Which findings should be published?[*]

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July 13, 2018

Abstract

Given a scarcity of journal space, what is the socially optimal rule for whether an empirical finding should be published? If the goal of publication is to inform the public about a policy-relevant state, journals should publish extreme results. For specific objectives, the optimal rule takes the form of a one- or two-sided test comparing the point estimate to the prior mean. Dynamic considerations may additionally justify the publication of precise null results. If one insists that standard inference remain valid conditional on publication, however, publication must not select on the study's findings (but may select on the study's design).

1 Introduction

Not all empirical findings get published. Journals may be more likely to publish findings that are statistically significant, as documented in studies such as Franco et al. (2014), Brodeur et al. (2016), and Andrews and Kasy (2017). They may also be more likely to publish findings that are surprising, or conversely ones that confirm some prior belief. Whatever its form, selective publication distorts statistical inference. If only estimates with large effect sizes were to be written up and published, say, then

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published studies would systematically overstate true effects. Such publication bias has been offered as one explanation for the perceived replication crisis in the social and life sciences.[1]

In response to these concerns, there have been calls for reforms in the direction of non-selective publication. One proposal is to promote statistical practices that de-emphasize statistical significance, for instance by banning "stars" in regression tables. Another proposal is for journals adopt Registered Reports, in which pre-registered analysis plans are reviewed and accepted prior to data collection (see Nosek and Lakens (2014) or Chambers et al. (2014)). Registered Reports guarantee that publication will not select at all on findings – after a plan is accepted, the journal is committed to publishing the study and the researcher has no flexibility over which results to write up.

In this paper we seek the optimal rule for determining whether a study should be published, given both its design and its findings. Our analysis is from an instrumental perspective: the value of a study is that it informs the public about some policy-relevant state of the world before the public chooses a policy action. In this framework, we will show that non-selective publication is not in fact optimal. Some findings are more valuable to publish than others. Put differently, we will find a trade-off between policy relevance and statistical credibility.

In a world without constraints, the first-best rule would be for all results – or even better, all raw data – to be published. This paper solves for a second-best publication rule. We take as given that there is some constraint on the share of studies that will be published, or equivalently that there is some opportunity cost of publication. (After formally presenting the model, we discuss further interpretations of this cost, e.g., as arising from a public with limited attention.) In our model, if a submitted study is published, the public observes its findings and takes the optimal policy action given its updated belief. If a study is not published, the public never observes the study's results, and does not necessarily know that a study was conducted; the public then takes a default action. This default action in the absence of publication is

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[1]Worries about selective publication go back at least to Sterling (1959). Discussions of publication bias and other threats to the credibility and reproducibility of scientific output can be found in Ioannidis (2005), Ioannidis (2008), and in reviews including Simmons et al. (2011), Gelman and Loken (2014), and Christensen and Miguel (2016). Open Science Collaboration (2015) and Camerer et al. (2016) conduct large-scale replications of experimental studies in psychology and economics, giving insight into the extent to which published results are in fact reproducible.

based on a default belief, where we allow for either a Bayesian public whose default belief correctly accounts for publication bias or for a naive public whose default belief always remains at its prior. The optimal publication rule is the one that maximizes the public's expected payoff from the eventual policy choice, minus the publication cost.

The optimal publication rule defined in this manner selects on a study's findings. To understand why, note that there is no instrumental value from publishing a study with a "null result" that doesn't move the policy away from the default action. The same policy would have been chosen even if the study weren't published, so publishing would incur a cost without a benefit. The studies that are worth publishing are the ones that show that there is some payoff gain from taking an action other than the default.

For two canonical policy environments, we give precise characterizations of the optimal publication rules. In the first environment, the public makes a continuous policy decision, such as the choice of a tax rate, and has quadratic losses in matching the policy to its ideal point. The optimal publication rule then takes the form of a "two-sided test:" the journal publishes estimates that are sufficiently far above or below the prior mean. In the second environment, the public makes a binary policy choice, such as whether to implement a job training program. Here the journal uses a "one-sided test." For instance, in the absence of publication, the default action might be to not implement the program. The journal will then only publish studies with high estimates of the program's benefit, which convince the public to implement the program. The critical values of these one- and two-sided "tests" come from a cost-benefit calculation, rather than corresponding to a conventional level such as p = .05 against a null hypothesis of zero.

Moving beyond these two special cases, suppose that the public faces any "super-modular" policy decision in which its preferred policy action is monotonic in the state of the world. For example, the preferred investment in a public good increases in its expected return. Then it is generally true that it is more valuable to publish studies with more extreme results – that is, studies which lead to more extreme beliefs.

We wish to stress that the nature of our exercise is to solve for the socially optimal rule regarding whether to publish a study that has some given set of results. That is, our model does not consider the incentives of researchers or journals, and we are not attempting to characterize the equilibrium publication rule arising from a strategic

interaction of these agents. As discussed in Glaeser (2006), researcher incentives play an important role in the publication process. Researchers make choices over the topics they study and their study designs, and then may selectively submit or possibly even manipulate their findings.[2] We do explore one way in which researcher study design choices may respond to journal publication rules in Appendix A.1.

After characterizing optimal publication rules, we return to the distortions caused by selective publication. Consider a study that consists of a normally distributed point estimate paired with a standard error. It is immediate that common forms of inference are valid if the publication probability does not depend on the point estimate given the standard error. We show that the inverse is also true: Under *any* policy in which the publication probability depends on the point estimate, common forms of frequentist inference will be *in*valid conditional on publication. Point estimates are no longer unbiased, for instance, and uncorrected likelihood-based estimation will not be accurate. Moreover, when a study is not published, a naive public that maintains its prior will have a distorted belief relative to a Bayesian public that accounts for publication bias. If we desire that standard inference or naive updating be valid, we must impose a non-selective publication rule that does not depend at all on the point estimate (although journals may still publish studies with small standard errors over studies with large standard errors).

Putting these results together, we see that selectively publishing extreme results is better for policy-relevance but leads to distorted inference. To the extent that the current (selective) publication regime qualitatively resembles the optimal rules we derive, then, a move towards non-selective publication in order to improve statistical credibility might have costs as well as benefits.

An abstraction in the model described above is that it considers a "static" environment with a single paper to be published and a single action to be taken. One may also be interested in the longer-term implications of publication rules, as in McElreath and Smaldino (2015) and Nissen et al. (2016).[3] To get some insight into these

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[2]Furukawa (2018) looks at a model (without journals) in which researcher decisions to publish papers interact with a public policy choice, where in equilibrium researchers choose to publish papers with extreme results. Müller-Itten (2017) looks at a competition between journals for prestige, in which journals choose whether to publish a submitted study based on a signal of its quality while researchers choose which journal to submit to.

[3]McElreath and Smaldino (2015) and Nissen et al. (2016) provide dynamic models to study whether an academic publication process with publication bias will eventually converge to truthful estimates. Akerlof and Michaillat (2017) performs a similar exercise for a more evolutionary form

issues, we consider a dynamic extension to our model that appends a second period in which exogenous information arrives before another action is taken. The publication decision in the first period now affects the action in both periods. Just as before, we find a benefit of publishing extreme results. But we also find a benefit of publishing precise results – even precise null results that don't change the current action. To wit, publishing a precise result today helps avoid future mistakes arising from the noise in the information that has yet to arrive. We show that publishing null results is most valuable when this future information is neither too precise nor too imprecise.

Throughout the paper, our derivations of optimal publication rules rely on characterizing the value of information for specified decision problems. Most theoretical treatments of the value of information study the ex-ante value of an experiment, i.e., the expected value prior to the realization; see classic treatments in Blackwell (1953), Lehmann (1988), or Persico (2000). These ex-ante comparisons are relevant for a characterization of non-selective publication rules, as we explore in Section 4.3. However, we allow for publication to select on a study's findings. We are thus predominantly concerned with the ex-post value of information given an experiment's realization, as studied in Frankel and Kamenica (2018). The decision to reveal a signal, at a cost, based on its realization is also related to the analysis of the discretionary disclosure of positive and negative news in Verrecchia (1983) and follow-up work.

The rest of the paper is structured as follows. Section 2 introduces our basic model of publication. Section 3 shows how to solve for the optimal publication rules, and provides some characterizations of the solution. Section 4 addresses the distortions that arise from selective rather than non-selective publication. Section 5 presents a two-period version of the model. Proofs and several extensions are discussed in the Appendix, including the endogeneity of submitted study designs, the plausibility of submitted results, and alternative social objectives.

2   The model of publication

In Sections 2.1-2.3 we present our benchmark model of publication and in Section 2.4 we discuss interpretations of the publication cost.

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of the accumulation of academic knowledge.

2.1 Timing of the game

There is an uncertain state of the world whose value is relevant to some public policy decision. A study that reveals information about this state may or may not be submitted to a journal. If a study is submitted, the journal will decide whether to publish it. If it is published, the results of the study are observed by the public. Then the public chooses a policy.

Let $\theta \in \Theta \subseteq \mathbb{R}$ denote the state of the world, and suppose that there is a common prior π_0 on this state shared by the public and the journal. The probability that a study arrives is $q \in (0, 1]$, independent of θ. A study can be summarized by the pair of random variables (X, S). The variable S, with generic realization $s \in \mathcal{S}$, represents the study *design* and the variable X, with generic realization $x \in \mathcal{X}$, represents the study *finding*. The design S is drawn from the distribution F_S, independently of the state θ. The finding X is drawn from the distribution $F_{X|\theta,S}$, with pdf $f_{X|\theta,S}$ relative to some dominating measure. That is, the study design determines how the distribution of the finding depends on the state. A leading example is that of a normally distributed signal, with X in $\mathbb{R}$, S in $\mathbb{R}_{++}$, and $X|\theta, S \sim \mathcal{N}(\theta, S^2)$.

If a study arrives, it will be evaluated by a journal which observes the finding and design (X, S) and then decides whether to publish the study. The journal uses a publication rule $p : \mathcal{X} \times \mathcal{S} \rightarrow [0, 1]$ where $p(X, S)$ describes the probability that a study (X, S) is published. As a matter of terminology, we say that the journal or publication rule publishes a study when $p(X, S) = 1$ and does not publish a study when $p(X, S) = 0$. Let D be the random variable with generic realization $d \in \{0, 1\}$ indicating whether a study is ultimately published: $D = 0$ if no study is published (because no study arrived, or because one arrived but was not published) and $D = 1$ if a study arrived and is published.

After a study is published or not, the public's belief on θ updates to a posterior π_1. When no study has been published ($D = 0$), π_1 is equal to some *default belief* π_1^0. When a study has been published ($D = 1$), its design S and finding X are publicly observed, and π_1 is instead equal to the belief $\pi_1^{(X,S)}$. We describe the belief updating process in Section 2.2.

Finally, given updated beliefs π_1, the public takes a policy action $a \in \mathcal{A} \subseteq \mathbb{R}$ to maximize the expectation of a utility function $U : \mathcal{A} \times \Theta \rightarrow \mathbb{R}$.[4] Let $a^*(\pi_1) \in$

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[4]While we maintain the assumption that states θ and actions a are real numbers in order to facilitate economic interpretations of our results, our solution approach in Section 3.1 will apply to

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$\arg\max_a \mathbb{E}_{\theta \sim \pi_1}[U(a, \theta)]$ indicate the chosen action when the public holds beliefs π_1. We assume existence of this argmax for any relevant utility functions and posterior distributions, and we confirm existence for all of our examples. Let $a^0 = a^*(\pi_1^0)$ be the *default action*, i.e., the action taken under the default belief, whereas $a^*(\pi_1^{(X,S)})$ is the action taken if a study (X, S) is published.

Social welfare, corresponding to the shared objective of both the journal and public, is the action payoff net of a publication cost. Let $c > 0$ indicate the social cost of publication; we discuss interpretations of this cost in Section 2.4. The welfare $W(D, a, \theta)$ induced by publication D, chosen action a, and state of the world θ, is

$$W(D, a, \theta) = U(a, \theta) - Dc. \tag{1}$$

We take the study arrival and study design S to be exogenous. (Appendix A.1 explores an extension in which study arrival and design are endogenous to the publication rule.) The policy action a is mechanical given induced beliefs π_1. Thus, the key decision in the model is the choice of publication rule p. We will search for the publication rule that maximizes the ex-ante expectation of welfare, which we call the *optimal* publication rule.

2.2 Beliefs and belief updating

The public's belief updating from the prior π_0 to the posterior π_1 depends on whether a study is published or not.

Beliefs conditional on publication

If a study (X, S) is published, the posterior belief becomes $\pi_1 = \pi_1^{(X,S)}$. We assume that $\pi_1^{(X,S)}$ is derived according to Bayes' Rule given the signal $X \sim F_{X|\theta,S}$. By Bayes' Rule, and since θ is independent of S, the density of $\pi_1^{(X,S)}$ relative to the prior π_0 is given by

$$\frac{d\pi_1^{(X,S)}}{d\pi_0}(\theta) = \frac{f_{X|\theta,S}(X|\theta, S)}{f_{X|S}(X|S)}, \tag{2}$$

with $f_{X|S}$ the unconditional density of X under study design S and prior $\theta \sim \pi_0$.

The belief $\pi_1^{(X,S)}$ also represents the journal's interim belief when evaluating a

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general spaces Θ and $\mathcal{A}$.

paper for publication,; the public adopts this belief if the paper is published, and the default belief π_1^0 otherwise.

Default beliefs in the absence of publication

If a study is not published – meaning that either no study arrived, or that a study did arrive but was not published – then the public updates to a default belief π_1^0. We consider two distinct possibilities for updating in the absence of publication. Bayesian updating is the sophisticated rule that accounts for any selection induced by the publication process; naive updating is the unsophisticated rule which fails to account for selection. While Bayesian updating is "correct" in the fully specified model, we consider naive updating to be, in many cases, a realistic description of updating.[5] Note that the default belief π_1^0 depends on the publication rule p under Bayesian updating, but does not depend on the publication rule under naive updating.

Bayesian updating is the updating rule for a public that understands the fully specified model of the world and correctly accounts for unpublished studies. When no study is published, the public understands that this event could have occurred because no study arrived (probability $1-q$) or because a study arrived (probability q) and was unpublished (conditional probability $1 - p(X, S)$ given X and S, with $\theta \sim \pi_0$, $S \sim F_S$, and $X \sim F_{X|\theta,S}$). The public then updates beliefs on θ to π_1^0 according to Bayes rule.[6] Denote the Bayesian default belief under publication rule p by $\pi_1^{0,p}$; its density relative to the prior is given by

$$\frac{d\pi_1^{0,p}}{d\pi_0}(\theta) = \frac{1 - q\mathbb{E}[p(X, S)|\theta]}{1 - q\mathbb{E}[p(X, S)]}. \tag{3}$$

Naive updating is the updating rule for a public that ignores the possibility of unpublished studies. Under naive updating, the public's default belief when it does not see a publication, π_1^0, is equal to its prior: $\pi_1^0 = \pi_0$. One can interpret a naive public as having an incorrect model of the world – in the absence of

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[5]Enke (2017) evaluates the extent to which people learn from the (informative) absence of a signal, and finds in an experiment that subjects cluster around either fully accounting for selection or entirely neglecting it. More people can be prompted to account for selection by making the absence of a signal more salient, or by reducing cognitive distractions.

[6]If a publication rule publishes with probability one, and if the probability of study arrival q is also one, then nonpublication is a zero probability event and beliefs are not pinned down by Bayes' rule. As a convention, in that case we let the Bayesian default belief be equal to the prior π_0.

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seeing a publication, the public is unaware of the possibility that a study might have been submitted and rejected. Alternatively, this updating rule arises as the limiting Bayesian belief for the case of $q \to 0$, i.e., a fully rational public that did not expect a study to be submitted on this topic.

Leading examples of priors and signals

A typical state of the world θ estimated in an empirical economics study might be a demand or supply elasticity, the magnitude of a treatment effect, or the net benefit of implementing a program. Our leading example for a prior distribution will be the *normal prior*, in which case $\Theta = \mathbb{R}$ and π_0 is $\mathcal{N}(\mu_0, \sigma_0^2)$, with $\mu_0 \in \mathbb{R}$ and $\sigma_0 \in \mathbb{R}_{++}$.

Our leading example for the signal distribution is that of *normal signals*. Under normal signals, we assume $\mathcal{X} = \mathbb{R}$ and $\mathcal{S} \subseteq \mathbb{R}_{++}$. The conditional distribution of findings, $F_{X|\theta,S}$, is given by $\mathcal{N}(\theta, S^2)$. The finding X is interpreted as the point estimate for θ, with standard error of S. F_S then describes the ex-ante distribution of standard errors.

When there is a normal prior and normal signals, the public's posterior belief on θ after publishing a study is given by

$$\pi_1^{(X,S)} = \mathcal{N}\left(\tfrac{\sigma_0^2}{S^2+\sigma_0^2} X + \tfrac{S^2}{S^2+\sigma_0^2} \mu_0, \tfrac{S^2\sigma_0^2}{S^2+\sigma_0^2} \right). \tag{4}$$

Note that the study design S, as defined here, summarizes the informational content of the finding X. There are several reasons why the variable S in the normal signals example might be larger than a study's reported standard error, which only captures sampling variation. One additional source of error is from limited external validity, in which the estimated parameter differs from and is only partially informative about the policy parameter of interest. Another source of error is a violation of the identifying assumptions required for the study's internal validity. We discuss some considerations that may arise when S is not fully observed by the journal in Appendix A.2.

2.3 Welfare maximization

The value of publication in our model arises from a "policy relevance" objective. The choice to publish rather than not publish a study (X, S) has a direct effect of

switching the policy action from a^0 to $a^*(\pi_1^{(X,S)})$ at a cost c. Under Bayesian updating, the choice of a publication rule may also have an indirect effect on the default action taken when no study is published.

The optimal publication rule $p(X, S)$ is the one that maximizes the ex-ante expectation of welfare. The ex-ante expected welfare given a publication rule p and a default action a^0 is equal to

$$EW(p, a^0) = \mathbb{E}\Big[qp(X, S)(U(a^*(\pi_1^{(X,S)}, \theta) - c) + \big(1 - qp(X, S)\big)U(a^0, \theta)\Big], \quad (5)$$

where the expectation is taken with respect to $\theta \sim \pi_0$, $S \sim F_S$, and $X \sim F_{X|\theta,S}$.[7] Under naive updating, the optimal p maximizes expected welfare $EW(p, a^*(\pi_0))$ over the choice of p. Under Bayesian updating, the optimal p maximizes $EW\left(p, a^*\left(\pi_1^{0,p}\right)\right)$ over the choice of p.

Leading examples of utility functions

Two leading utility functions we consider are *quadratic loss* and *binary action* utility.

The quadratic loss utility function has $\mathcal{A} = \mathbb{R}$ and $U(a, \theta) = -(a - \theta)^2$. This is a canonical utility function for a public that makes a continuous policy decision a, with the state θ representing the public's uncertain ideal point. Under quadratic loss utility, the maximizing action choice given belief π_1 is $a^*(\pi_1) = \mathbb{E}_{\theta \sim \pi_1}[\theta]$. The subjective expectation of the action utility is $\mathbb{E}_{\theta \sim \pi_1}[U(a^*(\pi_1), \theta)] = -\text{Var}_{\theta \sim \pi_1}[\theta]$.

The binary action utility function has $\mathcal{A} = \{0, 1\}$ and $U(a, \theta) = a \cdot \theta$. Here there is a binary decision, such as a choice to implement a program or not, where the state θ represents the net benefit of implementation. In that case $a^*(\pi_1) = \mathbf{1}(\mathbb{E}_{\theta \sim \pi_1}[\theta] > 0)$, where $\mathbf{1}$ is the indicator function (taking the action to be $a = 0$ at indifference). The subjective expectation of the action utility is $\mathbb{E}_{\theta \sim \pi_1}[U(a^*(\pi_1), \theta)] = \max\{0, \mathbb{E}_{\theta \sim \pi_1}[\theta]\}$.

We see that for both of these example utility functions, action choices depend only on mean beliefs. As a matter of notation, when working with either utility function, let μ_0, μ_1^0, and $\mu_1^{(X,S)}$ denote the means of the distributions π_0, π_1^0, and $\pi_1^{(X,S)}$.

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[7]If the publication rule p implies default belief π_1^0 under a specified updating rule, then the expected welfare it induces is $EW(p, a^*(\pi_1^0))$. However, (5) also gives us the formula for calculating the counterfactual payoff from an arbitrary default action a^0 that may not be the one implied by p and the updating rule.

2.4 Interpretation of the publication cost

As mentioned in the introduction, this paper begins with the observation that not all research findings get published. One can interpret our paper as solving for the optimal publication rule conditional on a fixed share of studies to be published. Specifically, an optimization problem with a constraint on the share of studies that can be published would be equivalent to an unconstrained problem with an appropriate shadow cost of publication; we can take c to be such a shadow cost.

Of course, there are real publication costs beyond any (presumably negligible) physical costs of printing and/or web hosting. On the journal side, the editor or referees may be responsible for verifying that the results are in fact as claimed. The editor can decide whether to desk reject the paper based on its claimed results. But if the editor wants to proceed, then before the paper can be published, peer reviewers have to put in work to confirm that the analysis behind its results is correct and should be trusted. On the researcher side, after one has determined the main results, there still is a cost of writing an article and preparing it for submission. These social costs would all be captured in the c term.

In a more "reduced form" manner, the publication cost might also represent an opportunity cost of the public's attention. For a public with limited ability to process information, publishing one study can pull attention from others. Relatedly, high-ranking journals do have a genuine limit on the number of papers they publish and reject a large share of submissions (see Card and DellaVigna (2013)). To the extent that publications in high-ranking journals receive disproportionate attention and influence, one can interpret our analysis as characterizing which papers should be published in these top journals.[8]

3 Optimal publication rules

This section begins with a general characterization of how to solve for optimal publication rules, covering both naive and Bayesian updating. We then apply this solution

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[8]One can make this "top journal" interpretation of our model formally precise up to a rescaling of costs. Assume that a study is first submitted to a top journal that has opportunity cost c of publication. If that journal does not publish the study then it ends up published in a low-ranked journal with publication cost 0. The public will become aware of a study at a top journal with probability $\bar{r} \in (0, 1]$, and a study at a low-ranked journal with probability $\underline{r} < \bar{r}$. The public's beliefs go to $\pi_1^{(X,S)}$ if they become aware of a study, and π_1^0 otherwise.

to derive optimal rules for our two leading example utility functions. Finally, we generalize the qualitative results to a broader class of utility functions.

3.1 Solving the model

Recall that after a paper has been submitted, the journal observes (X, S) and has interim belief π given by $\pi = \pi_1^{(X,S)}$. At this interim belief, the journal evaluates the expected payoff from publication (leading to public belief $\pi_1 = \pi$ and action $a^*(\pi)$) and from nonpublication (leading to public belief $\pi_1 = \pi^0$ and action a^0). Denote by $\Delta(\pi, a^0)$ the gross interim benefit[9] – not including publication costs – of publishing a study that induces interim belief π given default action a^0:

$$\Delta(\pi, a^0) = \mathbb{E}_{\theta \sim \pi}[U(a^*(\pi), \theta) - U(a^0, \theta)]. \tag{6}$$

The objective is to maximize the ex-ante welfare, not the interim welfare. But we can rewrite ex-ante welfare $EW(p, a^0)$ of a publication rule p and default action a^0 from (5) as the expected utility under the default action, plus the expected net interim benefit of publication:

$$EW(p, a^0) = \mathbb{E}\Big[qp(X,S)(U(a^*(\pi_1^{(X,S)}), \theta) - c) + \big(1 - qp(X,S)\big)U(a^0, \theta)\Big]$$
$$= \mathbb{E}[U(a^0, \theta)] + q\mathbb{E}\Big[p(X,S)(\Delta(\pi_1^{(X,S)}; a^0) - c)\Big]. \tag{7}$$

Say that publication rule p is *interim optimal* given default action a^0 if it (almost surely) publishes a study when $\Delta(\pi_1^{(X,S)}, a^0) > c$ and does not publish when $\Delta(\pi_1^{(X,S)}, a^0) < c$. By linearity of the expectations operator, we see from (7) that welfare is separable across realizations of (X, S). Hence, fixing the default action a^0, a publication rule maximizes expected welfare $EW(p, a^0)$ over choice of p if and only if it is interim optimal.[10] For concreteness, let $p^{I(a^0)}$ be the interim optimal publication

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[9]Frankel and Kamenica (2018) provide general characterizations of these gross interim benefit functions – the value of information, in an ex-post sense – across decision problems.

[10]In a game with different timing in which the journal could not commit to a publication rule, one might define a publication rule p and default belief π_1^0 as constituting a Bayes Nash equilibrium if they jointly satisfy (i) p is interim optimal given default action $a^0 = a^*(\pi_1^0)$, and (ii) the default belief π_1^0 is equal to $\pi_1^{0,p}$, the one induced by Bayesian updating given the publication rule p. Our notion of optimality under Bayesian updating does not impose (i); nevertheless, Lemma 2 below clarifies that (i) will in fact be satisfied for an optimal publication rule. Hence, any optimal publication rule would induce a Bayes Nash equilibrium, but the converse need not hold.

rule given a^0 that deterministically publishes at indifference:

$$p^{I(a^0)} = \begin{cases} 1 & \text{if } \Delta(\pi_1^{(X,S)}, a^0) \geq c. \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Under naive updating, the optimal publication rule solves $\max_p EW(p, a^0)$ subject to $a^0 = a^*(\pi_0)$. In particular, a^0 does not depend on p, so expected welfare is maximized by choosing an interim optimal publication rule. That is, a publication rule is optimal under naive updating if and only if it is interim optimal given default action $a^0 = a^*(\pi_0)$.

Under Bayesian updating, the optimal publication rule solves $\max_p EW(p, a^0)$ subject to $a^0 = a^*(\pi_1^{0,p})$. Solving this program requires taking into account the fact that a^0 changes with p. However, one can simplify the problem by observing that for any fixed p, the induced Bayesian default action $a^0 = a^*(\pi_1^{0,p})$ maximizes expected welfare $EW(p, a^0)$ over choice of a^0. This is because

$$\arg\max_{a^0} EW(p, a^0) = \arg\max_{a^0} \mathbb{E}[(1 - qp(X,S))U(a^0, \theta)] = \arg\max_{a^0} \mathbb{E}[U(a^0, \theta)|D = 0]$$
$$= \arg\max_{a^0} \mathbb{E}_{\theta \sim \pi_1^{0,p}}[U(a^0, \theta)],$$

where the last equality holds by Bayesian updating, because the conditional distribution of θ given $D = 0$ is equal to $\pi_1^{0,p}$. Therefore the Bayesian optimal publication rule p equivalently solves $\max_p \max_{a^0} EW(p, a^0)$. This maximization program corresponds to a sequential game of common interest, with both players aiming to maximize the same objective. Moreover, it holds that $\max_p \max_{a^0} EW(p, a^0) = \max_{a^0} \max_p EW(p, a^0) = \max_{p,a^0} EW(p, a^0)$. Put differently, in a sequential game of common interest, the value is the same regardless of which player moves first. The value is also equal to the "planner's solution" maximizing the objective over the joint choice of p and a^0.[11] Lemma 1 formally states this conclusion.

Lemma 1. *Under Bayesian updating, let p be an optimal publication rule and let $a^0 = a^*(\pi_1^{0,p})$ be the induced default action. Then for any publication rule p' and any action a', it holds that $EW(p, a^0) \geq EW(p', a')$.*

One immediate implication from Lemma 1 and the above discussion is that even

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[11]In a simultaneous game of common interest, the planner's solution is one equilibrium, but there may also be lower-payoff equilibria.

under Bayesian updating, p is interim optimal given the induced default action. Hence, the optimal policy is interim optimal regardless of the updating rule:

Lemma 2. *Under either naive or Bayesian updating, let p be an optimal publication rule, π_1^0 be the induced default belief, and $a^0 = a^*(\pi_1^0)$ be the default action. Then p is interim optimal given a^0.*

Lemma 2 tells us that in searching for optimal publication rules, it is sufficient to consider only those rules that are interim optimal with respect to some default action. Indeed, at a default action a^0, it is without loss of generality to focus on the specific interim optimal rule $p^{I(a^0)}$ – all interim optimal publication rules give the same payoff.[12] Going forward we adopt the convention of restricting attention to $p^{I(a^0)}$ in the class of interim optimal rules, and referring to this rule as "the" interim optimal publication rule given a^0.

The following result provides a recipe for finding the optimal publication rule, summarizing the implications of Lemmas 1 and 2.

Proposition 1.

1. *Suppose that the updating rule is naive, in which case $\pi_1^0 = \pi_0$ and $a^0 = a^*(\pi_0)$. Then the interim optimal publication rule given this default action, $p^{I(a^*(\pi_0))}$, is optimal.*

2. *Suppose that the updating rule is Bayesian, in which case $\pi_1^0 = \pi_1^{0,p}$ and $a^0 = a^*(\pi_1^{0,p})$ under publication rule p.*

   (a) *Let $\hat{a} \in \arg\max_{a \in \mathcal{A} \ s.t. \ a = a^*\left(\pi_1^{0,p^{I(a)}}\right)} EW\left(p^{I(a)}, a\right)$. Then the interim optimal publication rule given this default action, $p^{I(\hat{a})}$, is optimal.*

   (b) *Let $\hat{a} \in \arg\max_{a \in \mathcal{A}} EW\left(p^{I(a)}, a\right)$. Then the interim optimal publication rule given this default action, $p^{I(\hat{a})}$, is optimal.*

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[12]More precisely, $EW(p, a^0)$ is constant across all interim optimal p for a given a^0. So for naive updating, where the default action is fixed in advance, the payoff of all interim optimal publication rules is the same. For Bayesian updating, the default action varies with the publication rule. However, for any optimal publication rule inducing some default action, it holds that all interim optimal publication rules given that same default action are also optimal, even if they induce different default actions. To see this, let p be an optimal publication rule that induces default action a^0 (implying that p is interim optimal given a^0). Let p' be any other interim optimal rule given a^0 – for instance, $p' = p^{I(a^0)}$ – and let $a' = a^*(\pi_1^{0,p'})$ be the default action induced by p'. Then $EW(p', a^0) \geq EW(p, a^0)$ by interim optimality of p'; $EW(p', a') \geq EW(p', a^0)$ by the fact that $a' \in \arg\max_a EW(p', a)$; and $EW(p, a^0) \geq EW(p', a')$ by Lemma 1. Hence, these inequalities are all equalities. The payoff from publication rule p', $EW(p', a')$, is equal to the payoff from the optimal publication rule p, $EW(p, a^0)$, and so p' is optimal as well.

Proposition 1 part 1 formally restates the solution under naive updating. The optimal publication rule is the interim optimal rule given the naive default action.

For Bayesian updating, considered in Proposition 1 part 2, our characterization of the optimal publication rule is less direct. We provide two alternative maximization programs that can be solved to find the optimum. (Depending on the setting, one or the other may be more straightforward to apply.) Rather than maximizing over the original function space of publication rules, we are able to simplify the problem by maximizing over the action space, or a subset thereof. Specifically, each action induces an interim optimal publication rule, and the optimal publication rule is given by the induced interim optimal rule that yields the highest payoff.

To understand part 2a of Proposition 1, recall that just as each action induces an interim optimal publication rule, so too does each publication rule induce a Bayesian default action. Lemma 2 establishes that the optimal publication rule is interim optimal with respect to its induced default action. In other words, the default action is a "fixed point" of the mapping from actions to publication rules and back to actions. Therefore, when searching for an optimal publication rule, it is sufficient to maximize over interim optimal rules that are induced by some fixed point default action.

Unfortunately, solving for the set of fixed point default actions might not be straightforward. Part 2b of Proposition 1 gives us a version of the result that does not require solving for fixed points. Instead, we can maximize over the full action space. Moreover, while the payoff of the publication rule $p^{I(a)}$ that is interim optimal with respect to action a is generally given by $EW(p^{I(a)}, a^*(\pi_1^{0,p^{I(a)}}))$ – requiring us to solve for the Bayesian default action induced by $p^{I(a)}$ – the proposition states that we need only evaluate the simpler expression $EW(p^{I(a)}, a)$. To see this, consider two cases. In the case that a happens to be an action induced by some optimal policy, the payoff of the optimal policy $p^{I(a)}$ is in fact given by $EW(p^{I(a)}, a)$; see footnote 12. And in the case that a is not an action induced by some optimal policy, the payoff $EW(p^{I(a)}, a)$ will be below that of the optimum; see Lemma 1. So this program recovers the correct maximum.

3.2 Quadratic Loss Utility

Under quadratic loss utility, welfare is $W(D, a, \theta) = -(a - \theta)^2 - Dc$ for $a \in \mathcal{A} = \mathbb{R}$. The public chooses an action equal to its posterior mean belief about the state. So

when the default action is a^0, the gross interim benefit of publishing a study (X, S) inducing belief $\pi_1^{(X,S)}$ with mean $\mu_1^{(X,S)}$ evaluates to $(\mu_1^{(X,S)} - a^0)^2$. The interim optimal publication rule is therefore

$$
p^{I(a^0)}(X, S) = \begin{cases} 1 & \text{if } \left| \mu_1^{(X,S)} - a^0 \right| \geq \sqrt{c} \\ 0 & \text{otherwise} \end{cases}.
\tag{9}
$$

Lemma 2 establishes that this is the form of the optimal publication rule for a^0 equal to μ_1^0, the mean of the appropriate default belief. A study is published if and only if its results move the posterior mean by a sufficient amount in either direction relative to the default mean.

With naive updating, the optimal publication rule is given by plugging $a^0 = \mu_0$ into (9); see Proposition 1 part 1. For Bayesian updating, the default action a^0 is endogenous to the publication rule. The following proposition provides a condition that allows us to explicitly solve for the optimal publication rule under Bayesian updating, which given this condition is the same as under naive updating.

Proposition 2. *Suppose that there is quadratic loss utility, and that conditional on a study arriving the distribution of the interim mean* $\mu_1^{(X,S)}$ *is single-peaked and symmetric about the prior mean* μ_0.[13] *Then the optimal publication rule under Bayesian updating is the same as under naive updating: publish if and only if* $\left| \mu_1^{(X,S)} - \mu_0 \right| \geq \sqrt{c}$.

To prove this result, we show that under single-peakedness and symmetry, the prior mean is the *only* fixed point default action under Bayesian updating. So by Proposition 1 part 2a it must be the default action for the optimal policy.

Under the publication rule p of Proposition 2, the induced Bayesian default belief $\pi_1^{0,p}$ will not be equal to the naive default belief π_0. However, they will both have the same mean, and therefore they lead to the same default action.

Normal prior and normal signals

We now explicitly characterize optimal publication rules under a normal prior and normal signals. This prior and signal structure satisfies the hypotheses of Proposition

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[13]To be precise, by single-peaked and symmetric, we mean that (i) the distribution of the random variable $\mu_1^{(X,S)}$ has a pdf that is symmetric about μ_0; and (ii) for any $\mu' < \mu'' \leq \mu_0$ it holds that if the pdf evaluated at μ' is strictly positive, then the pdf evaluated at μ'' is strictly larger than at μ'. (Symmetry implies the same result for $\mu_0 \leq \mu'' < \mu'$.)
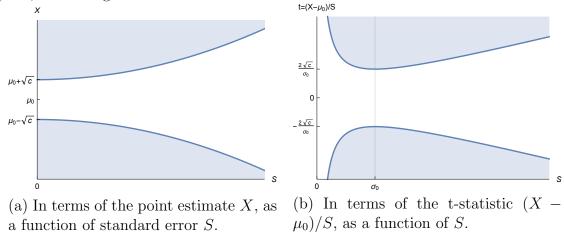
2: the distribution of posterior means is single-peaked and symmetric about μ_0 for each realized design S, and therefore across realizations of S as well. Hence, the journal optimally publishes under either updating rule if and only if $|\mu_1^{(X,S)} - \mu_0| \geq \sqrt{c}$. In combination with the updating formula (4) we get the following corollary.

Corollary 1. *Suppose that there is a normal prior, normal signals, and quadratic loss utility. Then under either Bayesian or naive updating, the optimal publication rule publishes if and only if* $|X - \mu_0| \geq \left(1 + \frac{S^2}{\sigma_0^2}\right)\sqrt{c}.$

This publication rule corresponds to a "two-sided test" in which the journal publishes if the point estimate is sufficiently high or sufficiently low. Equivalently, we can restate the publication rule in terms of a two-sided test for the t-statistic $(X - \mu_0)/S$: the journal publishes if and only if $\frac{|X-\mu_0|}{S} \geq \left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)\sqrt{c}$. See Figure 1.

The form of a two-sided test is of course familiar from the null-hypothesis significance testing paradigm. However, we wish to highlight two ways in which our policy is distinct from two-sided tests as they are traditionally applied. First, we compare the point estimate X to the prior mean, not to some other point, e.g., a null hypothesis of $\theta = 0$. Second, the cutoff for publication is not given by a conventional value, such as a t-statistic of 1.96 corresponding to a p-value of .05. The cutoff is determined by a cost-benefit analysis. We can take comparative statics on the value of this cutoff:

Figure 1: Optimal publication region (shaded) for quadratic loss utility, normal prior, normal signals.

(a) In terms of the point estimate X, as a function of standard error S.

(b) In terms of the t-statistic $(X - \mu_0)/S$, as a function of S.

Corollary 2. *Under the publication rule from Corollary 1,*

1. *The publication cutoff* $\left(1 + \frac{S^2}{\sigma_0^2}\right)\sqrt{c}$ *in terms of the difference of the point esti-mate from the prior mean is independent of the study arrival probability q and the mean μ_0. It is larger when the standard error S is larger, the prior variance σ_0^2 is smaller, or the cost of publication c is larger.*
2. *The publication cutoff* $\left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)\sqrt{c}$ *in terms of the magnitude of the t-statistic is nonmonotonic and convex in the standard error S: it has a minimum at $S = \sigma_0$ and goes to infinity as $S \to 0$ or $S \to \infty$.*

A given point estimate of X moves beliefs more, and thus makes publication more likely (in the sense of a smaller cutoff value for $|X - \mu_0|$), when the standard error S is smaller or when the prior uncertainty σ_0 is larger. Likewise, publication is more likely when the cost of publication c is lower.

When deciding to publish at a given t-statistic, rather than point estimate, we find a different and non-monotonic comparative static as a function of S. (For other parameters, the comparative statics in terms of the t-statistic would be identical to those on the point estimate.) For a precise study with a low standard error or an imprecise one with a high standard error, the journal requires a high t-statistic to be willing to publish; for a study of intermediate precision, the journal publishes at a lower t-statistic. To gain intuition, suppose the prior mean of θ is $\mu_0 = 0$ and the prior standard deviation is $\sigma_0 = 1$. The journal will publish a study if it moves the interim mean sufficiently far from 0. Consider studies that might arrive with a given t-statistic $t = X/S$, say, $t = 4$ (corresponding to a standard two-sided p-value of $\lesssim .0001$ against a null of $\theta = \mu_0$). If a very precise study arrives with a t-statistic of 4, then it must have had a small point estimate, and so it moves the mean very little: a study with a point estimate of $X = .04$ and a standard error of $S = .01$ moves the interim mean to $\simeq .039996$. As we begin to scale up the point estimate and standard error while keeping $t = 4$, the mean moves higher: $X = 4$ and $S = 1$ leads to a mean of 2. However, when we increase the point estimate and standard error further, the mean falls back towards 0, because the result becomes too noisy to move beliefs much. With $X = 400$ and $S = 100$, the mean is back down to $\simeq .039996$. The journal would be most inclined to publish the middle result out of these three possibilities.

In other words, fixing the "statistical significance" as measured by the t-statistic, the change in mean first grows and then declines in the "practical significance" as measured by the magnitude of the point estimate. Generally, fixing a t-statistic $t = (X - \mu_0)/S$, the difference from the interim mean to the prior mean at standard

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error S (with a corresponding point estimate of $X = \mu_0 + \sigma_0 t$) is given by $t\frac{\sigma_0^2 S}{\sigma_0^2 + S^2}$. The mean moves furthest at a standard error S equal to σ_0, the standard deviation of the prior. The change in mean falls towards zero at high standard errors because the interim mean is a weighted average of the prior mean and the point estimate, with weights proportional to the inverse of the variances. While the point estimate increases linearly with the standard error at a given t-statistic, the weight on the point estimate decreases quadratically.

One could easily incorporate additional parameters into the model to get further comparative statics. For instance, consider the publication rule for research questions that are more or less "important." Modeling importance as a payoff coefficient v in which $U(a, \theta) = -v \cdot (a - \theta)^2$, a more important policy with higher v is mathematically equivalent to a lower publication cost c. So, the more important a state of the world is for policy, the more willing the journal should be to publish at a given point estimate or t-statistic. Similarly, if one decomposes S into a combination of sampling error and imperfect external validity, the journal is less willing to publish a point estimate when either the sampling error is higher or when the external validity is worse.

3.3 Binary Action Utility

Under binary action utility, welfare is given by $W(D, a, \theta) = a\theta - Dc$ for $a \in \mathcal{A} = \{0, 1\}$. The public chooses action $a = 0$ if its posterior mean belief about the state is weakly less than 0, and action $a = 1$ if the posterior mean is positive. So when the default action is a^0, the gross interim benefit of publishing a study (X, S) inducing belief $\pi_1^{(X,S)}$ with mean $\mu_1^{(X,S)}$ evaluates to 0 if either $a^0 = 0$ and $\mu_1^{(X,S)} \leq 0$, or $a^0 = 1$ and $\mu_1^{(X,S)} \geq 0$; and to $|\mu_1^{(X,S)}|$ otherwise. The interim optimal publication rule is therefore

$$p^{I(a^0)}(X, S) = \begin{cases} 1 & \text{if } a^0 = 0 \text{ and } \mu_1^{(X,S)} \geq c, \\ & \quad \text{or } a^0 = 1 \text{ and } \mu_1^{(X,S)} \leq -c \cdot \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

Once again, Lemma 2 establishes that this is also the form of the optimal publication rule, for the appropriate default action.

Here, a study is published only if it moves the posterior mean belief sufficiently far in one direction. If the default action is low, then the journal only publishes studies

that move beliefs up; if the default action is high, then the journal only publishes studies that move beliefs down. Contrasting to the quadratic loss utility function, the posterior mean must move sufficiently far not relative to the mean of the default belief, but relative to the belief – normalized to 0 – at which the public is indifferent between the two actions.

With naive updating, we get the optimal publication rule by plugging the naive default action into (10): $a^0 = 0$ if $\mu_0 \leq 0$, and $a^0 = 1$ if $\mu_0 > 0$. See Proposition 1 part 1. The following result gives a condition under which we can explicitly solve for the Bayesian optimal policy, and for which it is equal to the naive optimal policy. For the statement of the result, it is convenient to normalize the prior mean of θ to be less than zero, meaning that the naive default action will be $a^0 = 0$. The result then holds when the ex-ante distribution of interim expectations on the state is sufficiently "left-leaning" relative to $\theta = 0$.[14]

Proposition 3. *Let $\mu_0 \leq 0$. Suppose that there is binary action utility, and that conditional on a study arriving the distribution of the interim mean satisfies $P(\mu_1^{(X,S)} \leq -k) \geq P(\mu_1^{(X,S)} \geq k)$ for all $k > 0$. Then the optimal publication rule under Bayesian updating is the same as under naive updating: publish if and only if $\mu_1^{(X,S)} \geq c$.*

The distributional assumption of Proposition 3 is strictly weaker than that of Proposition 2: given a prior mean $\mu_0 \leq 0$, any symmetric distribution of the interim mean $\mu_1^{(X,S)}$ is guaranteed to satisfy the condition of Proposition 3 even if it is not single-peaked. That said, one can not establish this result by applying Proposition 1 part 2a and showing that the naive default action is the unique Bayesian fixed point default action, as we did with Proposition 2. It can be the case – even under the stricter distributional conditions of Proposition 2 – that both actions are fixed points. In particular, it can be that with a low default action, the journal would only publish high signals, and after nonpublication the public would take a low action in response; while with a high default action, the journal would only publish low signals, and the public would take a high action in response. To prove Proposition 3, we instead apply Proposition 1 part 2b. We directly confirm that the interim optimal publication rule with default action $a^0 = 0$ gives a higher payoff than with default action $a^0 = 1$.

Under this publication rule (and unlike the quadratic loss publication rule from Proposition 2), the mean of the induced Bayesian default belief $\pi_1^{0,p}$ will be lower than

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[14] An analogous result for $a^0 = 1$ and a sufficiently "right-leaning" distribution holds when the prior mean is above zero.

the mean of the naive default belief π_0. High results are published and low results are not, and so to a Bayesian the absence of publication is suggestive of a low state. Regardless, though, both default beliefs lead to the same default action of $a^0 = 0$ and thus the same (interim) optimal publication rule.
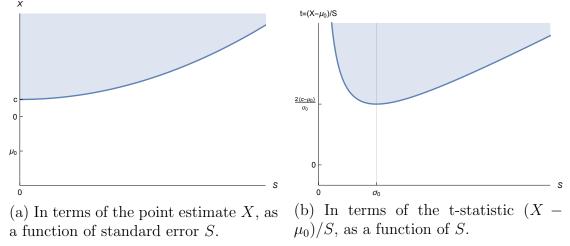
Normal prior and normal signals

We next explicitly characterize optimal publication rules under a normal prior and normal signals. Without loss of generality, assume that the prior mean is $\mu_0 \leq 0$. This prior and signal structure satisfies the hypotheses of Proposition 3, meaning that the journal optimally publishes under either updating rule if and only if $\mu_1^{(X,S)} \geq c$. Plugging in the updating formula (4):

Corollary 3. *Suppose that there is a normal prior with $\mu_0 \leq 0$, normal signals, and binary action utility. Then under either Bayesian or naive updating, the optimal publication rule publishes a study if and only if $X \geq \left(1 + \frac{S^2}{\sigma_0^2}\right) c - \frac{S^2}{\sigma_0^2}\mu_0$.*

This publication rule corresponds to a "one-sided test" in which a paper is published if the point estimate is sufficiently high. In terms of the t-statistic, the journal publishes if and only if $\frac{X-\mu_0}{S} \geq \left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)(c - \mu_0)$. See Figure 2.

Figure 2: Optimal publication region (shaded) for binary action utility, normal prior, normal signals.

(a) In terms of the point estimate X, as a function of standard error S.

(b) In terms of the t-statistic $(X - \mu_0)/S$, as a function of S.

Corollary 4. *Under the publication rule from Corollary 3,*

1. *The publication cutoff $\left(1 + \frac{S^2}{\sigma_0^2}\right) c - \frac{S^2}{\sigma_0^2} \mu_0$ in terms of the point estimate is independent of the study arrival probability q. It is decreasing in the mean μ_0. It is larger when the standard error S is larger, the prior variance σ_0^2 is smaller, or the cost of publication c is larger.*

2. *The publication cutoff $\left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right) (c - \mu_0)$ in terms of the t-statistic is nonmonotonic and convex in the standard error S: it has minimum at $S = \sigma_0$ and goes to infinity as $S \to 0$ or $S \to \infty$.*

The comparative statics of publication with respect to the standard error S, the prior variance σ_0^2, and the cost of publication c are essentially the same as those from the quadratic loss publication rule (see Corollaries 1 and 2). However, the two policies depend differently on the prior mean. Suppose we fix a point estimate $X > 0$ and we consider prior means $\mu_0 < 0$. With quadratic loss utility, increasing μ_0 towards 0 would make the journal less willing to publish: there will be a smaller difference $X - \mu_0$, and therefore the posterior mean will be closer to the prior mean. With binary actions, increasing μ_0 towards 0 makes the journal more willing to publish: the posterior mean will be higher in absolute terms, indicating that the benefit of switching from $a = 0$ to $a = 1$ is higher.

3.4 General utility functions

In the above two examples of utility functions, we found that it was optimal to publish results that were in some sense "extreme" or "surprising" – ones that moved beliefs a lot. In this section we show that this conclusion generalizes beyond the examples of quadratic loss and binary action utility.

We start with a simple observation: if a study is a "null result," meaning that it does not move the optimal action away from the default, then the gross interim benefit of publication is zero. Thus, the optimal publication rule never publishes null results.

Observation 1. Fix a default action a^0 and an interim belief π^I. If $a^*(\pi^I) = a^0$, then $\Delta(\pi^I, a^0) = 0 < c$.

Next, say that a utility function $U : \mathcal{A} \times \Theta \to \mathbb{R}$ is *supermodular* if for all $\underline{a} < \overline{a}$ and $\underline{\theta} < \overline{\theta}$, it holds that $U(\overline{a}, \overline{\theta}) + U(\underline{a}, \underline{\theta}) \geq U(\underline{a}, \overline{\theta}) + U(\overline{a}, \underline{\theta})$. Let $\geq_{FOSD}$ denote the first order stochastic dominance partial ordering on distributions. Under supermodular

utility functions, the public takes higher actions when it believes that the state is higher (in the sense of FOSD); quadratic loss and binary action utilities are both supermodular. Proposition 4 formalizes a sense in which it is generally optimal to publish extreme results when utility is supermodular.

Proposition 4. *Let U be supermodular. Let beliefs π', π'', and π''' satisfy $\pi''' \geq_{FOSD} \pi'' \geq_{FOSD} \pi'$. Then for any default action a^0, it holds that $\Delta(\cdot, a^0)$ is quasiconvex in the sense that $\Delta(\pi'', a^0) \leq \max\{\Delta(\pi', a^0), \Delta(\pi''', a^0)\}$.*

Suppose that some subset of study realizations leads to FOSD ordered beliefs. The proposition states that it is better to publish either a study that leads to a high belief or to a low belief than a study that leads to an intermediate belief. By Lemma 2, then, the journal optimally publishes studies that lead to sufficiently low or high beliefs, but not beliefs in the middle.[15] (See Appendix B.1 for examples illustrating how the conclusion of Proposition 4 can change when either supermodularity or FOSD ordering of beliefs is relaxed.)

Combining Proposition 4 and Observation 1, we further see that the published results will be ones that lead to extreme beliefs *relative to the default*. More precisely, fix some updating rule and some optimal publication policy leading to default belief π_1^0. Plugging the unpublished low belief $\pi' = \pi_1^0$ into Proposition 4, we see that if the journal publishes a study leading to the higher belief π'', it must also publish one leading to the even higher belief π'''. Likewise, plugging in the unpublished high belief $\pi''' = \pi_1^0$, we see that if the journal publishes the lower belief π'' then it also publishes the even lower belief π'.

One sufficient condition for the interim belief from study realization (x'', s'') to be FOSD higher than that of (x', s') is for the realizations to satisfy the monotone likelihood ratio property (MLRP) in θ. We say that (x'', s'') and (x', s') satisfy MLRP if the ratio $\frac{f_{X|\theta,S}(x''|\theta,s'')}{f_{X|\theta,S}(x'|\theta,s')}$ is increasing in θ. When findings at a given study design satisfy MLRP, we can apply Proposition 4 to derive the following corollary characterizing the study findings X that are optimally published. The corollary holds independently of any assumptions on the prior belief π_0.

Corollary 5. *Fix either updating rule. Let the utility function U be supermodular. Furthermore, suppose that $\mathcal{X} \subseteq \mathbb{R}$, and that at some given study design $S = s$ it holds*

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[15]To be precise, no published study leads to beliefs that are in between those of two unpublished studies. But on a given chain of ordered beliefs, the journal might publish only high results, only low results, all results, or no results.

that for any $x'' > x'$ in $\mathcal{X}$, (x'', s) and (x', s) satisfy MLRP. Then under the optimal policy, at $S = s$ a study is published if and only if X lies outside of an interval. That is, given s, there exists an interval $I^s \subseteq \mathbb{R}$ such that (X, s) is published if and only if $X \in \mathcal{X} \setminus I^s$.[16]

Under normal signals, for instance, for any $x'' > x'$ and any s, the realizations (x'', s) and (x', s) satisfy MLRP. So regardless of the prior, if there are normal signals and a supermodular utility function, the optimal publication rule at any given standard error will publish point estimates outside of an interval.

4   Selective and non-selective publication

In the previous section we characterized optimal publication rules from the perspective of policy-relevance. A key conclusion was that these welfare-maximizing publication rules tend to selectively publish extreme findings and do not publish moderate findings. This conclusion contrasts with calls for reform aimed at eliminating selection; such calls are motivated by the statistical distortions and the lack of replicability that selective publication can cause.

Section 4.1 reviews how selective publication distorts standard inference in the framework of our model. Selective publication leads to biased estimators, size distortions of confidence sets and tests, and the invalidity of naive updating. This analysis builds on earlier work on how standard inference from published results will be inaccurate when publication is based on a statistical significance filter, for instance in Rosenthal (1979) and Ioannidis (2005). Section 4.2 presents the novel result that, if we desire that standard inference be valid, then the publication rule must not select on findings at all.

The fact that selective publication distorts inference means that there is a trade-off between policy-relevance and credibility. The policy-relevance criterion pushes towards selectively publishing extreme results, while a desire to maintain the credibility of standard inference pushes instead towards non-selective publication. We conclude the section by characterizing the publication rules that maximize (policy-based) welfare subject to a constraint that publication may not select on findings.

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[16]We do not rule out the possibility that the interval I^s may be empty, in which case all studies are published; or that I^s may contain the full set $\mathcal{X}$, in which no studies are published.

4.1 Publication bias

Recall that the signal X is drawn from the distribution $F_{X|\theta,S}$ with density $f_{X|\theta,S}$; our leading example was $X|\theta, S \sim \mathcal{N}(\theta, S^2)$. Conventional statistical inference on θ using the estimator X would be based on this distribution. However, under publication rule $p(X, S)$, the distribution of X conditional on publication ($D = 1$) is different. The corrected density is

$$f_{X|\theta,S,D=1}(x|\theta, s) = \frac{p(x, s)}{\mathbb{E}[p(X, S)|\theta, S = s]} \cdot f_{X|\theta,S}(x|\theta, s). \tag{11}$$

Inference that ignores the selectivity of the publication rule will lead to distorted conclusions.

To illustrate this point, consider the following example. Let X be a normal signal with standard error of $S = 1$, so $X|\theta \sim \mathcal{N}(\theta, 1)$. For this signal, the conventional unbiased estimator of θ would be X, and the conventional confidence set with a nominal 95% coverage probability would be $[X - 1.96, X + 1.96]$. But suppose that results are only published when $|X| > 1.96$. Then, conditional on publication, X is a biased estimator of θ, and the interval $[X - 1.96, X + 1.96]$ does not have 95% coverage probability at every θ. Indeed, the probability of $\theta \in [X - 1.96, X + 1.96]$ is actually equal to zero at $\theta = 0$. The top left panel of Figure 3 plots the bias of X as a function of θ, defined as $\mathbb{E}[X - \theta|\theta, D = 1]$, and the top right panel of Figure 3 plots the coverage probability $P(\theta \in [X - 1.96, X + 1.96]|\theta, D = 1)$.[17]

Selective publication similarly impacts any form of likelihood-based inference. Maximum likelihood estimation conditional on S, likelihood ratio tests, and related methods must be adjusted for selection if the ratio $f_{X|\theta,S,D=1}(x|\theta, s)/f_{X|\theta,S}(x|\theta, s)$ varies with θ. As we see from (11), this ratio varies with θ whenever the publication probability $\mathbb{E}[p(X, S)|\theta, S = s]$ is not constant in θ. For the example with $X|\theta \sim \mathcal{N}(\theta, 1)$ in which a study is only published if $|X| > 1.96$, the publication probability falls as θ gets closer to 0; see the bottom left panel of Figure 3.

Selective publication also implies that naive updating in the absence of publication yields distorted beliefs. In general, recall that the ex-ante probability of observing no publication conditional on θ is given by $1 - q \cdot \mathbb{E}[p(X, S)|\theta]$. So the relative density

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[17]Similar figures for bias and coverage probability can be found in Andrews and Kasy (2017).
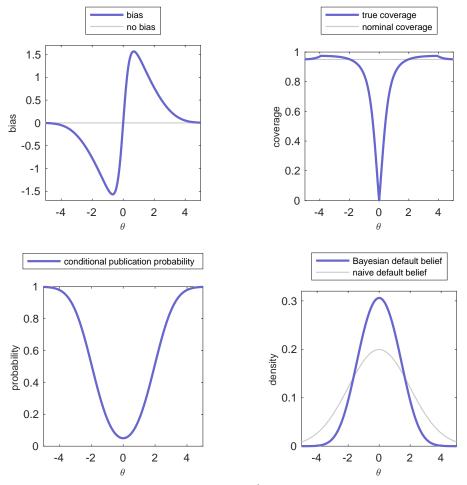
Figure 3: Distortions due to selective publication.

These plots are generated assuming $X|\theta \sim \mathcal{N}(\theta, 1)$ and that the conditional publication probability is given by $\mathbf{1}(X > 1.96)$. The top left panel plots the bias of X as an estimator of θ, conditional on publication. The top right panel plots the coverage probability of $[X - 1.96, X + 1.96]$ as a confidence set for θ, conditional on publication. The bottom left panel plots the probability of publication conditional on θ and on a study being submitted. The bottom right panel plots the Bayesian default belief relative to the naive default belief in the absence of publication, further assuming a prior of $\theta \sim \mathcal{N}(0, 4)$ and a probability $q = 1$ of study arrival.

of the Bayesian default belief to the prior is given by

$$\frac{d\pi_1^{0,p}}{d\pi_0}(\theta) = \frac{1 - q \cdot \mathbb{E}[p(X, S)|\theta]}{1 - q \cdot \mathbb{E}[p(X, S)]}. \tag{12}$$

The naive default belief (equal to the prior) differs from the Bayesian default belief whenever the publication probability $\mathbb{E}[p(X,S)|\theta]$ varies with θ.

The bottom right panel of Figure 3 compares the density of the Bayesian default belief (the posterior absent publication) to that of the naive default belief for our running example, assuming a prior of $\theta \sim N(0,4)$ and a study arrival probability of $q = 1$. When no publication is observed, a Bayesian who understands the data generating process knows that there may have been a study with $X \in [-1.96, 1.96]$ that was submitted but went unpublished. Hence the Bayesian default belief places a higher probability on θ close to 0, the center of the nonpublication interval, and a correspondingly lower probability on θ far from 0. Indeed, Abadie (2018) demonstrates how a failure to pass a standard statistical significance threshold can be extremely informative when studies are precise. Hence, if publication is based on a statistical significance test, the Bayesian default belief can greatly diverge from the naive one.

4.2 Characterizing validity and non-selectivity

Before turning to our main results in this section, our first result formalizes the straightforward observation that, when publication does not select on findings, traditional inference goes through and naive updating is valid. Say that a publication rule p is *non-selective* if $p(x,s)$ is constant in x for each s.[18] Note that non-selective publication rules do not condition publication on the finding X, but they may condition publication on the design S, which is independent of the state.

Lemma 3. *Suppose that the publication rule is non-selective and that $P(D = 1) > 0$. Then $f_{X|\theta,S,D=1}(x|\theta,s) = f_{X|\theta,S}(x|\theta,s)$, and thus the following properties hold.*

1. Frequentist unbiasedness. *If the estimator $\widehat{g} : \mathcal{X} \times \mathcal{S} \to \mathbb{R}$ for the estimand $g : \Theta \times \mathcal{S} \to \mathbb{R}$ satisfies $\mathbb{E}[\widehat{g}(X,S)|\theta, S = s] = g(\theta,s)$ for all θ, s, then $\mathbb{E}[\widehat{g}(X,S)|\theta, S = s, D = 1] = g(\theta,s)$ for all θ, s.*

2. Frequentist size control. *Fix a level $\alpha \in (0,1)$ and consider a confidence set C mapping from $\mathcal{X} \times \mathcal{S}$ to subsets of Θ. If $P(\theta \in C(X,S)|\theta, S = s) \geq 1 - \alpha$ for all θ, s, then $P(\theta \in C(X,S)|\theta, S = s, D = 1) \geq 1 - \alpha$ for all θ, s.*

3. Publication probability constant in state. *The publication probability $\mathbb{E}[p(X,S)|\theta, S = s]$ is constant in θ for all s.*

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[18]Formally, we mean by this statement that $p(x,s)$ is constant in x almost surely over realizations of X, i.e., that $P\big(p(X,s) = \mathbb{E}[p(X,s)|S = s]|\theta, S = s\big) = 1$ for all θ. Nothing changes if $p(x,s)$ may vary with x on sets of X that can only occur with zero probability given $\theta, S = s$.

4. Bayesian validity of naive updating. *The Bayesian default belief $\pi_1^{0,p}$ is equal to the naive default belief, i.e., the prior π_0.*

One interpretation of part 4 of the lemma, the Bayesian validity of naive updating, is as follows. Consider a "partially sophisticated" public which is aware that studies may sometimes go unpublished, but which does not know the study arrival rate q or the distribution of study designs F_s (and may not even have a well-specified prior over these objects). Such a public understands that naive updating can lead to distorted beliefs but it does not know how to correct this distortion. Under a non-selective publication rule, the public can in fact be confident in updating naively: for any q and any F_S, the Bayesian updating rule would be equal to the naive one.

Our next set of results establishes a sense in which, if one desires the above properties, then a publication rule cannot select on findings. For these results, we restrict to the class of normal signals.

First, we show that if the point estimate is an unbiased estimator for θ, or if the publication probability is constant in the state for every realization of S, then the publication rule must be non-selective. Likewise, if the public is not certain of the distribution of study designs F_S and it seeks a publication rule for which naive updating is guaranteed not to yield distorted beliefs, then the publication rule must be non-selective.[19] Given that non-selective publication also implies these properties (Lemma 3), it follows that non-selective publication is equivalent to any of these three properties. We will return to the size control property below, in Proposition 6.

Proposition 5. *Suppose that there are normal signals and suppose that there is an open set $\Theta_0 \subseteq \mathbb{R}$ contained in the support of the prior distribution of θ. Then the following statements are equivalent:*

1. Non-selective publication. *The publication decision $p(x, s)$ is constant in x for each s.*

2. Frequentist unbiasedness. *The expectation $\mathbb{E}[X|\theta, S = s, D = 1]$ is equal to θ for $\theta \in \Theta_0$ and for all s.*

3. Publication probability constant in state. *The publication probability $\mathbb{E}[p(X, S)|\theta, S = s]$ is constant over $\theta \in \Theta_0$ for each s.*

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[19] As discussed above, a partially sophisticated public might also be uncertain about the probability of study arrival q. However, the value of q does not affect whether naive beliefs are distorted. The Bayesian default belief is equal to the prior for some given value of $q \in (0, 1]$ if and only if it is equal to the prior for all values of $q \in (0, 1]$.

4. *Bayesian validity of naive updating. For all distributions F_S on $\mathcal{S}$, the Bayesian default belief $\pi_1^{0,p}$ is equal to the prior π_0.*

Notice that, fixing a distribution F_S of study designs, naive and Bayesian updating are equal as long as the publication probability is constant in the state *unconditional* on S. Part 3 of Proposition 5 imposes a stronger condition, that the publication probability is constant *conditional* on any realization of S. This stronger condition is equivalent to the requirement in part 4 that naive and Bayesian updating are equal not just for a given distribution F_S, but for all possible distributions F_S. Under this condition, a "partially sophisticated" public could confirm the validity of naive updating without knowing the distribution of study designs.

The first main step in proving Proposition 5 is to establish that non-selective publication is implied by the publication probability being constant in the state (part 3). This result follows from the completeness of distributions in the normal location family; see, for instance, Theorem 6.22 in Lehmann and Casella (1998). One formulation of completeness for our setting is that for any fixed standard error $S = s$ and any function $g(x)$ for which $\mathbb{E}[g(X)|\theta, S = s]$ is constant in θ (over an open set), it holds that $g(x)$ is almost everywhere constant. Applying this result to the function $g(x) = p(x, s)$, we see that if the conditional publication probability at state θ is constant in θ, then the publication probability $p(x, s)$ cannot vary with x, establishing non-selectivity.

The next step is to show that the Bayesian validity of naive updating (part 4) implies a constant publication probability in the state, which holds for general signal structures. Finally, we show that unbiasedness of the point estimate X (part 2) also implies a constant publication probability, which is a result more specific to normal signals.

Completeness holds for all exponential families of full rank (Lehmann and Casella, 1998, Theorem 6.22). Thus, constant publication probability in the state is equivalent to non-selective publication for signal distributions $F_{X|\theta,S}$ derived not just from the normal but also, for instance, the Binomial, Poisson, Beta, Dirichlet, Chi-squared, and Gamma distributions.

For signal distributions outside of the exponential families, the result need not hold. For instance, consider a uniform signal distribution in which $X|\theta, S \sim \text{Uniform}[\theta - S, \theta + S]$. Then any publication rule that is periodic with period $2S$, such as $p(X, S) = 0.5 + 0.5 \cdot \cos(\pi X/S)$, will have publication probability $\mathbb{E}[p(X, S)|\theta, S = s]$ constant

over θ even though $p(x, s)$ varies with x. Moreover, even with normal signals, we require an open set of states for the results to hold. If the set of states is $\Theta = \{0, 1\}$, then any publication rule that is symmetric in X about $1/2$ will lead to a constant publication probability across the two states.

Turning to frequentist size control of confidence sets, we get a weaker result than the equivalences of Proposition 5. We show that any publication rule that publishes point estimates outside of an interval – as do the publication rules of Corollary 1, Corollary 3, and (more generally) Corollary 5 – will necessarily fail to control the size of confidence sets. In particular, conditional on publication, there will be some state θ for which the coverage probability of the standard confidence interval $[X - zS, X + zS]$ is lower than the nominal level of $\Phi(z) - \Phi(-z)$.

Proposition 6. *Suppose that $\Theta = \mathbb{R}$, there are normal signals, and that the publication rule is given by $p(x, s) = \mathbf{1}(x \notin I(s))$ for some nondegenerate interval $I(s) \subsetneq \mathbb{R}$. Fix $z > 0$. Then for any $s \in \mathcal{S}$, there exists $\theta' \in \Theta$ such that*

$$P\big(\theta' \in [X - z \cdot s, X + z \cdot s]|\theta = \theta', S = s, D = 1\big) < \Phi(z) - \Phi(-z).$$

The restriction to rules that publish outside of an interval is necessary for this result. In Appendix B.2 we show that for any fixed $z > 0$ there do in fact exist other forms of selective publication rules for which the coverage probability of $[X - zS, X + zS]$ is equal to the nominal level. Note that if one required that the coverage probability of $[X - zS, X + zS]$ be equal to $\Phi(z) - \Phi(-z)$ for all z, rather for a single level of $z > 0$, it would immediately follow that the publication rule must be non-selective.

4.3 Optimal non-selective publication rules

One interpretation of Proposition 7 is that, under our leading example of normal signals, if we wish standard inference to remain valid, then we must restrict ourselves to non-selective publication rules. What is the *optimal non-selective* publication rule – the rule that maximizes the policymaker's utility subject to the constraint of being non-selective?

When the journal is not allowed to screen on the point estimate X, the only remaining option is to screen on the standard error S. In that case, regardless of the

prior or the utility function, the journal should publish studies with smaller standard errors over those with larger standard errors. The result follows from the fact that, conditional on a standard error $S = s$, it holds that $X \sim \mathcal{N}(\theta, s^2)$ is a Blackwell more informative signal of the state θ when s is smaller.

Proposition 7. *Suppose that there are normal signals. Then there exists $\bar{s} \geq 0$ for which the optimal non-selective publication rule publishes a study if and only if $S \leq \bar{s}$.*[20] *The rule is the same under naive and Bayesian updating.*

Extending beyond normal signals, any time study designs S can be ordered by Blackwell informativeness, the optimal non-selective publication rule would be to publish all studies with a sufficiently informative design.

Under a normal prior and quadratic loss utility – and maintaining the assumption of normal signals – we can explicitly solve for the optimal non-selective publication rule. If $\sigma_0^2 \geq c$ (high prior uncertainty, low costs), then a study is published if $S \leq \bar{s}$, with $\bar{s} = \sigma_0 \sqrt{\frac{\sigma_0^2}{c} - 1}$; and if $\sigma_0^2 < c$ (low prior uncertainty, high costs) then no study is published.[21] See Figure 4.
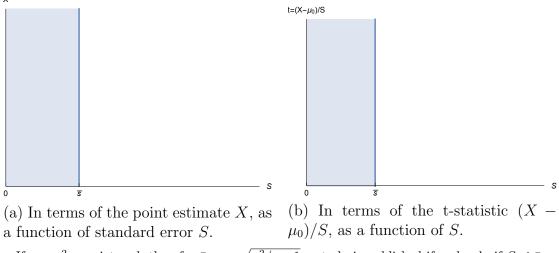
5 A dynamic model

The model of Section 2 takes there to be a single source of information about the state of the world: a study that may be published or not. After the publication decision is made, a policy action is taken, and the game is over. By contrast, if additional studies were to arrive in the future, the public might eventually receive information that would convince it to change its policy. The journal would have to make the decision of whether to publish a study today based on its expectations of what these future studies might reveal.

In order to explore some of these dynamic considerations, this section considers a *two-period model*. As before, there is an unknown policy-relevant state of the world θ, which we take to be persistent over time. The original model of publication and policy choice is the first period of the game. The new second period captures, in reduced

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[20]For $\bar{s} = 0$, no study would be published, not even an arbitrarily precise one.

[21]If a non-selective publication rule is used and no publication is observed, then the default belief will be π_0 (under either updating rule) and so the expected welfare will be $-\text{Var}_{\theta \sim \pi_0}[\theta] = -\sigma_0^2$. The expected welfare of non-selectively publishing conditional on S can be solved for as $\sigma_0^2 \cdot \frac{S^2}{S^2 + \sigma_0^2} - c$. So the optimal non-selective publication rule publishes a study if $\sigma_0^2 \cdot \frac{S^2}{S^2 + \sigma_0^2} - c \geq -\sigma_0^2$.

Figure 4: Optimal non-selective publication region (shaded) for quadratic loss utility, normal prior, normal signals.

(a) In terms of the point estimate X, as a function of standard error S.

(b) In terms of the t-statistic $(X - \mu_0)/S$, as a function of S.

If $c < \sigma_0^2$, as pictured, then for $\bar{s} = \sigma_0 \sqrt{\sigma_0^2/c - 1}$ a study is published if and only if $S \leq \bar{s}$. If instead $c \geq \sigma_0^2$, no studies are published.

form, the impact of future studies: additional exogenous information arrives and the public takes another action.[22] That is, only in the first period is there a publication decision to be made, and it is made before the second period information is realized. If the study is published then it affects the beliefs, and therefore the actions, in both periods.

Set-up of the two-period model. At the start of the game, the common prior over θ is π_0. In the first period, a study is submitted to a journal with probability q. If the study arrives, it has finding and design (X_1, S_1) with $S_1 \sim F_{S_1}$ and $X_1 \sim F_{X_1|\theta,S_1}$. The study is published with probability $p(X_1, S_1)$, and the public's induced belief is π_1. Belief updating to $\pi_1 = \pi_1^{(X_1,S_1)}$ given publication outcome $D = 1$ or to $\pi_1 = \pi_1^0$ given publication outcome $D = 0$ is as before, with the possibility of either naive or Bayesian updating in the absence of a publication. Then the action a_1 is taken, with $a_1 = a^*(\pi_1) \in \arg\max_a \mathbb{E}_{\theta \sim \pi_1}[U(a, \theta)]$.

Next, in the second period, an exogenous signal $X_2 \sim F_{X_2|\theta}$ (independent of (X_1, S_1) given θ) is publicly observed. Beliefs update according to Bayes' Rule from prior π_1 to posterior π_2. (The information structure in period 2, summarized by $F_{X_2|\theta}$,

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[22]Solving a richer dynamic model with a sequence of publication decisions, in which the information revealed at later periods is endogenous to what has been previously published, is beyond the scope of this paper.

is commonly known at the start of the game.) Finally, the action a_2 is taken, with $a_2 = a^*(\pi_2) \in \arg\max_a \mathbb{E}_{\theta \sim \pi_2}[U(a, \theta)]$.

Social welfare is a weighted sum of action payoffs, minus a cost of publication $c > 0$ which is incurred if a study is published:

$$W(D, a_1, a_2, \theta) = \alpha U(a_1, \theta) - Dc + (1 - \alpha)U(a_2, \theta). \tag{13}$$

The parameter $\alpha \in [0, 1)$ describes the first-period payoff weight, relative to a $1 - \alpha$ weight on the second period. The *dynamically optimal* publication rule p maximizes the ex-ante expectation of the above welfare.

In this section, we will restrict attention to quadratic loss utility; we explore binary action utility in Appendix B.3. Moreover, we restrict attention to *normal signals*. As before, normal signals means that the first period signal takes the form $X_1 \sim \mathcal{N}(\theta, S_1^2)$ for $X_1 \in \mathbb{R}$, and $S_1 \in \mathbb{R}_{++}$. It also now means that the second-period signal distribution is normal, with $F_{X_2|\theta}$ equal to $\mathcal{N}(\theta, s_2^2)$ for $s_2 \in \mathbb{R}_{++}$.
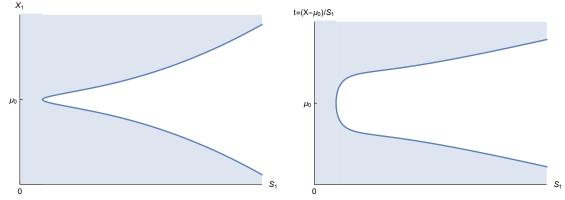
Let us reiterate that, under this model, the standard error of the second-period signal s_2 is a parameter that is known at the start of the game. Our interpretation is that s_2 would be low (i.e., precise) when the journal expects that other high quality studies on the topic in question will soon be performed. The parameter s_2 would be high (i.e., imprecise) when the journal expects future studies on the topic to be performed infrequently, or to be of low quality.

Characterizing the optimal publication rule. We begin our analysis by deriving the dynamically optimal publication rule under normal priors and naive updating. For this special case we can get an explicit formula for the optimum; see an illustration of what this publication rule can look like in Figure 5.

Proposition 8. *In the two-period model with normal priors, normal signals, quadratic loss utility, and naive updating, the dynamically optimal publication rule is to publish a study (X_1, S_1) if and only if the gross interim benefit is greater than or equal to c. The gross interim benefit is given by*

$$\frac{\sigma_0^4(s_2^4 + 2\alpha\sigma_0^2 s_2^2 + \alpha\sigma_0^4)}{(\sigma_0^2 + S_1^2)^2(\sigma_0^2 + s_2^2)^2}(X_1 - \mu_0)^2 + (1 - \alpha)\frac{\sigma_0^8 s_2^4}{(\sigma_0^2 + S_1^2)(\sigma_0^2 + s_2^2)^2(\sigma_0^2 S_1^2 + \sigma_0^2 s_2^2 + S_1^2 s_2^2)}. \tag{14}$$

Figure 5: Dynamically optimal publication region (shaded) for quadratic loss utility, normal prior, normal signals; naive updating.

(a) In terms of the point estimate X_1, as a function of standard error S_1.

(b) In terms of the t-statistic $(X_1 - \mu_0)/S_1$, as a function of S_1.

As pictured, parameters are such that sufficiently precise results are published. Under different parameter values such as a higher cost c, a study with a null result of $X_1 = \mu_0$ would not be published even with a perfectly informative design of $S_1 \simeq 0$.

The dynamic benefit of publication expressed in (14) is a sum of two terms. The first term in (14) scales with $(X_1 - \mu_0)^2$. This term represents a benefit of publishing *extreme* findings, similar to the benefit of publication in the single-period problem. These findings move the public's mean beliefs – and therefore its policy actions – further from the prior.

The second term in (14) expresses a benefit of publication that is new to the two-period model. This term does not depend on X_1 and it is decreasing in the standard error S_1: it gives a benefit of publishing *precise* results, independently of their point estimate. The value comes from the fact that publishing a precise result in period 1 can help reduce mistakes in period 2. (The benefit of publishing extreme findings, given by the first term of (14), is also higher for more precise studies.)

To gain intuition about this benefit of publishing precise results, consider the benefit of publishing a "null result," i.e., a study with a point estimate exactly equal to the prior mean: $X_1 = \mu_0$. Publishing such a study doesn't change the period 1 action payoff because the period 1 action will be $a_1 = \mu_0$ either way. But there is a period 2 benefit. The period 2 posterior mean will be some convex combination of μ_0 (the mean of π_1) and X_2. If the study at period 1 is not published then the belief π_1 will be less precise, leading the period 2 mean to place a lower weight on μ_0 and a higher weight on X_2. Since $X_2 \sim \mathcal{N}(\theta, s_2^2)$ is an imperfect, noisy signal, failing to

publish a null result in period 1 leads to excess *variance* of the second period action.

We next give comparative statics on the benefit of publishing null results.

Corollary 6. *Under the hypotheses of Proposition 8, the gross interim benefit of publishing a result* (X_1, S_1) *with* $X_1 = \mu_0$, *given by*

$$(1 - \alpha)\frac{\sigma_0^8 s_2^4}{(\sigma_0^2 + S_1^2)(\sigma_0^2 + s_2^2)^2(\sigma_0^2 S_1^2 + \sigma_0^2 s_2^2 + S_1^2 s_2^2)},$$

is:

1. *decreasing in* α, *going to 0 as* $\alpha \to 1$;
2. *increasing in* σ_0, *going to 0 as* $\sigma_0 \to 0$;
3. *decreasing in* S_1, *going to 0 as* $S_1 \to \infty$;
4. *nonmonotonic and quasiconcave in* s_2, *approaching 0 as* $s_2 \to 0$ *or* $s_2 \to \infty$.

The comparative static on α is straightforward. The benefit of publishing a null result – which increases payoffs only in the second period – is larger when the relative weight on the second period is larger. To understand the comparative statics on σ_0 and S_1, recall that the benefit arises from reducing second-period mistakes by making π_1 more precise relative to the prior π_0. There is a smaller benefit to increasing the precision of π_1 when the prior uncertainty (as measured by σ_0) is lower. And there is a larger increase in precision, and therefore a bigger benefit of publication, when the first period study is itself more precise (smaller S_1). Indeed, the journal might publish precise null results in the two-period model, but it will still not publish imprecise nulls.

The more subtle part of Corollary 6 is the comparative static on s_2, the informativeness of the second-period signal, in part 4. The benefit of publishing a null result is that it helps prevent the noisy signal X_2 from moving the public's mean belief to an incorrect value. But when the second-period signal is extremely precise ($s_2 \simeq 0$), there is no problem to be solved: the signal X_2 will reveal the state very precisely, and so to the extent that X_2 moves beliefs, it moves them to the truth. And when the second-period signal is extremely imprecise ($s_2 \simeq \infty$), there is also no problem: with high probability, observing X_2 will barely move beliefs. The period 2 studies that may cause mistakes by *moving the public's belief* to an *incorrect* value are those with an intermediate level of precision.

Moving beyond naive updating and normal priors, we do not have an explicit characterization of the dynamically optimal publication rule. But we can generalize

some of the key implications of the two-period model to an arbitrary prior and up-dating rule. First, we establish that there is a positive value of publishing any result that changes the public's belief distribution from the default – even a null result that doesn't move the mean. Second, we show that the value of publishing null results goes to zero when the future information, parametrized by s_2, becomes very precise or very imprecise.

To guarantee this last result, we impose a mild sufficient condition on the distribution π_0. Say that a belief π is *bounded by Pareto tails with finite variance* if there exist $K > 0$, $C > 0$, and $\gamma > 3$ such that for θ outside of the interval $[-K, K]$, π admits a density, and this density is bounded above by $C|\theta|^{-\gamma}$.[23]

Proposition 9. *Consider the two-period model with normal signals and quadratic loss utility. Given some prior π_0 with finite variance, let π_1^0 be the induced default belief either from naive updating, or from Bayesian updating under some publication rule and some $q < 1$. Consider the gross interim benefit of publishing a study $(X_1, S_1) = (x_1, s_1)$ that induces period-1 interim belief of $\pi_1^I = \pi_1^{(x_1, s_1)}$.*

  1. For any fixed s_2, this benefit is strictly positive as long as $\pi_1^I \neq \pi_1^0$.
  2. Suppose further that π_1^0 and π_1^I have the same mean. Then:
    (a) This benefit goes to zero as s_2 goes to 0.
    (b) Under the additional assumption that π_0 is bounded by Pareto tails with finite variance, this benefit goes to zero as s_2 goes to infinity.

Assuming that the study arrival probability is $q < 1$ imposes some regularity on the Bayesian default belief by guaranteeing that it places some weight on π_0. This condition is not invoked for part 1, but is useful in the proof of part 2.

6   Conclusion

In Sections 2-5, we analyzed optimal publication rules supposing that the social value of publication was derived from a policy-relevance objective. Broadly speaking, we argued for the publication of extreme results. Extreme results are more valuable to publish than moderate results because they move public beliefs, and therefore public

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[23]As indicated by the terminology, the Pareto distribution with pdf decaying at a rate of $\theta^{-\gamma}$ has finite variance if and only if $\gamma > 3$ (corresponding to a standard Pareto shape parameter, usually denoted α, strictly greater than 2). Any distribution with compact support, with normal tails, or with exponentially decaying tails is bounded by Pareto tails with finite variance.

policies, further from the defaults in the absence of publication. To conclude the paper, we discuss how our analysis might change if we were to reconsider some of our main assumptions.

One assumption maintained throughout the paper was that the arrival of studies submitted to journals is exogenous. Appendix A.1 considers an extension in which researchers may alter their study designs in response to the publication rule. Specifically, the researcher chooses whether to perform a study, and if so, at what precision. The researcher receives a benefit if the study is published. Her cost of performing the study depends on its precision, e.g., a higher cost for an experiment with a larger sample size. Given the researcher's incentives, we find that the journal optimally adjusts the publication rule in two ways: the journal rejects imprecise studies regardless of their findings, and it becomes more willing to publish studies that are sufficiently precise. This modified publication rule induces the researcher to conduct studies at an increased precision.

In Appendix A.2, we discuss the possibility that study designs may not be perfectly observed – a study may be a less reliable signal of the state than is indicated by its reported standard error. If that is the case, we may need to qualify our claim about publishing extreme results: it would still be optimal to publish results that moved beliefs further, but those results might not be the ones with the most extreme point estimates. Extreme point estimates may be considered to be "implausible," suggesting problems with the study rather than an extreme state.

Finally, Appendix A.3 looks for publication rules that maximize social objectives other than policy-based welfare. When the social objective is to learn the true state of the world, independently of any policy problem, we show that the form of the optimal publication rule may be essentially unchanged from our earlier analysis. However, when the social objective is to publish accurate results that are as close as possible to the truth, the publication rule can flip; the journal now wants to publish moderate rather than extreme results.

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Online Appendix

A   Extensions

A.1   A model with researcher incentives

Thus far, we have taken submissions to the journal to be exogenous. In reality submissions come about from a sequence of decisions by researchers: which topics to work on, what designs S to choose, and which findings X to actually write up and submit. In solving for an optimal journal publication rule, one ought to take into account the researchers' endogenous response to the incentives provided. To illustrate, this section presents a stylized *model with incentives* that explores a publication-motivated researcher's choices of whether to conduct a study and how to design that study.

Our analysis here complements some other recent theoretical investigations of how researcher or experimenter design choices may respond to incentives. In our example, the researcher's type will be commonly known and the design of a (submitted) study will be publicly observable, as in Henry and Ottaviani (2017) or the main analysis of McClellan (2017). Tetenov (2016) and Yoder (2018) study how a principal can screen across heterogeneous experimenters with privately known types. Libgober (2015) considers a setting in which study findings are observable, but the study design that led to a finding may be obscured.

Set-up.   There is a single researcher who takes a research topic as given. There is a common prior $\theta \sim \pi_0$ shared by all parties: the researcher, the journal, and the public.

The timing of the game is as follows. First, the journal publicly commits to a publication rule p for studies on this topic. Then the researcher chooses whether to

conduct a study and, if so, what study design S to use; the researcher will submit the results of any study to a journal. Then the game proceeds as in Section 2. If a study (X, S) is submitted it is published with probability $p(X, S)$, and finally the public updates its belief and takes a policy action. The key distinction from the original model is that the study submission probability q and the distribution of study designs F_S are now endogenous to the publication rule p.

To keep the analysis simple, we will restrict attention to naive updating. We will also focus on a normal signal structure, with $S \in \mathbb{R}_{++}$ and $X|\theta, S \sim \mathcal{N}(\theta, S^2)$. Outside of the normal signal structure, our results would qualitatively hold for any class of signals in which study designs S were ordered by Blackwell informativeness.

The researcher's problem. The researcher observes the publication rule p and then decides whether to conduct a study. If she does conduct a study then she chooses its standard error $S \in (0, \infty)$.

Normalize the researcher's outside option payoff from not conducting a study to 0. If a study is conducted, the researcher values its publication, but pays a cost that depends on the precision of the study. Specifically, the researcher gets a benefit of 1 for getting a study published, independently of the study's results. The researcher pays a cost $\kappa(S)$ for conducting a study with standard error S, with $\kappa : (0, \infty) \to \mathbb{R}_+$. (Assumptions such as $\kappa'(S) < 0$ would be natural – the researcher pays more for an experiment with a larger sample size, say – but we do not actually need to impose any conditions on the cost function for the results that follow.) So the researcher's ultimate payoff if she conducts a study with standard error S and publication outcome D is

$$D - \kappa(S).$$

Denote the researcher's expected payoff from conducting a study with standard error $S = s$, given journal publication rule p, by $V(s, p)$:

$$V(s, p) = \mathbb{E}_{\theta \sim \pi_0, X \sim \mathcal{N}(\theta, s^2)}[p(X, s)] - \kappa(s).$$

The researcher's participation constraint for being willing to conduct a study is

$$\max_{s \in (0, \infty)} V(s, p) \geq 0, \tag{P}$$

where we assume that the maximum is attained. Conditional on conducting a study, the researcher's choice of standard error S is determined by the incentive compatibility condition

$$S \in \arg \max_{s \in (0, \infty)} V(s, p). \tag{IC}$$

As before, we will assume that an argmax exists for any relevant p, without giving explicit conditions on primitives to guarantee that this will be the case.

The journal's problem. Let the journal maximize the expectation of welfare W given by the policy payoff minus any cost of publication:

$$W = U(a, \theta) - Dc.$$

That is, we suppose that the journal does not place any weight on the researcher's utility. Furthermore, assume that the public updates naively, so that the public's default action is fixed at $a^0 = a^*(\pi_0)$.

The journal's objective function takes the same form as in the original model, with the key distinction that the arrival of studies is no longer exogenous to the publication rule p. First, the study submission probability q depends on p: $q = 1$ if the participation constraint (P) is satisfied, and $q = 0$ otherwise. Second, conditional on participation, the standard error S depends on p through the incentive compatibility condition (IC). As is standard, assume that the researcher resolves indifferences in favor of the journal's preferences. The journal's problem is to choose an *incentive-optimal* publication rule p that maximizes expected welfare subject to these endogenous responses.

Observe that, conditional on the arrival of a study, the journal's gross interim benefit of publication is unchanged from its earlier definition in (6). A study that induces a journal interim belief of $\pi_1^{(X,S)}$ when the public's default action is $a^0 = a^*(\pi_0)$ yields gross interim benefit of $\Delta(\pi_1^{(X,S)}, a^*(\pi_0))$.

In the original model with exogenous study submission, the journal's optimal policy was given by the interim-optimal publication rule in which a study is published if and only if $\Delta(\pi_1^{(X,S)}, a^*(\pi_0)) \geq c$. Let us impose the assumption that the researcher would in fact be willing to participate if the journal were to use this interim-optimal publication rule and would submit a study with $S = s^{\text{int}}$. This assumption will simplify both the solution and the exposition of our results.

Assumption 1. *The participation constraint* (P) *is satisfied under the interim-optimal publication rule* $p = p^{I(a^*(\pi_0))}$. *Let* $s^{\text{int}} \in \arg\max_s V\left(s; p^{I(a^*(\pi_0))}\right)$ *be the researcher's choice of study design in response to the interim-optimal publication rule.*

Characterizing the optimal publication rule.

Proposition 10. *Consider the model with incentives under normal signals and naive updating, and suppose that Assumption 1 holds. Then there exist* $\bar{s} \leq s^{\text{int}}$, $\lambda \geq 0$, *and* $\rho \in [0, 1]$ *such that the following rule* p *is incentive-optimal:*

$$p(X, S) = \begin{cases} 1 & \text{if } S = \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) > c - \lambda, \\ & \quad \text{or if } S < \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) \geq c \\ \rho & \text{if } S = \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) = c - \lambda \\ 0 & \text{otherwise} \end{cases}.$$

Given this rule, the researcher chooses to conduct a study with $S = \overline{s}$.

The form of the optimal rule – at least at the chosen study design $S = \overline{s}$ – is very similar to the interim-optimal rule that was used in the model without incentives. A study is published if the gross interim benefit is sufficiently high.

However, the journal distorts publication from the interim-optimal rule in two ways. First, the journal does not publish any studies with standard error $S > \overline{s}$. The researcher is therefore induced to invest additional resources into the precision of studies and to reduce S from s^{int} to $\overline{s}$. Second, at $S = \overline{s}$ the journal relaxes the interim benefit threshold for publication from c to $c - \lambda$ in order to encourage researcher participation. Without that relaxation, a researcher might decide that a study at $S = \overline{s}$ would be too costly to conduct given its low likelihood of being published. (While in equilibrium the researcher never chooses $S < \overline{s}$, the journal has no reason to distort the publication rule at those more precise designs.)

In the original model without incentives, a journal which internalized all costs and benefits of publication would not need commitment power: ex-ante payoffs were maximized by publishing according to what was interim-optimal after receiving a study. Having added researcher incentives, the two distortions now require two forms of journal commitment. The journal commits not to publish imprecise studies, even if such a study was conducted and turned out to have extremely striking results. This commitment is never actually tested on the equilibrium path, though – imprecise studies are not conducted. The journal also commits to publish studies with weak findings when they have the appropriate precision. This second form of commitment is tested, as these studies are submitted (and published) in equilibrium.

One key simplification of this model of incentives is the assumption that there is no heterogeneity across researchers. This fact guarantees that researchers would always choose to conduct a study with a single standard error, known in advance. In a richer model, we would expect publication rules to reward more precise studies with higher publication probabilities in a more continuous manner than what we found here.

A.2 Imperfectly observed study designs

In determining whether to publish a study, a journal cares about the study's true information content. It may not be enough to treat the *reported* standard error as the variable S in our model of normal signals. As previously discussed, one concern is external validity: the parameter being estimated in the study may only be a proxy for the policy parameter of interest. Another concern is that the study may be internally flawed: a study with a misspecified model or an unconvincing identification strategy may report a very small standard error without actually being close to the truth.

When the study design is imperfectly observed, the point estimate can itself be informative as to the study's precision. To be concrete, assume that there are normal priors with mean normalized to 0 and there are normal signals, so that $\theta \sim \mathcal{N}(0, \sigma_0^2)$

and $X \sim \mathcal{N}(\theta, S^2)$. But now assume that the realization of $S \sim F_S$ is *unobserved* by the journal and the public. As noted in Subramanyam (1996), observing a point estimate with a larger magnitude $|X|$ leads to higher beliefs on the unobserved noise S. In our application, a small point estimate would suggest that the study design was precise, while a large point estimate would be suggestive of some hidden noise. The extreme realization might be attributed to a violation of the identifying assumptions, to a coding error, or to some other unseen flaw.

Continuing the example with X but not S observed by the journal and public, and with $\theta \sim \mathcal{N}(0, \sigma_0^2)$ and $X \sim \mathcal{N}(\theta, S^2)$, suppose further that there is quadratic loss utility. The journal makes a publication decision based on the posterior mean of θ, now conditional on X but not S:

$$\mu_1^{(X)} = \mathbb{E}[\theta|X] = \mathbb{E}[\mathbb{E}[\theta|X, S]|X] = \mathbb{E}\left[\frac{\sigma_0^2}{S^2 + \sigma_0^2}\Big|X\right] \cdot X.$$

The journal wants to publish if the interim benefit $(\mu_1^{(X)} - \mu_1^0)^2$ exceeds the publication cost c. A higher belief on S due to a larger point estimate $|X|$ translates into a lower weight $\mathbb{E}\left[\frac{\sigma_0^2}{S^2 + \sigma_0^2}\Big|X\right]$ on the point estimate. Indeed, when the prior on S is sufficiently dispersed, $\mathbb{E}\left[\frac{\sigma_0^2}{S^2 + \sigma_0^2}\Big|X\right]$ can decrease fast enough that $\mathbb{E}[\theta|X]$ is nonmonotonic and falls to 0 as X goes to infinity. (In addition to Subramanyam (1996), see discussion of this issue in Dawid (1973), O'Hagan (1979), and Harbaugh et al. (2016).) An intermediate point estimate would therefore move an observer's mean belief more than a very large, "implausible," point estimate would. Let us restate that our results in Section 3 support publishing "extreme results" in the sense of results that *lead to extreme beliefs*. If extreme signal realizations are written off as implausible, then they would not lead to extreme beliefs and thus should not be published.

A related possibility is that the study design S, capturing the true informational content of the study's findings, is better observed by the journal than by the public. After all, the journal editor and referees are experts who are charged with carefully evaluating the quality of a paper; a policymaker reading the study might not have this expertise. Consider a model where the journal observes (X, S) when making a publication decision, while if a paper is published the public sees only X. In such a model, the public can make an inference on the quality of the study design from the fact that the study was published. Publication implies that the journal had chosen to certify the study as clearing the bar of peer review. Suppose additionally that even unpublished studies are publicly available as working papers or preprints. In this case the only role of "publication" by a journal is certification or signaling value. A formal analysis of optimal publication rules in such an environment is an interesting topic for future research.

A.3 Alternative objectives

A.3.1 A Learning Objective

Separate from any decision problem, the public might value more precise knowledge of the state of the world out of purely academic interest. One natural way of measuring the precision of beliefs is by looking at the variance. We formalize a *learning* objective by supposing that the public seeks a publication rule that minimizes the expected variance of the posterior beliefs π_1. Formally, under the learning objective we replace the earlier relevance welfare function $W(D, a, \theta)$ from (1) with

$$W(D, \pi_1) = -\operatorname{Var}_{\theta \sim \pi_1}[\theta] - Dc, \tag{15}$$

where $c > 0$ continues to represent the social opportunity cost of publication. The *learning-optimal* publication rule p is the one which maximizes the ex-ante expectation of (15).

When considering learning, we restrict to Bayesian updating by the public. Therefore the subjective distribution of beliefs is consistent with the distribution under the true model, and the public's subjective variance is indeed accurate given its information.

There is a clear connection between learning and relevance under Bayesian updating. The posterior expectation of the quadratic loss relevance utility, which is correct in expectation, is minus the posterior variance. That corresponds to the learning welfare. So the learning-optimal policy is exactly the policy that maximizes the quadratic loss relevance objective under Bayesian updating, as in Section 3.2, regardless of assumptions about signals or priors. In order to maximize learning and minimize uncertainty over the state of the world, then, it remains optimal to publish only those studies which induce extreme posteriors. This gives an alternative interpretation of the previous results that were motivated by decision problems.

A.3.2 An Accuracy Objective

When there are normal signals, we naturally interpret the finding X as a point estimate of the state θ. Under an *accuracy* objective, a journal seeks to publish point estimates that are as close as possible to the true state of the world. These estimates can be thought of as the ones that would be the most "replicable" by future studies. Letting $\Theta = \mathcal{X} = \mathbb{R}$, we formalize our accuracy objective by replacing welfare from (1) with

$$W(D, \theta, X) = D \cdot (-(X - \theta)^2 + b), \tag{16}$$

where $b > 0$ indicates the shadow benefit of publication; if no study arrives, welfare is normalized to zero. For simplicity, we assume a quadratic loss from publishing values of X further from θ. (We consider a generalized loss function below.)

If the goal is to publish accurate or replicable results, a non-selective rule will do better than publishing only extreme values – but a different selective rule can do even better. Let the *accuracy-optimal* publication rule be the one maximizing the ex-ante expectation of this welfare function.

Under the accuracy objective, publication depends only on the belief $\pi_1^{(X,S)}$. The accuracy-optimal rule publishes a study $(X, S) = (x, s)$ if the interim expected welfare from (16) is greater than 0, i.e., if

$$\mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[(x - \theta)^2] \leq b. \tag{17}$$

We can explicitly solve for this rule when there are normal signals and normal priors: publish if $(X - \mu_0)^2 \leq \left(1 + \frac{\sigma_0^2}{S^2}\right)\left(b + b\frac{\sigma_0^2}{S^2} - \sigma_0^2\right)$.[24] At any standard error S, it is accuracy-optimal to publish studies with the point estimate X in a symmetric interval about μ_0; see Figure 6. (At sufficiently high standard errors, it may be the case that no studies are published.)

In other words, the accuracy-optimal publication rule has the opposite form as the publication rule maximizing quadratic loss relevance: at a given standard error, it publishes moderate findings and does not publish extreme ones. By the same token, publishing only extreme findings at a given standard error would minimize accuracy. This is because point estimates closer to the prior mean are thought (under the interim belief) to be closer to the true state. For intuition, recall that the distance of the point estimate from the interim mean, $X - \mu_1^{(X,S)}$, is linear in the distance of the point estimate from the prior mean, $X - \mu_0$. Of course, the accuracy-optimal publication rule is still partially aligned with the earlier (relevance-)optimal rules in that it publishes a larger range of point estimates when standard errors are smaller.
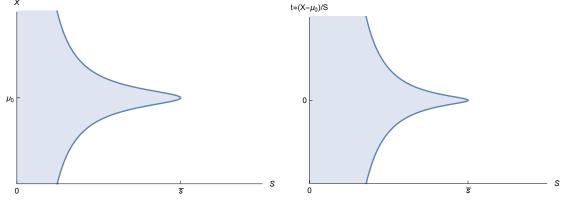
Just as the relevance-optimal rule is bad for accuracy, so too is the accuracy-optimal rule bad for relevance. For a fixed standard error and for a fixed share of studies to be published, the rule of publishing only moderate point estimates would actually minimize quadratic loss utility – and would therefore also be the worst for the learning objective.[25] A non-selective publication rule would be intermediate on both quadratic loss relevance and on accuracy.

Without giving an explicit characterization, the same qualitative result of publishing moderate results to maximize accuracy would hold if we were to generalize the accuracy objective (16) beyond a quadratic cost of distance. Consider a generalized

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[24]As a first step, one can rewrite (17) as $\mathrm{Var}_{\theta \sim \pi_1^{(x,s)}}[\theta] + (x - \mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[\theta])^2 \leq b$. We then plug in the variance and expectation from (4) to derive the publication rule above.

[25]As described in Section 3.1, we solved for the rule that maximized quadratic loss utility for Bayesian updating by first showing that the problem was equivalent to $\max_p \max_{a^0} EW(p, a^0)$; rearranging the order of maximization let us conclude that the globally optimal p was also interim-optimal given a^0. To solve for the policy that minimizes quadratic loss utility (at a fixed and commonly known standard error), one solves $\min_p \max_{a^0} EW(p, a^0)$. By a minimax theorem, one can rearrange the order of minimization and maximization and conclude that the globally pessimal p is also interim-pessimal given a^0, and the interim-pessimal policy is to publish moderate results.

Figure 6: Accuracy-optimal publication region (shaded) for quadratic distance, normal prior, normal signals.

(a) In terms of the point estimate X, as a function of standard error S.

(b) In terms of the t-statistic $(X - \mu_0)/S$, as a function of S.

If $b < \sigma_0^2$, as pictured, then no studies are published for $S > \bar{s}$, with $\bar{s} = \frac{\sigma_0 \sqrt{b}}{\sqrt{\sigma_0^2 - b}}$. If instead $b \geq \sigma_0^2$, then an interval of X containing $[\mu_0 - (b - \sigma_0^2), \mu_0 + (b - \sigma_0^2)]$ would be published for any S.

accuracy objective of

$$W(D, \theta, X) = D \cdot (-\delta((X - \theta)^2) + b), \tag{16'}$$

for a strictly increasing function $\delta(\cdot)$. (An arbitrary increasing function of $(X - \theta)^2$ is equivalent to an arbitrary increasing function of $|X - \theta|$.) One can establish that under normal signals and normal priors, the generalized accuracy-optimal policy maximizing $(16')$ takes the same form as with a quadratic cost: at a given standard error, either point estimates in a symmetric interval around μ_0 are published, or no point estimates are published. See Appendix B.4 below.

B Additional Results

B.1 Optimal publication without supermodularity and FOSD

Proposition 4 argues that extreme results should be the ones to be published, assuming that the utility function is supermodular. Extreme results are defined in relation to the FOSD ordering of the induced beliefs. In this subsection we demonstrate the importance of the two conditions of supermodularity and FOSD ordering. First, we provide a simple example to show that when the utility function does not satisfy supermodularity, the publication region can consist of beliefs inside rather than outside of an interval. Second, we illustrate that even under a supermodular utility function,

it is not necessarily the case that beliefs inducing more extreme actions are more valuable to publish when the beliefs are not ordered by FOSD.

B.1.1 Without supermodularity

Let $\mathcal{A} = \{0,1\}$ and $\Theta = \mathbb{R}$. Let the utility function be given by $U(0,\theta) = 0$ and $U(1,\theta) = k - \theta^2$ for some constant $k > 0$.[26] This utility function is not supermodular, as action 0 is optimal at (known) states θ for which $\theta \leq -\sqrt{k}$ or $\theta \geq \sqrt{k}$ while action 1 is optimal at states θ in $[-\sqrt{k}, \sqrt{k}]$.

Assume that there is a normal prior and normal signals, so that the interim belief after observing a signal X is given by $\pi_1^{(X,S)} = \mathcal{N}(\mu_1, \sigma_1^2)$, with

$$\mu_1 = \frac{\sigma_0^2}{S^2 + \sigma_0^2} X + \frac{S^2}{S^2 + \sigma_0^2} \mu_0$$
$$\sigma_1^2 = \frac{S^2 \sigma_0^2}{S^2 + \sigma_0^2}.$$

At any given standard error $S = s$, these interim beliefs are FOSD ordered in X. The expected utility of action $a = 1$ after observing study (X, S) is therefore given by $\mathbb{E}_{\theta \sim \pi_1^{(X,S)}}[k - \theta^2] = k - \mu_1^2 - \sigma_1^2$. Action $a = 1$ would be taken if $\mu_1^2 + \sigma_1^2 < k$, and action $a = 0$ would be taken if $\mu_1^2 + \sigma_1^2 > k$.

So, if the default action is $a^0 = 0$, take some $S = s$ such that $\sigma_1^2 = \frac{s^2 \sigma_0^2}{s^2 + \sigma_0^2} < k$. Then the gross interim benefit of publishing a study (X, s) will be

$$\Delta(\pi_1^{(X,s)}, 0) = \max\left\{ k - \left(\frac{\sigma_0^2}{s^2 + \sigma_0^2} X + \frac{s^2}{s^2 + \sigma_0^2} \mu_0 \right)^2 - \frac{s^2 \sigma_0^2}{s^2 + \sigma_0^2}, 0 \right\}.$$

For a low enough cost c that the journal would publish some result at $S = s$, the interim optimal publication rule would be to publish only those values of X inside of a bounded interval. If the absolute value of the default mean is larger than $\sqrt{k}$, then the default action is in fact guaranteed to be $a^0 = 0$. Thus, in this case the optimal publication rule also publishes findings inside of an interval.

B.1.2 Supermodularity without FOSD

One might conjecture that as long as the utility function satisfies supermodularity, then the conclusion of Proposition 4 applies not just to beliefs π', π'', π''' ordered by FOSD, but to beliefs that are ordered by the induced actions $a^*(\pi)$. The following provides a counterexample.

Let the state space and action space both be given by $\mathcal{A} = \Theta = \{0, 1, 2\}$. Define

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[26]Note that while the action space here is binary, this utility function differs from the "binary action" utility. That utility function was without loss of generality on the binary action space only until one adds assumptions on priors and signal distributions.

the utility function U by

$$U(a, \theta) = \begin{array}{c} \\ a \end{array} \begin{array}{c|ccc} & \multicolumn{3}{c}{\theta} \\ & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 0 \\ 1 & -10 & 10 & 11 \\ 2 & -15 & 6 & 20 \end{array}$$

This utility function is supermodular. Higher beliefs lead to higher actions, and if the state is known to be θ with certainty, then the corresponding optimal action is $a = \theta$.

Next, let $a^0 = 0$; let π' be state $\theta = 0$ with certainty; let π'' be state $\theta = 1$ with certainty; and let π''' be a 50% probability of $\theta = 0$ and a 50% probability of $\theta = 2$. It holds that $a^*(\pi') = 0$, $a^*(\pi'') = 1$, and $a^*(\pi''') = 2$, but these beliefs are not ordered by FOSD. The gross interim benefits of publishing studies leading to these beliefs are given by $\Delta(\pi', 0) = 0$, $\Delta(\pi'', 0) = 10$, and $\Delta(\pi''', 0) = 2.5$. Hence, it is most valuable to publish the study leading to the belief π'' that induces the intermediate action.

B.2 Size control for selective publication rules

Let there be normal signals, and fix $z > 0$. In this subsection we show how to construct selective publication rules for which the coverage probability of the confidence interval $[X - zS, X + zS]$ is equal to $\Phi[z] - \Phi[-z]$ for all θ. This exercise demonstrates that while non-selectivity is sufficient for confidence intervals (at a fixed standard error radius about the point estimate) to control size, it is not necessary.

Case of S=1: Normalizing $S = 1$, let the distribution of the finding X be given by $X \sim \mathcal{N}(\theta, 1)$ and the publication probability be given by $p(X)$. Then the coverage probability of a confidence interval of the form $[X - z, X + z]$ is given by

$$P(\theta \in [X - z, X + z]) = \frac{\int p(\theta + \epsilon) \mathbf{1}(\epsilon \in [-z, z]) \varphi(\epsilon) d\epsilon}{\int p(\theta + \epsilon) \varphi(\epsilon) d\epsilon}.$$

This coverage probability is equal to its nominal level, $\Phi(z) - \Phi(-z)$, for all θ, if and only if

$$\int p(\theta + \epsilon) \left[\mathbf{1}(\epsilon \in [-z, z]) - (\Phi(z) - \Phi(-z))\right] \varphi(\epsilon) d\epsilon = 0 \text{ for all } \theta.$$

Taking the Fourier transform $\mathscr{F}$ of this expression, and recalling that the Fourier transform maps convolutions into products, the above expression is equivalent to the condition

$$\mathscr{F}(p(\cdot)) \cdot \mathscr{F}\left([\mathbf{1}(\cdot \in [-z, z]) - (\Phi(z) - \Phi(-z))] \varphi(\cdot)\right) \equiv 0.$$

If the coverage probability is equal to its nominal level, we thus get that $\mathscr{F}(p(\cdot))$ has to equal zero everywhere except possibly at points where $\mathscr{F}([\mathbf{1}(\cdot \in [-z, z]) - (\Phi(z) - \Phi(-z))]\varphi(\cdot)) = 0$. Reversely, by the Fourier inversion theorem,[27] this condition is also sufficient for the coverage probability to be equal to its nominal level.

The Fourier transform $\mathscr{F}([\mathbf{1}(\cdot \in [-z, z]) - (\Phi(z) - \Phi(-z))]\varphi(\cdot))$ is real-valued, even, and continuous. Let t^* be any zero of this Fourier transform. Then for any publication rule of the form $p(x) = r_0 + r_1 \cdot \sin(t^* \cdot x) + r_2 \cdot \cos(t^* \cdot x)$ we get that nominal size control is satisfied. (Of course, one must ensure that the publication probability is bounded between 0 and 1.) We can also take linear combinations of these functions over different roots t^*. These are the only publication rules with nominal size control.

While we cannot obtain analytic solutions, at any z we can numerically solve for such roots. For instance, for $z = 1.96$, solutions include $t^* \simeq 2.11045, 3.49544$, etc. So under either of the publication rules $p(x) = .5 + .5\cos(2.11045x)$ or $p(x) = .5 + .5\cos(3.49544x)$, for example, the probability of $\theta \in [X - 1.96, X + 1.96]$ conditional on publication would be 95% at all θ.

General case: Fixing z, suppose that $p(x)$ is some publication rule that satisfies nominal coverage for $S = 1$. Then $p(x, s) = p(x/s)$ achieves nominal coverage for $S = s$.

B.3   Two-period model with binary actions

Consider the two-period model with normal priors, normal signals, and naive updating. Recall that we assumed that social welfare is given by

$$\alpha U(a_1, \theta) - Dc + (1 - \alpha)U(a_2, \theta),$$

where $\alpha U(a_1, \theta)$ is the $t = 1$ payoff of action a_1 (taken after X_1 might or might not have been published), and $(1 - \alpha)U(a_2, \theta)$ is the $t = 2$ payoff of action a_2 (taken after the observation of X_2).

Proposition 8 in Section 5 presented the gross interim benefit of publication – and therefore the optimal publication rule – for that setting under quadratic loss utility. Here, we will illustrate how some conclusions can change under binary action utility. We focus on characterizing how the interim benefit of publication varies as a function of the point estimate of the first-period study, X_1.

First, recall the quadratic loss analysis. With quadratic loss utility, the benefit of publishing towards the $t = 1$ payoff – that is, the expected increase in $\alpha U(a_1, \theta)$ – is quadratic in $(X_1 - \mu_0)$, giving a symmetric benefit of publishing more extreme results in either direction. The benefit of publishing towards the $t = 2$ payoff – the expected increase in $(1 - \alpha)U(a_2, \theta)$ – has one term that is quadratic in $(X_1 - \mu_0)$ and another

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[27]https://en.wikipedia.org/wiki/Fourier_inversion_theorem

term that is positive and constant in X_1. There is a benefit of publishing any result, including a null result with $X_1 = \mu_0$, and an additional benefit of publishing more extreme results. These disaggregated benefits are illustrated in panel (a) of Figure 7.

Now consider the model with binary action utility. The public's optimal action is $a = 0$ when its posterior mean is negative and $a = 1$ when its posterior mean is positive. Assume that $\mu_0 < 0$, and recall that we consider the case of naive updating, so the default action at $t = 1$ under nonpublication is $a = 0$. In that case the benefit towards the $t = 1$ payoff is $\alpha\mu_1^{(X_1,S_1)}$ if $\mu_1^{(X_1,S_1)} > 0$ and is 0 otherwise.[28] Since $\mu_1^{(X_1,S_1)}$ increases linearly with X_1, the benefit is zero at every X_1 from minus infinity through some positive number, and it increases linearly for larger X_1. See the blue curve in panel (b) of Figure 7.

Conditional on (X_1, S_1) and on X_2, the realized benefit of publication towards the $t = 2$ payoff is $(1-\alpha)|\mu_2^{(X_1,S_1),(X_2)}|$ if $\mu_2^{(X_1,S_1),(X_2)}$ and $\mu_2^{0,(X_2)}$ are of different signs, and is zero otherwise. The publication decision is made at $t = 1$, and so the benefit is evaluated by taking expectation over X_2 (under the $t = 1$ interim beliefs $\pi_1^{(X_1,S_1)}$). See the orange curve in panel (b) of Figure 7 for an illustration of this expected $t = 2$ benefit. As we see, this $t = 2$ benefit is somewhat subtle.

The first thing to note is that the expected $t = 2$ payoff is strictly positive everywhere except $X_1 = 0$. The $t = 2$ benefit of publishing a result with $X_1 = 0$ is zero (as is the $t = 1$ benefit) because a study reporting $X_1 = 0$ never changes the period 2 action. The action depends on the sign of the mean, and a study with $X_1 = 0$ moves the posterior mean closer to zero without changing the sign.
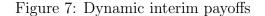
Moving away from $X_1 = 0$, there is a positive $t = 2$ benefit of publishing a result X_1 with an intermediate positive or negative value. Publishing a positive finding avoids the public's mistake of taking the action $a = 0$, in accord with its priors, when the unpublished period-1 study would actually indicate that the state is positive. Publishing a negative finding avoids the public's mistake of taking $a = 1$ after a positive finding in the second period, when the period-1 study would have indicated a negative state. Figure 7 shows that these costs are asymmetric (a conclusion we see in other numerical examples): there is a larger cost of failing to publish a study with a positive result, one that goes against the public's prior.
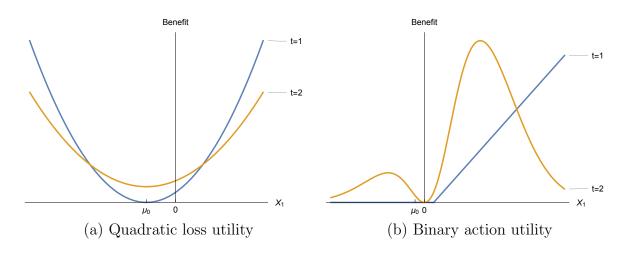
Finally, as X_1 gets more extreme in either direction, the $t = 2$ payoff benefit approaches zero. This is because "$t = 2$" is defined as the time after some additional information has arrived.[29] And an extreme X_1 is suggestive of an extreme state, meaning that the period 2 signal is very likely to reveal whether the state is positive or negative. For instance, if X_1 has a very large positive value, then we expect X_2

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[28] We follow the notational convention of the proof of Proposition 8 here, in which $\mu_1^{(X_1,S_1)}$ is the period 1 mean belief conditional on observing the period 1 study; $\mu_2^{(X_1,S_1),(X_2)}$ is the period 2 mean conditional on observing both studies; and $\mu_2^{0,(X_2)}$ is the period 2 mean after observing the second study if the first was not published.

[29] If there is a longer expected wait before new studies arrive and actions are updated, that corresponds in our model to a larger weight α on the $t = 1$ payoff.

to have a very large positive value as well. So publishing this study would give a $t = 1$ benefit by moving the first period action from $a_1 = 0$ to $a_1 = 1$. But the public will take $a_2 = 1$ in the second period regardless of whether the first period study is published.

Figure 7: Dynamic interim payoffs

(a) Quadratic loss utility   (b) Binary action utility

For both examples, we set $S_1 = 2$, $\sigma_0 = 2$, $\mu_0 = -1$, and $s_2 = 2$. The relative weight coefficient on the first period, α, is chosen to make the curves of similar scale as graphed; increasing α scales up the $t = 1$ benefit relative to that at $t = 2$. For quadratic loss utility, we have chosen $\alpha = .3$, with X_1 ranging from -5 to 3. For binary action utility, we have chosen $\alpha = .05$, with X_1 ranging from -10 to 15.

B.4   Generalized accuracy objectives

The generalized accuracy objective was defined in Appendix A.3.2 as

$$W(D, \theta, X) = D \cdot (-\delta((X - \theta)^2) + b) \tag{16$'$}$$

for δ some increasing function. To maximize this objective under normal priors and normal signals, it is optimal to publish studies with moderate results, i.e., ones with point estimates X inside of an interval centered at the prior mean μ_0:

Proposition 11. *Let there be normal priors and normal signals. The publication rule maximizing the generalized accuracy objective* (16$'$) *takes the following form: at $S = s$, either no studies (X, s) are published, or there exists k such that a study (X, s) is published if and only if $(X - \mu_0)^2 \leq k$.*

C Proofs

Proof of Lemma 1. Follows from arguments in the text. ☐

Proof of Lemma 2. Follows from arguments in the text. ☐

Proof of Proposition 1. Follows from arguments in the text. ☐

Proof of Proposition 2. By Proposition 1 part 2a, it suffices to show that $a = a^*(\pi_1^{0,p^{I(a)}})$ is uniquely solved by $a = \mu_0$ – in other words, that $a^0 = \mu_0$ is the unique fixed point when we map default actions to interim optimal publication rules, and then map publication rules back to default actions.

Conditional on a study arriving when the default action is a^0, the journal will not publish a study if $\mu(X, S)$ lies in the interval $(a^0 - \sqrt{c}, a^0 + \sqrt{c})$ (see Equation 9). Let $\bar{\mu}(a^0)$ indicate $\mathbb{E}[\theta | \mu_1^{(X,S)} \in (a^0 - \sqrt{c}, a^0 + \sqrt{c})]$, the expected state conditional on a study arriving and not being published. If this expectation is undefined due to the event $\mu_1^{(X,S)} \in (a^0 - \sqrt{c}, a^0 + \sqrt{c})$ occurring with zero probability, let $\bar{\mu}(a^0) = \mu_0$.

The mean of the default belief – and therefore the implied default action – conditional on nonpublication will be a convex combination of $\bar{\mu}(a^0)$ (with weight q) and μ_0 (weight $1 - q$). Therefore, to show that $a^0 = \mu_0$ is the unique fixed point, it is sufficient to show the following three items: (i) for $a^0 = \mu_0$, it holds that $\bar{\mu}(a^0) = a^0$; (ii) for any $a^0 < \mu_0$, it holds that $\bar{\mu}(a^0) > a^0$; and (iii) for any $a^0 > \mu_0$, it holds that $\bar{\mu}(a^0) < a^0$. (If we had assumed $q < 1$ then it would be sufficient to show (ii) and (iii) with weak inequalities.)

Item (i) follows from the fact that $\mu_1^{(X,S)}$ is symmetric about μ_0, and therefore it remains symmetric when this random variable is truncated outside of the interval $(\mu_0 - \sqrt{c}, \mu_0 + \sqrt{c})$. The proofs of items (ii) and (iii) will be identical to each other, up to the direction of inequalities, so let us focus on proving (ii). Fix $a^0 < \mu_0$. First, if there is a zero probability that $\mu_1^{(X,S)} \in (a^0 - \sqrt{c}, a^0 + \sqrt{c})$, then $\bar{\mu}(a^0) = \mu_0 > a^0$ and we are done. Otherwise, notice that symmetry about μ_0 combined with single-peakedness means that the pdf of $\mu_1^{(X,S)}$ is larger at $a^0 + k$ than at $a^0 - k$ for any $k > 0$, with the inequality being strict for any ϵ such that either pdf value is nonzero. Hence the mean of $\mu_1^{(X,S)}$ conditional on being in the interval $(a^0 - \sqrt{c}, a^0 + \sqrt{c})$ is strictly above the midpoint a^0. That completes the proof of item (ii). ☐

Proof of Corollary 1. As discussed in the text, this prior and signal structure satisfy the hypotheses of Proposition 2, and so the optimal rule is to publish if $|\mu_1^{(X,S)} - \mu_0| \geq \sqrt{c}$. By the normal updating formula (4), $\mu_1^{(X,S)} = \frac{\sigma_0^2}{S^2+\sigma_0^2} X + \frac{S^2}{S^2+\sigma_0^2}\mu_0$, and so

$$|\mu_1^{(X,S)} - \mu_0| = \frac{\sigma_0^2}{S^2+\sigma_0^2}|X - \mu_0| = \left(1 + \frac{S^2}{\sigma_0^2}\right)^{-1} |X - \mu_0|.$$ ☐

Proof of Corollary 2. The only comparative static that is not immediate is that for the t-statistic cutoff, $\left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)\sqrt{c}$, with respect to S. Taking straightforward limits confirms that the cutoff goes to infinity as $S \to 0$ and $S \to \infty$. The derivative of the

cutoff with respect to S is $\left(-\frac{1}{S^2} + \frac{1}{\sigma_0^2}\right)\sqrt{c}$, and the second derivative is $\frac{2\sqrt{c}}{S^3}$. Since the second derivative is positive, the cutoff is convex over $S \in \mathbb{R}_{++}$ and is minimized at the point where the first derivative is 0, which is $S = \sigma_0^2$. $\qquad\square$

Proof of Proposition 3. By Proposition 1 part 2b, it suffices to show that the payoff under default action $a^0 = 0$ is higher than under default action $a^0 = 1$, i.e., that $EW\left(p^{I(0)}, 0\right) \geq EW\left(p^{I(1)}, 1\right)$. The interim optimal publication rule $p^{I(a^0)}$ is given by (10): for $a^0 = 0$, publish if $\mu_1^{(X,S)} \geq c$, and for $a^0 = 1$, publish if $\mu_1^{(X,S)} \leq -c$. Expanding out $EW(p, a^0)$ from (5) for each possible value of a^0,

$$EW(p^{I(0)}, 0) = q\mathbb{E}\left[\begin{cases} \mu_1^{(X,S)} - c & \text{if } \mu_1^{(X,S)} \geq c \\ 0 & \text{if } \mu_1^{(X,S)} < c \end{cases}\right]$$

$$EW(p^{I(1)}, 1) = q\mathbb{E}\left[\begin{cases} \mu_1^{(X,S)} & \text{if } \mu_1^{(X,S)} > -c \\ -c & \text{if } \mu_1^{(X,S)} \leq -c \end{cases}\right] + (1-q)\mu_0.$$

Taking the difference,

$$EW(p^{I(0)}, 0) - EW(p^{I(1)}, 1) = q\mathbb{E}\left[\begin{cases} -c & \text{if } \mu_1^{(X,S)} \geq c \\ -\mu_1^{(X,S)} & \text{if } \mu_1^{(X,S)} \in (-c, c) \\ c & \text{if } \mu_1^{(X,S)} \leq -c \end{cases}\right] - (1-q)\mu_0. \tag{18}$$

We seek to show that this difference is nonnegative. Since $\mu_0 \leq 0$ by assumption, it is sufficient to show that the expectation term is nonnegative.

To show that the expectation term is nonnegative, first define a weakly increasing function $l : \mathbb{R} \to \mathbb{R}_+$ as follows:

$$l(k) = \begin{cases} 0 & \text{if } k \leq 0 \\ k & \text{if } k \in (0, c) \\ c & \text{if } k > c \end{cases}.$$

The expectation term in (18) can be rewritten as $\mathbb{E}[l(-\mu_1^{(X,S)})] - \mathbb{E}[l(\mu_1^{(X,S)})]$, and so it is sufficient to show that this difference is nonnegative.

Next, observe that the distribution of $-\mu_1^{(X,S)}$ first order stochastically dominates that of $\mu_1^{(X,S)}$:

$$P(-\mu_1^{(X,S)} \leq k) = 1 - P(\mu_1^{(X,S)} \leq -k)$$
$$\leq 1 - P(\mu_1^{(X,S)} \geq k)$$
$$= P(\mu_1^{(X,S)} \leq k),$$

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where the inequality comes from the assumption of $P(\mu_1^{(X,S)} \leq -k) \geq P(\mu_1^{(X,S)} \geq k)$. By FOSD, then, the expectation of $l(-\mu_1^{(X,S)})$ is weakly larger than the expectation of $l(\mu_1^{(X,S)})$, completing the proof. $\square$

Proof of Corollary 3. As discussed in the text, this prior and signal structure satisfy the hypotheses of Proposition 3, and so the optimal rule is to publish if $\mu_1^{(X,S)} \geq c$. By the normal updating formula (4), $\mu_1^{(X,S)} = \frac{\sigma_0^2}{\S^2 + \sigma_0^2} X + \frac{S^2}{S^2 + \sigma_0^2} \mu_0$. Rearranging, we see that $\frac{\sigma_0^2}{\S^2 + \sigma_0^2} X + \frac{S^2}{S^2 + \sigma_0^2} \mu_0 \geq c$ if and only if $X \geq \left(1 + \frac{S^2}{\sigma_0^2}\right) c - \frac{S^2}{\sigma_0^2} \mu_0$. $\square$

Proof of Corollary 4. The only comparative static that is not immediate is that for the t-statistic cutoff, $\left(\frac{1}{S} + \frac{S}{\sigma_0^2}\right)(c - \mu_0)$, with respect to S. The argument for this result follows identically as the argument for the analogous result in the proof of Corollary 2. $\square$

Proof of Proposition 4. Let $a' = a^*(\pi')$, $a'' = a^*(\pi'')$, and $a''' = a^*(\pi''')$. Moreover, recall that for any actions $\underline{a} \leq \bar{a}$ and any distributions $\underline{\pi} \leq_{FOSD} \bar{\pi}$, supermodularity implies that

$$\mathbb{E}_{\theta \sim \bar{\pi}}[U(\underline{a}, \theta)] + \mathbb{E}_{\theta \sim \underline{\pi}}[U(\bar{a}, \theta)] \leq \mathbb{E}_{\theta \sim \underline{\pi}}[U(\underline{a}, \theta)] + \mathbb{E}_{\theta \sim \bar{\pi}}[U(\bar{a}, \theta)]. \quad (19)$$

Now consider the two exhaustive cases of $a^0 \leq a''$ and $a^0 \geq a''$.
    If $a^0 \leq a''$, then

$$\mathbb{E}_{\theta \sim \pi'''}[U(a^0, \theta)] + \mathbb{E}_{\theta \sim \pi''}[U(a'', \theta)] \leq \mathbb{E}_{\theta \sim \pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta \sim \pi'''}[U(a'', \theta)]$$
$$\leq \mathbb{E}_{\theta \sim \pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta \sim \pi'''}[U(a''', \theta)]$$
$$\Rightarrow \mathbb{E}_{\theta \sim \pi''}[U(a'', \theta)] - \mathbb{E}_{\theta \sim \pi''}[U(a^0, \theta)] \leq \mathbb{E}_{\theta \sim \pi'''}[U(a''', \theta)] - \mathbb{E}_{\theta \sim \pi'''}[U(a^0, \theta)]$$
$$\Rightarrow \Delta(\pi'', a^0) \leq \Delta(\pi''', a^0),$$

where, on the first line, the first inequality follows from (19) and the second inequality follows from the fact that $a''' = a^*(\pi''')$. The second line then rearranges terms from the left-hand side and the right-hand side of the first line.
    Alternatively, if $a^0 \geq a''$, then by a similar argument

$$\mathbb{E}_{\theta \sim \pi'}[U(a^0, \theta)] + \mathbb{E}_{\theta \sim \pi''}[U(a'', \theta)] \leq \mathbb{E}_{\theta \sim \pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta \sim \pi'}[U(a'', \theta)]$$
$$\leq \mathbb{E}_{\theta \sim \pi''}[U(a^0, \theta)] + \mathbb{E}_{\theta \sim \pi'}[U(a', \theta)]$$
$$\Rightarrow \mathbb{E}_{\theta \sim \pi''}[U(a'', \theta)] - \mathbb{E}_{\theta \sim \pi''}[U(a^0, \theta)] \leq \mathbb{E}_{\theta \sim \pi'}[U(a', \theta)] - \mathbb{E}_{\theta \sim \pi'}[U(a^0, \theta)]$$
$$\Rightarrow \Delta(\pi'', a^0) \leq \Delta(\pi', a^0). \quad \square$$

Proof of Corollary 5. Follows from arguments in the text. $\square$

Proof of Lemma 3. As stated, when publication is non-selective, the distribution of $X|\theta, S = s, D = 1$ is identical to the distribution $X|\theta, S = s$ for every s. Parts 1

and 2 follow immediately from that observation. Part 3 follows from the definition of non-selective publication: $p(x, s)$ constant in x implies that $\mathbb{E}[p(X, S)|\theta, S = s]$ is equal to that same constant. To show part 4, recall that the independence of S and θ implies that if $\mathbb{E}[p(X, S)|\theta, S = s]$ is constant for each s, then it is constant in expectation across S, and so $\mathbb{E}[p(X, S)|\theta]$ is constant as well. The result then follows from (12). □

Proof of Proposition 5.
- Part $1 \Rightarrow$ all other parts: Non-selectivity of part 1 implies the other parts by Lemma 3. Specifically, part $1 \Rightarrow$ part 2 by part 1 of Lemma 3 (plugging in $\widehat{g}(X, S) = X$ and $g(\theta, S) = \theta$); part $1 \Rightarrow$ part 3 by part 3 of Lemma 3; and part $1 \Rightarrow$ part 4 by part 4 of Lemma 3.
- Part $3 \Rightarrow$ part 1: Fixing $S = s$, recall that X is a complete statistic for θ in the normal location model when Θ_0 contains an open set in $\mathbb{R}$; see for instance Theorem 6.22 in Lehmann and Casella (1998). Completeness means that for any measurable function $g : \mathcal{X} \to \mathbb{R}$, if $\mathbb{E}[g(X)|\theta, S = s] = 0$ for all $\theta \in \Theta_0$, then $P(g(X) = 0|\theta, S = s) = 1$ for all $\theta \in \Theta_0$. Apply this definition to $g(x) = p(x, s) - \mathbb{E}[p(X, s)|S = s]$. Assuming part 3, that the publication probability is constant over $\theta \in \Theta_0$, it holds that the expectation of $g(X)$ is 0 for all $\theta \in \Theta_0$, and hence that $p(X, s) = \mathbb{E}[p(X, s)|S = s]$ with probability 1 given θ and $S = s$, establishing part 1.
- Part $2 \Rightarrow$ part 3: To simplify notation, consider without loss of generality the case $s = 1$. Then the unbiasedness condition $\mathbb{E}[X|\theta, S = 1, D = 1]$ can be written as

$$\frac{\int x\varphi(x - \theta)p(x, 1)dx}{\int \varphi(x - \theta)p(x, 1)dx} = \theta.$$

Equivalently, using the fact that $\varphi'(x) = -x \cdot \varphi(x)$,

$$0 = \int (x - \theta)\varphi(x - \theta)p(x, 1)dx$$
$$= -\int \varphi'(x - \theta)p(x, 1)dx$$
$$= \partial_\theta \left[\int \varphi(x - \theta)p(x, 1)dx \right]$$
$$= \partial_\theta \mathbb{E}[p(X, S)|\theta, S = 1].$$

If the last line is equal to 0 then $E[p(X, S)|\theta, S = 1]$ is constant over θ in any open set contained in the support. The same argument applies for all other values of S. □
- Part $4 \Rightarrow$ part 3: Restating (12), the relative density of the Bayesian default

belief to the prior is given by

$$\frac{d\pi_1^{0,p}}{d\pi_0}(\theta) = \frac{1 - q \cdot \mathbb{E}[p(X,S)|\theta]}{1 - q \cdot \mathbb{E}[p(X,S)]}.$$

The Bayesian default belief is equal to the prior when, under the prior $\theta \sim \pi_0$, this relative density is almost surely constant in θ (in which case the ratio is identically equal to 1). In other words, it holds when $\mathbb{E}[p(X,S)|\theta]$ is almost surely constant in θ. Moreover, note that $\mathbb{E}[p(X,S)|\theta]$ must be continuous in θ since the signal density function $f_{X|\theta,S}(x|\theta,s)$ is a smooth function of θ for all x,s. Hence, if the Bayesian default belief is equal to the prior, then $\mathbb{E}[p(X,S)|\theta]$ must be constant in θ over the support of the prior.

Now, highlighting the dependence of this publication probability on the distribution F_S,

$$\mathbb{E}[p(X,S)|\theta] = \int_{s \in \mathcal{S}} \mathbb{E}[p(X,S)|\theta, S = s]dF_S(s).$$

We see that the LHS of this equation is constant over θ in the support of the prior for all distributions F_S if and only if, for all s, $\mathbb{E}[p(X,S)|\theta, S = s]$ is constant over θ in the support. (If there exists s' such that $\mathbb{E}[p(X,S)|\theta, S = s']$ varies in θ, then the distribution F_S placing all probability mass on s' will have $\mathbb{E}[p(X,S)|\theta]$ vary in θ.) So if the Bayesian default belief is equal to the prior for all F_S, then the publication probability is constant over θ in Θ_0 for all s.

Proof of Proposition 6. Without loss of generality, fix $s = 1$. First consider the case of a bounded interval $I(1)$. Then there exist θ' (the midpoint of the interval) and $y > 0$ (the radius) such that $I(1) = [\theta' - y, \theta' + y]$. If $y > z$,

$$P(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, D = 1) = 0,$$

and the result follows. If $y \leq z$, applying the law of iterated expectations and letting $C = 1$ denote the event of study submission,

$$\Phi(z) - \Phi(-z) = P(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, C = 1)$$
$$= P(D = 0|\theta = \theta', S = 1, C = 1) \cdot P(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, C = 1, D = 0)$$
$$+ P(D = 1|\theta = \theta', S = 1, C = 1) \cdot P(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, C = 1, D = 1).$$

Conditional on a study submitted but not published, it holds that $X \in [\theta' - y, \theta' + y]$, and therefore since $y \leq z$ that $\theta' \in [X - z, X + z]$:

$$P(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, C = 1, D = 0) = 1.$$

Therefore $\Phi(z) - \Phi(-z)$ is equal to a weighted average of 1 and $P(\theta' \in [X - z, X + z]|\theta = \theta', S = 1, D = 1, C = 1)$ – with positive weights on both – and hence $P(\theta' \in$

$[X - z, X + z]|\theta = \theta', S = 1, D = 1, C = 1) < \Phi(z) - \Phi(-z)$, yielding the desired result.

Consider finally the case of unbounded $I(1)$. If $I(1) = (-\infty, y]$ for some y, then for $\theta' < y - z$,

$$P(\theta' \in [X - z \cdot s, X + z \cdot s]|\theta = \theta', S = s, D = 1) = 0 < \Phi(z) - \Phi(-z).$$

A symmetric argument holds for $I(1) = [y, \infty)$ and $\theta' > y + z$, concluding our proof. □

Proof of Proposition 7. As stated in the text, the result follows from the fact that the signal $X|S = s$ distributed according to $\mathcal{N}(\theta, s^2)$ is a Blackwell more informative signal of θ when s is smaller. Blackwell more informative signals have higher expected value to a decisionmaker regardless of utility function U or prior π_0. Moreover, as $s \to \infty$, the signal becomes uninformative and the expected benefit of publication goes to 0, which is below $c > 0$. □

Proof of Proposition 8. As with the one-period problem, the optimal publication rule is the one that is interim optimal given the naive default belief $\pi_1^0 = \pi_0$.

Suppose a study (X_1, S_1) arrives at period 1. Let μ_1^0 indicate the posterior mean at period 1 in the absence of publication, and $\mu_1^{(X_1,S_1)}$ the posterior mean at period 1 if the study is published. Let $\mu_2^{0,(X_2)}$ indicate the posterior mean at period 2 if the study had not been published and then the second period signal is observed to be X_2, and $\mu_2^{(X_1,S_1),(X_2)}$ the posterior mean at period 2 if the study had been published. We can calculate these posterior means as follows:

$$\mu_1^0 = \mu_0$$
$$\mu_1^{(X_1,S_1)} = \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{S_1^2}} \left(\frac{\mu_0}{\sigma_0^2} + \frac{X_1}{S_1^2} \right)$$
$$\mu_2^{0,(X_2)} = \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{s_2^2}} \left(\frac{\mu_0}{\sigma_0^2} + \frac{X_2}{s_2^2} \right)$$
$$\mu_2^{(X_1,S_1),(X_2)} = \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{S_1^2} + \frac{1}{s_2^2}} \left(\frac{\mu_0}{\sigma_0^2} + \frac{X_1}{S_1^2} + \frac{X_2}{s_2^2} \right)$$

Consider the interim stage, at which (X_1, S_1) has been observed by the journal and not yet published, and hence at which the journal has interim belief $\pi_1^1(X_1, S_1)$. From this interim perspective, publication has a cost of c. It then delivers a benefit towards the first-period action payoff, and a benefit towards the second-period action payoff.

The benefit of publication towards the first-period payoff is $\alpha(\mu_1^{(X_1,S_1)} - \mu_1^0)^2$, which

simplifies to

$$\alpha(\mu_1^{(X_1,S_1)} - \mu_1^0)^2 = \alpha \frac{\sigma_0^4}{(\sigma_0^2 + S_1^2)^2}(X_1 - \mu_0)^2 \tag{20}$$

The period 2 action if the study is published is $\mu_2^{(X_1,S_1),(X_2)}$, and is $\mu_2^{0,(X_2)}$ otherwise. Hence, conditional on X_2, the benefit of first-period publication towards the second-period payoff is $(1-\alpha)(\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)})^2$. At the interim stage, then, the expected second-period payoff is the expectation of that value over the random variable X_2, given beliefs $\theta \sim \pi_1^{(X_1,S_1)}$ and $X_2 \sim \mathcal{N}(\theta, s_2^2)$. Writing out this expectation and simplifying,

$$\mathbb{E}\left[(1-\alpha)\left(\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)}\right)^2 | X_1, S_1\right]$$
$$= (1-\alpha)\left(\mathbb{E}\left[\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)}|X_1,S_1\right]^2 + \mathrm{Var}\left[\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)}|X_1,S_1\right]\right) \tag{21}$$

Next observe that, given X_1 and S_1, the conditional distribution of X_2 is

$$X_2|(X_1,S_1) \sim \mathcal{N}\left(\mu_1^{(X_1,S_1)}, \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{S_1^2}} + s_2^2\right).$$

Plugging this conditional distribution into the various terms of (21) and simplifying,

$$(1-\alpha)\mathbb{E}\left[\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)}|X_1,S_1\right]^2 = (1-\alpha)\left(\frac{\sigma_0^2 s_2^2}{(\sigma_0^2 + S_1^2)(\sigma_0^2 + s_2^2)}\right)^2 (X_1 - \mu_0)^2 \tag{22}$$

$$(1-\alpha)\,\mathrm{Var}\left[\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)}|X_1,S_1\right] = (1-\alpha)\frac{\sigma_0^8 s_2^4}{(\sigma_0^2 + S_1^2)(\sigma_0^2 + s_2^2)^2(\sigma_0^2 S_1^2 + \sigma_0^2 s_2^2 + S_1^2 s_2^2)} \tag{23}$$

The gross interim payoff of publication is the sum of the right-hand sides of (20), (22), and (23). To get the form stated in the proposition, we add up the coefficients on $(X_1 - \mu_0)^2$ in (20) and (22):

$$\alpha\frac{\sigma_0^4}{(\sigma_0^2 + s_1^2)^2} + (1-\alpha)\left(\frac{s_2^2 \sigma_0^2}{(\sigma_0^2 + s_1^2)(\sigma_0^2 + s_2^2)}\right)^2 = \frac{\sigma_0^4(s_2^4 + 2\alpha\sigma_0^2 s_2^2 + \alpha\sigma_0^4)}{(\sigma_0^2 + S_1^2)^2(\sigma_0^2 + s_2^2)^2}. \qquad \square$$

Proof of Corollary 6.
1. This result is immediate.

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2. The derivative of the benefit with respect to σ_0 evaluates to

$$(1-\alpha)\frac{2s_2^4\sigma_0^7(S_1^4\sigma_0^4 + 2s_2^4(S_1^2 + \sigma_0^2)(2S_1^2 + \sigma_0^2) + s_2^2(5S_1^4\sigma_0^2 + 4S_1^2\sigma_0^4))}{(s_2^2 + \sigma_0^2)^3(S_1^2 + \sigma_0^2)^2(S_1^2\sigma_0^2 + s_2^2S_1^2 + s_2^2\sigma_0^2)^2}$$

which is positive. As $\sigma_0 \to 0$, the numerator goes to 0 while the denominator goes to a positive limit.

3. The derivative of the benefit with respect to S_1 evaluates to

$$-(1-\alpha)\frac{2s_2^4 S_1\sigma_0^8(2S_1^2\sigma_0^2 + \sigma_0^4 + 2s_2^2S_1^2 + 2s_2^2\sigma_0^2)}{(s_2^2 + \sigma_0^2)^2(S_1^2 + \sigma_0^2)^2(S_1^2\sigma_0^2 + s_2^2S_1^2 + s_2^2\sigma_0^2)^2}$$

which is negative. As $S_1 \to \infty$, the numerator is constant while the denominator goes to infinity.

4. The derivative of the benefit with respect to s_2 evaluates to

$$(1-\alpha)\frac{2\sigma_0^8 s_2^3(-s_2^4(S_1^2 + \sigma_0^2) + s_2^2\sigma_0^2(S_1^2 + \sigma_0^2) + 2S_1^2\sigma_0^4)}{(s_2^2 + \sigma_0^2)^3(S_1^2 + \sigma_0^2)(S_1^2\sigma_0^2 + s_2^2S_1^2 + s_2^2\sigma_0^2)^2}.$$

This has the sign of $-s_2^4(S_1^2 + \sigma_0^2) + s_2^2\sigma_0^2(S_1^2 + \sigma_0^2) + 2S_1^2\sigma_0^4$. This expression is a concave quadratic in s_2^2, which is positive at $s_2^2 = 0$ and maximized at $s_2^2 = \frac{\sigma_0^2}{2} > 0$. In particular, the derivative in s_2 is positive and then negative. As $s_2 \to 0$, the numerator goes to zero while the denominator goes to a positive limit. As $s_2 \to \infty$, the numerator increases at a rate of s_2^4 while the denominator increases at a rate of s_2^6, so the ratio goes to 0.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

Proof of Proposition 9. The proof of part 1 holds for any distributions $\pi_1^I \neq \pi_0$. For part 2, the proofs rely on the fact that both distributions arise from the same prior π_0 (implying, for instance, that they share a common support), and that $q < 1$ if updating is Bayesian.

1. Write mean beliefs at the first period when a study (X_1, S_1) is published or not by $\mu_1^{(X_1,S_1)}$ and μ_1^0, and in the second period conditional on X_2 by $\mu_2^{(X_1,S_1),(X_2)}$ and $\mu_2^{0,(X_2)}$. The gross interim benefit of publishing a study (X_1, S_1) can be expressed as follows as the first-period action benefit plus the expected second-period action benefit:

$$\alpha(\mu_1^{(X_1,S_1)} - \mu_1^0)^2 + (1-\alpha)\mathbb{E}_{\theta\sim\pi_1^{(X_1,S_1)}, X_2\sim\mathcal{N}(\theta,s_2^2)}[(\mu_2^{(X_1,S_1),(X_2)} - \mu_2^{0,(X_2)})^2]$$

The first term, the first-period action benefit, is nonnegative (and strictly positive when the means $\mu_1^{(X_1,S_1)}$ and μ_1^0 differ). So it suffices to show that when $\pi_1^{(X_1,S_1)} \neq \pi_1^0$, the second term, the expected second-period action benefit, is strictly positive. In turn, it suffices to show that when $\pi_1^{(X_1,S_1)} \neq \pi_1^0$, there

exists X_2 for which $\mu_2^{(X_1,S_1),(X_2)} \neq \mu_2^{0,(X_2)}$. The second-period action benefit is nonnegative and is continuous in X_2, and X_2 has full support given any first-period interim belief $\pi_1^{(X_1,S_1)}$. So if the second-period action benefit is strictly positive at some X_2, then it is strictly positive in expectation.

The claim thus follows if we can show that, if $\mu_2^{(X_1,S_1),(X_2)} = \mu_2^{0,(X_2)}$ holds for all X_2, then $\pi_1^{(X_1,S_1)} = \pi_1^0$.

Without loss of generality, normalize $s_2 = 1$, so that $X_2 \sim \mathcal{N}(\theta, 1)$. Define

$$m(x; \pi) = \mathbb{E}_{\theta \sim \pi}[\theta | X_2 = x]$$

as the posterior mean of θ under π when $X_2 = x$. We seek to show that if $m(x; \pi) = m(x; \tilde{\pi})$ for almost all $x \in \mathbb{R}$, then $\pi = \tilde{\pi}$.

Taking φ to be the PDF of the standard normal, define $\pi * \varphi$ to be the marginal density of X_2 given $\theta \sim \pi$, which always exists:

$$(\pi * \varphi)(x) = \int_{\mathbb{R}} \varphi(x - \theta) d\pi(\theta).$$

It then holds that

$$\frac{\partial \log((\pi * \varphi)(x))}{\partial x} = \frac{1}{(\pi * \varphi)(x)} \frac{\partial (\pi * \varphi)(x)}{\partial x} = \frac{\int_{\mathbb{R}} \varphi'(x - \theta) d\pi(\theta)}{\int_{\mathbb{R}} \varphi(x - \theta) d\pi(\theta)} = \frac{\int_{\mathbb{R}} (\theta - x) \varphi(x - \theta) d\pi(\theta)}{\int_{\mathbb{R}} \varphi(x - \theta) d\pi(\theta)}$$

$$= \mathbb{E}_{\theta \sim \pi}[\theta | X_2 = x] - x = m(x; \pi) - x \qquad (24)$$

where the last equality on the first line follows from the identity $\varphi'(x) = -x\varphi(x)$. (This equation is also known as "Tweedie's formula.") Integrating the left- and right-hand sides yields

$$(\pi * \varphi)(x) = C \cdot \exp\left(\int_0^x (m(x; \pi) - x) dx\right)$$

for a constant of integration C pinned down by the fact that $\pi * \varphi$ integrates to 1. The same formula holds for $\tilde{\pi} * \varphi$, replacing π by $\tilde{\pi}$ on the right-hand side. We can therefore conclude that if $m(x; \pi) = m(x; \tilde{\pi})$ for all x, then $(\pi * \varphi)(x) = (\tilde{\pi} * \varphi)(x)$ for all x as well.

So, suppose that $m(x, \pi) = m(x, \tilde{\pi})$ for all $x \in \mathbb{R}$, and hence that $(\pi * \varphi)(x) = (\tilde{\pi} * \varphi)(x)$. For any distribution π of θ, denote its characteristic function (Fourier transform) by $\psi_\pi(t) = E_{\theta \sim \pi}[\exp(it\theta)]$. The fact that $\pi * \varphi = \tilde{\pi} * \varphi$ implies

$$\psi_\pi(t) \cdot \exp(-t^2/2) = \psi_{\tilde{\pi}}(t) \cdot \exp(-t^2/2)$$

for all t, where $\exp(-t^2/2)$ is the characteristic function of the standard normal distribution. This holds because the Fourier transform maps convolutions of random variables into products of their characteristic functions. It immediately

follows that $\psi_\pi(\cdot) = \psi_{\tilde{\pi}}(\cdot)$, since $\exp(-t^2/2)$ is different from 0 for all t, so that the characteristic function of π is equal to the characteristic function of $\tilde{\pi}$. Equality of their characteristic functions implies equality of π and $\tilde{\pi}$, by Lemma 2.15 in Van der Vaart (2000).

2. Let μ_1 denote the shared mean of π_1^0 and of π_1^I. Throughout this proof, it will be convenient to highlight the dependence of the distribution of the signal X_2 on the standard error parameter s_2, and so we will write the signal as $X_2^{(s_2)}$. In particular, $X_2^{(s_2)}|\theta \sim \mathcal{N}(\theta, s_2^2)$. Furthermore, let

$$m(x; \pi, s_2) = \mathbb{E}_{\theta \sim \pi}[\theta | X_2^{(s_2)} = x]$$

be the public's period-2 expectation of θ given period-1 belief π followed by period-2 observation $X_2^{(s_2)} = x$. As a final notational point, in this proof and the proofs of the corresponding Lemmas, any integral is to be interpreted as a definite integrals over the domain $\mathbb{R}$ unless otherwise specified.

Since the two beliefs π_1^0 and of π_1^I yield the same period 1 action of $a_1 = \mu_1$, the interim gross benefit of publishing the study is the expected benefit in the second-period, which can be written as

$$(1-\alpha)\mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^I, s_2) - m(X_2^{(s_2)}; \pi_1^0, s_2)\right)^2\right]. \qquad (25)$$

We seek to show that, under the appropriate conditions, the expression (25) goes to zero as $s_2 \to 0$ (for part 2a) and as $s_2 \to \infty$ (for part 2b).

Lemma 4. *If distribution π has a finite mean and variance, then*

$$\lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi}\left[\left(m(X_2^{(s_2)}; \pi, s_2) - X_2^{(s_2)}\right)^2\right] = 0.$$

Lemma 5. *If distribution π has mean μ_1 and is bounded by Pareto tails with finite variance, then*

$$\lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi}\left[\left(m(X_2^{(s_2)}; \pi, s_2) - \mu_1\right)^2\right] = 0. \qquad (26)$$

We will apply Lemma 4 to show part 2a of the Proposition and Lemma 5 to show part 2b.

Before proceeding, it is valuable to establish one other preliminary result.

Lemma 6. *Given any π_1^0 and π_1^I as derived under the hypotheses of Proposition 9, there exists $C' > 0$ such that for all $s_2 > 0$ and all functions $y : \mathbb{R}_+ \to \mathbb{R}_+$, it holds that $\mathbb{E}_{\theta \sim \pi_1^I}\left[y\left(X_2^{(s_2)}\right)\right] \le C'\mathbb{E}_{\theta \sim \pi_1^0}\left[y\left(X_2^{(s_2)}\right)\right]$.*

We now proceed to the proofs of each part.

(a) First observe that the distributions π_1^0 and π_1^I both have a finite variance. To see that this holds for π_1^I, recall that $\pi_1^I = \pi_1^{(x_1, s_1)}$ is a posterior distribution updated after observing a normal signal $(X_1, S_1) = (x_1, s_1)$. The posterior distribution (from any prior) after observing a normal signal has a finite variance. To see that this holds for π_1^0, recall that π_1^0 arises as a default belief from the prior π_0 with a finite variance. In the case of naive updating, $\pi_1^0 = \pi_0$, so the result is immediate. In the case of Bayesian updating, observe from (3) that $\frac{d\pi_1^0}{d\pi_0}(\theta) \leq \frac{1}{1-q}$ for all θ, and therefore $\pi_0 \geq (1-q)\pi_1^0$; so if π_1^0 had an infinite variance, then so too would π_0.

Plugging $\pi = \pi_1^I$ into Lemma 4, we have that

$$\lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^I, s_2) - X_2^{(s_2)}\right)^2\right] = 0.$$

Applying Lemma 6, we also have that there exists a constant $C' > 0$ such that

$$0 \leq \lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^0, s_2) - X_2^{(s_2)}\right)^2\right] \leq \lim_{s_2 \to 0} C' \mathbb{E}_{\theta \sim \pi_1^0}\left[\left(m(X_2^{(s_2)}; \pi_1^0, s_2) - X_2^{(s_2)}\right)^2\right].$$

Plugging $\pi = \pi_1^0$ into Lemma 4, we have that the right-hand expression is equal to 0. Hence,

$$\lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^0, s_2) - X_2^{(s_2)}\right)^2\right] = 0.$$

In other words, both $m(X_2^{(s_2)}; \pi_1^I, s_2)$ and $m(X_2^{(s_2)}; \pi_1^0, s_2)$ converge to $X_2^{(s_2)}$ in mean-square as $s_2 \to 0$ under $\theta \sim \pi_1^I$. Therefore they converge to each other in mean-square, establishing the desired conclusion that the expression (25) goes to 0 as $s_2 \to 0$, as long as the three variables $m(X_2^{(s_2)}; \pi_1^I, s_2)$, $m(X_2^{(s_2)}; \pi_1^0, s_2)$, and $X_2^{(s_2)}$ are all square-integrable under $\theta \sim \pi_1^I$.

The three variables are indeed square-integrable, as they each have finite means and variance. To see that, observe that the posterior mean $m(X_2^{(s_2)}; \pi_1^I, s_2)$ has mean equal to μ_1 and, by the Law of Total Variance, variance less than $\mathrm{Var}_{\theta \sim \pi_1^I}$: the variance of the posterior mean given some signal is bounded above by the variance of the prior. The other posterior mean variable $m(X_2^{(s_2)}; \pi_1^0, s_2)$ has a finite mean and variance under the distribution $\theta \sim \pi_1^0$ by the same arguments, and therefore finite mean and variance under the distribution $\theta \sim \pi_1^I$ by Lemma 6.[30] Finally, the mean

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[30]To see that $m(X_2^{(s_2)}; \pi_1^0, s_2)$ has a finite mean under $\theta \sim \pi_1^I$, recall $\mathbb{E}_{\theta \sim \pi_1^0}[m(X_2^{(s_2)}; \pi_1^0, s_2)]$ is finite if and only if $\mathbb{E}_{\theta \sim \pi_1^0}\left[\left|m(X_2^{(s_2)}; \pi_1^0, s_2)\right|\right]$ is finite; and the latter being finite implies by Lemma

of $X_2^{(s_2)}$ is μ_1 and the variance is $\mathrm{Var}_{\theta \sim \pi_1^I}(\theta) + s_2^2$.

(b) First observe that the distributions π_1^0 and π_1^I are both bounded by Pareto tails with finite variance since they arise from the prior π_0 that is bounded by Pareto tails with finite variance. To see that this holds for π_1^I, recall that $\pi_1^I = \pi_1^{(x_1, s_1)}$ is a posterior distribution updated after observing a normal signal $(X_1, S_1) = (x_1, s_1)$. It holds that $\frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}$ is equal to a constant times $\varphi(\frac{x_1 - \theta}{s_1})$, and hence the tails of π_1^I decay at a rate at least as fast as those of π_0. To see that this holds for π_1^0 in the case of naive updating, $\pi_1^0 = \pi_0$, and so the result is immediate. To see that this holds for π_1^0 in the case of Bayesian updating, observe from (3) that $\frac{d\pi_1^0}{d\pi_0}(\theta) \leq \frac{1}{1-q}$ for all θ, and therefore $\pi_0 \geq (1-q)\pi_1^0$; so if π_1^0 were not bounded by Pareto tails with finite variance, then neither would π_0.

Plugging $\pi = \pi_1^I$ into Lemma 5, we have that

$$\lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^I, s_2) - \mu_1\right)^2\right] = 0.$$

Applying Lemma 6, we also have that there exists a constant $C' > 0$ such that

$$0 \leq \lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^0, s_2) - \mu_1\right)^2\right] \leq \lim_{s_2 \to \infty} C' \mathbb{E}_{\theta \sim \pi_1^0}\left[\left(m(X_2^{(s_2)}; \pi_1^0, s_2) - \mu_1\right)^2\right].$$

Plugging $\pi = \pi_1^0$ into Lemma 5, we have that the right-hand expression is equal to 0. Hence,

$$\lim_{s_2 \to \infty} \mathbb{E}_{\theta \sim \pi_1^I}\left[\left(m(X_2^{(s_2)}; \pi_1^0, s_2) - \mu_1\right)^2\right] = 0.$$

In other words, both $m(X_2^{(s_2)}; \pi_1^I, s_2)$ and $m(X_2^{(s_2)}; \pi_1^0, s_2)$ converge to μ_1 in mean-square as $s_2 \to 0$ under $\theta \sim \pi_1^I$. Therefore they converge to each other in mean-square, establishing the desired conclusion that the expression (25) goes to 0 as $s_2 \to 0$, as long as they are both square-integrable under $\theta \sim \pi_1^I$; that was established in the proof of the previous part. $\square$

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6 that $\mathbb{E}_{\theta \sim \pi_1^I}\left[\left|m(X_2^{(s_2)}; \pi_1^0, s_2)\right|\right]$ and hence $\mathbb{E}_{\theta \sim \pi_1^I}[m(X_2^{(s_2)}; \pi_1^0, s_2)]$ are finite. Call $\tilde{\mu}$ the mean of $m(X_2^{(s_2)}; \pi_1^0, s_2)$ under $\theta \sim \pi_1^I$; the fact that $m(X_2^{(s_2)}; \pi_1^0, s_2)$ has a finite variance under $\tilde{\theta} \sim \pi_1^0$ means that $\mathbb{E}_{\theta \sim \pi_1^0}[m(X_2^{(s_2)}; \pi_1^0, s_2) - \tilde{\mu})^2]$ is finite, and thus by Lemma 6 $\mathbb{E}_{\theta \sim \pi_1^I}[m(X_2^{(s_2)}; \pi_1^0, s_2) - \tilde{\mu})^2]$ is also finite.

Proof of Lemma 4. First observe that

$$\mathbb{E}_{\theta \sim \pi}[(X_2^{(s_2)} - \theta)^2] = s_2^2$$

$$\Rightarrow \lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi}[(X_2^{(s_2)} - \theta)^2] = 0. \tag{27}$$

Next recall that for any s_2 and any realization $X_2^{(s_2)} = x$, the posterior mean of the updated belief, $m(x; \pi_1^I, s_2)$, minimizes the expected square distance to θ:

$$m(x; \pi, s_2) \in \arg \min_{g_{s_2}: \mathbb{R} \to \mathbb{R}} \mathbb{E}_{\theta \sim \pi}[(g_{s_2}(x) - \theta)^2 | X_2^{(s_2)} = x]$$

$$\Rightarrow \mathbb{E}_{\theta \sim \pi}[(m(x; \pi, s_2) - \theta)^2 | X_2^{(s_2)} = x]$$

$$\leq \mathbb{E}_{\theta \sim \pi}[(g_{s_2}(x) - \theta)^2 | X_2^{(s_2)} = x] \, \forall g_{s_2}.$$

Since this inequality holds for each realization $X_2^{(s_2)} = x$, it also holds in expectation:

$$\mathbb{E}_{\theta \sim \pi}[(m(X_2^{(s_2)}; \pi) - \theta)^2] \leq \mathbb{E}_{\theta \sim \pi}[(g_{s_2}(X_2^{(s_2)}) - \theta)^2] \, \forall g_{s_2}.$$

Plugging in $g_{s_2}(x)$ equal to the identity function x,

$$0 \leq \mathbb{E}_{\theta \sim \pi}[(m(X_2^{(s_2)}; \pi, s_2) - \theta)^2] \leq \mathbb{E}_{\theta \sim \pi}[(X_2^{(s_2)} - \theta)^2].$$

Taking the limit as $s_2 \to 0$ as in (27), the right-hand side of the above expression converges to 0, and hence

$$\lim_{s_2 \to 0} \mathbb{E}_{\theta \sim \pi}[(m(X_2^{(s_2)}; \pi, s_2) - \theta)^2] \to 0. \tag{28}$$

So we see that $m(X_2^{(s_2)}; \pi, s_2)$ and $X_2^{(s_2)}$ both converge to θ in mean-square as $s_2 \to 0$. We can conclude that $m(X_2^{(s_2)}; \pi, s_2)$ converges to $X_2^{(s_2)}$ in mean-square, and hence we have proven our result, if $m(X_2^{(s_2)}; \pi, s_2)$, $X_2^{(s_2)}$, and θ are all square-integrable under $\theta \sim \pi$. In turn it suffices to show that these random variables all have a finite mean and a variance. By assumption, the mean and variance of θ under π are finite. Then $X_2^{(s_2)}$ and $m(X_2^{(s_2)}; \pi, s_2)$ also share the mean of θ under π for all s_2. The variance of X_2 is given by $\text{Var}_{\theta \sim \pi}(\theta) + s_2^2$. Finally, the variance of $m(X_2; \pi, s_2)$ is bounded above by $\text{Var}_{\theta \sim \pi}(\theta)$ by the Law of Total Variance: the variance of the posterior mean given some signal is bounded above by the variance of the prior. $\square$

Proof of Lemma 5. Applying a transformation with $\lambda = 1/s_2$, let $\hat{X}_2^{(\lambda)} = \lambda X_2^{(1/\lambda)}$; $X_2^{(\lambda)}$ is equal to the t-statistic $X_2^{(s_2)}/s_2$. That is, $\hat{X}_2^{(\lambda)} | \theta \sim \mathcal{N}(\lambda\theta, 1)$, where $\hat{X}_2^{(\lambda)} | \theta$ has pdf at $\hat{x}$ of $\varphi(\hat{x} - \lambda\theta)$. Correspondingly, let

$$\hat{m}(\hat{x}; \pi, \lambda) = \mathbb{E}_{\theta \sim \pi}[\theta | \hat{X}_2^{(\lambda)} = \hat{x}]$$

be the public's period-2 expectation of θ given period-1 belief π followed by period-2 observation $\hat{X}_2^{(\lambda)} = \hat{x}$, i.e., given $X_2^{(1/\lambda)} = \hat{x}/\lambda$. This transformation will be convenient because as $s_2 \to \infty$ and $\lambda = 1/s_2 \to 0$, the variable $\hat{X}^{(\lambda)}|\theta$ approaches a standard normal, whereas $X_2^{(s_2)}|\theta$ approaches an improper distribution with infinite variance.

We seek to show that for any π with mean μ_1 that is bounded by Pareto tails with finite variance, it holds that

$$\lim_{\lambda \to 0} \mathbb{E}_{\theta \sim \pi}[(\hat{m}(\hat{X}^{(\lambda)}; \pi, \lambda) - \mu_1)^2] = 0. \tag{29}$$

Writing the expectation from (29) out in integral form,

$$\mathbb{E}_{\theta \sim \pi}[(\hat{m}(\hat{X}^{(\lambda)}; \pi, \lambda) - \mu_1)^2] = \int \int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) d\hat{x}.$$

By Lebesgue's dominated convergence theorem, to show (29), it suffices to show (i) for all $\hat{x}$, $\lim_{\lambda \to 0} \int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) = 0$; and (ii) there exists a "dominating" function $g : \mathbb{R}_+ \to \mathbb{R}_+$ that is Lebesgue-integrable, i.e., $\int g(\hat{x}) d\hat{x}$ is finite, such that for λ sufficiently small, $\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \leq g(\hat{x})$ for all $\hat{x}$.

Step 1: *Show that for all $\hat{x}$, $\lim_{\lambda \to 0} \int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) = 0$.*

It holds that

$$\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) = (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)$$

$$\leq (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \int \varphi(0) d\pi(\theta)$$

$$= (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(0).$$

So to show the desired result that $\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)$ converges to 0 for all $\hat{x}$, it suffices to show that $(\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2$ converges to 0 for all $\hat{x}$. In turn, it suffices to show that $\hat{m}(\hat{x}; \pi, \lambda)$ converges to μ_1 for any fixed $\hat{x}$. Writing $\hat{m}(\hat{x}; \pi, \lambda)$ in integral form,

$$\hat{m}(\hat{x}; \pi, \lambda) = \frac{\int \theta \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}{\int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)} \tag{30}$$

In the denominator of (30), for all θ, $\varphi(\hat{x} - \lambda\theta) \to \varphi(\hat{x})$ as $\lambda \to 0$. Moreover, $\varphi(\hat{x} - \lambda\theta) \leq \varphi(0)$ for all θ and λ, and $\int \varphi(0) d\pi(\theta) = \varphi(0) < \infty$. So $\varphi(0)$ is a dominating function for $\varphi(\hat{x} - \lambda\theta)$ that is integrable with respect to π_0, and hence by the dominated convergence theorem the denominator approaches $\int \varphi(\hat{x}) d\pi(\theta) = \varphi(\hat{x})$.

In the numerator of (30), for all θ, $\theta \varphi(\hat{x} - \lambda\theta) \to \theta \varphi(\hat{x})$ as $\lambda \to 0$. Moreover, $|\theta \varphi(\hat{x} - \lambda\theta)| \leq |\theta| \varphi(0)$ for all θ and λ, and $\int \theta \varphi(0) d\pi(\theta) = \varphi(0) \int |\theta| d\pi(\theta) < \infty$ because π has a finite mean. So $|\theta| \varphi(0)$ is a dominating function for $\theta \varphi(\hat{x} - \lambda\theta)$ that

is integrable with respect to π, and hence by the dominated convergence theorem the numerator approaches $\int \theta \varphi(\hat{x}) d\pi(\theta) = \mu_1 \varphi(\hat{x})$.

Taking the ratio, we have that $\hat{m}(\hat{x}; \pi, \lambda)$ converges to $\mu_1 \varphi(\hat{x})/\varphi(\hat{x}) = \mu_1$ as $\lambda \to 0$, completing this step.

Step 2: *Show that there exists a dominating function $g : \mathbb{R}_+ \to \mathbb{R}_+$ that is Lebesgue-integrable, such that for λ sufficiently small, $\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \leq g(\hat{x})$ for all $\hat{x}$.*

First, observe that

$$
\begin{aligned}
\int (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) &= (\hat{m}(\hat{x}; \pi, \lambda) - \mu_1)^2 \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&= \left(\frac{\int \theta \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}{\int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)} - \mu_1 \right)^2 \cdot \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&\leq \frac{\int (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}{\int \varphi(\hat{x} - \lambda\theta) d\pi(\theta)} \cdot \int \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \\
&= \int (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) \quad\quad\quad (31)
\end{aligned}
$$

where the inequality in the third line follows from Jensen's inequality: $(\mathbb{E}[\theta | \hat{X}^{(\lambda)} = \hat{x}] - \mu_1)^2 = (\mathbb{E}[\theta - \mu_1 | \hat{X}^{(\lambda)} = \hat{x}])^2 \leq \mathbb{E}[(\theta - \mu_1)^2 | \hat{X}^{(\lambda)} = \hat{x}]$.

So it suffices to find an integrable function g for which $g(\hat{x})$ is everywhere larger than (31) for all $\lambda \in (0, 1]$.

- *Constructing g for $\hat{x} \in [-2K, 2K]$.*
  The expression (31) is uniformly bounded above by $\int (\theta - \mu_1)^2 \varphi(0) d\pi(\theta) = \varphi(0) \operatorname{Var}_{\theta \sim \pi}(\theta)$. So, let

  $$
  g(\hat{x}) = \varphi(0) \operatorname{Var}_{\theta \sim \pi}(\theta) \text{ for } \hat{x} \in [-2K, 2K].
  $$

  It holds that $\int_{-2K}^{2K} g(\hat{x}) d\hat{x} = 4K\varphi(0) \operatorname{Var}_{\theta \sim \pi}(\theta) < \infty$.

- *Constructing g for $\hat{x} > 2K$.*
  Expanding out (31), we have

  $$
  \int (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta) = \underbrace{\int_{-\infty}^{\frac{\hat{x}}{2\lambda}} (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}_{A} + \underbrace{\int_{\frac{\hat{x}}{2\lambda}}^{\infty} (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}_{B}
  $$
  $$(32)$$

First let us bound the term labeled A in (32). For $\theta \leq \frac{\hat{x}}{2\lambda}$, it holds that $\hat{x} - \lambda\theta \geq \hat{x}/2$. Therefore, assuming further that $\hat{x} \geq 2K$ – and in particular

that $\hat{x} \geq 0$ – it holds that $\varphi(\hat{x} - \lambda\theta) \leq \varphi(\hat{x}/2)$. Hence,

$$\underbrace{\int_{-\infty}^{\frac{\hat{x}}{2\lambda}} (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}_{A} \leq \int_{-\infty}^{\frac{\hat{x}}{2\lambda}} (\theta - \mu_1)^2 \varphi(\hat{x}/2) d\pi(\theta)$$

$$\leq \int_{-\infty}^{\infty} (\theta - \mu_1)^2 \varphi(\hat{x}/2) d\pi(\theta)$$

$$= \varphi(\hat{x}/2) \operatorname{Var}_{\theta \sim \pi}(\theta).$$

Now we move to the term labeled B in (32). By the fact that π is bounded by Pareto tails with finite variance,

$$\underbrace{\int_{\frac{\hat{x}}{2\lambda}}^{\infty} (\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\pi(\theta)}_{B} \leq \int_{\frac{\hat{x}}{2\lambda}}^{\infty} C\theta^{-\gamma}(\theta - \mu_1)^2 \varphi(\hat{x} - \lambda\theta) d\theta$$

$$\leq \int_{\frac{\hat{x}}{2\lambda}}^{\infty} C\theta^{-\gamma}(\theta + |\mu_1|)^2 \varphi(\hat{x} - \lambda\theta) d\theta$$

$$\leq C \frac{(\frac{\hat{x}}{2\lambda} + |\mu_1|)^2}{(\frac{\hat{x}}{2\lambda})^{\gamma}} \int_{\frac{\hat{x}}{2\lambda}}^{\infty} \varphi(\hat{x} - \lambda\theta) d\theta$$

$$= C \frac{(\frac{\hat{x}}{2\lambda} + |\mu_1|)^2}{(\frac{\hat{x}}{2\lambda})^{\gamma}} \frac{1}{\lambda} (1 - \Phi(-\frac{\hat{x}}{2}))$$

$$= 2^{\gamma-2} C \lambda^{\gamma-3} \frac{(\hat{x} + 2\lambda|\mu_1|)^2}{\hat{x}^{\gamma}} \Phi(\frac{\hat{x}}{2})$$

$$\leq 2^{\gamma-2} C \frac{(\hat{x} + 2|\mu_1|)^2}{\hat{x}^{\gamma}} \text{ for } \lambda \in (0, 1]$$

The inequality in the third line follows because $\theta^{-\gamma}(\theta + |\mu_1|)^2$ is decreasing in θ over $\theta > 0$ for any $\gamma > 2$, so we increase the expression when we plug in the lowest value of θ, i.e., $\theta = \hat{x}/(2\lambda)$. The inequality in the last line follows because $\lambda^{\gamma-3}(\hat{x} + 2\lambda|\mu_1|)^2$ is increasing in λ over $\lambda > 0$ for any $\gamma > 3$, so we increase the expression relative to $\lambda \leq 1$ when we plug in $\lambda = 1$; and we also increase the expression when we replace $\Phi(\frac{\hat{x}}{2})$ by 1. These two observations about increasing and decreasing functions can be straightforwardly confirmed by taking derivatives.[31]

Putting the bounds on terms A and B together, let

$$g(\hat{x}) = \varphi(\hat{x}/2) \operatorname{Var}_{\theta \sim \pi}(\theta) + 2^{\gamma-2} C \frac{(\hat{x} + 2|\mu_1|)^2}{\hat{x}^{\gamma}} \text{ for } \hat{x} > 2K.$$

---

[31]The derivative of $\theta^{-\gamma}(\theta + |\mu_1|)^2$ with respect to θ evaluates to $-\theta^{-(1+\gamma)}(\theta + |\mu_1|)(|\mu_1|\gamma + (\gamma - 2)\theta) < 0$. The derivative of $\lambda^{\gamma-3}(\hat{x} + 2\lambda|\mu_1|)^2$ with respect to λ evaluates to $\lambda^{\gamma-4}(\hat{x} + 2|\mu_1|\lambda)((\gamma - 3)\hat{x} + 2|\mu_1|(\gamma - 1)\lambda) > 0$.

As established, $g(\hat{x})$ is larger than (31) for all $\lambda \leq 1$. Moreover, $\int_{2K}^{\infty} g(\hat{x})d\hat{x}$ is finite: the first term is an integral of a normal pdf, and the second term is an integral of an expression that decays to zero as $\hat{x}$ goes to infinity at a rate of $\hat{x}^{2-\gamma}$, with the exponent $2 - \gamma < -1$.

- *Constructing g for $\hat{x} < -2K$.*
  This case proceeds symmetrically to the construction for $\hat{x} > 2K$, now taking

$$g(\hat{x}) = \varphi(\hat{x}/2)\,\mathrm{Var}_{\theta \sim \pi}(\theta) + 2^{\gamma-2}C\frac{(|\hat{x}| + 2|\mu_1|)^2}{|\hat{x}|^{\gamma}} \quad \text{for } \hat{x} < 2K.$$

Just as with $\hat{x} > 2K$, when $\hat{x} < -2K$ we have that $g(\hat{x})$ is an upper bound for (31) when $\lambda \leq 1$, and $\int_{-\infty}^{-2K} g(\hat{x})d\hat{x}$ is finite.

We have now established that $g(\hat{x})$ is an upper bound for (31) for all $\lambda \leq 1$ and for all $\hat{x}$, and that $\int g(\hat{x})d\hat{x} < \infty$, concluding the proof. $\qquad\square$

Proof of Lemma 6. Define $f^I_{X_2^{(s_2)}}(x) = \frac{1}{s_2}\int \varphi(\frac{x-\theta}{s_2})d\pi_1^I(\theta)$ and $f^0_{X_2^{(s_2)}}(x) = \frac{1}{s_2}\int \varphi(\frac{x-\theta}{s_2})d\pi_1^0(\theta)$ to be the marginal densities of $X_2^{(s_2)}$ under the respective distributions on θ of π_1^I and π_1^0.

Step 1: *Show that there exists $C' > 0$ such that $\dfrac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)} \leq C'$ for all s_2.*

First observe that

$$\frac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)} = \frac{\int \varphi(\frac{x-\theta}{s_2})d\pi_1^I(\theta)}{\int \varphi(\frac{x-\theta}{s_2})d\pi_1^0(\theta)} = \frac{\int \varphi(\frac{x-\theta}{s_2})\frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)}d\pi_1^0(\theta)}{\int \varphi(\frac{x-\theta}{s_2})d\pi_1^0(\theta)} \leq \sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)}.$$

Next, recall that $\pi_1^I = \pi_1^{(x_1,s_1)}$, which is a posterior belief on θ given prior $\theta \sim \pi_0$ and some fixed signal realization $(X_1, S_1) = (x_1, s_1)$. Hence

$$\frac{d\pi_1^I(\theta)}{d\pi_0(\theta)} = \frac{\varphi(\frac{x_1-\theta}{s_1})}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}$$

$$\Rightarrow \sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)} \leq \frac{\varphi(0)}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}.$$

Under naive updating, $\pi_1^0 = \pi_0$, and thus $\sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)} = \sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}$, bounded by the finite constant $C' = \frac{\varphi(0)}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}$. (Recall that x_1 and s_1 are taken as constants here.) Under Bayesian updating with study arrival probability $q < 1$, (3) implies that $\frac{d\pi_0(\theta)}{d\pi_1^0(\theta)} \leq \frac{1}{1-q}$ for all θ, and therefore that $\sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_1^0(\theta)} = \sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}\frac{d\pi_0(\theta)}{d\pi_1^0(\theta)} \leq \frac{1}{1-q}\sup_{\theta} \frac{d\pi_1^I(\theta)}{d\pi_0(\theta)}$. Hence for Bayesian updating we have a bound $C' = \frac{1}{1-q}\frac{\varphi(0)}{\int \varphi(\frac{x_1-\theta'}{s_1})d\pi_0(\theta')}$.

In either case C' gives an upper bound on $\dfrac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)}$.

Step 2: *Show that* $\mathbb{E}_{\theta \sim \pi_1^I}\left[y\left(X_2^{(s_2)}\right)\right] \leq C'\mathbb{E}_{\theta \sim \pi_1^0}\left[y\left(X_2^{(s_2)}\right)\right].$

Rewriting expectations in integral form,

$$
\begin{aligned}
\mathbb{E}_{\theta \sim \pi_1^I}\left[y\left(X_2^{(s_2)}\right)\right] &= \int y\left(X_2^{(s_2)}\right) f^I_{X_2^{(s_2)}}(x)dx \\
&= \int y\left(X_2^{(s_2)}\right) \frac{f^I_{X_2^{(s_2)}}(x)}{f^0_{X_2^{(s_2)}}(x)} f^0_{X_2^{(s_2)}}(x)dx \\
&\leq \int y\left(X_2^{(s_2)}\right) C' f^0_{X_2^{(s_2)}}(x)dx \text{ (by Step 1)} \\
&= C'\mathbb{E}_{\theta \sim \pi_1^0}\left[y\left(X_2^{(s_2)}\right)\right]. \qquad \square
\end{aligned}
$$

Proof of Proposition 10. We first state a lemma that does not depend on Assumption 1.

Lemma 7. *In searching for an incentive-optimal publication rule, it is without loss of generality to restrict to rules $p(X, S)$ satisfying*

$$
p(X, S) = \begin{cases}
1 & \text{if } S = \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) > c - \lambda, \\
& \quad \text{or if } S < \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) \geq c \\
0 & \text{if } S > \bar{s}, \\
& \quad \text{or if } S = \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) < c - \lambda \\
& \quad \text{or if } S < \bar{s} \text{ and } \Delta(\pi_1^{(X,S)}, a^*(\pi_0)) > c
\end{cases}
$$

for some $\bar{s} \in (0, \infty)$ and λ in $\mathbb{R} \cup \{-\infty, \infty\}$ in which the researcher chooses $S = \bar{s}$ if she conducts a study.

It remains only to show that in the incentive-optimal contract of the form in Lemma 7, the researcher chooses to conduct a study; that $\bar{s} \leq s^{\text{int}}$; and that $\lambda \geq 0$.

The facts that the researcher conducts a study and that $\bar{s} \leq s^{\text{int}}$ both follow from Assumption 1.

First, Assumption 1 guarantees that the journal prefers to follow the interim-optimal rule – at which the researcher conducts a study with $S = s^{\text{int}}$, and the journal only publishes studies with a nonnegative interim net benefit – than any rule that publishes nothing at all. (In the model without incentives in which $q = 1$ and S is deterministically equal to s^{int}, publishing no studies is feasible, but the interim-optimal rule is preferred.) So the incentive-optimal rule will induce the researcher to conduct a study, meaning that the researcher must be choosing $S = \bar{s}$.

Second, fix any publication rule of the form in Lemma 7 with $\bar{s} = s^h$ and $\lambda = \lambda^h$,

for $s^h > s^{\text{int}}$. We claim that the publication rule of the same form with $\overline{s} = s^{\text{int}}$ and $\lambda = 0$ weakly improves payoffs. To see why this claim holds, note that the publication rule with $\overline{s} = s^h$ and $\lambda = \lambda^h$ would be weakly improved upon by one with $\overline{s} = s^{\text{int}}$ and $\lambda = 0$, supposing researcher participation. Recall that normal signals are Blackwell ordered by their standard errors: at standard error $S = s^{\text{int}}$, the findings X can be garbled into something informationally equivalent to findings from $S = s^h$. So some stochastic publication rule at $S = s^{\text{int}}$, combined with a garbling of these signals to the public, replicates the distribution of outcomes[32] that occur when a study arrives with $S = s^h$ and is published under the publication rule given by $\overline{s} = s^h$ and $\lambda = \lambda^h$. But the journal's payoffs given a study with $S = s^{\text{int}}$ are improved by removing the garbling to the public. Payoffs are further improved by publishing under the interim-optimal publication rule at $S = s^{\text{int}}$, which is exactly that given by a rule of the form in Lemma 7 with $\overline{s} = s^{\text{int}}$ and $\lambda = 0$. Finally, by Assumption 1, the publication rule with $\overline{s} = s^{\text{int}}$ and $\lambda = 0$ does indeed get researcher to conduct a study, since the interim outcome satisfies the researcher's participation constraint.

The final step is to show that $\lambda \geq 0$. This is because, for any publication rule of the form of Lemma 7, increasing λ increases the publication probability at $S = \overline{s}$. Hence, it makes the researcher better off if she chooses $S = \overline{s}$ and slackens her incentive constraints. Moreover, starting from $\lambda < 0$, increasing λ to 0 improves the journal's payoff, since again $\lambda = 0$ is interim optimal and hence optimal conditional on a study being submitted at $S = \overline{s}$. $\qquad\square$

Proof of Lemma 7. Take an arbitrary publication rule $\tilde{p}$. We will show that it can be replaced by a rule p of the desired form that weakly increases the journal's payoff.

First suppose that $\tilde{p}$ does not induce the researcher to conduct a study. Then define some p of the form in the statement of the Lemma by setting $\overline{s}$ arbitrarily and setting $\lambda = 0$. If the publication rule p induces the researcher not to participate, then the journal's payoffs are unchanged from $\tilde{p}$. If the rule p induces the researcher to conduct a study with standard error $S = s$, then the journal's payoffs are weakly higher than before, since under p the journal never publishes studies that give negative net interim payoff.

So, for the rest of the proof, assume that $\tilde{p}$ does in fact induce the researcher to conduct a study with S equal to some level $\overline{s}$. We show that there exists λ such that we can replace $\tilde{p}$ with a publication rule p satisfying the following properties and weakly improve the journal's payoff:

1. At $s > \overline{s}$, $p(x, s) = 0$:

   Let $p(x, s) = \tilde{p}(x, s)$ at $s \leq \overline{s}$ and 0 at $x > \overline{s}$. The publication rule p gives the researcher the same payoff from choosing $S = \overline{s}$ and weakly reduces her payoff from choosing other values of S, and so under p the researcher's behavior is unchanged. She continues to conduct a study with $S = \overline{s}$ and the journal's

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[32]I.e., the probability of publication at each state, and the joint distribution over public actions and states conditional on publication.

payoff given the choice of $S = \bar{s}$ is also unchanged.

2. At $s = \bar{s}$, $p(X, \bar{s}) = 1$ if $\Delta(\pi_1^{(X,\bar{s})}, a^*(\pi_0)) > c - \lambda$, and $p(X, \bar{s}) = 0$ if $\Delta(\pi_1^{(X,\bar{s})}, a^*(\pi_0)) < c - \lambda$:

   Let $p(x, s) = \tilde{p}(x, s)$ at all $s \neq \bar{s}$. Denote the probability of publication under $\tilde{p}$ at $S = \bar{s}$, given by $\mathbb{E}[\tilde{p}(X, S)|S = \bar{s}]$, by $y \in [0, 1]$. If $y = 0$ then $\tilde{p}$ is equivalent to a publication rule p of the appropriate form with $\lambda = \infty$. If $y = 1$ then $\tilde{p}$ is equivalent to a publication rule p of the appropriate form with $\lambda = -\infty$. For interior y, define $p(\cdot, \bar{s})$ so as to maximize the journal's payoff subject to accepting a share y of papers at this standard error. To do so, first set $\lambda \in \mathbb{R}$ as the supremum over values of l such that $P(\Delta(\pi_1^{(X,\bar{s})}, a^*(\pi_0)) > c - l|S = \bar{s}] \leq y$. Next, let $p(x, \bar{s}) = 0$ if $\Delta(\pi_1^{(x,\bar{s})}, a^*(\pi_0)) < c - \lambda$ and let $p(x, \bar{s}) = 1$ if $\Delta(\pi_1^{(x,\bar{s})}, a^*(\pi_0)) > c - \lambda$. Finally, if $\Delta(\pi_1^{(x,\bar{s})}, a^*(\pi_0)) = c - \lambda$, set $p(x, \bar{s})$ such that the publication probability at $S = \bar{s}$, $\mathbb{E}[p(X, S)|S = \bar{s}]$, is equal to y. (This last step is only relevant if $\Delta(\pi_1^{(X,\bar{s})}, a^*(\pi_0)) = c - \lambda$ with positive probability at $S = \bar{s}$.)

   The publication rules p and $\tilde{p}$ publish with the same probability as each other conditional on any choice S by the researcher. Hence, the researcher continues to be willing to pick $S = \bar{s}$. Moreover, given the constraint of publishing with probability y at $S = \bar{s}$, the journal's expected payoff given a researcher choice of $S = \bar{s}$ is maximized by p. Hence, the journal weakly prefers p to $\tilde{p}$ if the researcher is to choose $S = \bar{s}$.

3. At $s < \bar{s}$, $p(x, s) = 1$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) \geq c$ and $p(x, s) = 0$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) < c$:

   Let $p(x, s) = \tilde{p}(x, s)$ at $s \geq \bar{s}$; at $s < \bar{s}$, let $p(x, s) = 1$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) \geq c$ and $p(x, s) = 0$ if $\Delta(\pi_1^{(x,s)}, a^*(\pi_0)) < c$.

   Under publication rule p, the researcher will either continue to choose $S = \bar{s}$ or will switch to $s' < \bar{s}$. If the researcher continues to choose $S = \bar{s}$, then the journal's payoffs are as before. If the researcher now chooses $s' < \bar{s}$, we claim that the journal must be weakly better off. (This argument exactly follows an argument in the proof of Proposition 10.) To show the claim, recall that normal signals are Blackwell ordered by their standard errors: at standard error $S = s'$, the finding X can be garbled into something informationally equivalent to a finding from $S = \bar{s}$. So some stochastic publication rule at $S = s'$, combined with a garbling of these signals to the public, replicates the distribution of outcomes (probability of publication at each state, and joint distribution over public actions and states conditional on publication) that occur when a study arrives with $S = \bar{s}$ and is published under the publication rule given by $p(X, \bar{s})$. But the journal's payoffs given a study that has been published with $S = s'$ are improved by removing the garbling to the public. Payoffs are further improved by publishing under the interim-optimal publication rule at $S = s'$, which is exactly that under p.

The only remaining item to prove is that it is without loss of generality to suppose

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that if the researcher chooses to conduct a study, she chooses $S = \bar{s}$; applying step 3 above could possibly have changed the researcher's choice of S to something below $\bar{s}$. However, iterating step 1 (with $\bar{s}$ redefined to the new choice of S) recovers a publication rule of the appropriate form in which the researcher does choose $S = \bar{s}$. $\qquad \square$

Proof of Proposition 11. Recall that under normal priors and normal signals, the variance of $\pi_1^{(X,S)}$ is independent of X. So fix $S = s$, and without loss of generality normalize the variance of $\pi_1^{(X,s)}$ to 1. Then given $X = x$ and $\theta \sim \pi_1^{(x,s)}$, the distribution of a random variable $Y = (x - \theta)^2$ is a noncentral chi-squared distribution with noncentrality parameter λ (equal to $(x - \mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[\theta])^2$) that increases in $(x - \mu_0)^2$. The variable Y has CDF over realizations y given by $1 - Q_{1/2}(\sqrt{\lambda}, \sqrt{y})$ for Q the Marcum Q-function.[33] By Sun et al. (2010) Theorem 1(a), $Q_{1/2}(\sqrt{\lambda}, \sqrt{y})$ strictly increases in its first term $\sqrt{\lambda}$, implying that the distribution of $(x - \theta)^2$ under $\pi_1^{(x,s)}$ increases in the sense of FOSD as $(x - \mu_0)^2$ increases. Hence $\mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[\delta((x - \theta)^2)]$ increases in $(x - \mu_0)^2$. A study $(X, S) = (x, s)$ is published if and only if $\mathbb{E}_{\theta \sim \pi_1^{(x,s)}}[\delta((x - \theta)^2)] \leq b$, so at standard error $S = s$ studies are published only if $(X - \mu_0)^2$ is sufficiently small. $\qquad \square$

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[33]See Wikipedia for details: `https://en.wikipedia.org/wiki/Noncentral_chi-squared_distribution`.