Quantitative Methods in Economics
Causality and treatment effects

Maximilian Kasy

Harvard University, fall 2016

8) The Regression Discontinuity Design

(cf. “Mostly Harmless Econometrics,” chapter 6)

Sharp Regression Discontinuity Design

- Sometimes assignment for treatment D is determined based on whether a unit exceeds some threshold c on a variable X (called the forcing variable or running variable):

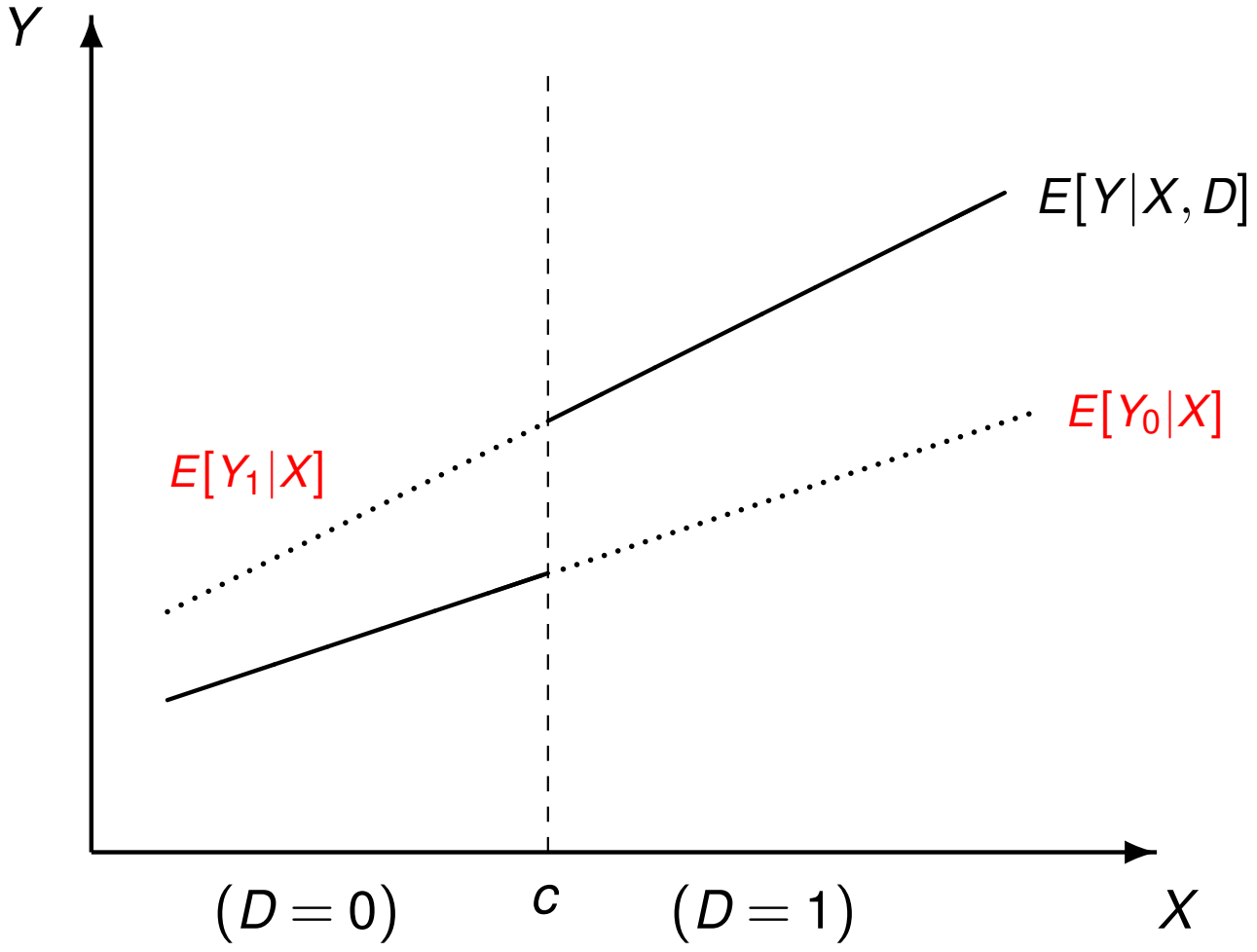
$$D_i = 1\{X_i \geq c\} \quad \text{so} \quad D_i = \begin{cases} D_i = 1 & \text{if } X_i \geq c \\ D_i = 0 & \text{if } X_i < c \end{cases}$$

- Design arises often from administrative decisions, where the allocation of units to a program is partly limited for reasons of resource constraints, and sharp rules rather than discretion by administrators is used for allocation.
- Usually X is correlated with the outcomes Y so comparing treated and untreated does not provide causal estimates.
- But we can use the discontinuity in $E[Y|X]$ at the cutoff value $X = c$ to estimate the effect of D on Y for units with $X = c$.

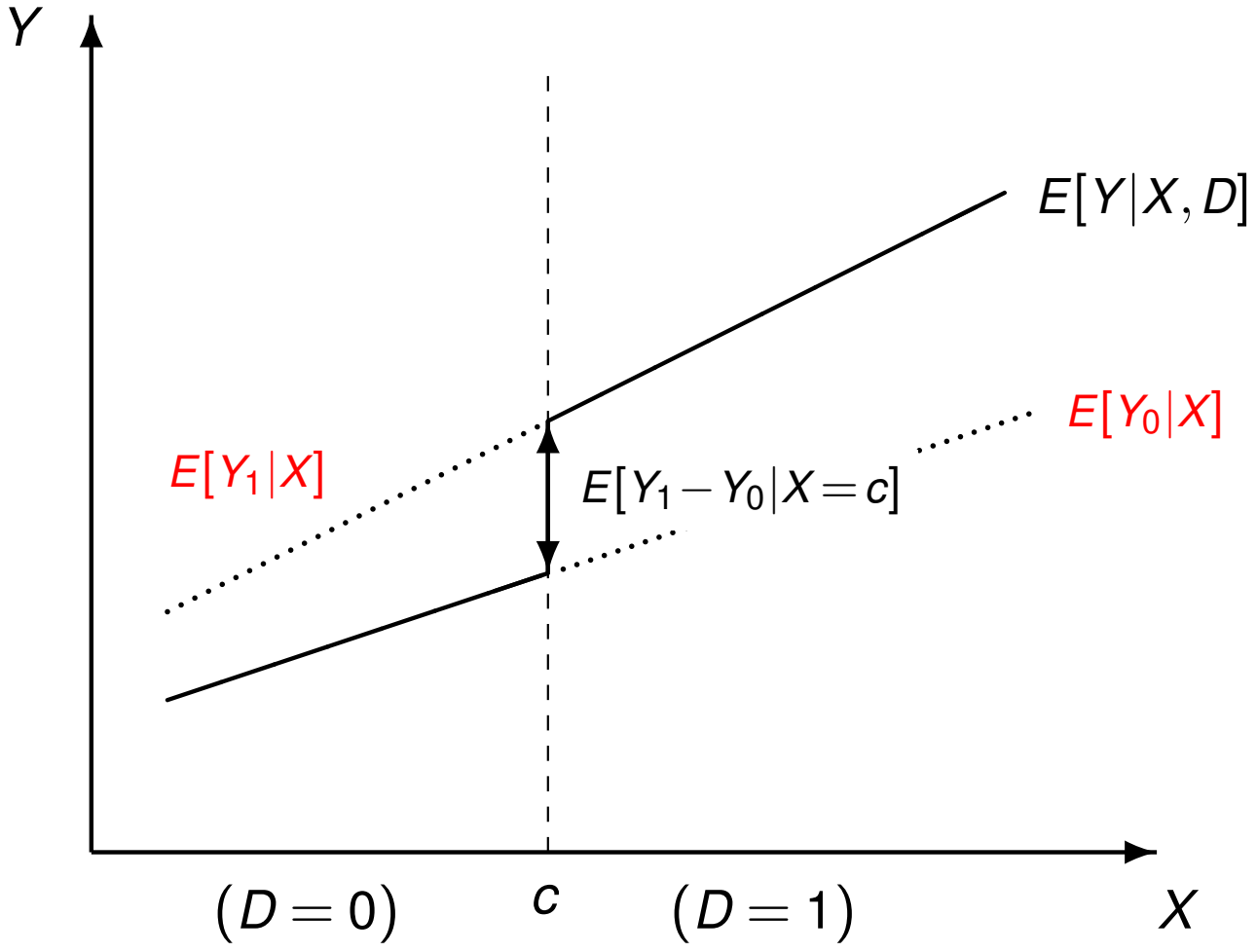
Sharp Regression Discontinuity Design

- RDD is a fairly old idea (Thistlethwaite and Campbell, 1960) but this design experienced a renaissance in recent years.
- Thistlethwaite and Campbell (1960) study the effects of college scholarships on later students' achievements.
- Scholarships are given on the basis of whether or not the student's test score is larger than some cutting value.
  - Treatment D is scholarship
  - Forcing variable X is SAT score with cutoff c
  - Outcome Y is subsequent college grades
  - Y_0 denotes potential grades without the scholarship
  - Y_1 is potential grades with the scholarship
- Y_1 and Y_0 are correlated with X: on average, students with higher SAT scores obtain higher college grades.
- However, if $E[Y_1|X]$ and $E[Y_0|X]$ are continuous functions of X at $X = c$, then we can attribute any discontinuity in $E[Y|X]$ at $X = c$ to the effect of the treatment.

Sharp RDD: Graphical Interpretation



Sharp RDD: Graphical Interpretation



Sharp RDD: Identification

Identification Assumption

$E[Y_1|X]$ and $E[Y_0|X]$ are continuous at $X = c$

Identification Result

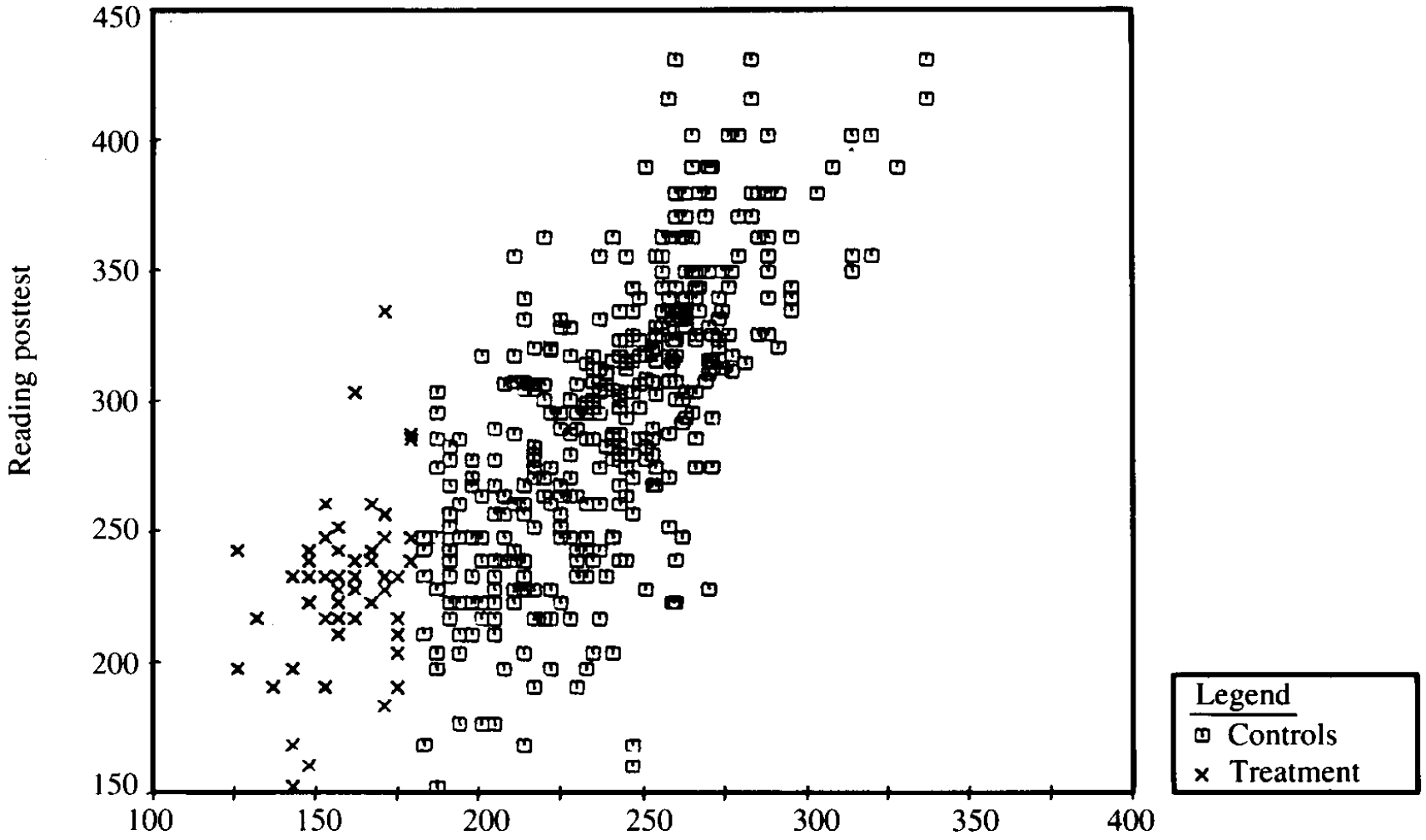
The treatment effect is identified at the threshold as:

$$\begin{aligned} E[Y_1 - Y_0|X = c] &= E[Y_1|X = c] - E[Y_0|X = c] \\ &= \lim_{x \downarrow c} E[Y|X = x] - \lim_{x \uparrow c} E[Y|X = x] \end{aligned}$$

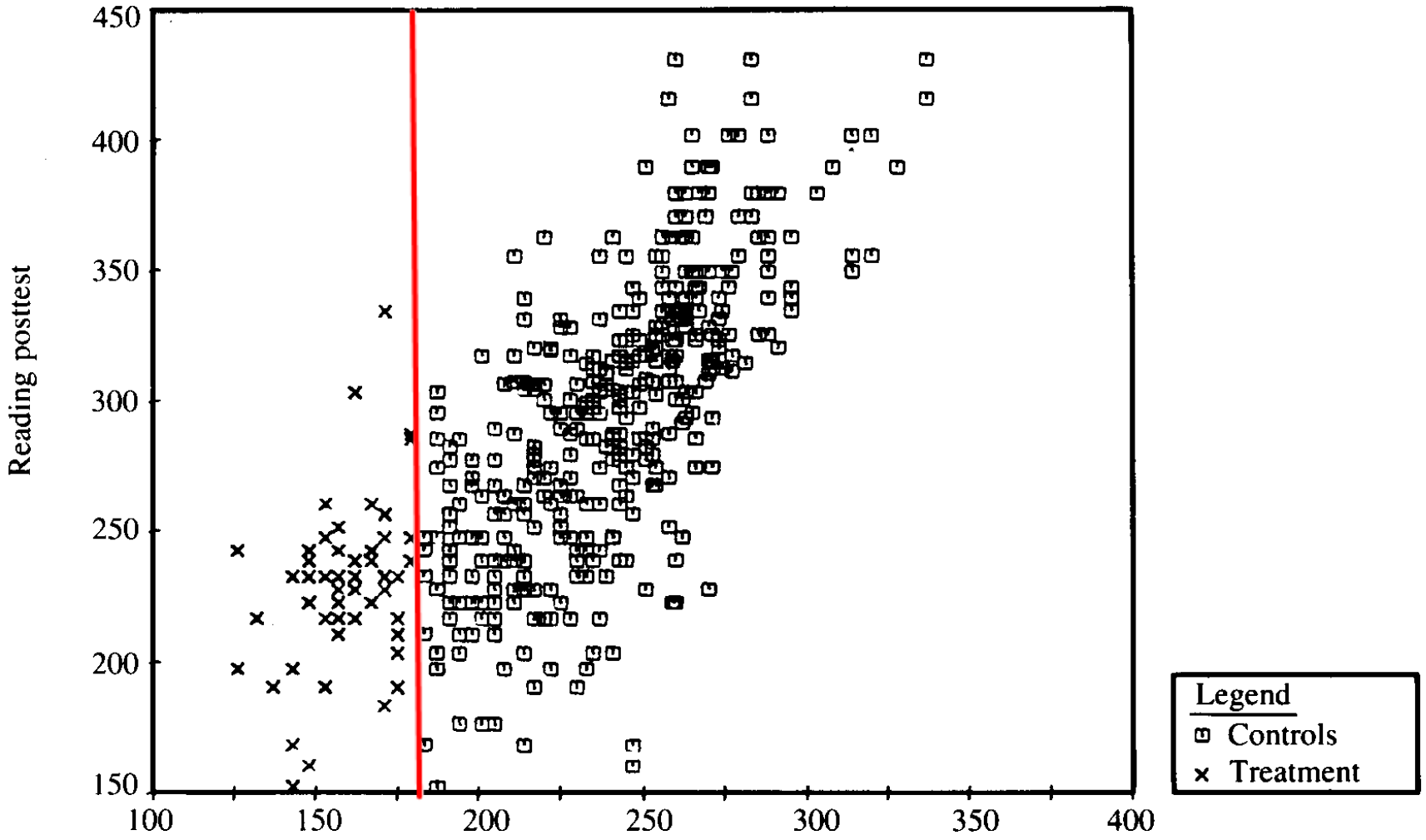
Continuity is a natural assumption but could be violated if:

- There are differences between the individuals who are just below and above the cutoff that are not explained by the treatment (e.g., the same cutoff is used to assign some other treatment)
- Subjects can manipulate the running variable in order to gain access to the treatment or to avoid it

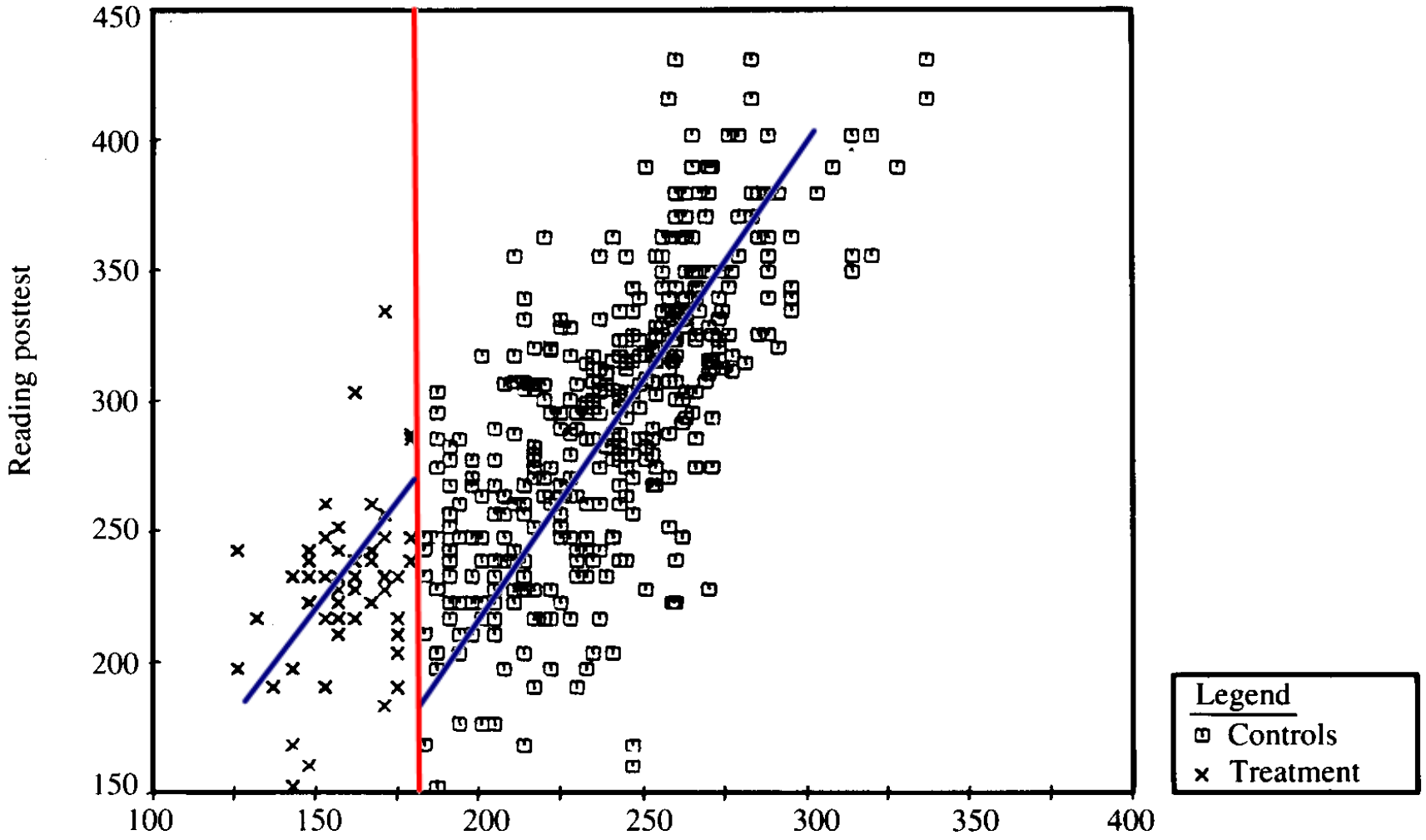
A Compensatory Reading Program



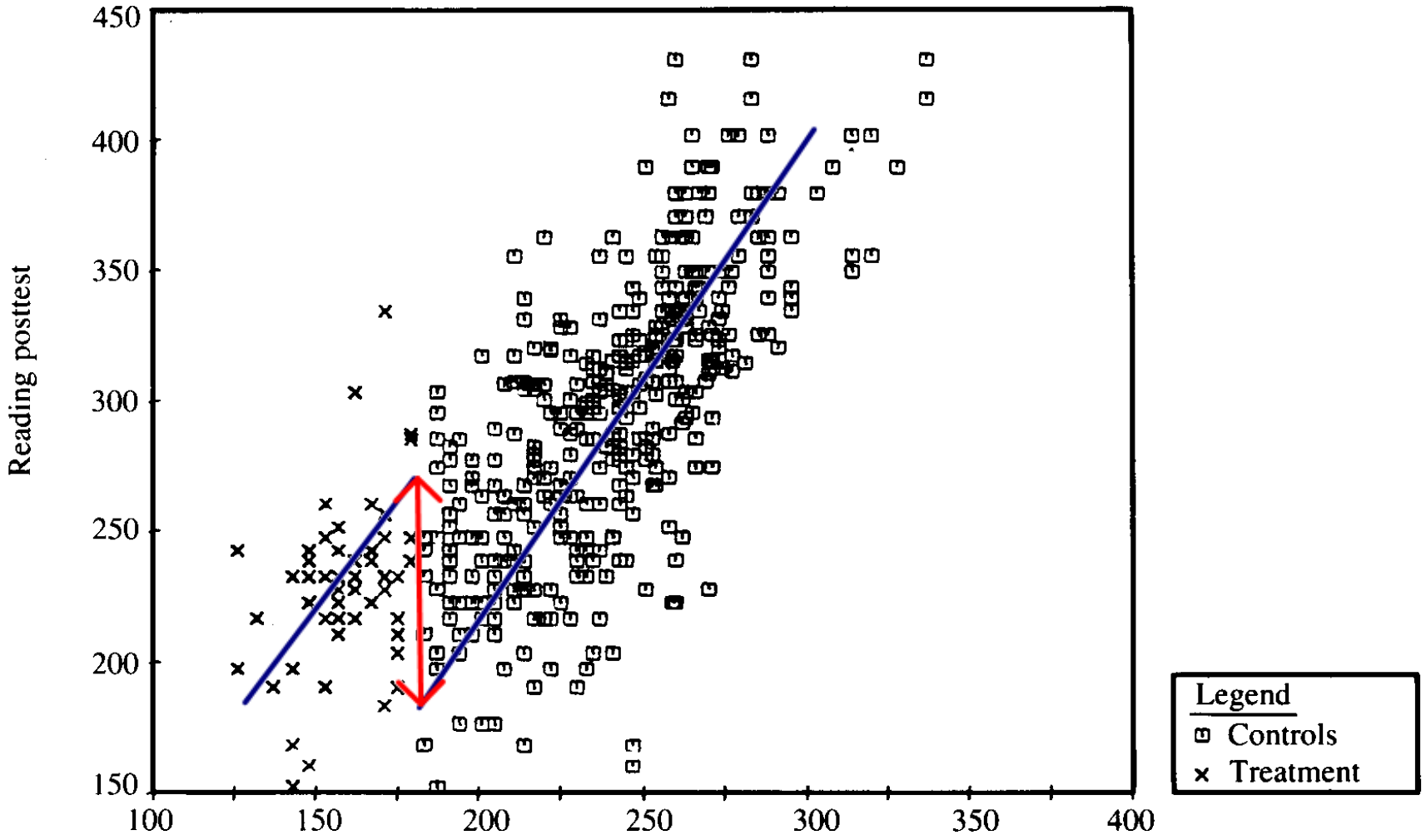
A Compensatory Reading Program



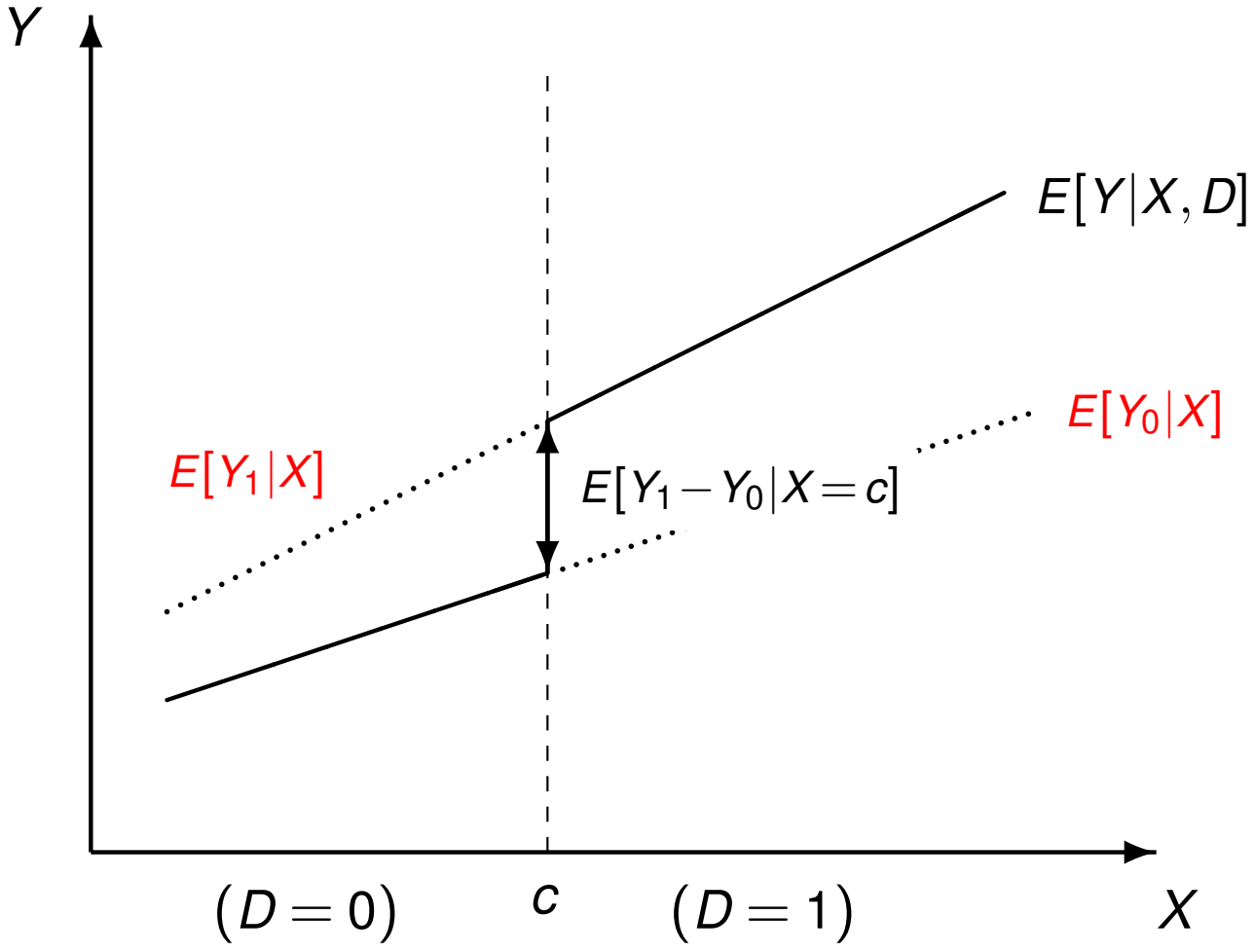
A Compensatory Reading Program



A Compensatory Reading Program



Sharp RDD Estimation: Linear Case



Sharp RDD Estimation: Linear Case

- $E[Y_0|X]$ and $E[Y_1|X]$ are distinct linear functions of X, so the average effect of the treatment $E[Y_1 - Y_0|X]$ varies with X:

$$E[Y_0|X] = \mu_0 + \beta_0 X, \qquad E[Y_1|X] = \mu_1 + \beta_1 X.$$

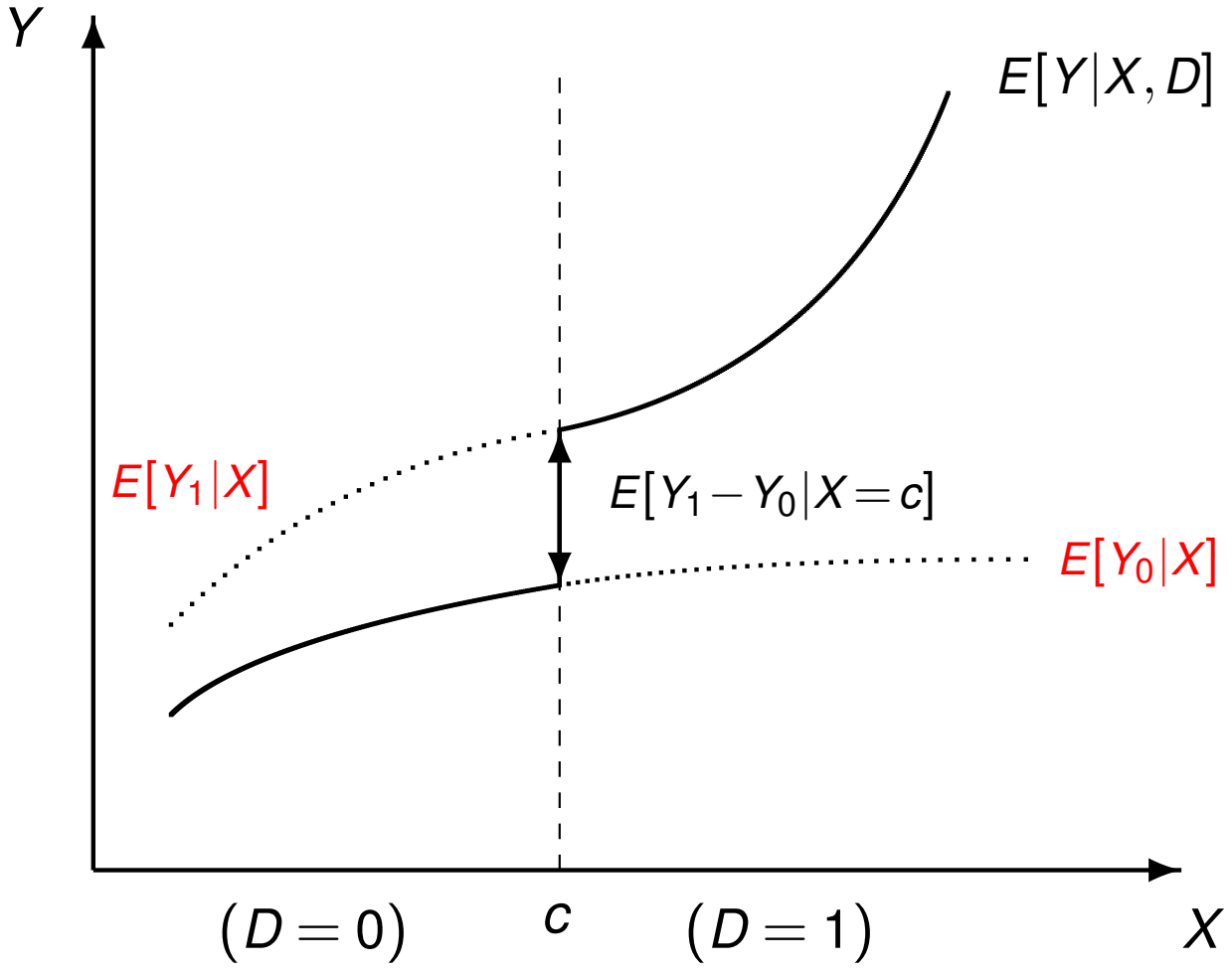
  So $E[Y_1 - Y_0|X] = (\mu_1 - \mu_0) + (\beta_1 - \beta_0)X$.

- Then, it is easy to show that:

$$\begin{aligned} E[Y|X,D] =& (\mu_0 + \beta_0 c) + \beta_0 (X - c) \\ &+ \Big((\mu_1 - \mu_0) + (\beta_1 - \beta_0)c\Big) D + (\beta_1 - \beta_0)\Big((X - c) \cdot D\Big). \end{aligned}$$

- Regress Y on D, $(X - c)$ and the interaction $(X - c) \cdot D$. Then, the coefficient of D reflects the average effect of the treatment at $X = c$.

Sharp RDD Estimation: Non-Linear Case



Sharp RDD Estimation: Non-Linear Case

- $E[Y_0|X]$ and $E[Y_1|X]$ are distinct non-linear functions of X and the average effect of the treatment $E[Y_1 - Y_0|X]$ varies with X.
- Include quadratic and cubic terms in $(X - c)$ and their interactions with D in the equation.
- The specification with quadratic terms is

$$\begin{aligned} E[Y|X,D] =& \mu + \gamma_1(X-c) + \gamma_2(X-c)^2 \\ &+ \alpha D + \delta_1(X-c)\cdot D + \delta_2(X-c)^2 \cdot D. \end{aligned}$$

  The specification with cubic terms is

$$\begin{aligned} E[Y|X,D] =& \mu + \gamma_1(X-c) + \gamma_2(X-c)^2 + \gamma_3(X-c)^3 + \alpha D \\ &+ \delta_1(X-c)\cdot D + \delta_2(X-c)^2 \cdot D + \delta_3(X-c)^3 \cdot D. \end{aligned}$$

- In both cases $\alpha = E[Y_1 - Y_0|X = c]$.

Compensatory Reading Program (Trochim, 1990)

- Evaluation of a compensatory reading program conducted among second-graders in Providence (RI) in the late 70's
- Comprehensive Test of Basic Skill was administered to students in 1978. Those who scored below certain cutoff (179) were assigned to a compensatory reading program
- Y: a second test administered to the same students in 1979
- X: is the pre-test score
- D is assignment to the compensatory reading program

Compensatory Reading Program (Trochim, 1990)

Table 1. Estimates for initial model, second-grade reading program, Providence, RI school district, 1978-79

Variable	b	SE(b)	p
Constant	216.27346	9.02495	<.001
Linear (x_i)	.88212	.49993	.078
Program effect (Z_i)	21.69478	17.24679	.209
Linear interaction ($x_i z_i$)	-2.52099	2.85144	.377
Quadratic (x_i^2)	.00816	.00774	.292
Quadratic interaction ($x_i^2 z_i$)	-.13967	.13471	.300
Cubic (x_i^3)	-.00003	.00003	.266
Cubic interaction ($x_i^3 z_i$)	-.00171	.00173	.324

$R^2 = .56919$

Compensatory Reading Program (Trochim, 1990)

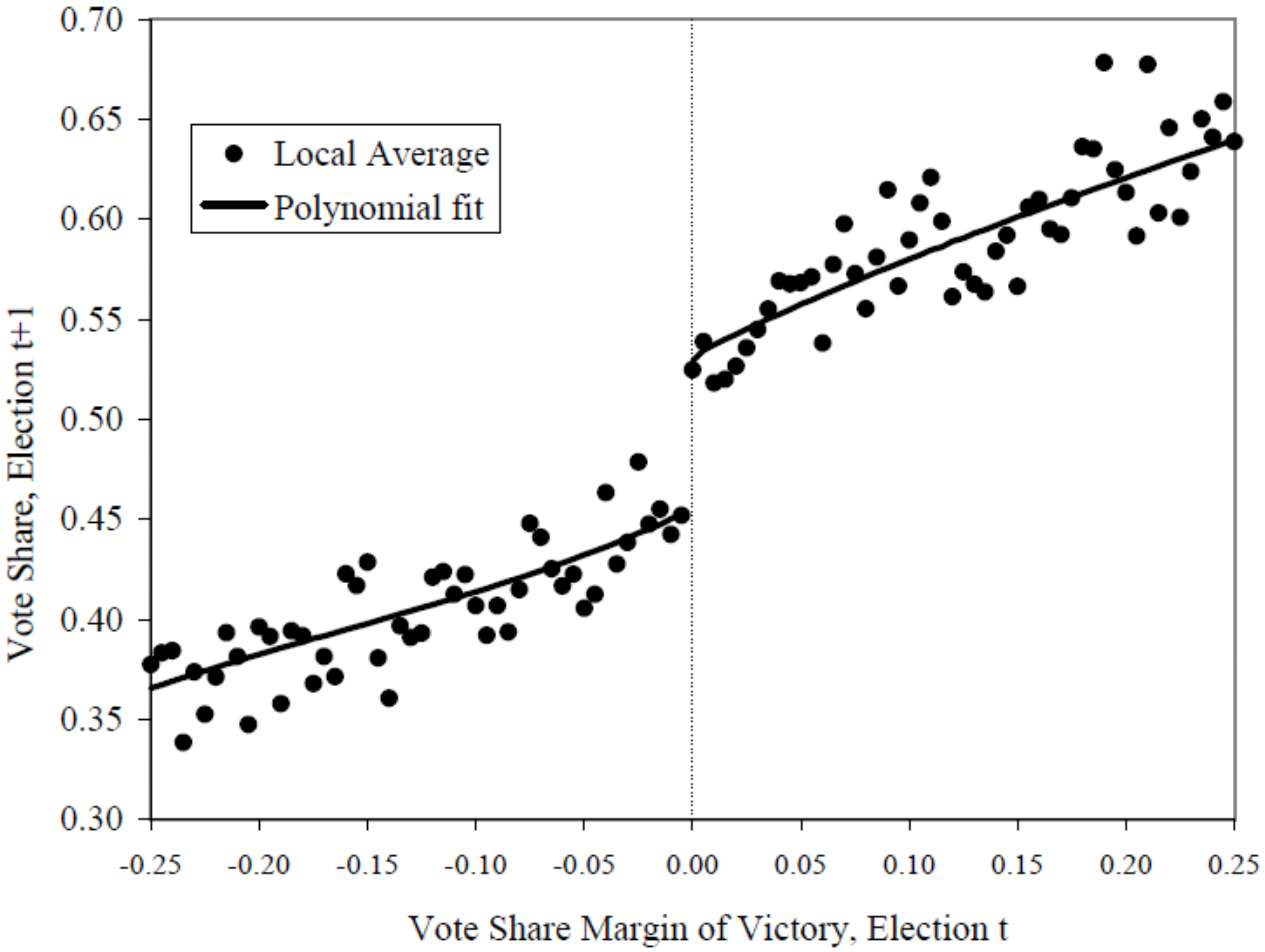
Table 2. Estimates for revised models, second-grade reading program, Providence, RI school district, 1978-79

Variable	b	SE(b)	p
Revision 1: cubic terms eliminated			
Constant	209.1262	6.3448	<.001
Linear (x_i^*)	1.3919	.2006	<.001
Program effect (Z_i)	37.4535	13.5511	.006
Linear interaction ($x_i^* z_i$)	-.4602	1.2013	.702
Quadratic (x_i^{*2})	-.0003	.0014	.836
Quadratic interaction ($x_i^{*2} z_i$)	-.0023	.0246	.925
	R^2 = .56706		
Revision 2: cubic and quadratic terms eliminated			
Constant	210.11	4.22	<.001
Linear (x_i^*)	1.35	.06	<.001
Program effect (Z_i)	35.84	10.06	<.001
Linear interaction ($x_i^* z_i$)	-.51	.39	.193
	R^2 = .56702		
Revision 3: linear term only			
Constant	210.87	4.18	<.001
Linear (x_i^*)	1.34	.06	<.001
Program effect (Z_i)	44.61	7.50	<.001
	R^2 = .56542		

Party Incumbency Advantage (Lee, 2008)

- Incumbent parties and candidates enjoy great electoral success in the U.S. and other countries
- Measuring incumbent advantage is difficult because “better” parties or candidates may be consistently favored by the electorate
- Lee (2008) uses the Regression Discontinuity Design to study party incumbency advantage in the U.S.
- The data come from elections to the U.S. House of Representatives (1946 to 1998)

Party Incumbency Advantage (Lee, 2008)



RDD: Robustness and Falsification Checks

1. Robustness: Are results sensitive to alternative specifications?
2. Balance Checks: Do other covariates W jump at the cutoff?
3. Placebo Tests: Do jumps occur at placebo cutoffs $c^\star$?
4. Sorting: Do units sort around the cutoff?

RDD: Robustness

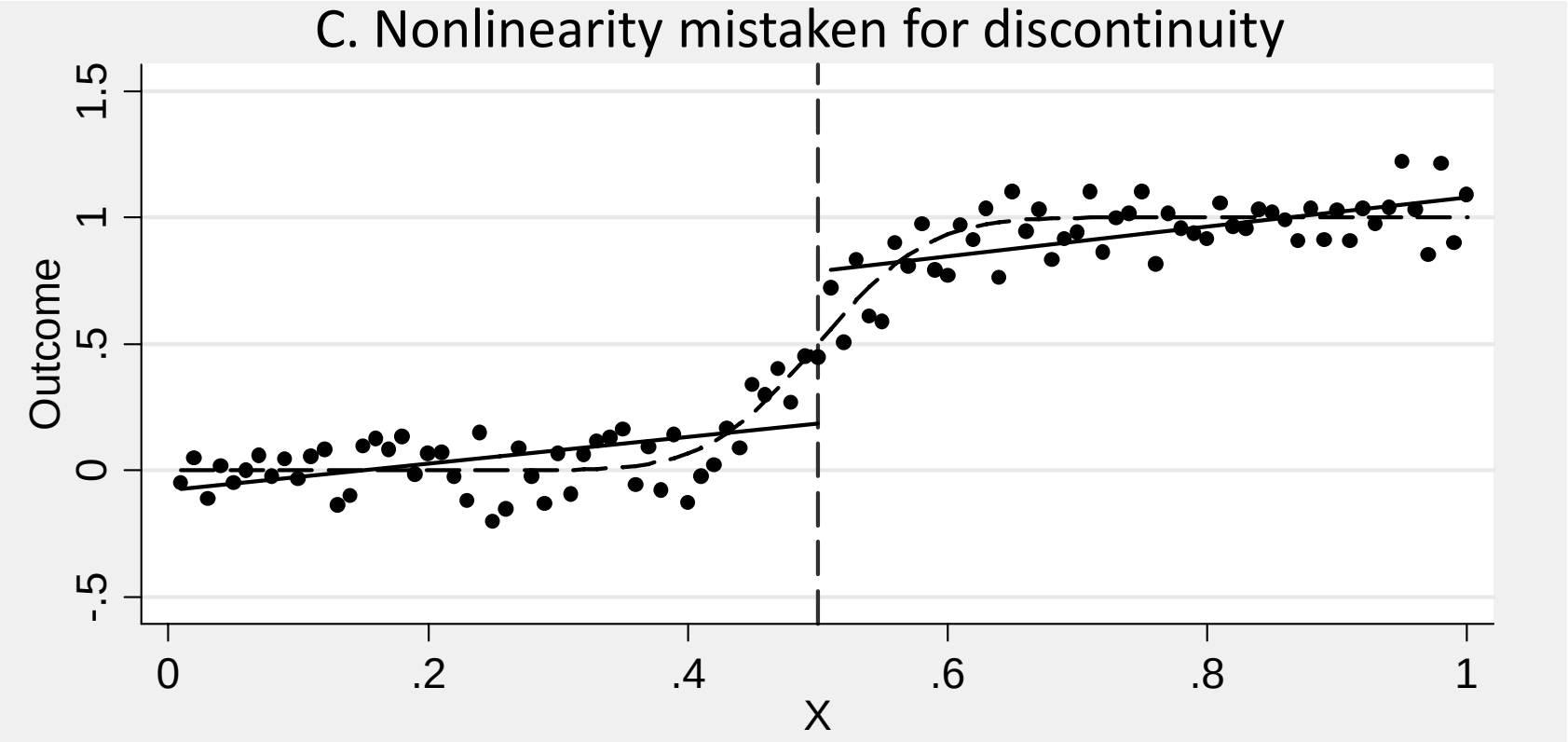


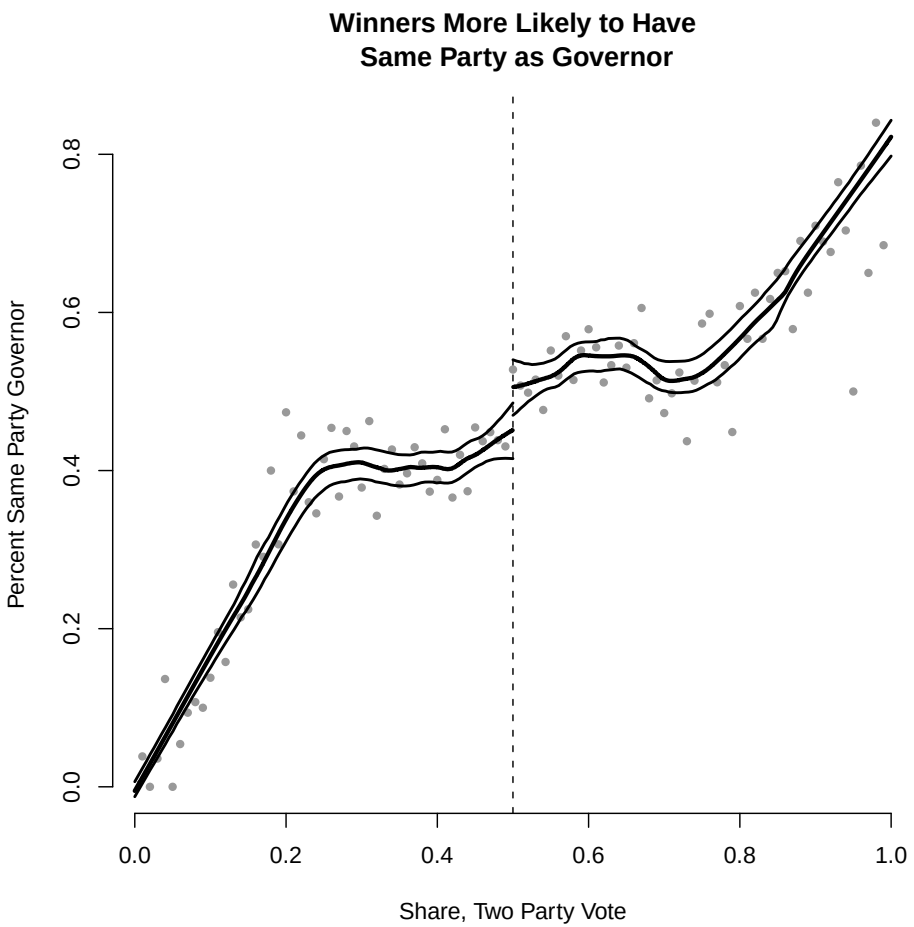
Figure form Angrist and Pischke (2008)

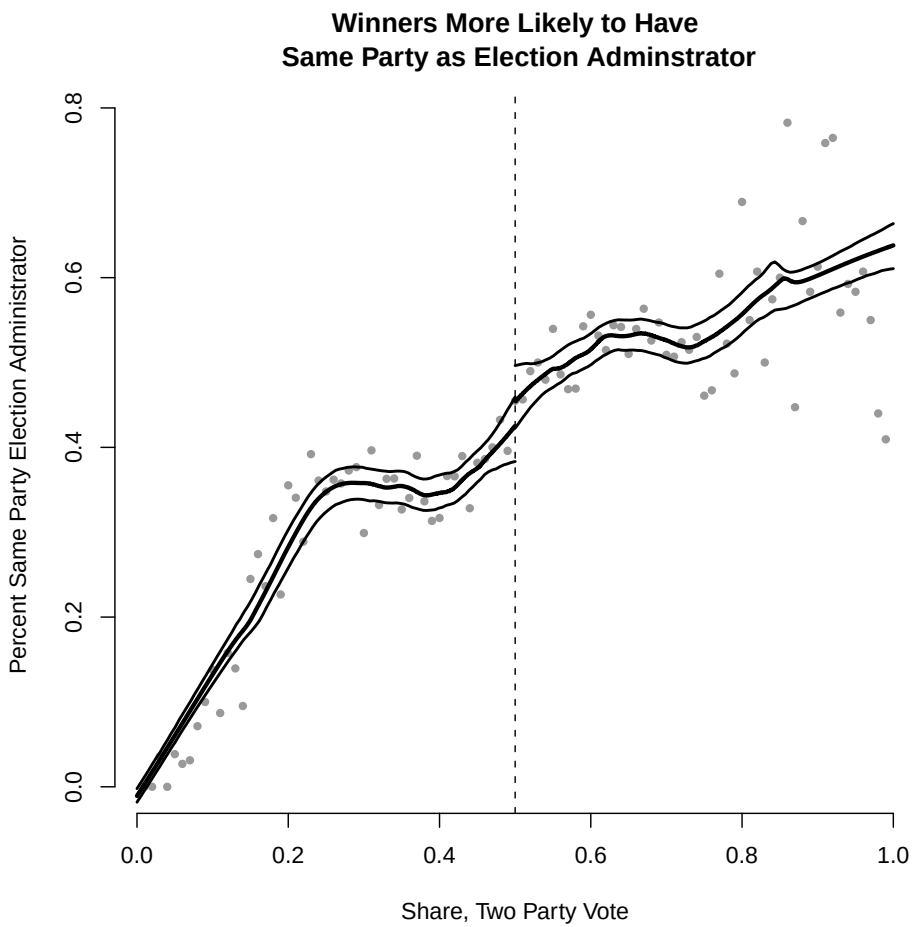
- A miss-specified functional form can lead to a spurious jump
- Check sensitivity to more flexible specifications

RDD: Balance Checks

- Lee (2008) uses the regression discontinuity design to estimate party incumbency advantage in U.S. House elections. However, ...
- Grimmer et al. (2010) find that winners of close House elections are more likely to be of the same party as the State Governor and the Secretary of State
- Caughey and Sekhon (2010) find that bare winners tend to enjoy substantial financial advantage over bare losers.?
- Close elections may in fact be quite predictable!

RDD: Balance Checks (Grimmer et al., 2010)





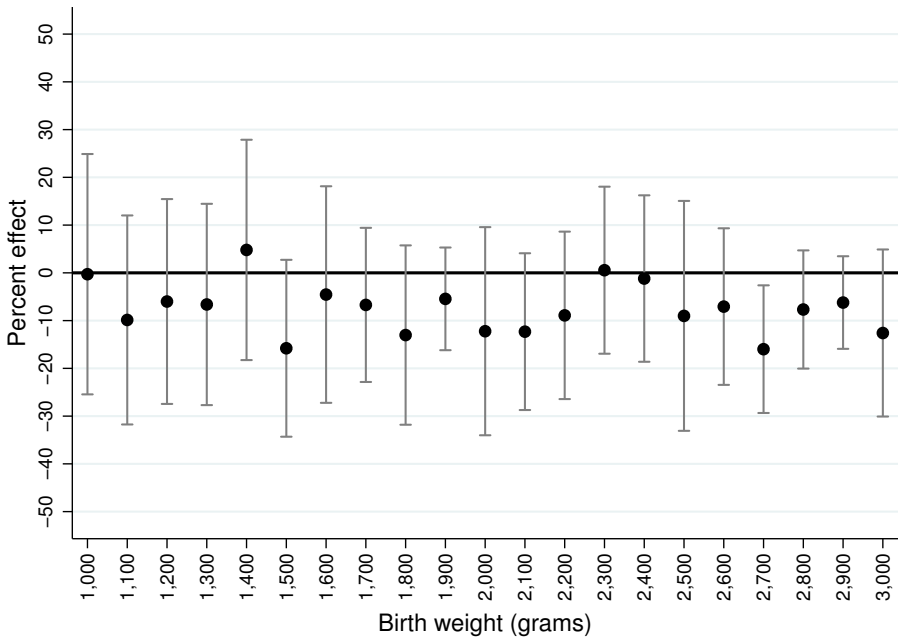
RDD: Placebo Tests

- Almond et al. (2010) use a medical definition of "very low birth weight"$<$ 1500 grams, to estimate the effect of additional medical care on newborns
- They find that newborns just below the 1500 grams cutoff receive additional treatment and survive with higher probability than newborns just above the cutoff
- However, Barreca et al. (2010) find evidence of non-random rounding at 100-gram multiples of birth weight
- Newborns of low socioeconomic status, who tend to be less healthy, are disproportionately represented at 100-gram multiples (balance check)
- As a result, newborns with birth weights just below each 100-multiple have more-favorable mortality outcomes than newborns with birth weights just above the cutoffs

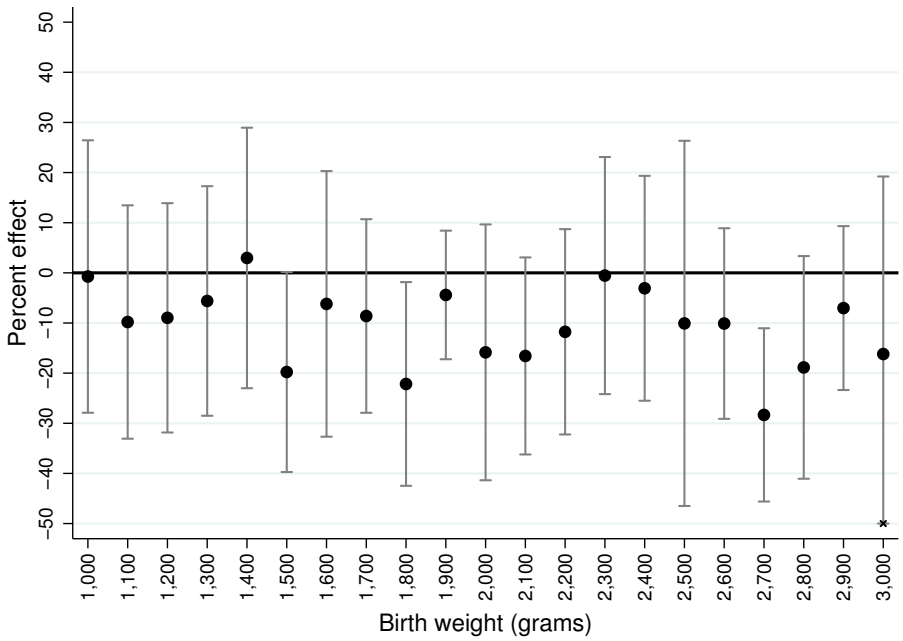
RDD: Placebo Tests (Barreca et al., 2010)

Estimated Impacts of Having Birth Weight $< Z$

Panel A: One-Year Mortality



Panel B: 28-Day Mortality

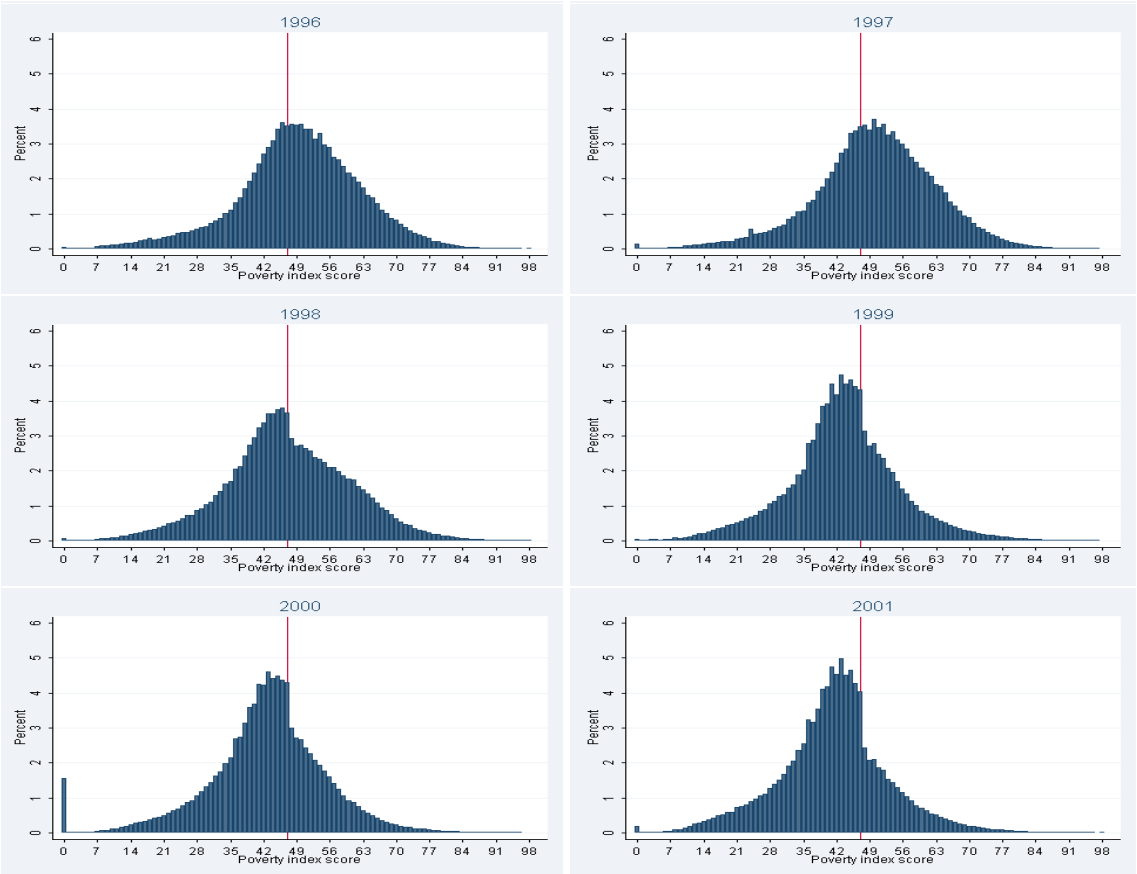


RDD: Sorting/Bunching

- Subjects or program administrators may invalidate the continuity assumption if they strategically manipulate X to be just above or below the cutoff
- This is a concern especially if the exact value of the cutoff is known to the subjects in advance
- This type of behavior, if it exists, may create a discontinuity in the distribution of X at the cutoff (i.e., "bunching" to the right or to the left of the cutoff)
- A formal test is provided by McCrary (2008)

RDD: Sorting/Bunching (Camacho and Conover, 2010)

Example: Manipulation of a poverty index in Colombia. A poverty index is used to decide eligibility for social programs. The algorithm to create the poverty index becomes public during the second half of 1997.



Fuzzy Regression Discontinuity Design

- Cutoff does not perfectly determine treatment but creates a discontinuity in the probability of receiving the treatment
- For example:
  - The probability of being offered a scholarship may jump at a certain SAT score (above which the applications are given "special consideration")
  - Incentives to participate in a program may change discontinuously at a threshold, but the change is not powerful enough to move all units from nonparticipation to participation
- For units close to the cutoff we can use

$$Z_i = \begin{cases} 1 & \text{if } X_i \geq c \\ 0 & \text{if } X_i < c \end{cases}$$

  as an instrument for D_i.
- We estimate the effect of the treatment for compliers: those units (close to the discontinuity, $X_i \simeq c$) whose treatment status, D_i, depends on Z_i.

Fuzzy Regression Discontinuity Design

- The idea is that for units that are very close to the discontinuity Z_i can act as an instrument
- The LATE parameter is:

$$\lim_{\substack{c-\varepsilon \leq X \leq c+\varepsilon \\ \varepsilon \to 0}} \left(\frac{E[Y|Z=1] - E[Y|Z=0]}{E[D|Z=1] - E[D|Z=0]} \right),$$

or

$$\frac{\lim_{x \downarrow c} E[Y|X=x] - \lim_{x \uparrow c} E[Y|X=x]}{\lim_{x \downarrow c} E[D|X=x] - \lim_{x \uparrow c} E[D|X=x]}$$

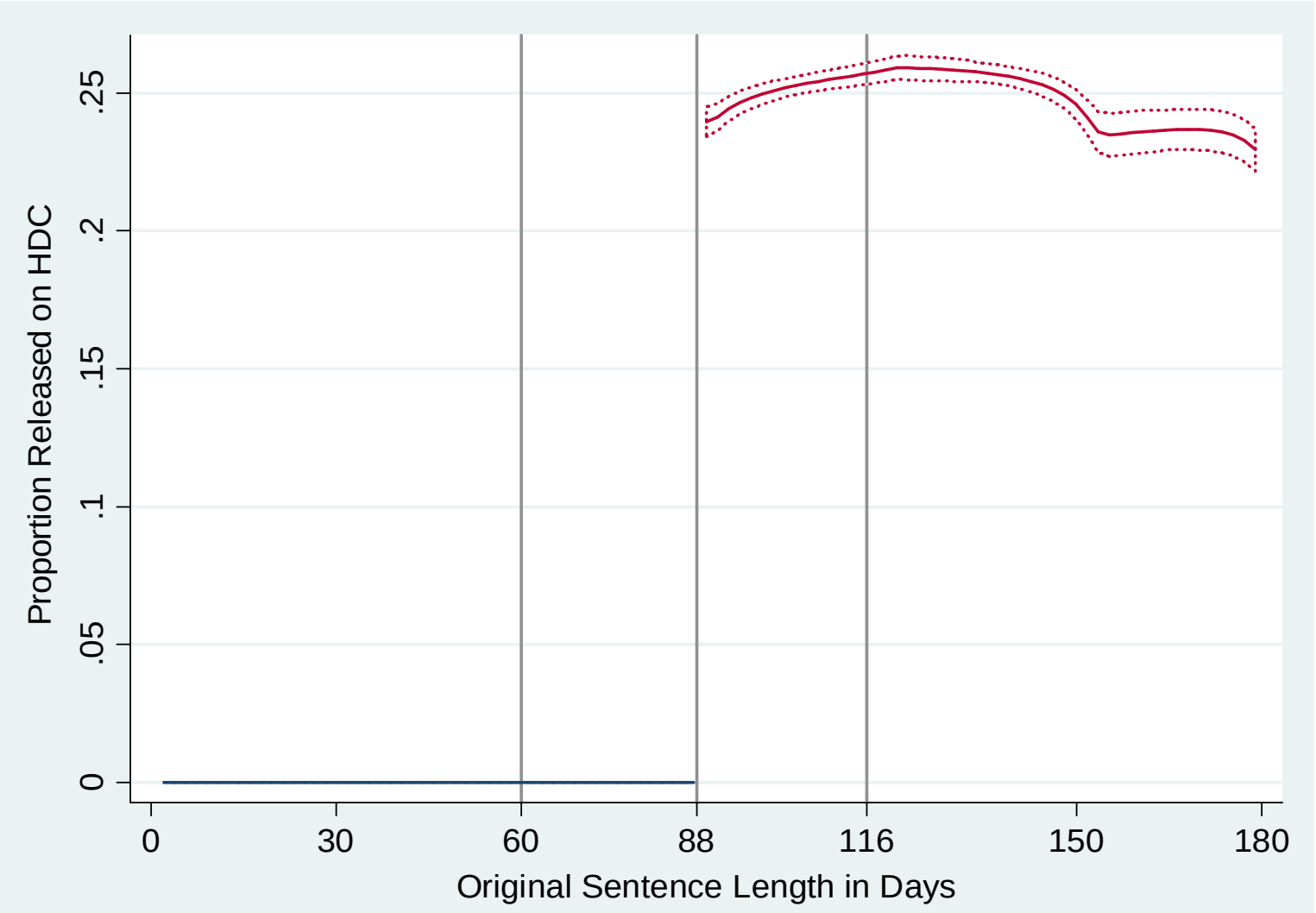
- This suggests:
  1. Run a sharp RDD for Y
  2. Run a sharp RDD for D
  3. Divide your estimate in step 1 by your estimate in step 2
- Many authors just run instrumental variables for those units with $X \simeq c$

Early Release Program (Marie, 2009)

- Prison systems in many countries suffer from overcrowding and high recidivism rates after release.
- Some countries use early discharge of prisoners on electronic monitoring.
- Difficult to estimate impact of early release program on future criminal behavior: best behaved inmates are usually the ones to be released early.
- Marie (2008) considers the Home Detention Curfew (HDC) program in England and Wales.
- This is a fuzzy RDD: Only offenders sentenced to more than three months (88 days) in prison are eligible for HDC, but not all of those are offered HDC.

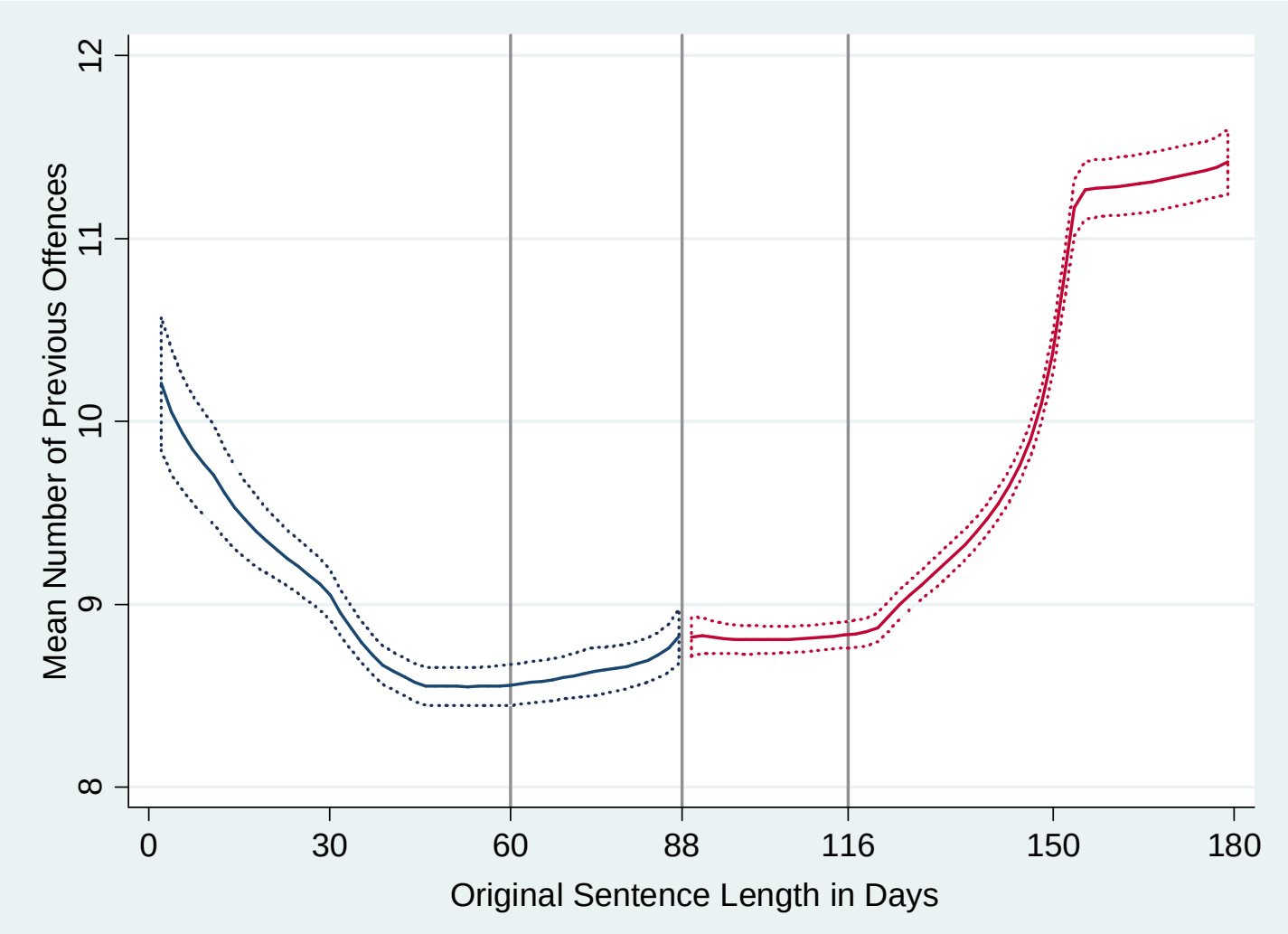
Early Release Program (Marie, 2009)

Figure 2 : Proportion Released on HDC by Original Sentence Length



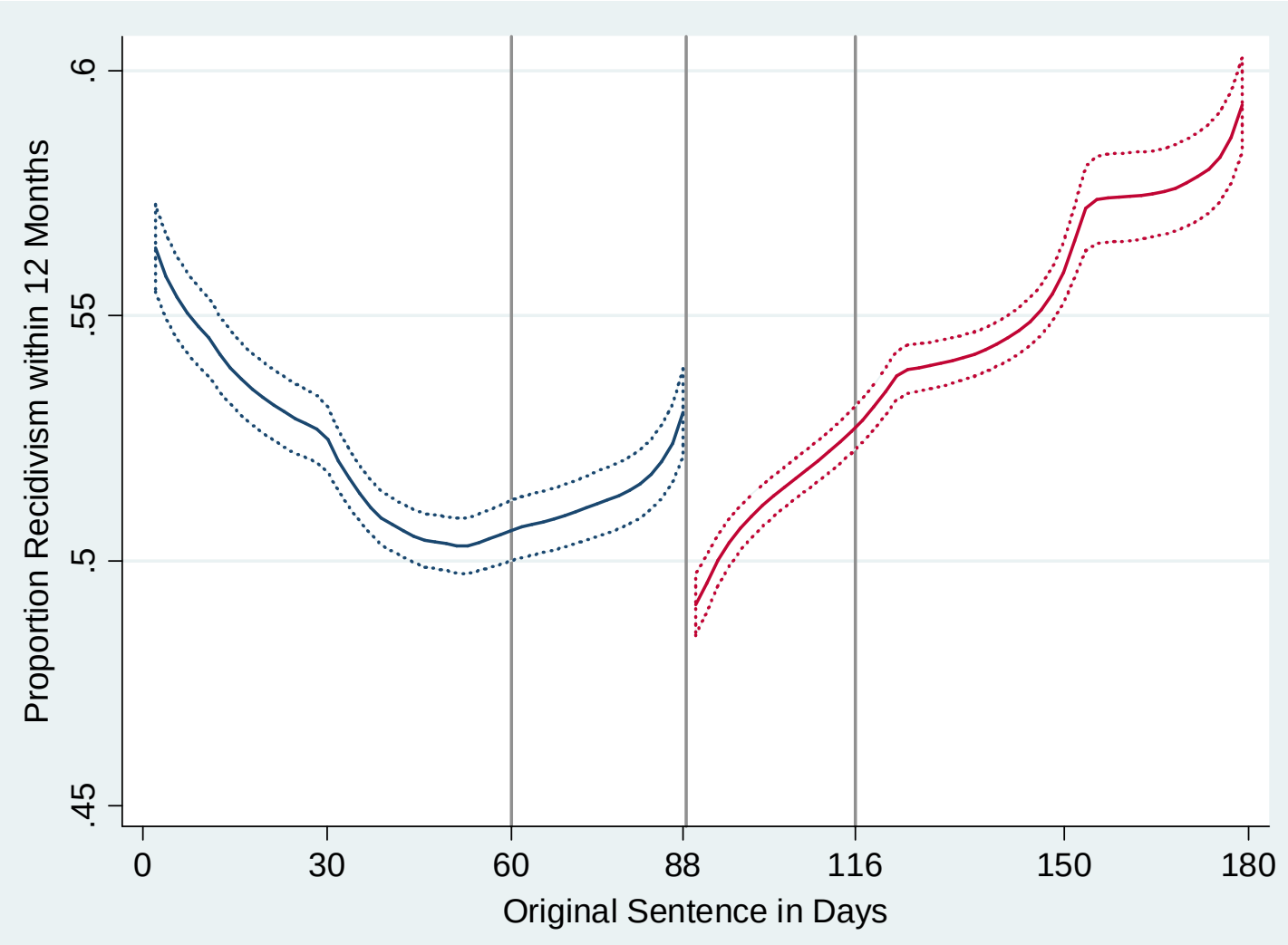
Early Release Program (Marie, 2009)

Figure 3 : Number of Previous Offences by Original Sentence Length



Early Release Program (Marie, 2009)

Figure 4: One Year Recidivism Rate by Original Sentence Length



Early Release Program (Marie, 2009)

Table 5: RD Estimates of HDC Impact on Recidivism

Panel A: Recidivism Within 12 Months of Release	**Estimation on Individuals Sentenced to Between 58 and 118 Days: +/- 4 Weeks**		
	(1)	**(2)**	**(3)**
Discontinuity of HDC Participation Around Threshold ($HDC^+ - HDC^-$)	.242 (.003)	.243 (.003)	.237 (.003)
Difference in Recidivism Around Threshold ($Rec^+ - Rec^-$)	-.023 (.005)	-.022 (.005)	-.016 (.005)
Estimated Effect of HDC on Recidivism Participation ($Rec^+ - Rec^-$)/ ($HDC^+ - HDC^-$)	*-.094* *(.020)*	*-.090* *(.018)*	*-.066* *(.018)*
Controls	No	Yes	Yes
Prison Fixed Effects	No	No	Yes
Sample Size	41,761	41,761	41,761

Panel B: Recidivism Within 24 Months of Release	**Estimation on Individuals Sentenced to Between 58 and 118 Days: +/- 4 Weeks**		
	(1)	**(2)**	**(3)**
Discontinuity of HDC Participation Around Threshold ($HDC^+ - HDC^-$)	.242 (.003)	.243 (.003)	.237 (.003)
Difference in Recidivism Around Threshold ($Rec^+ - Rec^-$)	-.019 (.005)	-.019 (.004)	-.013 (.005)
Estimated Effect of HDC on Recidivism Participation ($Rec^+ - Rec^-$)/ ($HDC^+ - HDC^-$)	*-.079* *(.020)*	*-.077* *(.018)*	*-.053* *(.019)*
Controls	No	Yes	Yes
Prison Fixed Effects	No	No	Yes
Sample Size	41,761	41,761	41,761

Note: Robust standard errors in parenthesis. The estimation is based on individuals sentenced to between 59 and 118 days. The controls included in column (2) are: gender, age, ethnic minority, breached in the past number previous offences, month and year of release dummies, and the type of crime incarcerated for (8 types). The same model with 126 prison establishment fixed effects is reported in column (3).