under which an increase in the utilitarian surplus will necessarily come along with a reduced number of matched agents.

Proposition 4. *Suppose that all firms have a sufficiently large number of gaps in their interview offers in the LCr and the surplus $U(\cdot, \cdot)$ is concave with a concave first derivative on the student ability parameter. When the interviewing regime moves from LCr to HCr then without excess interviewing capacity for the firms, the number of matched agents in the overall match weakly decreases. If the surplus strictly increases, the number of matched agents strictly decreases.*

The details of the proofs of all the propositions above are presented in the appendix Section B.2.

6 Applications and Extensions

In this section, we extend the main model along three directions and discuss the impact of correlated fitness factors, diversity in student preferences over firms, and marginal costs of interviewing on social surplus and individual welfare.

6.1 Correlated fitness factors

We have maintained the assumption that the firm specific fitness factors are independent of each other and of the ability parameter. In many economic settings, it is possible that a student who is found misfit for a particular firm will be misfit for some other firm with a higher probability. In the first extension, we relax the independence assumption of fitness factors across firms. In the second subsection, we will discuss a few possibilities for the correlations in fitness factors. Fitness factor can be correlated with the student ability or dependent on the firm identity.

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[36]A firm can always choose to interview the same set of students it was interviewing in the LCr but possibly may decide not to, which implies that the firm is weakly better off.

6.1.1 Fitness factor for a firm correlated with fitness factors for other firms

We assumed in our earlier discussion that the probability that a student is found fit for any firm is equal to p. This was true for both the unconditional probability and the probability conditional, say, on being fit for some other firm. We maintain the assumption about the unconditional probability. However, we now move towards a correlated setting.

If we assume a positive correlation in the firm fitness factors, a student found fit for a particular firm will be fit for another firm with a probability higher than p. With the same unconditional probabilities of being fit (p), the probability that a student, who was found misfit by some firm, is found fit by another firm will be lower than p.[37] Consider a symmetric setting where the correlation between all firms is identical. The fitness factors are binary variables and we can measure the correlation using the phi coefficient.[38] In a symmetric setting, we assume that all firm pairs have the same phi coefficient. The correlation matrix for the fitness factors for all the firms will be represented by a $F \times F$ matrix where all the off-diagonal elements will be equal.

However, in the current setting we also care about correlations with more than one variable beyond what can be captured in the usual correlation matrix, e.g. the probability that a student is fit for firm 3 knowing that she is found misfit for firms 1 and 2, etc. The correlation in these conditional relationships is relevant for us. Although a fully general setting requires a specification of an exponential number (in F) of parameters which are internally consistent, the setting simplifies significantly under symmetry. We can summarize the correlation information in a $F-$dimensional vector $\tilde{p} = \begin{bmatrix} p_{[0]} & p_{[-1]} & p_{[-2]} & \cdots & p_{[-(F-1)]} \end{bmatrix}'$ where $p_{[-i]}$ is the factor by which the probability of

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[37]Consider a simple example where the unconditional probability that a student is found fit for a firm is 0.5. Moreover if the student is found fit for one firm, then the probability that the student is found fit for the second firm is 0.8. The unconditional probability of finding the student fit for firm 2 can also be evaluated as the following. Probability (student is fit for firm 1) $\times$ Probability (student is fit for firm 2 given that she is fit for firm 1) $+$ Probability (student is misfit for firm 1) $\times$ Probability (student is fit for firm 2 given that she is misfit for firm 1) $= 0.5 \times 0.8 + 0.5 \times x = 0.5$. This suggests that if the student is found misfit for firm 1 then she will be found fit for firm 2 only with probability 0.2.

[38]Suppose we have two random variables, fitness factors for two firms M and N and the different probabilities are summarized below.

	Fit for N	Misfit for N	Sum
Fit for M	p_{11}	p_{10}	$p_{1\cdot}$
Misfit for M	p_{01}	p_{00}	$p_{0\cdot}$
Sum	$p_{\cdot 1}$	$p_{\cdot 0}$	1

The statistical measure of association, the **phi coefficient**, for the fitness factors can be calculated as follows.

$$\phi = \frac{p_{11}p_{00} - p_{10}p_{01}}{\sqrt{p_{1\cdot}p_{0\cdot}p_{\cdot 1}p_{\cdot 0}}}$$

a student being fit is reduced given that the student is misfit for i other firms.[39] For instance, a student found misfit by 3 firms is found fit for a fourth firm with probability $p_{[-3]} \times p$.

Consider an example with 3 firms where the correlation vector $\tilde{p} = [1 \ 0.8 \ 0.64]$. The probability that a student is found fit for firm 2 if she is a misfit for firm 1 is $0.8p$ and vice versa. Similarly the probability of finding a student as a fit for any given firm (say firm 3) if she is a misfit for the other two firms is $0.64p$.[40]

The distribution over student types is again over $\{\succ_S\} \times [0,1] \times \{-1,-1\}^F$ where $\succ_S$ is the common preference of all students over the firms. This distribution G^{corr} agrees with the correlation in the fitness factors as we described above and thus our economy now becomes $E^{corr} = [G^{corr}, q, U]$.

We focus on essentially unique equilibria and maintain all our other assumptions about strategies and utilities for the firms and students. We discuss the extension of the equilibrium existence result, theorem 1, here. We relegate the discussion about the analogues of Proposition 2, Proposition 3 and Proposition 4 to Appendix Sections B.3 and B.4.

Theorem 2. *In an economy E^{corr}, where agents face an interviewing constraint k, there exists an equilibrium and it is essentially unique.*

We provide the detailed proof in appendix Section B.1. The existence of uniqueness is intuitively very similar to Theorem 1. The iterative evaluation of the firm strategies now factors in the correlation vector that we described above but proceeds as before. The first firm decides its interview offers so that it interviews just enough students to fill the number of job openings it has or if that is not possible, until its interviewing capacity is reached. The presence of correlation does not impact the choice made by the first firm. Firm 2 faces two regions of students. One where the

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[39] Suppose we are interested in finding the probability that a student is fit for firm j given that she is a misfit for some i firms. Due to the symmetry of the setting, the identity of the i firms is irrelevant and the only parameter that will feature in the analysis is $p_{i]}$. Also note that $p_{[0]} = 1$ but we include it for the sake of completeness.

[40] The **phi coefficient**, for the fitness factors for two firms in this economy will be 0.2. It is calculated as follows.

Probability of the students being misfit for one firm and fit for the other $(p_{10}$ or $p_{01}) = (1-p)p_{[-1]}p$

Probability of the student being fit for both firms $(p_{11}) = p - (1-p)p_{[-1]} \times p$

Probability of the student being misfit for both firms $(p_{00}) = (1-p)(1 - p_{[-1]} \times p)$

$$\phi = \frac{(p - (1-p)p \times p_{[-1]})((1-p)(1 - p \times p_{[-1]})) - (1-p)^2 p^2 p_{[-1]}^2}{p(1-p)}$$

$\phi = 1 - p_{[-1]} = 0.2$

It agrees with the intuition that smaller the value of p_{-1} greater the correlation between the fitness factors. A value of $p_{-1} < 1$ indicates that the correlation is positive and $p_{-1} > 1$ indicates a negative correlation between the fitness factors.

students have an interview offer from the first firm and another consisting of the remaining students who do not. The students in the region with an interview from the better firm are available only when they are found a misfit with the first firm and now further due to the presence of correlation with smaller probability pp_{-1}. Firm 2 can evaluate its interview offers optimally given the choices made by firm 1. Thus, the strategies for all the firms can be evaluated to prove the existence of an equilibrium. Moreover, since the argument proceeds as in the earlier case with iterated elimination of dominated strategies, the equilibrium is essentially unique. This suggests, and we prove in the appendix Sections B.3 and B.4, that the original analysis applies in this case.

6.1.2 Fitness factor of firms dependent on the student ability and the firm's identity

The fitness factor for a firm was assumed to be independent of the student ability and the firm's identity. We now consider situations where the probability of finding a student fit for some firm depends on the ability of the student and/or the rank of the firm. We consider the following three cases.

1. The fitness probability is a function of the student ability and is given by a function $p : \Theta_e \to [0, 1]$.

2. The probability depends on the firm identity but not on the student ability and can be summarized by an F-dimensional vector. The ith component of the vector is p_i which corresponds to the probability of being fit for firm i.

3. The probability depends on both student ability and firm identity and be presented by F functions where the ith function corresponds to firm i's fitness probability function, given by $p_i : \Theta_e \to [0, 1]$.

The existence result extends very easily when the fitness factor for a firm-student pair remains just dependent on the student ability but still independent across firms, i.e. $p(e^\theta)$. The student decisions about which interview offers to accept continue to remain straightforward and hence the interviewing strategy for the firms can be solved using iterative reasoning. These arguments hold even when the probability of a student being fit is dependent on the firm identity but decreases for less desirable firms. This assumption still maintains the simplicity of student decisions when faced with many interview offers, i.e. pick the best interview offers from any pool. We get the results about the existence of an essentially unique equilibrium, the increase in surplus and the decrease in the number of matched students for at least one firm following a move from a low-capacity regime to a high-capacity regime in these settings. To avoid repetition, we state and prove

these in appendix Section B.7. These arguments also naturally extend when the fitness factor depends on both the firm identity and the student ability as long as the probability of being fit for a given student decreases with the decreasing firm desirability, i.e. $p_M(e^\theta)$ decreases as the firm desirability decreases. Note that, when the fitness factor varies with firm identity in any other way or specifically increases for less desirable firms, the iterative reasoning will not help us for the equilibrium existence.[41] We do not investigate this issue any further in this work.

6.2 More general student preferences

We have maintained the assumptions that students unanimously agree about the ranking over all firms and that the firms agree entirely about the student ability parameter. We relax these assumptions and investigate equilibrium existence. We also present the result about decrease in welfare with increased interviewing capacities for the students.

Student preferences often assume more complex forms than complete agreement over firm rankings. In some cases, there is a broad agreement about the 'tier' to which a particular firm belongs and everyone in the market agrees on the ordering of the tiers. However, within a particular tier there might be idiosyncratic preferences among the students. Similarly, firm's ex-ante evaluations for students can have some correlation but need not be exactly identical. In the current discussion, we focus on student preferences which are 'tiered.' We discuss the related case where the students' ability evaluations differ across firms in Appendix Section A.5 as the results are similar. We start by defining block-correlated preferences.

Definition 12. The marginal distribution of G over $\Theta^\succ$, i.e. the distribution of student preferences, is *block correlated* if there exists a partition $F_1, F_2, \cdots, F_B$ of the firms such that

  1. If firm $i \in F_b$, $i' \in F_{b'}$ and $b < b'$ then all students prefer i over i'.

  2. Each student's preference within each block are uniform and independent.

Coles et al. [2013] use a variant of block-correlated preferences and demonstrate the value of signaling in a one-to-one matching market. We use these preferences for the students and establish the existence of an equilibrium. The student preferences are not identical. They are drawn from permutations of possible preferences over firms, say $\Theta^\succ$. The distribution G is over $\Theta^\succ \times [0, 1] \times \{-1, 1\}^F$.

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[41]Although the iterative reasoning fails, it is straightforward to analyze the equilibrium existence as we did in the example in Section 1.2.

Our definition for block correlated preferences differs from that in Coles et al. [2013] as we do not impose the restriction that the firm preferences over students have to be uniform and independent. We continue with our earlier assumption that the preferences of firms are defined by the two components–ability and firm-specific fitness factor, i.e. e^θ and f^θ. The type of a student also includes her preference over the firms which is not trivially unique as it was in the earlier setting. Each student θ has a type $(\succ^\theta, e^\theta, f^\theta)$ where $\succ^\theta$ is drawn from block-correlated preferences over the firms. The distribution of the student types is such that the ability value is drawn over $[0, 1]$ uniformly and independently with respect to the other elements of the student type, including the preference over firms. The fitness-factor for any given firm is also independently drawn and is 1 (indicating a fit) with probability p and -1 (indicating a misfit) with the complimentary probability $1 - p$. We now define a rank of a firm within a particular block which will be helpful for the exact specification of utilities for the students.

Definition 13. The *rank within a block* of a firm i under preference $\succ$, which we denote as $\rho(\succ, i)$, is defined as the difference between the rank of i on $\succ$ and the sum of the number of firms in better blocks than the block to which i belongs. If $i \in F_b$ then $\rho(i, \succ) = $ rank of i under $\succ - \Sigma_{i=1}^{b-1} B_i$.

Intuitively, the rank within a block provides the position of a firm under the preference only within its block. Consider an example with 4 firms labeled 1 through 4 where they belong to either of the two blocks B_1 and B_2. The first block $B_1 = \{1, 2\}$ and $B_2 = \{3, 4\}$. The student preferences are with equal probability any ordering from the following set $\{1234, 2134, 1243, 2143\}$. Consider the preference ordering 2143 which implies that the student's preference is $2 \succ 1 \succ 4 \succ 3$. The rank within the block for firm 4 is 1, i.e. $\rho(\succ, 4) = 1$.

In our earlier discussion, we had assumed that a firm M and a fit student θ generate a surplus of $2U(M, e^\theta)$. The firms were ordered and all the students agreed on their preferences over these firms. Now the students agree on the tiers. The role of the firms' rank is taken by the rank of the tier in this more general setting. In other words, in our main model there were F blocks and each block had a single firm. Now we assume that the expected surplus generated by a firm in block F_b and a fit student with ability e^θ is given by $2U(b, e^\theta)$. The firm always gets half of this surplus, $U(b, e^\theta)$. However, the utility a student gets from a match is slightly different as per the students' idiosyncratic preference over firms within a block. More precisely, the student utility is the sum of $U(b, e^\theta)$ and an idiosyncratic element given by $\epsilon(\rho(f, \succ_i), b)$ where $\epsilon(\cdot, \cdot) : F^2 \to \mathbb{R}$ has the following properties.

1. The average of $\epsilon(x, b)$ is 0 over all firms in block F_b, i.e. if $x \in \{1, 2, \cdots, B_b\}$ where B_b is the number of firms in block F_b then $\Sigma_{x=1}^{B_b} \epsilon(x, b) = 0$.

2. $\epsilon(x, b)$ is decreasing in x.

We can now describe the economy as $E^{BC} = [G^{BC}, q, U, \epsilon]$. The timing of the model is the same as we discussed in our main model in Figure 6. We can now prove the following theorem about equilibrium existence.

Theorem 3. *In a block correlated economy E^{BC}, there exist infinitely many equilibria and all of them have the following features.*

1. *The equilibria are essentially identical to each other or*

2. *The equilibria differ in the firm interviewing strategies such that any given student always gets the same number of interview offers from a given block.*

We provide the detailed proof in Appendix Section B.5. The existence follows almost similarly to the existence in the main theorem except that here we evaluate the strategies iteratively for each block at a time rather than iteratively solving them for each firm. The uniqueness is up to the number of firms in a given block that extend their interview offers to a student for a given value of e^θ. This leads to the multiplicity of equilibria.

When the students have different preferences, interviewing capacities act as a mechanism to sort the students in different interview positions as per their preferences. When the capacity increases, there may be reduced sorting and this could decrease the welfare. The following proposition captures this intuition.

Proposition 5. *When the student preferences are block-correlated and the interviewing capacity for students is increased, the overall surplus of the match does not always increase.*

We only need to provide an example where the overall surplus decreases with an increase in capacity. The example in Section 1.3 serves this purpose and we skip the proof. This serves as a caution that increasing capacities for students may not be enough to increase the social welfare in such settings.

However, when the students do not agree about the preferences over firms, there is room for students to signal their preferences to the firms [Coles et al., 2010]. If a student can signal the first ranked firm about her preference then the student will benefit from getting an interview offer from her favorite firm, if the firm responds to such signals. Moreover, a firm will find it optimal to respond to such a signal as it will ensure that the firm's interview spots are used in the most effective manner. This is possibly, a win-win situation. However, due to the potential cheap talk

nature of these signals, a student has incentives to send such signals to every firm and that in turn will cause the signal to lose its value.

Limited credible signaling can solve the problems associated with the cheap talk nature of the signals. It is analyzed in Coles et al. [2013] in a one-to-one matching setting between firms and students where the firms make up to one offer to a particular student and the students have a chance to send up to one signal to a firm before the firms send their offers. However, their analysis is silent about the impact when the firms also agree to some degree on the desirability of the students as we would expect in many situations. This question is pertinent especially given the insight in Kushnir [2013] where introduction of signaling harmfully impacts overall welfare. In his setup the student preferences are identically aligned with very high probability close to 1 and are idiosyncratic with the complementary probability. Our discussion here also elucidates upon the robustness of these insights when firms can extend more than one (interview) offers. When there are more interview offers than the recruiting capacity for each firm, a firm can choose to respond to some 'signalers' and some 'non-signalers.' We show that signaling improves welfare in our setting under certain conditions and present it in Appendix Section B.6

6.3 Marginal Interviewing Cost Setup

In the discussion so far, we assumed that the interviewing constraints manifested as capacities for the students and firms on the number of interviews. We now present an example similar to the one in Section 1.1 with marginal interviewing costs. It is natural to consider settings where the marginal cost is increasing. We show that our insights hold under convex interviewing costs.

Consider two firms 1 and 2 and a continuum of students. A student's type is given by (e^θ, f^θ) where e^θ is the ability of the student drawn uniformly from $[0, 1]$. The fitness factor of a student with firm 1 is 1 with probability 0.5 and that with firm 2 is 1 also with probability 0.5. The two fitness factors are independent for all students. All students prefer firm 1 over firm 2. Each firm has a hiring quota of 0.2 mass of students. The marginal cost for firm M to interview the kth mass of students is given by $c_M(k) = \frac{0.6}{M}k^2$ and aggregate cost for firm M to interview k mass of students is $C_M(k) = \int_0^k c_m(t)dt = \frac{0.2}{M}k^3$. The interviewing costs for the students can be given by a function $c : \{1, 2\} \to \mathbb{R}$ where $c(n)$ represents the total cost of interviewing with n firms.[42] The surplus generated by a student with ability e^θ and firm M is given by $2U(M, e^\theta)$. However, the net surplus will be the surplus after accounting for the interviewing costs.

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[42]This can easily be transformed in to a marginal interviewing cost but for the ease of exposition we describe them as aggregate costs for the students. However it is clear that this is not a setting where interviewing constraints appear as capacities.
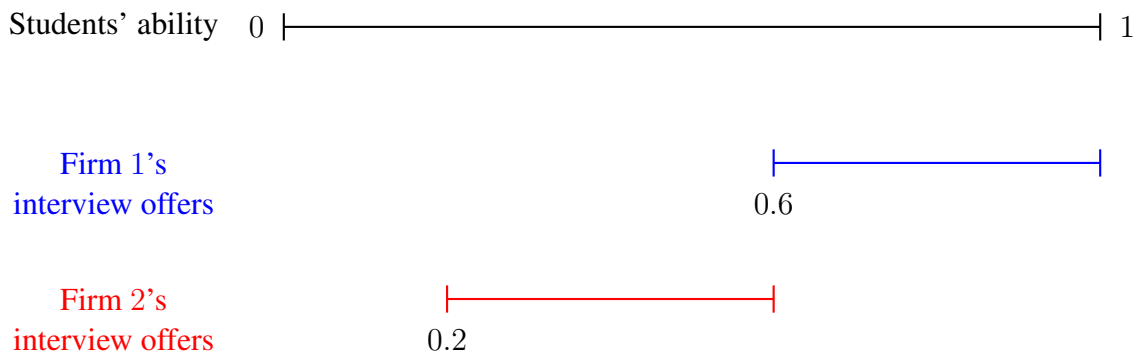
Figure 7: Interview offers by firms 1 and 2 when students' interviewing costs are high.

$$U(k, e^\theta) = \begin{cases} \frac{e^\theta}{k} & \text{if the student is a fit for firm } k, \\ -\infty & \text{if the student is a misfit for firm } k. \end{cases}$$

We start our analysis by assuming that the student interviewing costs are high. Here, the cost of interviewing with the one firm is 0.04 and that with the two firms is 0.2. Thus, a student considers the marginal cost of $0.2 - 0.04 = 0.16$ when she decides if she wants to interview with 2 firms instead of 1 firm. Note that this is not to say that the student pays different costs based on the firm identity (firm 1 versus firm 2). It only depends on the number of interviews the students decides to take up. It is straightforward to verify that the interview offers in this setting will be as shown in Figure 7.[43] Firm 1 extends interview invitation to students with ability in $[0.6, 1]$ and firm 2 extends interviews to those with ability $[0.2, 0.6)$.

Now, we evaluate the impact of reducing the interviewing costs for students. Suppose these costs of interviewing are reduced to a half of their values. The students' cost of interviewing for one firm is 0.02 and for two firms is 0.1. Thus, a student considers the marginal cost of $0.1 - 0.02 = 0.08$ when she decides if she wants to interview with 2 firms instead of just 1 firm. The interviewing strategies for the firms are described in Figure 8.[44] Firm 1 continues with the

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[43]Note that due to the interviewing costs for students, even the best students will not accept an interview offer from firm 2, if they are already being interviewing with firm 1. The expected share of surplus if a student accepts an interview offer from firm 2 $=$ Probability (misfit for firm 1) $\times$ Probability (fit for firm 2) $\times U(e^\theta, 2) = 0.5 \times 0.5 \times \frac{e^\theta}{2} < 0.16$ for all e^θ. The firms' optimal decisions are explained in appendix Section A.6.

[44]With the reduced interviewing cost for the students at least some students who get an interview offer from firm 1 will take up firm 2's interview offers. The expected surplus from an interview offer from firm 2 is $0.125e^\theta$ as explained in the footnote above. All students with $e^\theta \geq \frac{0.08}{0.125} = 0.64$ can accept the second interview offer from firm 2, should one be made. Firm 2 extends interview offers to students with ability in $[0.353, 0.6) \cup [0.705, 1]$ which are roughly represented above. The number of matched agents to firm 2 is slightly less than its required number 0.2. We explain the optimal decisions by firm 2 and calculation about the reduced number of matched agents in the appendix

Students' ability　0 ├────────────────────────────────────┤ 1

Firm 1's
interview offers
　　　　　　　　　　　　　├──────────────────┤
　　　　　　　　　　　　　0.6

Firm 2's
interview offers
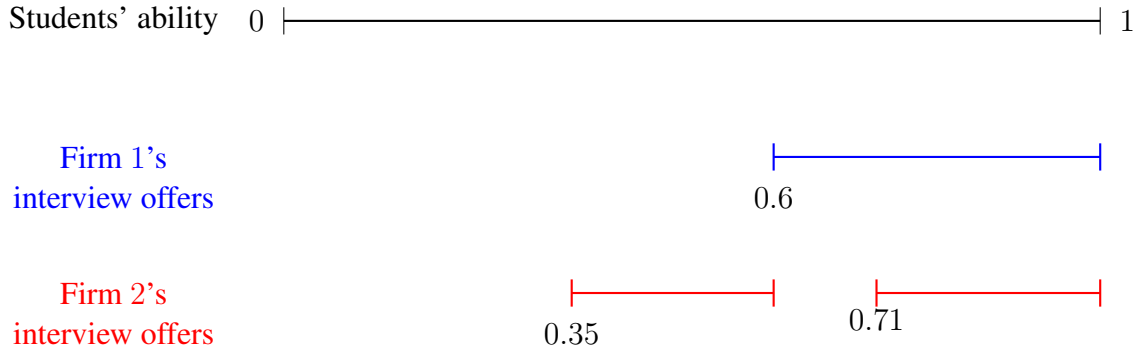　　　　　　├───────────┤　├───────────┤
　　　　　　0.35　　　　　0.71

Figure 8: Interview offers by firms 1 and 2 when students' interviewing costs are low. Firm 1 interviews the best students available. Firm 2, however, chooses not to and strategically extends interview offers with a gap in the middle.

same interviewing strategy and firm 2 has a different interview offers with a gap in the middle where some students with ability greater than 0.6 are not interviewed by firm 2. The optimality of these interview offers is discussed in Section A.6. Thus, when the firms' marginal costs of interviewing are increasing, we get similar results.

7　Conclusion

Interviewing processes are organized in a variety of ways for the entry-level markets, which are the focus of this work. We established a tractable model of interviewing in a many-to-one matching framework to generate results that are in line with anecdotal evidence. The assumption about a continuum of students gave us a convenient (essentially) unique equilibrium. We found that some firms have gaps in their interview offers. Moreover, when the interviewing capacities increase for one side of the market and there is enough agreement about preferences before the interviewing process, the utilitarian surplus increases. This, however, does not bring a Pareto improvement if the interviewing constraints matter for the firms and students. An increase in interviewing capacity for one side of the market, e.g. students, improves the welfare for agents on the opposite side and the agents at the top on the same side of the market. However lower ranked students are worse off due to an increase in students' interviewing capacity.

An increase in interviewing capacity can be accompanied with a lower number of matched agents. If a social planner is concerned about the number of matched agents, she may choose to keep the capacities low for the market participants. This insight helped us contextualize the capac-

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Section A.6.

ity constraints in place for the graduating students in Indian management schools. This reduction in the number of matched agents also provides a caution for an asymmetric reduction in interviewing costs when the objective is to match as many market participants as possible while improving the overall welfare.[45] In the context of residency markets, reducing interviewing constraints with the help of technology or using coordination where it reduces the costs for both sides of the market will be advantageous.

We check the robustness of our results by extending the main model along various dimensions like existence of correlation in the firm fitness factors and diversity of ex-ante student or firm preferences over the opposite side. At the same time, we also qualify the general insight that reduced frictions improves utilitarian surplus and prove that this does not hold if there are information asymmetries. There are situations where increased student interviewing capacities reduced not only the number of matched agents but also the aggregate surplus from matching.

Our analysis offers several policy-relevant guidelines. First, if there is enough agreement about preferences before the interviewing process, we can recommend the social planner or market designer to coordinate efforts to increase the number of contacts made for both sides of the market. This will raise surplus as agents on both sides will be able to interact with more agents and have a more complete evaluation of the market. In some markets such an intervention is feasible as a central organization coordinates at least some parts of the process.[46] Second, credible signaling mechanism can help mitigate the risk of a lower number of matched agents as it will help the coordination aspect of interviewing especially with idiosyncratic preferences. Third, if the lower-ranked students and firms can be nudged towards a greater number of interviews, it will be welfare-enhancing because these are the agents who are left unmatched due to interviewing constraints.

As interviewing constraints change for the participants of various labor markets either due to a centralized interviewing process or due to remote interviews being conducted via a video conference, a market designer needs to improve the benefits for both sides of the market appropriately to bring in Pareto improvements. The details of the market we consider matter and our current exercise underscores this point.

We now focus on aspects of the model which we did not discuss earlier and will represent some of the future directions for this work. First, the fitness factor was a digital signal with either a fit or a misfit value. If the fitness factor is a continuous variable instead, we conjecture that all the results of the model continue to hold. Second, the dependence of firm fitness factors on the firm identity

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[45]This insight holds even when a social planner cares about the utilitarian welfare if the agents are risk-averse.

[46]For example, the American Association for Colleges of Podiatric Medicine (AACPM) organizes a centralized interviewing process for podiatry graduates to find residency positions.

in non-trivial ways is an important extension to bring the model closer to reality. Third, although there is agreement about the introduction of signaling before interviewing stages to improve the overall welfare, there is no guidance about the optimal number and nature of these. The model we analyzed can be used to take up simulations and add insights on this front.

We think that interviewing is an interesting area of matching which needs theoretical and empirical investigation to ensure better outcomes in matching markets. We provide a step in that direction through this work to understand the *black-box* of the interviewing phenomenon.

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	Podiatry Match	College Admissions	IIT-B placements
Positions advertised	Program registration on CASPR website (April - July)	Implicit (and selective)	Companies send Job Application Forms (JAF)
Applications	Future residents apply (September, October)	Applications sent via Common App /Universal App	Student apply on the placement website
Interview Offers	Interview lists due from the programs (late November)	Not Applicable	Companies inform the placement cell about the interview lists
The interviewing process	Centralized Residency Interviewing Program (CRIP) (Early January)	Implicitly decided by the colleges	Companies visit campus on assigned days to conduct interviews
Job offers	Programs list their preferences in an order	Colleges send admissions/ waitlist/reject decisions	Final list of candidates
Final matching	Candidates submit their rank order lists to the centralized system and a match is announced	Students accept one offer	Candidates decide to accept and compete only for better firms, if any

Table 2: Various interactions comparable to the National Residency Match Program process in the National Podiatry match, the college admissions process and the engineering graduate placements process in Indian Institute of Technology-Bombay

A   Discussion

A.1   Interactions in Matching Markets

The two sides of the matching market interact over an extended period of time. The process is broadly the same across different settings and is summarized in Table 2.

A.2   Costly Interviewing affects the market outcomes

We investigate whether interviewing constraints at all affect market wide outcomes. To that end, we compare the NRMP survey data [National Resident Matching Program Ⓡ, Data Release and Research Committee, b] from the residency directors and applicants. Residency directors indicated that on an average, of the 856 applications received for 7 advertised positions, 119 candidates were sent an interview invitation and 96 were actually interviewed. Similar data from the survey of doctors [National Resident Matching Program Ⓡ, Data Release and Research Committee, a] indicates that of the 15 interview offers, the candidates (who were matched at the end of the process) attended only 11 of them. This suggests that the impact of interviewing capacities are non-trivial.

The programs do not interview all applicants and candidates do not accept all the interview invitations. Moreover, each year through the secondary match process organized as the Supplemental Offer Acceptance Procedure (SOAP), most of the positions that remain unfilled in the main process, match with a resident who is unmatched in the main process. In the NRMP main Match 2015, of the 30212 positions, 1306 remained unfilled. Out of the 34905 applicants, 8025 applicants were left completely unmatched. Through the Supplemental Offer Acceptance Procedure, 1129 positions were filled [National Resident Matching Program ®, Results and Data]. This provides some directional evidence that interviewing constraints matter.

A.3 Timing of the market

In many markets, the timing of various transactions is coordinated and the participants comply with these required timelines. The residency and fellowship matchings that take place through the SF match is yet another example of coordinated timings by various specialties.[47] The repeated nature of these interactions and advantages a centralized application and rank ordering system provides to the programs is sufficient for their continued participation. As a result even the participants follow the timeline and the guidelines.

In some markets, the timing is not particularly easy to coordinate and in fact repeated efforts to get these in place have failed. A notable example of this is the clerkship market where graduating law students in the U.S seek clerkship positions with various Judges in the different Circuits. We refer an interested reader to Avery et al. [2001, 2007] for a detailed account of this market. We do not expect to capture this market through our discussion in the example and the main model.

In some markets the timing of interviewing is crucial even when there is coordination. To consider an example, closer to home, the economics job market for graduating economists has centralized interviewing which represents only the first stage of the process. The later stages which include fly-outs and closer interviews that take place in a decentralized manner and only a very few candidates are invited for this process. Arguably the reason is the immense cost on getting the entire faculty to attend the job market candidates seminars and form preferences over them. This market clears top-down where the departments lower in the rank order, make their decisions after the top departments have already conducted their fly-outs and extended their interview offers. We abstract away from such considerations in our model.

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[47]See https://www.sfmatch.org/MatchCalendar.aspx. Last accessed July 15th, 2015.

A.4 Example in Section 1.3: Surplus calculation

Recall that the sum of students' utilities is the same along with the sum of the idiosyncratic components related to the students' preferences over the firms. In the low-capacity regime, all the students who match get a utility of ϵ as they match with their first preference firm, if they do. Of the 0.36 mass of students who are interviewed by the two firms combined, 0.18 mass gets an offer and hence the aggregate idiosyncratic component is 0.36ϵ. In the high-capacity regime, the top 0.06 mass of students get interviewed by both firms and 50% of them get an offer from their first preference firm and a further 25% of them get an offer from their second choice firm, while the remaining 25% are found misfit for both firms. The sum of idiosyncratic components for these top students is $0.06[0.5\epsilon - 0.25\epsilon] = 0.015\epsilon$. However, the remaining students are randomly shared by the two firms and there is no sorting along the student preferences and hence on average the matched students get a 0 idiosyncratic utility. Thus, the total idiosyncratic utility for the students is $0.015\epsilon < 0.36\epsilon$, obtained in the low-capacity regimes. Thus, even the sum of student welfare is lower under the high-capacity regime. However, note that the students at the top in the ability range $[0.94, 1]$ get a higher expected utility and are matched with greater probability while the students in the ability range $[0.64, 0.7)$ are worse off in terms of the expected utility. Moreover, the sum of gains is lower than the sum of losses in this case unlike the first example.

A.5 Analysis of an economy where firms' evaluations are not identical

In this discussion, we will just focus on the result about reduction in overall welfare when the students' interviewing costs are reduced when firms' evaluations of student abilities are correlated but not identical. We do not discuss the equilibrium existence result as it follows very closely from the proof of Theorem 1.

Proposition 6. *When the ability of students is evaluated differently by each firm, a reduction in student interviewing costs does not necessarily increase the overall quality of matching.*

Proof We present an example to prove this.

Consider an economy with 2 firms and a continuum of students of mass 1. The two firms are labeled 1 and 2. Each student's type $\theta = (e^\theta, f^\theta)$ is drawn from a cumulative distribution F over $\Theta = [0, 1]^2 \times \{-1, 1\}^2$. The first component $e^\theta = (e_1^\theta, e_2^\theta)$ represents the ability vector. Each firm i only sees the $i - th$ component of the ability for the students who apply. More precisely, it can not see the value for e_j^θ for $j \neq i$. The fitness factor is similar to the fitness factor in the main model. We assume that the distribution of e_i^θ is uniform over $[0, 1]$ and we assume a
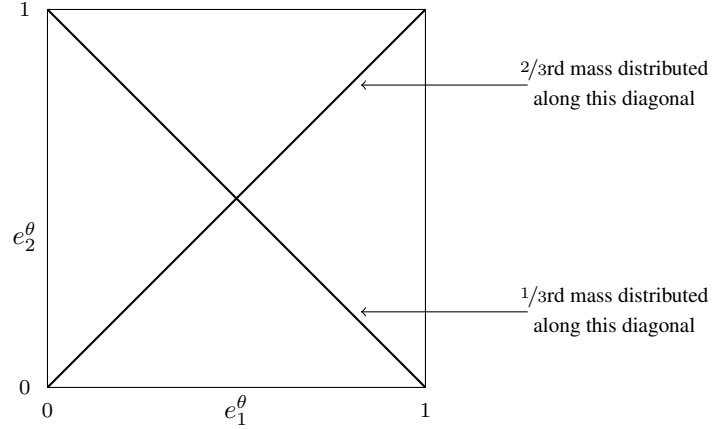
Figure 9: The student distribution along the two ability parameters corresponding to each firm, e_1^θ and e_2^θ.

specific form of correlation between e_1^θ and e_2^θ as shown in figure 9. We assume that a $\rho = 2/3$ mass of students is uniformly distributed along the 'correlated' diagonal, i.e. $e_1^\theta = e_2^\theta$ and the remaining $1/3$ is distributed along the $e_1^\theta = 1 - e_2^\theta$ diagonal. Each student is fit for a firm with probability 0.5 independent of everything else. A fit firm-student pair generates a utility given by $2U(i, e_i^\theta) = \frac{1}{i} V(e_i^\theta)$ where $V(x) = x - \frac{x^3}{3}$.

In the low-capacity regime, a student can interview with only 1 firm and this changes to 2 in the high-capacity regime. As is always the case, the choice of regime does not impact the firms ranked $\leq k_{LC}$. The first firm always interviews the best 0.24 mass according to its own criterion, i.e. all students with ability, $e_1^\theta \in [[0.76, 1]$ as shown in figure 10. In the low-capacity regime, $2/3rd$ of the top 0.24 mass according to firm 2, is in firm 1's interview offer region. These students lying on the correlated diagonal of the distribution will reject an interview offer from firm 2. Hence, firm 2 extends interview offers to all students with ability $e_2^\theta \geq 0.6$. Only a third of the top 0.24 and all the remaining 0.16 mass students will accept firm 2's interview offers. Note that firm 2 is successful in hiring some of the top students with ability $e_2^\theta \geq 0.76$.

In the high-capacity regime all students can accept firm 2's interview offers. However a fraction (specifically $2/3rd$ fraction) of the top 0.24 mass students are also interviewed by firm 1 and hence available only if they are found a misfit with it. Firm 2 evaluates the effective probability of being able to hire a student in the top 0.24 ability spectrum as follows.
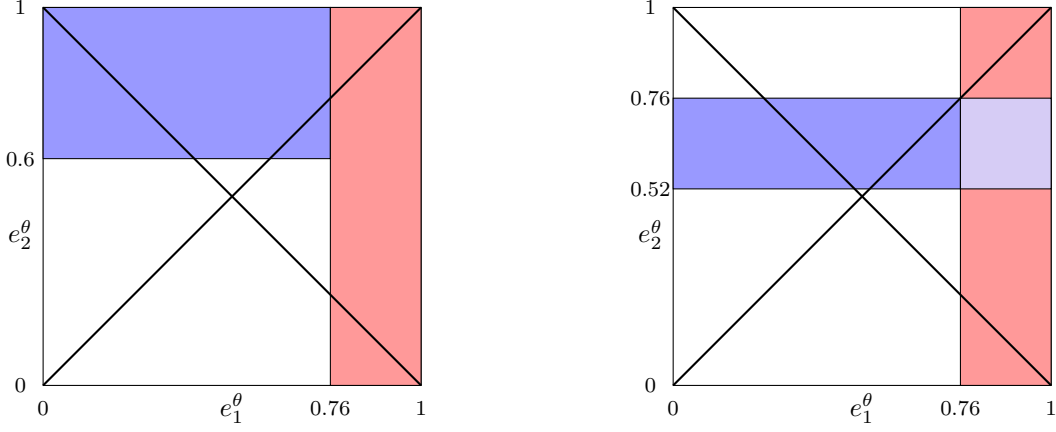
(a) The interview offers by the firms in the low-capacity regime.

(b) The interview offers by the firms in the high-capacity regime.

Figure 10: Firm 1 interview offers are in red and those by firm 2 are in blue.

Fraction of students hired by firm 2 in the overlap region

$$= \frac{2}{3} \times \text{Prob(a student is a misfit with firm 1)} \times \text{Prob(a student is a fit for firm 2)}$$

$$+ \frac{1}{3} \times \text{Prob(a student is a fit for firm 2)}$$

$$= \frac{2}{3} \times 0.5 \times 0.5 + \frac{1}{3} \times 0.5 == \frac{1}{3}$$

If firm 2 were to interview students such that $e_2^\theta \in (0.24, 0.76)$ then it will be able to hire each of these students with probability 0.5 as none of them have an interview offer from firm 1. Firm 2 in fact chooses to interview students between $e_2^\theta \in [0.52, 0.76)$ as $0.5U(2, 0.52) > \frac{1}{3}U(2, 1)$.[48] Note that firm 2 chooses not to interview any student in the top ability region and hence does not hire any student with ability $e_2^\theta \geq 0.76$. It is clear that when firm 2 hires a strictly dominated distribution on the ability parameter of the students and hence the utility for firm 2 is strictly lower.

In this case the student surplus is exactly equal to the firm surplus and hence the overall quality of matching goes down when the students' interviewing costs are reduced.   □

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[48]Recall that $U(i, e^\theta) = \frac{1}{i}V(e^\theta)$ where $V(x) = x - \frac{x^3}{3}$ for firm i and a student with ability e_2^θ

A.6 Example in Section 6.3: Surplus calculation for firm 2

In the high interviewing cost regime, we know that the students who have an interview offer from firm 1 will not accept firm 2 interview offers. To check for the optimality of firms' interviewing decisions, we take a two step procedure. First, we can verify if the firms' interviewing decisions justify the cost they incur. Second, we verify if there is a profitable deviation by interviewing some other set of students. Both firms decides to interview 0.4 mass of students. The marginal cost of interviewing for firm 2 when it interview 0.4 mass of students is $\frac{0.6}{2}(0.4)^2 = 0.048$. The expected surplus from interviewing the student at ability $e^\theta = 0.2$ is $0.5 \times \frac{0.2}{2} = 0.05$. The marginal decision at the smallest ability student by firm 2 is positive. At all other students the surplus received is higher and the costs are lower for both firms. This proves that the all the interviewing decisions justify the cost incurred by the firms. Since both firms are able to recruit the required number of candidates and are doing the best they can given the interviewing constraints faced by the students.

We also want to evaluate the firms' optimal decisions when the interviewing costs for students are reduced so that the cost is 0.02 to interview with 1 firm and 0.1 to interview with 2 firms. We explained above that any student with ability $e^\theta \geq 0.64$ will find it worthwhile to take up the second interview offer from firm 2, if one is extended.[49]

Let us verify that firm 2's decisions are optimal. If firm 2 can interview all the students with ability $[0.35, 0.6) \cup [0.7, 1]$,[50] the firms' hiring quota will be met. We will rather prove that firm 2 actually chooses to interview a smaller region than this because the marginal cost of interviewing the students when they are interviewing 0.55 mass of students is greater than the surplus share from interviewing students with ability 0.35 or 0.7. This will be sufficient to prove that the optimal regions will be a strict subset of the region specified above and optimally so. The marginal cost of interviewing 0.55 mass of students for firm $2 = \frac{0.6}{2} \times (0.55)^2 = 0.9075$. However, the expected share of surplus for firm 2 from interviewing the marginal student of ability 0.35 or 0.7 is given by $Prob(fit) \times V(2, 0.35) = 0.5 \times \frac{0.35}{2}$ and Prob(misfit for firm $1 \times$ Prob(fit for firm $2 \times V(2, 0.7) = 0.5 \times 0.5 \times \frac{0.7}{2}$. These values are $0.0875 <$ the marginal cost we inferred above. This proves that firm 2 will actually reduce its interview offers in both regions till the marginal cost of interviewing is just equal to the surplus share from the marginal student. This will necessarily reduce the number of matched agents with firm 2.

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[49]The expected surplus for a student with ability of 0.64 is Probability(misfit for firm 1)$\times$ Probability (fit for firm 2) $\times \frac{0.64}{2} = 0.08$.

[50]The number of matches when the interview offers are as specified above will be 0.25×0.5 from the students in the lower ability region and 0.3×0.25 from the students with interview offers from firm 1. This adds up to 0.2, the hiring quota for firm 2.

B Proofs

B.1 Proofs of Theorem 1 and Theorem 2

We prove the more general case, Theorem 2 and observe that our economy in the main model, E is the special case of the correlated economy E^{corr} where the correlation is 0. More precisely, if we set the $F-$dimensional vector $\tilde{p} = [1 \ 1 \ 1 \ \cdots \ 1]$, the correlated economy boils down to our economy in the main model. Thus, Theorem 1's proof follows from the proof of Theorem 2.

Recall that in this setting, the students agree about their preferences over the firms. The marginal distribution over ability is uniform over $[0, 1]$. The fitness factors are correlated and it is summarized by $\tilde{p} = [p_0 \ p_{-1} \ p_{-2} \ \cdots \ p_{-(F-1)}]$. We prove theorem 2 with a series of lemmas in the following steps.

Step i) For all preferences, there exists a unique stable matching and truth-telling is optimal for both firms and students.

Step ii) Each firm has a unique nondegenerate interview offer strategy solvable by iterated elimination of dominated strategies under assumption 1 for all firms.

Step iii) The firm and student preferences after nondegenerate interview offers result in a unique nondegenerate stable matching.

The next two lemmas will prove step i) above and also prove Lemmas 1, 2, and 3.

Lemma 5. *The firms and students find truth-telling optimal to the mechanism which implements a student-proposing deferred acceptance algorithm.*

Proof All students agree on the preference ranking of the firms. We know from Dubins and Freedman [1981] that students have a dominant strategy of truth-telling under student-proposing deferred acceptance algorithm.[51]

Consider the strategic choice for a firm. Note that when students completely agree on their preferences over firms, the student-proposing deferred acceptance algorithm boils down to a serial dictatorship algorithm where the firms make their choice in the order of their ranking. We know that in a serial dictatorship the only time the preferences of a firm matter is when it has to choose a set of students and hence truth-telling is optimal.

Lemma 6. *For any preference reporting by the firms where students report the preferences truthfully, there exists a unique stable matching.*

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[51]This is true even in more general settings than the one we consider of complete agreement over firm ranking.

Proof The existence of a unique nondegenerate stable matching follows very closely from the existence and uniqueness result due to Azevedo and Leshno [2015] for the true complete preferences. However we define stability in terms of the preferences that firms and students realize at the end of the interviewing process. Hence, we present a simple proof in our context.

From above we know that the outcome of the student-proposing deferred acceptance algorithm coincides with the serial dictatorship outcome where firms go in the order of their desirability from the students' perspective. The outcome is necessarily a stable matching with respect to the reported preference which follows from Gale and Shapley [1962].

To prove the uniqueness, suppose there is another stable matching which differs from the match outcome above. Consider the best firm i which has a different assignment in this stable matching as compared to the SD outcome above. Clearly this firm can not be assigned to a student who was assigned to a better firm in the SD outcome.[52] Either firm i has a new student who was assigned to a lower ranked firm i' than itself in the SD outcome or the firm has an empty spot. Serial dictatorship gave firm i a chance to act before firm i'. This is inconsistent with stability of the Serial dictatorship outcome, which we know from above is, in fact stable. This gives us the required contradiction and there is no firm which has a different match as compared to the SD outcome above. Due to the property of stable matchings, there is no student who has a different assignment. Thus, there does not exist another stable matching outcome proving the required uniqueness.

We now present the proof of step ii) and Lemma 4.

Proof of Lemma 4 Consider the interviewing strategy by firm 1. Firm 1 with a distaste for interviewing will extend interview offers to all those students whom it can hire following a desirable outcome in the interviewing process. A student facing an interview offer from firm 1 knows that under assumption 1 she will get a final job offer with probability p. Any other firm's interview offer will result in an offer with equal or lesser probability. The student's best response to this is to always accept an interview offer from firm 1. Thus, the best firm extends interview offers just enough to fill its hiring capacity unless the interviewing capacity is met before that. The students never regret accepting an interview offer from firm 1 and possibly rejecting some other firm's interview offer later in the process because before the interviewing process the unconditional fitness probability across firms is identical and independent of ability of the student, i.e. p.

Firm 2 knows the interview offers of firm 1 which are uniquely pinned down above. Firm 2 can choose to interview any set of students but just enough to ensure that it will be willing to extend an offer to each and every student found fit, knowing that some of them will be hired away by firm 1. Under the distaste for interviewing property, firm 2's interview offers result in a final offer with

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[52]If this is the case then firm i will not be the best firm which has a different assignment.

probability pp_{-1}. Any other firm's offers will convert in a final offer with the same probability if it is the next best firm and with lower probability if it is not. Hence, a student facing an interview offer and having excess interviewing capacity will always choose to accept the interview offer.

This argument can be made for any firm i knowing the interview offers for all firms better than itself and knowing that the students indeed accept the best offers up to their interviewing capacity. Thus, there is a unique nondegenerate interview offer strategy for the firms.

The exact interview offers from each firm can be iteratively found as done in the above proof to ensure that the interview offers meet the following three conditions.

1. Firm i's interview offers are optimal given the interviews offers by firms 1 through $i-1$.

2. None of the students will have to reject the interview from firm i due to their capacity constraints.

3. The interview offer set is nondegenerate and respects the capacity firm i has.

Note that there are possible redundancies in the interview offers by firms, where a firm can extend interview offers to students who do not have any interviewing capacity and those students in turn reject the offers made by such firms. We abstract away from such multiplicities. We know that a student will never regret having accepted a 'better' interview offer as per our discussion above.

The first step accounts for an existence of a stable matching. However, the equilibrium requires a nondegenerate stable matching outcome. From step ii) we know that there are unique nondegenerate interview offers from the firms. In the last step we indeed prove the connection that when the firm strategy includes only nondegenerate interview offers the resulting preferences lead to a nondegenerate stable matching. Suppose not and consider the best firm i for whom the interviewing strategy is not nondegenerate. Of the two possibilities suppose that there is a student θ who is not assigned to firm i although all her right neighbors are assigned to i and the student still does not form a blocking pair with that firm. This can essentially happen if that student was not interviewed by the firm. However, we can conclude that the student had interviewing capacity as the students to her right accepted an interview offer from i.[53] Thus, it must be the case that she did not receive an interview offer from firm i. This is ruled out by the first requirement of nondegenerate interview offers and hence our assumption is incorrect. Suppose that there is a student θ who is matched to a firm but none of her right neighbors are. The right neighbors will not form a blocking pair with

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[53]If this were not true, there exists at least one firm better than i which extended its interview offers to the current student but not to her right neighbors. The students on the right will not form a blocking pair with this better firm only if they were not interviewed. This contradicts the initial assumption that firm i was the best such firm.

this firm only if they were not interviewed. Similar argument leads us to the conclusion that the preferences that result after nondegenerate interview offers form nondegenerate stable matchings.

Now we can combine all the results above to present the proof for Theorem 2. We have assumed the application decision to be trivial by making applications costless. Thus, the following strategies comprise an equilibrium.

1. All students apply to all firms.

2. Each firm follows the interview offer strategy as described in the proof of step ii).

3. All students accept all the interview offers they receive.

4. A firm finds all the 'fit' students acceptable and lists them in the true order and all students list the firms in the true order.

The above strategies constitute an equilibrium follows from step i) and ii) and the stable matching that results is a nondegenerate stable matching due to step iii). This is an 'essentially' unique equilibrium because the only other strategies that survive the preference reporting stages are the ones that do not materially impact the equilibrium outcome or the ones where the firms add or remove finite number of degenerate mass sets of students at the interview offer or preference reporting stage. This later possibility will still give us an 'essentially' unique equilibrium.

B.2 Proofs of Proposition 2 and 4

The proofs of Propositions 2 and 4 follow from the proofs of Propositions 7 and 8 which establishes the more general case with correlated fitness factors in Section B.3. The first 4 steps outlined in the proof prove Proposition 2 and 7. This proof does not rely on any of the extra assumptions, specifically maximum diversity of interview offers, which are used in the proof of Proposition 4 and 8.

B.3 Proof of Propositions 7 and 8

We first state all the propositions and then present their proofs.

Proposition 7. *When the interviewing regime moves from LCr to HCr, the quality of the match weakly increases. If firms do not have any excess interview capacity and the quality strictly increases, the quantity strictly decreases at least for one firm.*

Proposition 8. *Suppose that all firms have maximum diversity in their interview offers in the LCr regime and the component of the student surplus $V(\cdot)$ is concave with a concave first derivative. When the interviewing regime moves from LCr to HCr, without excess interviewing capacity for the firms, the quantity of the overall match weakly decreases. If the quality strictly increases, the quantity of the matching strictly decreases.*

We present the proofs of Propositions 7 and 8 together. A reader interested only in the proof of Propositions 2 and 4 can read the proof using the assumption that the correlation vector $\tilde{p} = [p_0 p_{-1} p_{-2} \cdots p_{-(F-1)}] = [111 \cdots 1]$ for the general case we do not make any such assumption and state the proof below for the correlated economy. The results about an increase in quality and the existence of a firm with lower quantity follows from the first four steps of the proof and we do not use any of the extra assumptions used in Proposition 4 until after that.

Consider the low-capacity (LCr) and high-capacity (HCr) to be with student interviewing capacities of k_{LC} and k_{HC} respectively. We know that $k_{LC} < k_{HC}$ and the students can interview with more firms in the HCr. We keep the firm interviewing capacity fixed for simplicity and to make the comparisons across regimes stark.

We prove the following Lemmas.

Lemma 7. *If the interview offers are different between the HC and LC regimes, the best firm to extend different interview offers is ranked worse than k_{LC}. For this firm, the utility of the match outcome strictly increases and the number of positions filled strictly decreases in the HC regime.*

We call this firm, if it exists as firm i.

Proof of Lemma 7 We prove this in two steps.

Step 1) Compare the equilibrium under HC regime to that under LC regime and identify the best firm which has a different interview offer.

Step 2) For the best firm identified above, say i, identify the regions of students as it sees in different regimes and its choices.

Step 1 From Theorem 2,[54] we know that for every interviewing cost regime, there is an (essentially) unique equilibrium and hence, a unique match outcome. The equilibrium outcomes can be identified simply by the optimal interviewing strategy for each firm which respects the capacity for the firms and the students. Consider the best firm i which has different interview offers across the two regimes. Note that $i \geq k_{LC} + 1$. It is clear that for all firms weakly better than the k_{LC}th

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[54]for a reader interested in the proofs of Proposition 2 and 4 alone, this can read from Theorem 1 in stead.

firm the student interviewing capacity definitely does not matter in both the regimes. Thus, their choices are identical in both the regimes.

Step 2 Identify the regions of students as seen by firm i, $I_i(0), I_i(1), I_i(2), \cdots, I_i(k_{LC} - 1), I_i(\infty)$ under the LC regime. Each $I_i(j)$ for finite j stands for the set of students with j interview offers from firms better than firm i and $I_i(\infty)$ is the set of students who have no excess interviewing capacity as they already have k_{LC} interview offers. Note the following properties about these sets and their proofs below.

Sets are ordered. Any $\theta_i \in I_i(i)$ and $\theta_j \in I_i(j)$ such that $i < j$, we have $e^{\theta_i} < e^{\theta_j}$. **Sets $I_i(j)$ for all finite j have open right boundary and closed left boundary. Set $I_i(\infty)$ is a closed set. Sets for all finite i are nondegenerate.** Except possibly $I_i(\infty)$, all other sets have non-zero mass.

A reader not interested in the specifics of the proof of the above properties can skip this paragraph. Suppose the first property does not hold. Consider a specific i and j such that $i < j$ but there exists a $\theta_i \in I_i(i)$ and $\theta_j \in I_i(j)$ such that $e^{\theta_i} > e^{\theta_j}$. Student θ_i has fewer interview offers than student θ_j. Consider the lowest ranked firm $i' \succ i$ which does not interview student θ_i but interviews student θ_j and such a firm exists. We can replace the equilibrium strategy of i' to interview student θ_i and a small neighborhood to the right instead of θ_j and a small neighborhood on the right. The said firm will be strictly better off. We found a nondegenerate deviation for the said firm which gives us the required contradiction. Hence, the above sets are ordered as described above. The second property about the structure of the sets follows from the fact that the interview offers have a similar structure. The structure of the interviews is guaranteed due to the requirement on interview offers that they must be nondegenerate. The last property follows immediately from the previous observation.

Compare similar regions for the HC regime and let us call them $\bar{I}_i(0), \bar{I}_i(1), \bar{I}_i(2), \cdots, \bar{I}_i(k_{HC} - 1), \bar{I}_i(\infty)$. Since the interviewing strategies of all the better firms are unchanged, we essentially have

$$\bar{I}_i(i) = I_i(i) \quad \forall i \leq k_{LC} - 1$$

Moreover the region $\bar{I}_i(\infty) \subset I_i(\infty)$ and the new additions are sets $\bar{I}_i(k_{LC}), \bar{I}_i(k_{LC}+1), \cdots, \bar{I}_i(k_{HC} - 1)$. Since firm i chooses to extend interview offers to a new set of students it does strictly better than in the LC regime.[55] We have assumed that the firms have no excess interviewing capacity so the firm chooses to replace the students in the regions $I_i(i)$ with lower i by the ones in the high ability region, i.e. $\bar{I}_i(k_{LC}+)$, who can now take up the interview offers from this firm. Note that since firm i moves some of its interview offers from students with lesser interviews from better

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[55]As the interview offers have open right boundaries, the firm will move non-zero mass of interview offers to the new region and hence will do strictly better.

firms to students with more interview offers from better firms, the quantity of the match for firm i will be necessarily lower. $\square$

We now define a concept about a better set of students and use it in the next Lemma.

Definition 14. Compared to the LC regime, a firm sees a strictly *better set* of students in the HC regime, if the following holds.

- For all students θ with $\theta \in I_i(j)$ and $\theta \in \bar{I}_i(\bar{j})$, we have $\bar{j} \leq j$

- There exists a student θ with $\theta \in I_i(j)$ and $\theta \in \bar{I}_i(\bar{j})$, such that $\bar{j} < j$

Lemma 8. *If the interview offers are different between the LC and HC regime, all firms worse than firm i see strictly better set of students and get weakly better utility from matching in the HC regime.*

The above two Lemmas 7 and 8 taken together prove Propositions 7 and thus also Proposition 2.

Proof of Lemma 8 We prove this in two steps.

Step 1) Identify the impact on the subsequent firm(s) due to a different optimal strategy by firm i as the set of students it sees is strictly better.

Step 2) Identify the quality improvement for all firms.

Step 1 Consider the impact of firm i's strategy in the HC regime on the next firm's evaluations. Note that as compared to the LC regime in the HC regime, some students in the region $I_i(0)$ do not receive an interview offer from firm i. These students were a part of $I_{i+1}(1)$ in the LC regime but are a part of $\bar{I}_{i+1}(0)$ in the HC regime. This argument holds for a nondegenerate set of students in each of the regions (by continuity). Thus, the lower ranked firm sees a strictly better set of students.

Step 2 Whenever a firm sees a strictly better set of students, the firm will do at least as well as it was doing earlier even if it continues with the same strategy as it followed in the LC regime. This can be further strictly improved if the firm re-optimizes its interview offers. This argument can be continued for all subsequent firms and thus essentially all firms lower than i do strictly better under the HC regime. $\square$

From the above Lemmas, we know that if one firm has a different interviewing strategy then it will have a strictly better outcome. All the other firms will also have a weakly better match outcome. Thus, if the match outcomes are different under the two regimes then the quality of the match strictly improves. However, it is possible that no firm follows a different interviewing strategy and hence we get the weaker result that the match quality weakly improves when the
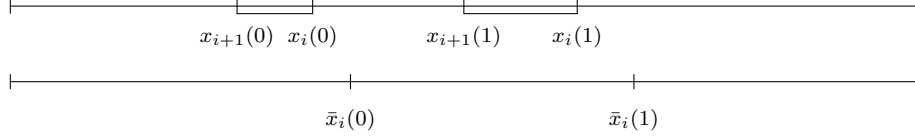
Figure 11: The optimal choices by firms i and $i + 1$ under different regimes.

regime changes from LC to HC. This proves Propositions 7 and Proposition 2. Note that this result only relied on the assumption that $U(i, e^\theta)$ is an increasing function of ability. This agrees with our intuition that a system moves towards a more efficient outcome if the frictions are reduced. □

Lemma 9. *If the interview offers are different between the LC and HC regimes and all firms have maximum diversity in their interview offers in the LC regime, then the utility of matching strictly increases for firm i and all firms worse than i. This is accompanied by a strict decrease in the number of positions filled for each of these firms.*

All the above Lemmas 7, 8, and 9 taken together essentially prove Propositions 8 and thus also Proposition 4.

Proof of Lemma 9 Consider the actual choices made by firm i from the different regions under different regimes. Let us call $x_i(0), x_i(1), x_i(2), \cdots, x_i(k)$ such that $x_i(j)$ is the ability of the worst student it interviews in region $I_i(j)$. Such a value exists because each set is open on the right but closed on the left. Similarly let us define $x_{i+1}(0), x_{i+1}(1), x_{i+1}(2), \cdots, x_{i+1}(k)$.

Given the actual choices by the firm and the fact that each firm's interview offers had maximum diversity, we know the following.[56]

$$U(i, x_i(0)) = \prod_{j=0}^{i}(1 - pp_j) \times U(i, x_i(i)) \quad \forall i \in 1, 2, 3, \cdots, k$$

$$U(i + 1, x_{i+1}(0)) = \prod_{j=0}^{i}(1 - pp_j) \times U(i + 1, x_{i+1}(i)) \quad \forall i \in 1, 2, 3, \cdots, k$$

Since the firms decide their interview offers as best response to better firm's decisions, we can further say that $x_{i+1}(i) \leq x_i(i) \ \forall i \in 1, 2, 3, \cdots, k$ as can be seen in Figure 11. We start our investigation of the impact of the change for firm i on the subsequent firms. Consider the

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[56]We need the maximum diversity assumption to ensure that these condition holds as equalities. If not, we would have $U(i, x_i(0)) \leq (1 - p)^i \times U(i, x_i(i)) \quad \forall i \in 1, 2, 3, \cdots, k$.

relationship between $x_i(0) - x_{i+1}(0)$ and $x_i(1) - x_{i+1}(1)$. Note that these points are optimal choices by firm f and $i + 1$. Due to the assumption on the functional form of $U(i, e^\theta) = h(i)V(e^\theta)$, the following holds too.

$$U(i + 1, x_{i+1}(0)) = (1 - pp_{-1}) \times U(i + 1, x_{i+1}(1))$$
$$V(x_{i+1}(0)) = (1 - pp_{-1}) \times V(x_{i+1}(1))$$

Consider an ϵ_0 move in $x_{i+1}(0)$ and an ϵ_1 move in $x_{i+1}(1)$ such that

$$V(x_{i+1}(0) + \epsilon_0) = (1 - pp_{-1}) \times V(x_{i+1}(1) + \epsilon_1)$$
$$\frac{V(x_{i+1}(0) + \epsilon_0) - V(x_{i+1}(0))}{\epsilon_0} = (1 - pp_{-1})\frac{V(x_{i+1}(1) + \epsilon_1) - V(x_{i+1}(1))}{\epsilon_1}\frac{\epsilon_1}{\epsilon_0}$$

For ϵ_0 sufficiently close to 0, ϵ_1 will also be close to 0 and we will get the following.

$$\epsilon_0 = (1 - pp_{-1})\frac{V'(x_{i+1}(1))}{V'(x_{i+1}(0))}\epsilon_1$$

We know that $x_{i+1}(1) > x_{i+1}(0)$ and $V(\cdot)$ is a concave function, so $V'(x_{i+1}(1)) < V'(x_{i+1}(0))$. Thus, we have $\epsilon_0 < \epsilon_1$.

This proves that at every point $x_{i+1}(0), x_{i+1}(1)$, the optimal points corresponding to $x_{i+1}(0) + \epsilon_0, x_{i+1}(1) + \epsilon_1$ will be such that $\epsilon_0 < \epsilon_1$. This argument will leads us to the following relationship.

$$x_i(0) - x_{i+1}(0) < x_i(1) - x_{i+1}(1)$$

To evaluate the impact of the move in optimal choices by firm i on the immediately next firm, consider the ϵ_i^0 change at $x_i(0)$ and ϵ_{i+1}^0 change at $x_{i+1}(0)$ corresponding to the 'same' ϵ_1 move at $x_i(1)$ and $x_{i+1}(1)$. This comparison will help us evaluate the optimal decision of firm $i + 1$ following the changes for firm i in the following way. Suppose firm i moves the lowest ability interview offer to a Δ_1 higher point. There is an extra set of students in the region $\bar{I}_{i+1}(0)$ due to this move. If this firm decides to naively just shift the interview regions to account for the newer students available, then the present comparison can shed some light on whether firm $i + 1$ should move even further in no interview region or should it replace its interview offers from that region with the students in the region with one interview (from better firms).

From the above analysis, we know that

$$\epsilon_i^0 = (1 - pp_{-1})\frac{V'(x_i(1))}{V'(x_i(0))}\epsilon_1$$

$$\epsilon_{i+1}^0 = (1 - pp_{-1})\frac{V'(x_{i+1}(1))}{V'(x_{i+1}(0))}\epsilon_1$$

$$\frac{\epsilon_i^0}{\epsilon_{i+1}^0} = \frac{\frac{V'(x_i(1))}{V'(x_i(0))}}{\frac{V'(x_{i+1}(1))}{V'(x_{i+1}(0))}}$$

Let us call $V'(\cdot)$ as $g(\cdot)$. We know that $g(x) > 0, g'(x) \leq 0, g''(x) \leq 0$. We also know $0 < x_i(0) - x_{i+1}(0) < x_i(1) - x_{i+1}(1)$

By Mean Value Theorem, we know that

$$g(x_{i+1}(0)) - g(x_i(0)) = |g'(x_0)|\Big(x_i(0) - x_{i+1}(0)\Big)$$

$$g(x_{i+1}(1)) - g(x_i(1)) = |g'(x_1)|\Big(x_i(1) - x_{i+1}(1)\Big)$$

where $x_0 \in (x_{i+1}(0), x_i(0))$ and $x_1 \in (x_{i+1}(1), x_i(1))$. Moreover, $x_i(0) \leq x_{i+1}(1)$ implies that $x_0 < x_1$. We know that $g'(\cdot) \leq 0$ throughout and $g''(\cdot) \leq 0$. The following follows from these properties of $g(\cdot)$

$$g'(x_0) \geq g'(x_1)$$

$$-g'(x_0) \leq -g'(x_1)$$

$$|g'(x_0)| \leq |g'(x_1)|$$

$$|g'(x_0)|(x_i(0) - x_{i+1}(0)) < |g'(x_1)|(x_i(1) - x_{i+1}(1))$$

$$g(x_{i+1}(0)) - g(x_i(0)) < g(x_{i+1}(1)) - g(x_i(1))$$

We further know that since $x_i(0) \leq x_i(1), g'(\cdot) \leq 0, g > 0$, we have $\frac{1}{g(x_i(0))} \leq \frac{1}{g(x_i(0))}$

The above two inequalities give us the following.

$$\frac{g(x_{i+1}(0)) - g(x_i(0))}{g(x_i(0))} < \frac{g(x_{i+1}(1)) - g(x_i(1))}{g(x_i(1))}$$

$$\Rightarrow \frac{g(x_{i+1}(0))}{g(x_i(0))} < \frac{g(x_{i+1}(1))}{g(x_i(1))}$$

$$\Rightarrow \frac{\frac{g(x_i(1))}{g(x_i(0))}}{\frac{g(x_{i+1}(1))}{g(x_{i+1}(0))}} < 1 \quad \Rightarrow \frac{\frac{V'(x_i(1))}{V'(x_i(0))}}{\frac{V'(x_i(1))}{V'(x_{i+1}(0))}} < 1$$

$$\frac{\epsilon_i^0}{\epsilon_{i+1}^0} < 1$$

We have thus proved that an ϵ_1 move (to the right) in both $x_i(1)$ and $x_{i+1}(1)$ corresponds to a larger change in $x_{i+1}(0)$ than in $x_i(0)$. By aggregating the small moves all throughout, we can say that if $x_i(1) - x_{i+1}(1) = \bar{x}_i(1) - \bar{x}_{i+1}(1) = \Delta_1$ then

$$\bar{x}_i(0) - x_i(0) < \bar{x}_{i+1}(0) - x_{i+1}(0)$$
$$\bar{x}_i(0) - \bar{x}_{i+1}(0) < x_i(0) - x_{i+1}(0)$$

We now explain the exact implication of our finding. If firm $i+1$ were to naïvely keep the 'same' mass in different regions of students, i.e. the mass of students with interviews in region $\bar{I}_{i+1}(0)$ (under the HC regime) is the same as the mass in region $I_{i+1}(0)$ (under the LC regime), then it will interview students θ with ability such that $\bar{x}_i(0) - [x_i(0) - x_{i+1}(0)] \le e^\theta < \bar{x}_i(0)$.

From above we know that

$$\bar{x}_i(0) - [x_i(0) - x_{i+1}(0)] < \bar{x}_{i+1}(0)$$
$$V(\bar{x}_i(0) - [\bar{x}_i(0) - x_{i+1}(0)]) < V(\bar{x}_{i+1}(0)) = (1-p)V(\bar{x}_{i+1}(1)).$$

This shows that firm $i+1$ will choose to shift some interview offers from the $\bar{I}_{i+1}(0)$ region to $\bar{I}_{i+1}(1)$ region. This will not only further increase the quality but it will decrease the quantity of the matching. This argument was not crucially dependent on region with 0 and 1 interviews, it is equally applicable to the regions with 1 and 2 interviews. So the firm will choose to shift more of its mass to the region with 2 interviews. This continues all the way and we get the result that the firm has weakly better quality and weakly lower quantity of matching. □

B.4 Proof of Proposition 3

We know from Lemma 7 that if the interview offers differ, the best firm, say firm i, to extend a different set of interviews will get a higher utility from matching (and will have a lower number of positions filled up). We also know that all firms worse than firm i see a strictly better set of students and get a weakly higher utility from Lemma 8. This essentially proves the firm welfare result in the above proposition. The existence of firm i proves that there is a non-empty set of firms which is strictly better off.

Consider firm i and the region $I_i(\infty)$ as explained in Step 1 of the proof of Lemma 7. Recall that $I_i(\infty)$ is the set of students who have no excess interviewing capacity as they already have k_{LC} interview offers. Consider the student with lowest ability who is a part of $I_i(\infty)$ and call her student $\hat{\theta}$. All students with ability above $e^{\hat{\theta}}$ have k_{LC} interview offers from firms better than i. By assumption, these firms do not change their interview offers. Moreover firm i (and possibly many subsequent firms) may extend interview offers to some of these students. Thus, these students do weakly better in terms of the expected utility from the match outcome and the probability of finding a match. The existence of a different nondegenerate interviewing strategy by firm i ensures that there is a non-zero mass of these students. Thus, all these students with interview offers from firms i and worse do strictly better as they get more interview offers than in the HC regime while keeping all the offers they received in the LC regime. Their welfare increases in terms of the expected utility from the match as well as the probability of being matched.

Now consider the firm, call it firm j, which has different interview offers between the two regimes, extends interview offers to students in $I_j(0)$,[57] and is the worst firm to be so.[58] All firms better than firm j who do not extend an interview offer to the students in $I_j(0)$ region students in the LC regime, face a strictly better set of students and hence will continue to not extend any interview offers to all the students in $I_j(0)$.

All firms worse than firm j who have a different interview offer between the two regimes, do not extend interview offers to the students in their $I.(0)$ region by definition of firm j. Consider a student $\tilde{\theta}_1$ which belongs to $I_j(0)$, gets an interview offer from one of the worse firms with different interview offers, and is such a student with the lowest ability. It is clear that $e^{\tilde{\theta}_1} \neq x_j(0)$ otherwise

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[57]Recall that the set $I_j(0)$ is the set of students who do not have an interview offer from any of the firms better than j. Moreover such a region always exists for all firms as the students are on the long side of the market.

[58]If no such firm exists, then we iteratively look for a firm which has different interview offers between the two regimes, extends interview offers to students in $I_j(m)$, and is the worst firm to be so, for different values of $m \in \{1, 2, 3, \cdots\}$ in the increasing order of numbers. Since firm i exists with different interview offers between the two regimes, we will certainly be able to find the firm j corresponding to the smallest m possible. We will provide the proof for $m = 0$ as it is easier to follow the intuition. The arguments stay valid should the smallest m in the above steps be larger than 0 merely by replacing the (0) with (m).

this firm would be better off by extending some interview offers to the students in $I_.(0)$ region by continuity. However, we have assumed that firm j is the lowest ranked firm to do so.

Consider the interviewing strategy for firm j in the HC regime which is different than that in the LC regime. The effective value of expending an interview slot is the lowest for students with lowest ability in $I_j(0)$ region, i.e. for the student with ability $x_j(0)$. Recall that a firm chooses to allocate its interview offers across different regions so that the effective values for the interview slots are equal to the student with ability $x_j(0)$ or larger. When faced with strictly better students firm j will first choose to eliminate the students with lowest effective value so as to extend interview offers to students with better effective value of interviewing. Hence there will be a non-zero mass of students with ability greater than $x_j(0)$ who will no get an interview offer in the HC regime although they did receive an offer in the LC regime.

Consider a student $\tilde{\theta}_2$ such that this student belongs to $I_j(0)$, gets an interview offer in both the regimes from firm j and there exists a non-empty set of students with ability $\in [x_j(0), e^{\tilde{\theta}_2})$, who do not have an interview offer in the HC regime although they were interviewed by firm j in the LC regime.

Label the student with the lower ability among $\tilde{\theta}_1$ and $\tilde{\theta}_2$ as $\tilde{\theta}$. Consider the set of students in $I_j(0)$ with ability strictly lower than $e^{\tilde{\theta}}$, call this set as X. The students in this set were necessarily not extended any interview offers from those worse firms who have different interview offers even in the LC regime. They will continue to not receive an offer even in the HC regime. All other firms worse than firm j continue to extend the same interview offers across the two regimes and hence are not considered in the welfare comparisons. We have already proved that all firms better than j will not extend any interview offers to students in $I_j(0)$ and $X \subseteq I_j(0)$. All students with ability in $[x_j(0), e^{\tilde{\theta}})$ are extended an interview offer from firm j in the LC regime but not in the HC regime.

Thus, we have proved that these students who belong to set X, get a weakly lower expected utility and also a weakly lower probability of finding a match. There also exists a non-zero mass of students who are strictly worse off.

Hence we have $e^{\theta_1} = e^{\hat{\theta}}$ and $e^{\theta_2} = e^{\tilde{\theta}}$ such that the following claims in proposition 3 hold.

1. All students with ability at or above e^{θ_1} are weakly better off and there exists a non-zero mass of students who are strictly better off, in terms of both–the expected utility from a match and the ex-ante probability of finding a match.

2. All students with ability strictly below e^{θ_2} are weakly worse off and there exists a non-zero mass of students who are strictly worse off, in terms of both–the expected utility from a match and the ex-ante probability of finding a match. $\square$

The same arguments work exactly for the correlated setting as well and we just state the relevant proposition and skip the proof.

Proposition 9. *When the interviewing regime shifts from LC to HC and the interviewing offers are different, there exist two threshold abilities e^{θ_1} and e^{θ_2} such that the following holds.*

1. *All students with ability at or above e^{θ_1} are weakly better off and there exists a non-zero mass of students who are strictly better off, in terms of the expected utility from a match as well as the ex-ante probability of finding a match.*

2. *All students with ability strictly below e^{θ_2} are weakly worse off and there exists a non-zero mass of students who are strictly worse off, in terms of both the metrics–the expected utility from a match and the ex-ante probability of finding a match.*

Moreover, all firms are weakly better off and there exists a non-empty set of firms which are strictly better off.

B.5   Proof of Theorem 3

The proof of Theorem 3 follows very closely to the proof of Theorems 1 and 2. However, the economy has block correlated preferences for the students and we present the relevant arguments here. Recall that the only difference in the current case from the main model is that the students preferences do not agree entirely but only over 'blocks' of firms.

We prove this in the following three steps as in Section B.1.

Step i)   For all preferences, there exists a unique stable matching and truth-telling is optimal for both firms and students.

Step ii)   Each block has a unique nondegenerate interview offer strategy solvable by iterated elimination of dominated strategies under assumption 1 for all firms. However the offers are not unique for each firm within the block.

Step iii)   The firm and student preferences after nondegenerate interview offers result in a unique nondegenerate stable matching.

In the current block correlated economy, we need to evaluate if the firms have incentives to tell the truth to a student-proposing deferred acceptance algorithm. Note that the preferences of the firms over students whom they find acceptable are exactly the same. There are differences about which students a firm might find acceptable (based on the fitness factors). We know that a firm with

responsive preferences might possibly alter the outcomes of the matching process if it initiates a rejection chain with a student who would have been otherwise acceptable. If such a rejection chain comes back to the firm with a better student, then we say that the firm has an incentive to initiate the rejection chain. Suppose such a firm exists, say firm i, and it rejects an acceptable student to initiate such a rejection chain. The student may apply to the next firm on her preference list, if such a firm exists. Any firm will tentatively accept such a student and will only reject a student with lower ability, if at all. Thus the rejection chain strictly goes in the direction of lower ability students. Even if the rejection chain comes back to firm i it would be with students who have worse ability. Thus, truth-telling is in fact a dominant strategy for the firms.

The uniqueness for the proof of step i) comes from the fact that any other stable matching that exists can be found using a rejection chain initiated by some firmImmorlica and Mahdian [2005]. From the above discussion, we know that no firm would be willing to initiate a rejection chain. Thus, there is no other stable matching.

The interviewing strategy can be solved sequentially for each block and is similar to our discussion for the main model where the strategies were found for each firm one by one. Consider the first block of firms. Each firm i wants to hire q_i mass of students and has an interviewing capacity of $k_i q_i$. If t of the B_1 firms[59] extend interview offers to some student θ, we can find the effective value of interviewing her as follows.

- A fraction $\frac{1}{t}$ will find firm i as the best amongst the ones who have extended interview offers. Such students will be found fit with probability p and be effectively available for i to hire with probability p.

- A fraction $\frac{1}{t}$ will find firm i to be the second best amongst the ones who have extended interview offers. Such students will be available only if found misfits by their respective best choice firms, i.e. $1 - p$ and hence are effectively available with probability $(1 - p)p$.

- $\cdots$

- A fraction $\frac{1}{t}$ of the students will find firm i to be the $\hat{t}$th best firm and hence will be effectively available for firm i with probability $(1 - p)^{\hat{t}-1}p$.

- $\cdots$

Thus the effective value for firm i of interviewing student θ who accepts t interview offers from B_1 will be given by $\frac{1}{t}\big(p + (1 - p)p + (1 - p)^2 p + \cdots + (1 - p)^{t-1}p\big)V(e^\theta)$

---

[59]We denote the cardinality of set B_1 with a slight abuse of notation as B_1.

Suppose x_{B_1}, x_{B_1-1}, $\cdots$, x_2, and x_1 are the abilities of the lowest ability students who get an interview offer from B_1, $B_1 - 1$, $\cdots$, 2, and 1 firm(s) respectively from block 1. Optimality requires that the following conditions hold.

$$\frac{1}{B_1}\big(p + (1-p)p + (1-p)^2 p + \cdots + (1-p)^{B_1-1}p\big)V(x_{B_1})$$
$$= \frac{1}{B_1 - 1}\big(p + (1-p)p + (1-p)^2 p + \cdots + (1-p)^{B_1-2})p\big)V(x_{B_1-1}) =$$
$$\cdots$$
$$= \frac{1}{2}\big(p + (1-p)p\big)V(x_2)$$
$$= pV(x_1)$$

It is important to note that the choice of the above ability points is uniquely identifiable subject to the following constraints.

1. All firms in the block either meet their interviewing capacity or hiring quota.

2. None of the firms interview more students than required to fill their capacity.

In the discussion above, we have implicitly assumed that the student interviewing capacity k_S is has played no role, i.e. $k_S \geq B_1$. If this is not true, the blocks with $> k_S$ interviews for the students can not exist. All firms will extend interview offers to this set of students and the students will pick the best k_S firms as per their preference order.[60]

We can continue with the strategic choices of the next block given the choices by the first block. The ability thresholds for B_2, $B_2 - 1$, $\cdots$, 1 offers from this block can be found using similar method. We also need to ensure that the students can accept those many interview offers, a condition which can possibly be relevant in some cases. The interview offers for each block can thus be iteratively found.

We did not specify the interview offers for each firm individually within any block. The exact strategies can be any of the (infinite number of) possibilities. As long as the specified number of firms, say t are extending the interview offers to those students who belong to a specific region, i.e. the students with ability between x_t and x_{t+1}, it will be an equilibrium. If less number of firms extend an interview offer, one of the firms will have a profitable deviation to instead extend

---

[60]Note that it will not be an equilibrium where the firms randomly extend interview offers so that only k_S offers are extended to the students. To find a profitable deviation, we only need to consider the firm who is getting the lowest ability students and knows that if it deviates to this very top regions, it will be better off as only those students will accept its interview offers who find it amongst the k_S best firms of the interview offers they have.

interviews in that region. If more than t firms extend interview offers to the said region, one of the firms will prefer to expend its interview slots elsewhere.

B.6 Introduction of a signaling stage

We continue with the block correlated economy E^{BC} described above in Section 6.2 and add a signaling stage to it. We allow for the students to send at most one signal. The timing of the game has more stages to account for the presence of signaling. It is as follows.

1. Student preferences over firms are realized and the ability parameter is revealed to the students.

2. Students choose whether to send a signal and if so, to which firm along with the applications.

3. Firms see the signals and applications from all the students who sent them.

4. Each firm decides to send interview offers to some students based on the applications and signals it received and the belief it forms based on the signals.

5. Students accept some interview offers and the accepted interviews take place.

6. Students and firms report their preferences to a central clearing authority and matching takes place as per the student-proposing deferred acceptance algorithm.

We now define the equilibrium of the modified game as follows. We denote $\mathcal{N}$ as the option for the students to not send a signal and with a slight abuse of notation also refer $\mathcal{N}$ as the possibility where a firm does not get any signals from any of the students.

Definition 15. An *equilibrium* of the signaling, interviewing and matching game is

1. a strategy of applications and signaling for each student, $\sigma_S : \Theta_e \times \Theta^\succ \to 2^F \times (F \cup \mathcal{N})$,

2. a belief for each firm about the preferences of the students who sent it a signal, $\mu_i(\cdot | S_i)$ for all $i \in F$ where $S_i \subseteq S \cup \mathcal{N}$ is the set of students who sent it a signal.

3. a strategy for each firm to extend interview offers, $\sigma_i : 2^{\Theta_e} \times S_i \to 2^{\Theta_e} \times S_i$ for all $i \in F$,

4. a strategy of interview acceptances for each student $\sigma_{\theta_e} : 2^F \to 2^F$ for all $\theta_e \in \Theta_e$, and

5. a set of preferences P_θ for all $\theta \in \Theta$ and P_i for all $i \in F$

   such that each firm and student find its/her strategies optimal given those of the other firms and students and a nondegenerate stable matching results.

66

Note that the equilibrium we focus on is in pure strategies for the students at the application and signaling stages. This implies that all students with a given ability and preference follow the same strategies. Moreover, we will restrict attention to symmetric strategies for students, i.e. $\sigma_S(\theta, \eta(\succ)) = \eta(\sigma_s(\theta, \succ))$ where $\eta(\cdot)$ is a permutation of the rank ordering over firms (which is consistent with block-symmetric preferences).

We now also add the following technical assumption on the idiosyncratic component of the students' utilities.[61]

Assumption 2. $\frac{(1-p)^{B_i-1}}{B_i}[U(i, e^\theta) - U(j, e^\theta)] \geq [\epsilon(1, i) - \epsilon(B_i, i)] + [\epsilon(1, j) - \epsilon(B_j, j)] \ \forall e^\theta$

This assumption ensures that the idiosyncratic utility for the students from any firm is a very small portion of the total utility. We will explain the exact role of this assumption when we use it.[62]

Bets-in-block (pure) strategies refer to pure strategies where students send a signal to the best firm in a block if the student decides to send a signal to a particular block. If the firms have best-in-block beliefs, i.e. a firm concludes that it is the best firm in its block for the student who sends a signal, the optimal response by the students is to send a signal to its best-in-block firm. Getting an interview offer from the best ranked firm is the best thing a signal can do in this case. The other possibilities that a firm think it is the second-best firm if a student send a signal can be ruled out using Cho and Kreps [1987] Intuitive Criterion. We only focus on best-in-block strategies in conjunction with best-in-block firms.

Remark 1. For all non-babbling symmetric equilibria of the signaling, interviewing and matching game, we have the following. Each student with ability e^θ sends a signal to the best firm in the block which meets the following two conditions.

1. The firms in the block respond to the signals from students with ability e^θ, and

2. It is the best block where all firms do not extend interview offers to the specific student

We focus on best-in-block pure strategies for the students and best-in-block beliefs for the firms.

Consider a student θ. This student has an option to send a signal to one of the blocks as we focus on pure strategies for the students. It will be wasteful to send a signal to the (best firm in

---

[61]Recall that $U(b, x)$ is the utility that any firm in block b gets when matched with a fit student of ability x. It is also the expected utility that the student gets from being matched with a random firm in block b. The idiosyncratic component of the student's utility is given by $\epsilon(t, b)$–corresponding to the firm which has a rank t within block b–which adds to the utility $U(b, x)$. See definition 13 for the exact meaning of rank within a block.

[62]Consider the example of 4 firms and 2 blocks. If the idiosyncratic utility is a much smaller portion as compared to the common value for each firm. In this case the assumption is satisfied.

the) block which comprises of firms who all send an interview offer to this student. Even without signaling the student can be sure of getting an interview offer from the best firm in such a block (at equilibrium). Consider the best block i such that all firms in the block do not send an interview offer to the student (at equilibrium). Also consider another such block j where all firms in that block do not send interview offers to the student with certainty. If such a j does not exist, by weak optimality of sending a signal only to such blocks, the student will send a signal to the best firm in block i and the proposition will be hold trivially.

We only need to consider the following three cases where we compare the option of sending a signal to block i best firm (option 1) with the option of sending a signal to block j best firm (option 2).[63]

I) It is an equilibrium where the student sends a signal to block i and not to block j.

II) There does not exist an equilibrium where the student sends a signal to block j.

III) It can not be the case that the student does not send a signal to block i or block j.

Case I If all students of ability e^θ send a signal to their respective best firms in block i, then each firm expects to get a signal from $\frac{1}{B_i}$ of these students. Each firm has the following coordinated strategy[64] for the students of this ability.

(i) Send an interview offer to all students who have sent a signal.

(ii) Send an interview offer to $\frac{t_i - 1}{B_i}$ of those students from whom it has not received a signal.

Due to the presence of continuum of students, this results in the following equilibrium offers for a student who sends a signal to this block i.

1. An interview offer from the firm it sent a signal.

2. Interview offers from $t_i - 1$ firms whom it did not send a signal.

The student with ability e^θ can choose to send a signal to the best firm in block i, i.e. take the equilibrium path of action. It can otherwise decide to send a signal to the best firm in block j. Let us evaluate the two options in terms of the expected utility the student can get. Suppose that there are t_x interview offers from firms which lie between blocks i and j.

---

[63]The remaining case where the student sends a signal to both block i and block j is impossible as we focus on symmetric pure strategies and hence not considered.

[64]We call this strategy coordinated as the firms 'virtually' coordinate on the equilibrium to ensure that each student has t_i and exactly t_i offers from the firms in this block. The equilibration process is of great interest but out of scope for the current discussion. If anything, the presence of signaling will ease the equilibration process.

Option 1 results in an interview offer from the best firm in block i, say i_1, with probability 1, $t_i - 1$ offers from other firms in $B_i \setminus \{i_1\}$ and interview offers from any t_j firms in block j. The relevant parts of the expected utility $(Util(opt1))$[65] from the above offers can be expressed as follows.

$$
\begin{aligned}
&[p + (1-p)p + \cdots + (1-p)^{t_i-1}p]U(i, e^\theta) \\
&+(1-p)^{t_i+t_x}(p + (1-p)p + \cdots + (1-p)^{t_j-1}p)U(j, e^\theta) \\
&\quad + [p\epsilon(1, i) + p(1-p)\epsilon(\cdot, i) \cdots] \\
&\quad + (1-p)^{t_i+t_x}(p\epsilon(\cdot, j) + p(1-p)\epsilon(\cdot, j) + \cdots)
\end{aligned}
$$

Option 2 results in $t_i - 1$ offers from any of the firms in B_i, t_j offers from any of the firms in block j, and also an offer from the best firm in block j (if it has not already sent an interview offer). The relevant parts of the expected utility $(Util(opt2))$[66] from the above offers can be expressed as follows.

$$
\begin{aligned}
&[p + (1-p)p + \cdots + (1-p)^{t_i-2}p]U(i, e^\theta) \\
&+(1-p)^{t_i-1+t_x}(p + (1-p)p + \cdots + (1-p)^{t_j-1}p)U(j, e^\theta) \\
&\quad + \frac{B_j - t_j}{B_j}(1-p)^{t_i-1+t_x+t_j}pU(j, e^\theta) \\
&\quad + [p\epsilon(\cdot, i) + p(1-p)\epsilon(\cdot, i) \cdots] \\
&\quad + (1-p)^{t_i+t_x}(p\epsilon(1, j) + p(1-p)\epsilon(\cdot, j) + \cdots) \\
&\quad + \frac{B_j - t_j}{B_j}(1-p)^{t_i-1+t_x+t_j}p\epsilon(\cdot, j)
\end{aligned}
$$

The expected utility expressions corresponding to each of the options above have four components. The first component has a series of probabilities multiplying $U(i, e^\theta)$. For option 1, the best in block firm i_1 extends an interview offer to this student and finds her fit with probability p. If she is found misfit for the first firm, she has a chance p of being found fit for the next firm which extends an interview offer to her. A total of t_i such firms extend an interview offer to this student. We know that the best firm i_1 extends an interview offer to the student. However, the identity of the remaining $t_i - 1$ firms is not known for sure. This also explains the third component of the expected utility expression. It has $p\epsilon(1, i)$ coming from firm i_1 and the remaining $\epsilon(\cdot, i)$ corresponding to the other firms. Similarly the second and fourth components correspond to the

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[65] The explanation for these expressions is provided below.
[66] The explanation for these expressions is provided below.

block-specific utility for interview offers from block j and the firm-specific components where the identity of none of the t_j firms is known for sure. It is important to note that the expected utilities corresponding to the jth block kick in only after all the better firms, i.e. $t_i + t_x$ in number, have found the student misfit. This explains the leading $(1-p)^{t_i+t_x}$ multiplying the block-specific and firm-specific expected utility components from block j.

The expected utility expression for the second option also has four components. The first and the third components have only $t_i - 1$ terms as the student gets only $t_i - 1$ offers from the firms who have not seen a signal from this student. The second component has the regular t_j terms corresponding to the t_j interview offers from the firms in block j. However, there is a probability $\frac{B_j - t_j}{B_j}$ that the best firm in this block, say i_1, was not going to make an offer and hence the other t_j firms were the ones making an interview offer. This leads to the case where the student gets $t_j + 1$ interview offers from this block.

We are now ready to put a bound on the difference between the expected utility of sending a signal to block i and that to block j.

$$
\begin{aligned}
Util(opt1) - Util(opt2) > {} & (1-p)^{t_i-1}pU(i, e^\theta) + (1-p)^{t_i+t_x+t_j-1}pU(j, e^\theta) \\
& + p\epsilon(1, i) + (1-p)\epsilon(B_i, i) + (1-p)^{t_i+t_x}\epsilon(B_j, j) \\
& - \left[(1-p)^{t_i-1}pU(j, e^\theta) + \frac{B_j - t_j}{B_j}(1-p)^{t_i+t_x+t_j-1}U(j, e^\theta) \right. \\
& \left. + \epsilon(1, i) + (1-p)^{t_i+t_x}\epsilon(1, j) \right] \\
\geq {} & (1-p)^{t_i-1}p[U(i, e^\theta) - U(j, e^\theta)] \\
& - \left[(1-p)(\epsilon(1, i) - \epsilon(B_i, i)) + (1-p)^{t_i+t_x}(\epsilon(1, j) - \epsilon(b_j, j)) \right]
\end{aligned}
$$

We know the following from our technical assumption about the utilities from different blocks.

$$
\frac{1}{B_i}(1-p)^{B_i}(U(i, e^\theta) - U(j, e^\theta)) \geq [\epsilon(1, i) - \epsilon(B_i, i)] + [\epsilon(1, j) - \epsilon(B_j, j)]
$$
$$
(1-p)^{t_i-1}(U(i, e^\theta) - U(j, e^\theta)) \geq (1-p)[\epsilon(1, i) - \epsilon(B_i, i)] + (1-p)^{t_i+t_x}[\epsilon(1, j) - \epsilon(B_j, j)]
$$
$$
Util((opt1) - Util(opt2) > 0
$$

Thus, there is no profitable deviation for the student from the equilibrium course of action.

Case II If all students of ability e^θ send a signal to their respective best firms in block j, then each firm in block j expects to get a signal from $\frac{1}{B_j}$ of these students. Each firm in block j has the following coordinated strategy[67] for the students of this ability.

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[67]We call this strategy coordinated as the firms 'virtually' coordinate on the equilibrium to ensure that each student

1. Send an interview offer to all students who have sent a signal.

2. Send an interview offer to $\frac{t_j-1}{B_j}$ of those students from whom it has not received a signal.

Block i firms send an interview offer to $\frac{t_i}{B_i}$ of the students in a coordinated manner. We now evaluate the two options that the student faces –sending a signal to the best firm from either block i or block j.

Option 1 results in interview offers from t_i random firms in block i and a possibly additional $(t_i + 1\text{th})$ interview offer (from this block) from the best firm, say i_1, (if it has not already sent an offer) and $t_j - 1$ interview offers from a random set of firms in block j. The relevant parts of the expected utility $(Util(opt1))$ from the above offers can be expressed as follows.

$$[p + (1-p)p + \cdots + (1-p)^{t_i-1}p]U(i, e^\theta) + \frac{B_i - t_i}{B_i}(1-p)^{t_i}pU(i, e^\theta)$$

$$+[\frac{t_i}{B_i}(1-p)^{t_i+t_x} + \frac{B_i - t_i}{B_i}(1-p)^{t_i+t_x+1}](p + (1-p)p + \cdots + (1-p)^{t_j-2}p)U(j, e^\theta)$$

$$+[p\epsilon(1, i) + p(1-p)\epsilon(\cdot, i) \cdots (1-p)^{t_i-1}p\epsilon(\cdot, i)] + \frac{B_i - t_i}{B_i}p(1-p)^{t_i}\epsilon(\cdot, i)$$

$$+ [\frac{t_i}{B_i}(1-p)^{t_i+t_x} + \frac{B_i - t_i}{B_i}(1-p)^{t_i+t_x+1}](p\epsilon(\cdot, j) + (1-p)p\epsilon(\cdot, i) + \cdots + (1-p)^{t_j-2}p\epsilon(\cdot, i))$$

Option 2 results in t_i interview offers from any of the firms in block i, an interview offer from the best firm in block j, say j_1, and $t_j - 1$ interview offers from any of the firms from the set $B_j \setminus \{j_1\}$. The relevant parts of the expected utility $(Util(opt2))$ from the above offers can be expressed as follows.

$$[p + (1-p)p + \cdots + (1-p)^{t_i-1}p]U(i, e^\theta)$$

$$+(1-p)^{t_i+t_x}(p + (1-p)p + \cdots + (1-p)^{t_j-1}p)U(j, e^\theta)$$

$$+[p\epsilon(\cdot, i) + p(1-p)\epsilon(\cdot, i) + \cdots + (1-p)^{t_i-1}p\epsilon(\cdot, i)]$$

$$+ (1-p)^{t_i+t_x}(p\epsilon(1, j) + (1-p)p\epsilon(\cdot, i) + \cdots + (1-p)^{t_j-1}p\epsilon(\cdot, i))$$

The expected utility expressions corresponding to each of the options above can be broken down in four components like we did in case I. The first component has a series of probabilities multiplying $U(i, e^\theta)$ corresponding to the t_i offers which the student gets. There is a possibility of a $t_i + 1\text{th}$ interview offer from this block if the t_i firms that were meant to extend the interview offers to her did not include the best in block firm, i_1. Note that this also explains the third component

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has t_i and exactly t_i offers from the firms in this block.

of the expected utility expression which accounts for the fact that i_1 will definitely interview the student and the identities of the other $t_i - 1$ or t_1 firms is not known for sure. The $t_j - 1$ interview offers from the jth block will result in the series of probabilities multiplying $U(j, e^\theta)$. However, the expected utilities corresponding to the jth block matter only after the t_i or $t_i + 1$ firms from block i and the t_x firms from blocks between i and j find the student a misfit. Due to the uncertainty about the exact number of offers from block i due to the off-equilibrium action from this student, we account for the different possibilities when evaluating the worth of interview offers from block j.

The expected utility expression for the second option also has four components. The first and the third components correspond to the t_i random offers from block i. The second component all the t_j terms corresponding to the t_j interview offers from the firms in block j including the best firm j_1. The fourth component accounts for the firm-specific utility from being matched to a firm within block j.

We are now ready to put a bound on the difference between the expected utility of sending a signal to block i and that to block j.

$$
\begin{aligned}
Util(opt1) - Util(opt2) > {} & \frac{B_i - t_i}{B_i}(1 - p)^{t_i}pU(i, e^\theta) \\
& + p\epsilon(1, i) + (1 - p)\epsilon(B_i, i) + (1 - p)^{t_i + t_x}\epsilon(B_j, j) \\
& - \Big[\frac{B_i - t_i}{B_i}(1 - p)^{t_i + t_x}pU(j, e^\theta) \\
& + \epsilon(1, i) + (1 - p)^{t_i + t_x}\epsilon(1, j)\Big] \\
\geq {} & \frac{B_i - t_i}{B_i}(1 - p)^{t_i}p[U(i, e^\theta) - U(j, e^\theta)] \\
& - [(1 - p)(\epsilon(1, i) - \epsilon(B_i, i)) + (1 - p)^{t_i + t_x}(\epsilon(1, j) - \epsilon(B_j, j))]
\end{aligned}
$$

We know the following from our technical assumption about the utilities from different blocks.

$$
\begin{aligned}
\frac{1}{B_i}(1 - p)^{B_i}(U(i, e^\theta) - U(j, e^\theta)) & \geq [\epsilon(1, i) - \epsilon(B_i, i)] + [\epsilon(1, j) - \epsilon(B_j, j)] \\
\frac{B_i - t_i}{B_i}(1 - p)^{t_i}(U(i, e^\theta) - U(j, e^\theta)) & \geq (1 - p)[\epsilon(1, i) - \epsilon(B_i, i)] \\
& \quad + (1 - p)^{t_i + t_x}[\epsilon(1, j) - \epsilon(B_j, j)] \\
Util(opt1) - Util(opt2) & > 0
\end{aligned}
$$

Thus, there is a profitable deviation for the student to send a signal to the best block firm i_1 instead of the conjectured equilibrium strategy of sending it to block j. This leads to a contradiction

and hence it is not an equilibrium to send a signal to this block.

Case III We want to prove that it can not be an equilibrium to waste the signal for a student and not send it to any firm. We will prove that the student has a profitable deviation of sending it to the best block firm. We will proceed with similar steps and compare the option of sending the signal to block i to that of not sending it at all.

Option 1 of sending a signal to the best firm in block i results in t_i interview offers from any of the firms in block i and a possibly additional ($t_i + 1$th) interview offer (from this block) from the best firm, say i_1, (if it has not already sent an offer).

Option 2 of not sending the signal at all results in t_i interview offers from any of the firms in block i.

It is clear to see that there is only a possible extra interview offer from a $t_i + 1$th firm and the expected utility of sending a signal to the ith block firm is greater than not sending it to any firm. This is true even if the interview capacity of the student binds because with probability $\frac{B_i = t_i}{B_i}$ the student's best choice firm does not send an interview offer to her without the signal.

We have thus proved that of the three possible scenarios, the only equilibrium is that of sending a signal to the best firm in block i which was the best block where all firms did not send an interview offer to the student. $\square$

Now with the student signaling strategies in our hand, we can focus on the equilibrium characterization and the impact of signaling on these markets. We will focus on non-babbling symmetric equilibria where at least some signals from some students are not ignored by the firms. This provides the result that signaling can achieve the sorting mechanism that was achieved under small interviewing capacity.

Proposition 10. *There exists at least one non-babbling symmetric signaling equilibrium. There exists a symmetric non-babbling signaling equilibrium such that the sum of students' utilities goes up, the sum of firms' utilities and the number of matched agents stay the same.*

Proof We know from Theorem 2's proof above that for all preferences that result from any equilibrium interviewing results in a unique stable matching and truth-telling is optimal. We will take that result and use it in our setting here with signaling as after the interviews have taken place the two settings are not different.

We will find the firm interview offers by solving them for each block iteratively. We will also solve for the signaling strategies of the students as we know that they send them to the best block firm of the best block for which all firms do not extend an offer to her and also respond to signals sent by students of her ability. We can come up with the equilibrium offers by iterated elimination of dominated strategies. From theorem 2's proof we know that the first block firms extend at least

$1, 2, 3, \cdots, B_1$ interview offers to students whose ability is greater than $x_1, x_2, \cdots, x_{B_1}$ respectively. Now consider the strategy of these students such that they send the best firm in this block a signal if they do not get an offer from all the firms.

A signal sent from a student has two impacts.

1. There is a direct effect of signaling that each firm recognizes that a student finds it best, expects a signal from $\frac{1}{B_1}$ of the students and send an interview offer to these students

2. There is an indirect effect of signaling on those firms whom she did not send a signal as those firms recognize that such a student has sent a signal to her best firm and will be interviewed by that firm.

The indirect effect manifests itself in a possibly different strategy for the firm about the cut-offs on the ability dimension if the students send signals to all the top firms. Consider the region where the students received two interview offers from the first block. Earlier a firms that interviews such a student and will be able to hire her with probability $(0.5)(p + (1 - p)p)$. The firms chose to extend 2 interview offers to all students with ability between x_2 and x_3. However, now consider the same region of students where the students send a signal to their respective best firms. A firm that does not receive a signal recognizes that the probability of hiring this student is $(1 - p)p$ which is lower than the probability of hiring for such a student when there was no signaling. The firm would optimally decide to extend its interview offers only to the students with ability $x_{2_{-1}}(> x_2)$. Similarly there will be different ability thresholds for extending interview offers for students who have not sent a signal, i.e. $x_{3_{-1}}$.

The following is an equilibrium.

1. Students with ability in the range of x_k and $x_{k_{-1}}$ do not send signals to block 1 firms and all firms in this block ignore any signals from these students.

2. Other students who do not receive B_1 offers from block B_1 send a signal to their best firm in this block.

3. Students extend interview offers as per the above strategy of sending an offer to everyone who sends a signal and also those above the thresholds even when they do not send a signal.

The match increases the utility of the students as they get weakly better matches. The firms continue to get the same number of students in all the regions and are not affected. Hence the welfare for the students goes up and that of the firms remains unchanged. The equilibrium number of matches also stays the same.

The equilibrium strategies are uniquely identified up to multiplicities within a block. The solution can be obtained iteratively while being mindful of the signaling strategy by the students of best-in-block firm from the best block which does not extend an interview offer for sure and responds to the signals. When compared with the results from Coles et al. [2013], the result about firm welfare might appear at odds. They prove that in all non-babbling signaling equilibria in their setting, the student welfare improves but the firm welfare is ambiguously affected. The ambiguous effect of signaling on the firms' welfare in Coles et al. [2013] was due to the competition effect when a particular firm pays attention to some student who was not very high up on its ranking but sent it a signal. However, the alignment of firm preferences on the ability parameter of the students rules out this effect and ensures that the overall welfare for the firms does not decrease. The number of matches stay the same as at this equilibrium the firms use the interview offers similar to those in the case of no-signaling but just align the offers more towards the students who signal.

The student welfare result follows in similar spirit because if anything the students are going to gain more from getting an offer from their best ranked firm within a block. However, the comparison is subtle because for some other symmetric non-babbling equilibria, the welfare for some students may go down with signaling. Although our current model does not have any student-specific idiosyncratic utility to the firms from employing students who rank it at the top of their list, it is an easy extension to include and we summarize this in the following Remark without providing its proof.

Remark 2. If the firm receive a part of the idiosyncratic component of the utility, there exists a non-babbling equilibrium where the sum of firms' utilities weakly increases when they pay attention to the signals from the students.

B.7 Fitness factor correlated with ability and firm identity

Let us call the economy where the fitness factor for students depend on the ability and firm identity as E^{fitab}. Specifically the economy is characterized by a function $p : F \times \Theta^e \to [0,1]$ where $p(i, e^\theta)$ is the probability of finding a student with ability e^θ) fit for firm i. We assume that $p(i, e^\theta)$ is decreasing in i.

Theorem 4. *In an economy E^{fitab} where the agents face an interviewing constraint $[k_F, k_S]$, there exists an equilibrium and it is essentially unique.*

The proof of this theorem will follow exactly the same way as proof of Theorem 2 presented in Section B.1. We draw attention to the differences and avoid presenting the entire proof.

Our results for decrease in quantity of matching and the quality and quantity tradeoff also follow almost identically with the same steps.

Step i) For all preferences, there exists a unique stable matching and truth-telling is optimal for both firms and students.

Step ii) Each firm has a unique nondegenerate interview offer strategy solvable by iterated elimination of dominated strategies under assumption 1 for all firms.

Step iii) The firm and student preferences after nondegenerate interview offers result in a unique nondegenerate stable matching.

In this setting the fitness factor is independent across firms but is possibly dependent on the identity of the firm and the student ability. Thus, the probability of finding a student θ fit for a particular firm i is given by $p(i, e^\theta)$. The information about these functions is common knowledge just as the value of p was known to all the market participants. Given the characteristics of $p(\cdot, \cdot)$ function, the student's choice remains the same when faced with more interview offers than they can accept, i.e. pick the best interview offers. This in turn simplifies the interview offer strategy decisions for the firms. The problem for each firm remains to find out the optimal region of student abilities which maximizes the expected value of spending those interview slots given the decision by better firms. In the discussion so far it always included the best students available who have a certain number of interviews. However, now the fitness factor is also dependent on the student ability and the student at the very top might be hired away with much higher probability and hence each firm needs to find the exact region where the value of its interview slots is maximized. This does not affect the process by which the firms decide their interview offers. The iterative procedure of deciding the interview offers continues to hold. The rest of the proof applies in this case too.

Proposition 11. *When the interviewing regime moves from LC to HC, the quality of the match weakly increases. If firms do not have any excess interview capacity and the quality strictly increases, the quantity strictly decreases at least for one firm.*

The move from LC to HC regimes also naturally extends itself to this more general setting. The quality improvement holds as the firms make strategic choices in the HC regime while declining the options available even under the LC regime. The decrease in quantity for the firm which extends different interview offers also follows in very similar spirit from our proof of Proposition 2