# Quantitative Methods in Economics
# Causality and treatment effects

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# 5) Difference-in-Differences

(cf. “Mostly Harmless Econometrics,” chapter 5)

# Selection on Unobservables

- Often there are reasons to believe that treated and untreated differ in unobservable characteristics that are associated to potential outcomes even after controlling for differences in observed characteristics.
- In such cases, treated and untreated may not be directly comparable, even after adjusting for observed characteristics.
- The difference-in-differences estimator uses the same strategy as the panel data fixed-effects estimators to get rid of unobserved confounders whose effects do not change in time.

## Motivating Example: The Mariel Boatlift

- How do inflows of immigrants affect the wages and employment of natives in local labor markets?
- Card (1990) uses the Mariel Boatlift of 1980 as a natural experiment to measure the effect of a sudden influx of immigrants on unemployment among less-skilled natives

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- Card (1990) uses the Mariel Boatlift of 1980 as a natural experiment to measure the effect of a sudden influx of immigrants on unemployment among less-skilled natives
- The Mariel Boatlift increased the Miami labor force by 7%
- Individual-level data on unemployment from the Current Population Survey (CPS) for Miami and four comparison cities (Atlanta, Los Angeles, Houston and Tampa-St. Petersburg)

# Motivating Example: The Mariel Boatlift

Differences-in-differences estimates of the effect of immigration on unemployment[a]

| | Group | Year | | |
|---|---|---|---|---|
| | | 1979 (1) | 1981 (2) | 1981–1979 (3) |
| | Whites | | | |
| (1) | Miami | 5.1 (1.1) | 3.9 (0.9) | − 1.2 (1.4) |
| (2) | Comparison cities | 4.4 (0.3) | 4.3 (0.3) | − 0.1 (0.4) |
| (3) | Difference Miami-comparison | 0.7 (1.1) | − 0.4 (0.95) | − 1.1 (1.5) |
| | Blacks | | | |
| (4) | Miami | 8.3 (1.7) | 9.6 (1.8) | 1.3 (2.5) |
| (5) | Comparison cities | 10.3 (0.8) | 12.6 (0.9) | 2.3 (1.2) |
| (6) | Difference Miami-comparison | − 2.0 (1.9) | − 3.0 (2.0) | − 1.0 (2.8) |

[a] Notes: Adapted from Card (1990, Tables 3 and 6). Standard errors are shown in parentheses.

# Difference-in-Differences Setup

Two groups:

- $D = 1$: treated units
- $D = 0$: control units

Two periods:

- $T = 0$: pre-treatment period
- $T = 1$: post-treatment period

Potential outcomes:

- $Y_{1i}(t)$: outcome unit $i$ attains in period $t$ if treated before $t$
- $Y_{0i}(t)$: outcome unit $i$ attains in period $t$ if not treated before $t$

# Difference-in-Differences Setup

Treatment effect for unit $i$ at time $t$ is

$$Y_{1i}(t) - Y_{0i}(t).$$

Observed outcomes $Y_i(t)$ are realized as

$$Y_i(t) = Y_{0i}(t)(1 - D_i(t)) + Y_{1i}(t)D_i(t).$$

Because the treatment occurs only after $t = 0$, we define

$$D_i = D_i(1).$$

It follows that,

$$Y_i(0) = Y_{0i}(0),$$
$$Y_i(1) = Y_{0i}(1)(1 - D_i) + Y_{1i}(1)D_i.$$

# Difference-in-Differences Result

Let

$$\alpha_{ATET} = E[Y_1(1) - Y_0(1)|D = 1].$$

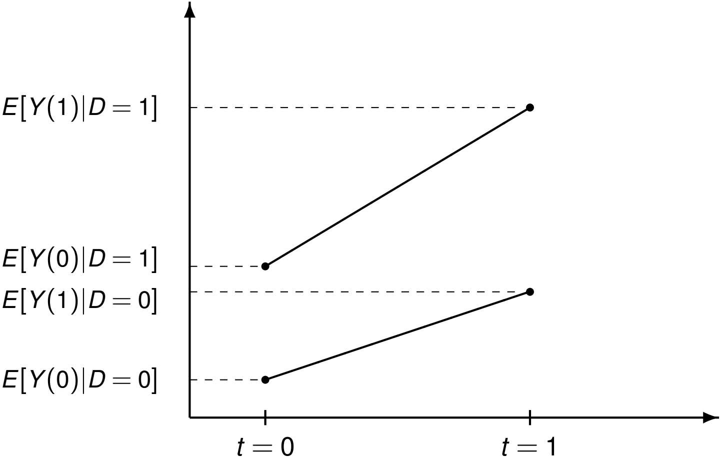
If the treated and non-treated would have exhibited the same trend in the absence of the treatment,

$$E[Y_0(1) - Y_0(0)|D = 1] = E[Y_0(1) - Y_0(0)|D = 0],$$

then:

$$\begin{aligned}\alpha_{ATET} = &\Big[E[Y(1)|D = 1] - E[Y(1)|D = 0]\Big] \\ &- \Big[E[Y(0)|D = 1] - E[Y(0)|D = 0]\Big].\end{aligned}$$

# Difference-in-Differences: Graphical Interpretation



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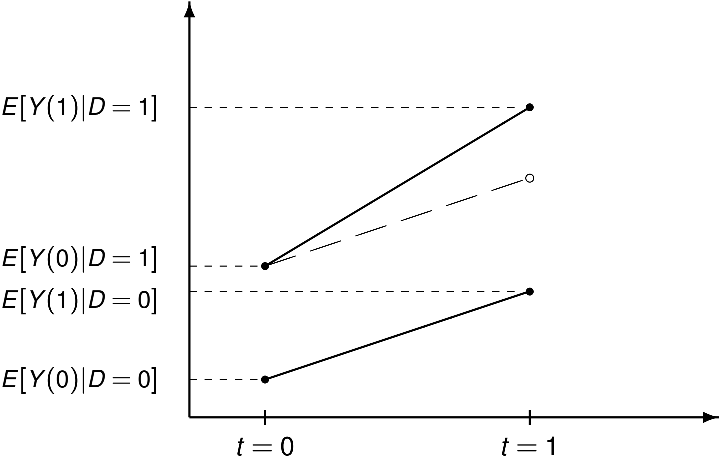

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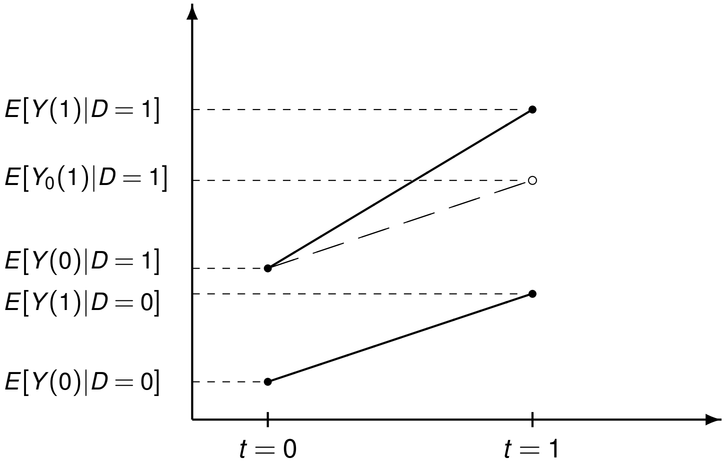

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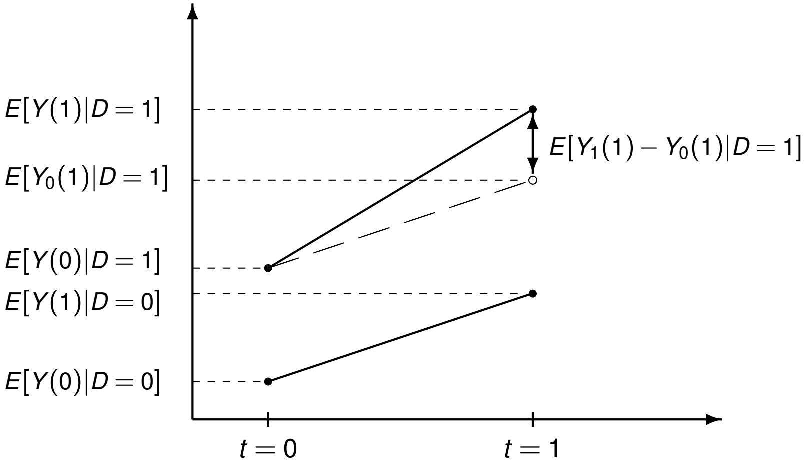

# Difference-in-Differences Estimators

1. Panel Data/Sample Means

$$\left\{\frac{1}{N_1}\sum_{D_i=1} Y_i(1) - \frac{1}{N_0}\sum_{D_i=0} Y_i(1)\right\} - \left\{\frac{1}{N_1}\sum_{D_i=1} Y_i(0) - \frac{1}{N_0}\sum_{D_i=0} Y_i(0)\right\}$$

$$= \left\{\frac{1}{N_1}\sum_{D_i=1}\{Y_i(1) - Y_i(0)\} - \frac{1}{N_0}\sum_{D_i=0}\{Y_i(1) - Y_i(0)\}\right\},$$

where $N_1$ is the number of treated individuals and $N_0$ is the number of non-treated individuals.

# Difference-in-Differences Estimators

2. Repeated Cross-Sections/Sample Means

Let $\{Y_i, D_i, T_i\}_{i=1}^{N}$ be the pooled sample (the two different cross-sections merged) where $T$ is a variable that indicates the period (0 or 1) in which the individual is observed.

A DID estimator is given by:

$$\left\{ \frac{\sum D_i \cdot T_i \cdot Y_i}{\sum D_i \cdot T_i} - \frac{\sum (1 - D_i) \cdot T_i \cdot Y_i}{\sum (1 - D_i) \cdot T_i} \right\} - \left\{ \frac{\sum D_i \cdot (1 - T_i) \cdot Y_i}{\sum D_i \cdot (1 - T_i)} - \frac{\sum (1 - D_i) \cdot (1 - T_i) \cdot Y_i}{\sum (1 - D_i) \cdot (1 - T_i)} \right\}.$$

# Difference-in-Differences Estimators

3. Repeated Cross-Sections/Regression

   The same estimator can be obtained using regression techniques. Consider the linear model:

$$Y_i = \mu + \gamma \cdot D_i + \delta \cdot T_i + \alpha \cdot (D_i \cdot T_i) + \varepsilon_i,$$

   where $E[\varepsilon|D,T] = 0$. Then, it is easy to show that

$$\begin{aligned} \alpha = \quad & \{E[Y|D=1,T=1] - E[Y|D=0,T=1]\} \\ - \quad & \{E[Y|D=1,T=0] - E[Y|D=0,T=0]\}. \end{aligned}$$

# Difference-in-Differences Estimators

Consider a regression version of the DID estimator including covariates:

$$Y_i = \mu + \gamma D_i + \delta T_i + \alpha D_i T_i + X_i' \beta + \varepsilon_i.$$

- introducing time-invariant $X$'s in this way is not helpful (they get differenced-out)
- time-varying $X$'s may be problematic because they are often affected by the treatment and may introduce endogeneity

More sensible: Interact time-invariant covariates with the time indicator:

$$Y_i = \mu + \gamma D_i + \delta T_i + \alpha D_i T_i + T_i X_i' \beta_1 + (1 - T_i) X_i' \beta_0 + \varepsilon_i$$

$\Rightarrow$ $X$ is used to explain differences in trends.

# Difference-in-Differences Estimators

4. Panel Data/Regression

   With panel data we can use regression with the dependent variable in first differences:

   $$\Delta Y_i = \delta + \alpha \cdot D_i + X_i' \beta + u_i,$$

   where $\Delta Y_i = Y_i(1) - Y_i(0)$, $\beta = \beta_1 - \beta_0$, and $u_i = \Delta \varepsilon_i$.

   $\Rightarrow$ Here $X$ also explains differences in trends.

# Difference-in-Differences Estimators

The fixed effects estimator generalizes DID in the context of panel data and multiple groups and time periods:

$$Y_{it} = \mu + \gamma_i + \delta_t + \alpha D_{it} + X_{it}'\beta + \varepsilon_{it}$$

- One intercept for each unit, $\gamma_i$, and time period, $\delta_t$
- Cluster standard errors at the subject unit level
- Can add unit specific time trends:

$$Y_{it} = \mu + \gamma_{0i} + \gamma_{1i} t + \delta_t + \alpha D_{it} + X_{it}'\beta + \varepsilon_{it}$$

with unit specific intercepts $\gamma_{0i}$ and unit specific trend coefficients $\gamma_{1i}$ that multiply time trend variable $t$.

# Difference-in-Differences: Threats to Validity

1. **Compositional differences**: In repeated cross-sections we do not want that the composition of the sample changes between periods.

   The distribution of $(D, X)$ should be similar for the pre-treatment and post-treatment periods.

2. **Non-parallel trends**: Different trends for treated and nontreated in the absence of the treatment.

   Falsification Test: Under the parallel trends assumption during the periods $t = -1, 0, 1$, we have:

   $$E[Y(0) - Y(-1)|D = 1] - E[Y(0) - Y(-1)|D = 0] = 0$$

   Apply DID estimator to $t = -1, 0$ and test if $\alpha = 0$

# DDD: Mandated Maternity Benefits (Gruber, 1994)

TABLE 3—DDD ESTIMATES OF THE IMPACT OF STATE MANDATES ON HOURLY WAGES

| Location/year | Before law change | After law change | Time difference for location |
|---|---|---|---|
| A. *Treatment Individuals: Married Women, 20–40 Years Old:* | | | |
| Experimental states | 1.547<br>(0.012)<br>[1,400] | 1.513<br>(0.012)<br>[1,496] | −0.034<br>(0.017) |
| Nonexperimental states | 1.369<br>(0.010)<br>[1,480] | 1.397<br>(0.010)<br>[1,640] | 0.028<br>(0.014) |
| Location difference at a point in time: | 0.178<br>(0.016) | 0.116<br>(0.015) | |
| Difference-in-difference: | −0.062<br>(0.022) | | |

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| B. *Control Group: Over 40 and Single Males 20–40:* | | | |
| Experimental states | 1.759<br>(0.007)<br>[5,624] | 1.748<br>(0.007)<br>[5,407] | −0.011<br>(0.010) |
| Nonexperimental states | 1.630<br>(0.007)<br>[4,959] | 1.627<br>(0.007)<br>[4,928] | −0.003<br>(0.010) |
| Location difference at a point in time: | 0.129<br>(0.010) | 0.121<br>(0.010) | |
| Difference-in-difference: | −0.008:<br>(0.014) | | |

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| **DDD:** | **−0.054**<br>**(0.026)** | | |

# Terrorism in Cities (Abadie and Dermisi, 2008)

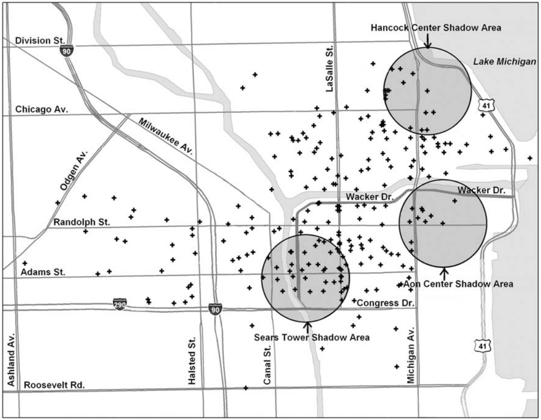


FIGURE I

Chicago's Central Business District Office Buildings and Shadow Areas

Crosses represent all Class A and Class B office buildings in Chicago's Central Business District. Shaded circles represent 0.3-mile radius "shadow areas" surrounding the three main Chicago landmark buildings: the Aon Center, the Hancock Center, and the Sears Tower.

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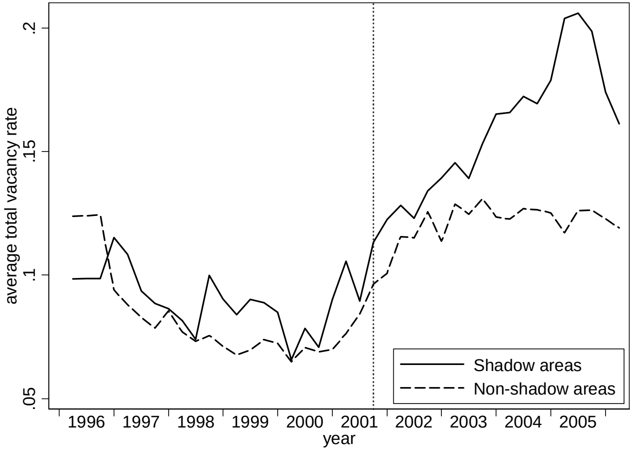


FIGURE II

Average Vacancy Rates in Shadow and Non-shadow Areas

# Terrorism in Cities (Abadie and Dermisi, 2008)

TABLE I

DESCRIPTIVE STATISTICS – MEANS AND STANDARD DEVIATIONS
(Class A and B office buildings in downtown Chicago)

| | (1) Entire sample | (2) Inside shadow areas | (3) Outside shadow areas | (4) Diff. (2)-(3) (s.e.) |
|---|---|---|---|---|
| *Characteristics of the buildings:* | | | | |
| shadow (= 1 if in shadow area, = 0 otherwise) | .27<br>[.45] | | | |
| Class A (=1 if Class A building, =0 if Class B building) | .21<br>[.41] | .44<br>[.50] | .13<br>[.34] | .31**<br>(.07) |
| distance to anchor (miles) | .46<br>[.26] | .19<br>[.08] | .56<br>[.24] | -.38**<br>(.02) |
| height (hundred feet) | 2.76<br>[2.46] | 4.43<br>[2.90] | 2.14<br>[1.94] | 2.29**<br>(.39) |
| number of stories | 19.77<br>[18.80] | 32.59<br>[21.67] | 14.96<br>[15.08] | 17.63**<br>(2.90) |
| rentable building area (sq. feet) | 353,683<br>[499,847] | 665,705<br>[604,842] | 236,675<br>[397,123] | 429,031**<br>(80,243) |
| *Vacancy rates (fraction):* | | | | |
| First quarter of 2001 | .0803<br>[.0949] | .0901<br>[.0903] | .0699<br>[.0989] | .0202<br>(.0174) |
| First quarter of 2006 | .1491<br>[.1306] | .1740<br>[.1302] | .1228<br>[.1266] | .0512**<br>(.0248) |
| *Rent per square foot (current USD):* | | | | |
| First quarter of 2001 | 30.40<br>[5.43] | 32.22<br>[5.59] | 28.08<br>[4.25] | 4.14**<br>(1.23) |
| First quarter of 2006 | 28.08<br>[5.97] | 29.09<br>[5.30] | 26.78<br>[6.54] | 2.31*<br>(1.28) |
| *Number of buildings in the sample* | 242 | 66 | 176 | |

# Terrorism in Cities (Abadie and Dermisi, 2008)

TABLE II

9/11 AND VACANCY RATES IN DOWNTOWN CHICAGO OFFICE BUILDINGS
(Fixed-effects estimates with clustered standard errors, 1996-2006)

Dependent variable: Building vacancy rate

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| shadow area×post-9/11 | .0303* (.0166) | | | |
| distance to anchor×post-9/11 | | -.0617* (.0362) | | |
| distance to non-shadow area×post-9/11 | | | .2302** (.0633) | |
| height×post-9/11 | | | | .0052** (.0022) |
| *R-squared* | .39 | .39 | .39 | .39 |
| *Number of observations* | 9,922 | 9,922 | 9,922 | 9,922 |

*Note:* The sample is a quarterly panel of Class A and Class B office buildings in the extended Chicago Central Business District between the second quarter of 1996 and the second quarter of 2006. See text of the article for the exact limits of the area of the City of Chicago included in our sample. Observations are weighted by the rentable area of the buildings. All specifications include building fixed effects and a full set of year×quarter dummies. Standard errors (in parentheses) are clustered at the building level.
* indicates statistical significance at the 10% level.
** indicates statistical significance at the 5% level.

# Terrorism in Cities (Abadie and Dermisi, 2008)

TABLE III

TIME SINCE 9/11 AND VACANCY RATES IN DOWNTOWN CHICAGO OFFICE BUILDINGS
(Fixed-effects estimates with clustered standard errors, 1996-2006)

Dependent variable: Building vacancy rate

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| shadow area×post-9/11 | -.0046 | | | |
| | (.0173) | | | |
| shadow area×quarters since 9/11 | .0037** | | | |
| | (.0017) | | | |
| distance to anchor×post-9/11 | | .0156 | | |
| | | (.0379) | | |
| distance to anchor×quarters since 9/11 | | -.0081** | | |
| | | (.0039) | | |
| distance to non-shadow area×post-9/11 | | | .0614 | |
| | | | (.0639) | |
| distance to non-shadow area×quarters since 9/11 | | | .0178** | |
| | | | (.0060) | |
| height×post-9/11 | | | | -.0003 |
| | | | | (.0022) |
| height×quarters since 9/11 | | | | .0006** |
| | | | | (.0002) |
| *R-squared* | .39 | .39 | .39 | .39 |
| *Number of observations* | 9,922 | 9,922 | 9,922 | 9,922 |

*Note:* The sample is a quarterly panel of Class A and Class B office buildings in the extended Chicago Central Business District between the second quarter of 1996 and the second quarter of 2006. See text of the article for the exact limits of the area of the City of Chicago included in our sample. Observations are weighted by the rentable area of the buildings. All specifications include building fixed effects and a full set of year×quarter dummies. Standard errors (in parentheses) are clustered at the building level.
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# Terrorism in Cities (Abadie and Dermisi, 2008)

TABLE V

REGRESSIONS USING PRE-9/11 DATA ONLY
(Fixed-effects estimates with clustered standard errors, 1996-2001)

Dependent variable: Building vacancy rate

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| shadow area×after 1998 | . 0120<br>(.0165) | | | |
| distance to anchor×after 1998 | | -.0313<br>(.0370) | | |
| distance to non-shadow area×after 1998 | | | .1017<br>(.0839) | |
| height×after 1998 | | | | .0026<br>(.0025) |
| *R-squared* | .48 | .48 | .48 | .48 |
| *Number of observations* | 5,324 | 5,324 | 5,324 | 5,324 |

*Note:* The sample is a quarterly panel of Class A and Class B office buildings in the extended Chicago Central Business District between the second quarter of 1996 and the third quarter of 2001. See text of the article for the exact limits of the area of the City of Chicago included in our sample. Observations are weighted by the rentable area of the buildings. All specifications include building fixed effects and a full set of year×quarter dummies. Standard errors (in parentheses) are clustered at the building level.
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