INTERNATIONAL ECONOMIC REVIEW
Vol. 30, No. 1, February 1989

BUYBACKS, EXIT BONDS, AND THE OPTIMALITY OF DEBT AND LIQUIDITY RELIEF*

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We compare forms of market-based debt relief with coordinated debt forgiveness on the part of creditors. Debtors benefit from these schemes to the extent that expected payments fall, but lose to the extent that they sacrifice current resources. Creditors' benefit when the buyback resources are "additional," and when debt reduction creates greater incentives for full repayment. The source of buyback resources is a critical determinant in assessing potential welfare gains. For liquidity-constrained debtors, debt reduction is not optimal for debtors or creditors. For these countries the optimal debt-relief package (from the *creditors'* perspective) will include an infusion of new lending.

1. INTRODUCTION

Most observers agree that the muddling-through strategy of dealing with problem debtors is at a crossroads. The debtor economies have suffered through reform and severe contraction, yet their prospects are little improved. For their part, creditors have been unable to overcome the free-rider problem and together provide the new lending needed to sustain investment and growth. Exposures have fallen slightly, but at a cost of a steady deterioration in the quality of outstanding loans.

This has prompted calls by some observers for muddling-out: partially writing down creditor claims to make way for business as usual. Their argument is that high debt levels act like a tax on investment incentives. Partial forgiveness would provide more stimulus to growth and adjustment, and to the return of capital flight, and therefore could increase debt service. To use Krugman's (1989) terminology, the debt is so high that countries are on the wrong side of the "debt-relief Laffer curve."[2] Few debts, however, have thus far been forgiven. One reason may be that it is not in creditors' interest to give up their chance for full repayment. But it is hard to be sure because the same free-rider problem that has crippled new lending will also block a coordinated write down.

A different group of observers has sought instead to fill the debt-reduction void through market-based schemes, such as buybacks, buyouts, and exit bonds. This unlikely group includes advocates of the debtors, who are frustrated by the free-rider problem and attracted by the voluntary nature of these schemes, credi-

* Manuscript from June 1988; revised October 1988.

[1] This is a revised version of NBER Working Paper no. 2675. I thank Rudi Dornbusch, Paul Krugman, and especially Jeff Sachs for sharing many of their insights on these issues, Mike Dooley, Jonathan Eaton, Jeff Frankel, Peter Kenen, Richard Portes and Ken Rogoff for comments, and the Alfred P. Sloan and Ford foundations for financial support.

[2] Sachs (1988a, 1988b) made this argument originally.

tors, who believe they are better off under these schemes than under a write down, and investment bankers, for whom a market made is a penny earned.

Yet these market-based schemes are not well understood. Important papers by Helpman (1987), Dooley (1988) and Krugman (1988) have clarified the analytics of some of the market-based proposals. Nevertheless, general conclusions about the similarities and differences between buybacks, buyouts, exit bonds and pure forgiveness have not yet emerged.[3]

In the first part of this paper, we compare several market-based debt reduction schemes with pure debt forgiveness by creditors. In particular, we study how the outcomes of these schemes depend on the source of debt-repurchase resources. We consider four different sources: the creditors themselves, exogenous foreign aid, the debtor's future income, and the debtor's current endowment. Whether or not market-based schemes benefit creditors and/or debtors depends on the opportunity cost of these resources and the disincentive effects of a large debt overhang. In practice, these schemes must be evaluated from both debtor and creditors' perspectives on a case-by-case basis to determine their merit. There is no automatic answer as to who gains and by how much.

Despite their differences, these market-based debt relief schemes, as well as pure debt forgiveness, rest on a common feature: the disincentives to invest created by a substantial debt overhang. How important is this "incentive constraint" likely to be in practice? Since 1982 investment has fallen on average by 5 percent of GNP, exactly offsetting the rise in the noninterest external surplus (which roughly measures the reduction in liquidity).[4] In the meantime, the debt itself has grown only slowly. Liquidity constraints, not incentive constraints, are probably most responsible for the low levels of investment in many problem debtors. It would therefore be surprising if debt reduction alone would always be the optimal stimulus to investment.

The second part of this paper studies the role of liquidity in the design of an optimal relief plan. We find that countries that are severely liquidity constrained are good candidates for a debt reduction that will benefit all. That is, these countries are more likely to be on the wrong side of the debt-relieft Laffer curve. However, they are also the countries that can benefit least from a write down, since current resources are already so dear. We then show that by offering some current liquidity, creditors can induce a greater investment response and yet forgive less. In liquidity-constrained countries, pure debt relief alone will raise, but not maximize, the value of creditors' claims. Thus, relative to pure debt relief, creditors' optimal arrangement will supply less forgiveness, but more liquidity, and in doing so will also make the debtor better off.

Taken together, the paper's two parts suggest that in some cases market-based debt-relief schemes are in *no one's* interest. In other cases they will be Pareto

[3] Helpman (1987) provides a very general analysis of debt/equity swaps and debt forgiveness. Dooley (1988) discusses the pricing of buybacks and simulates their welfare effects. Krugman (1988) incorporates incentive effects and shows that marginal buybacks and exit bond offerings are equivalent to unilateral debt relief. See also Williamson (1988) and Bulow and Rogoff (1988).

[4] See the discussion in Dornbusch (1988), particularly table 3.10.

improving, but not Pareto optimal. In order to evaluate seriously the benefits of these schemes, more attention must be paid to the source of debt-relief resources and severity of liquidity constraints.

The paper has the following structure.[5] Section 2 in the text presents a formal model which incorporates the investment incentive effects we wish to study. We derive the equilibria associated with several debt-relief schemes in Section 3. Section 4 considers the impact of liquidity relief on creditors' optimal choice of debt reduction. Section 5 concludes.

2. A MODEL WITH INVESTMENT INCENTIVE EFFECTS

Several authors, most notably Sachs (1988a, 1988b) and Krugman (1988, 1989), have argued that the disincentive effects of an inherited debt may allow partial forgiveness to benefit both the debtor and its creditors. In this section, we build a more formal model to trace out the incentive effects which are critical to the success of debt relief schemes. The approach is deliberately simple, but our basic conclusions are far more general.

We consider a two-period model similar to that in Froot, Scharfstein, and Stein (1988). The debtor country derives welfare from the discounted sum of the utility of consumption in periods 1 and 2:

$$W = U_1(C_1) + \beta C_2, \tag{1}$$

where U_1 satisfies the Inada conditions, and $U_1' > 0$ and $U_1'' < 0$. The world discount factor is 1, and $\beta < 1$. We choose this special formulation for welfare in order to separate clearly the effects of risk aversion and intertemporal substitutability. Welfare is linear in period-two consumption in order to abstract from the risk-sharing issues considered by Helpman (1987). Naturally, these issues are important, but they complicate the algebra without adding to the intuitions below.[6] A major disadvantage of linear welfare, however, is that it implies an infinite elasticity of intertemporal substitution. By allowing for concavity in period-one utility, we can explore the implications of finite intertemporal substitutability without forcing preferences to be risk-averse.

The country enters the model with an endowment E, and an inherited debt, D. In period zero, the country announces its plans for a buyback. In period one, there is a competitive auction among creditors in which they exchange old debt for the new securities. Also in period one the country chooses a level of investment, I, which yields period-two output of $\tilde{y} = f(I) + \varepsilon$, where f also satisfies the Inada conditions, $f' > 0, f'' < 0$, and ε is a random variable with support $[\underline{\varepsilon}, \bar{\varepsilon}]$.[7] In period two, the country must make a payment on its outstanding obligations, $D - x$, where x is the old debt retired less any new securities issued. Put differ-

[5] Readers who are unfamiliar with market-based debt relief may want to refer to Appendix 1 of the NBER working paper version. It contains a primer on how buybacks and exit bonds work.

[6] Indeed, some of the propositions below go through with trivial modification for concave period-two subutility.

[7] If the price of output is uncertain, then the randomness would enter multiplicatively, rather than additively. The analysis below goes through in either case.

ently, x is the amount of effective debt relief. The investment incentives we wish to study are sharpest if we make the "gunboat-technology" assumption that the entire output, $\tilde{y}$, can be confiscated by creditors in the event of default.[8] Period-two payments are then

$$R = \min (D - x, \tilde{y}). \tag{2}$$

Under these assumptions, the country chooses investment to maximize its objective function, taking x as given,

$$W^* = \max_I U_1(E - I) + \beta \mathrm{E}(\max (0, \tilde{y} - D + x)), \tag{3}$$

where E is the expectations operator.[9] The last term in Equation (3) follows directly from the repayment assumption in (2). In good states, the country pays off its debt and gets to consume whatever remains. In all other states, the country cannot fully meet its debt service requirements, so the investment project's output is confiscated.

The country's first-order condition for investment is given by,

$$f'(I^*) = \frac{U_1'}{\beta G}, \tag{4}$$

where $G = G(I^*, x) = \int_{\varepsilon^*}^{\bar{\varepsilon}} g(\varepsilon)\, d\varepsilon$ is the probability that the country will reap some surplus from the project, and $\varepsilon^* = D - x - f(I^*)$ defines the level of output that exactly pays off the outstanding obligations. In some states a marginal increase in output is confiscated, creating a disincentive to invest. The factor $1/G > 1$ measures the investment distortion, the extent to which the marginal product of investment is greater than at the country's first-best level.

Equation (4) defines implicitly the optimal level of investment as an increasing function of x, $I^* = I^*(x)$.[10] As the overall debt payment falls, additional investment raises period-two consumption in more states of the world. The debtor perceives this as a higher return on investment, and therefore invests more.[11]

[8] Qualitatively, our analysis relies only on the assumption that the country sacrifices an amount that increases with the value of output when default occurs. The results would also hold were we to assume that creditors can impose penalties on the debtor in proportion to the value of output, but cannot actually confiscate output.

[9] We assume the endowment E is small enough so the country would be a borrower at the world interest rate were it not credit rationed. This assumption is critical for the incentive effects to have an impact on investment. See the discussion in Section 3.4 and footnote 26 below.

[10] We assume that $f(I)$ is sufficiently concave to make this statement true. Applying the implicit function theorem to (4), and using (3) yields

$$\frac{dI^*}{dx} = \frac{-f'\beta g(\varepsilon^*)}{\beta G f'' + U'' + \beta G (f')^2} > 0,$$

where the denominator is the second-order condition for the problem in (3).

[11] The stylized model of this section could easily be expanded to include an export and import sector, and a government which collects debt payments from the private sector through domestic income taxes.

3. A MENU OF DEBT-RELIEF SCHEMES

Where do the initial resources needed to generate the effective relief, x, come from? We consider four sources: 1) partial forgiveness from creditors, 2) aid from foreign governments, 3) output from the debtor's investment project, and 4) the debtor's endowment.

3.1. *Pure Debt Relief.* Suppose that creditors agree to write down their collective claims, an action we call pure debt relief. We take their choice of x as exogenous, and assume that the debtor sets investment optimally ($I = I^*(x)$). We return to how x is determined after discussing the following Proposition:

PROPOSITION 1. *Under pure debt relief, the welfare of the debtor, the welfare of creditors (taken together), and the price of a dollar's worth of the remaining debt, can be completely described by the amount of effective debt relief, x:*

(i) *Debtor welfare is given by*

$$W^*(x) = U_1(\mathrm{E} - I^*(x)) + U_2(I^*(x), x), \tag{5}$$

where $U_2(I^*(x), x) = \beta E(\max(0, f(I^*) + \varepsilon - D + x))$ and $dW^*/dx > 0$;

(ii) *Creditors' collective welfare is the market value of their claims:*

$$V^*(x) = E(\min(f(I^*(x)) + \varepsilon, D - x)), \tag{6}$$

where

$$\frac{dV^*}{dx} = (1 - G)f'\left(\frac{dI^*}{dx}\right) - G, \qquad \frac{dV^*}{dx} \in [-1, \infty]. \tag{7}$$

(iii) *The price of a marginal unit of debt after the debt relief takes effect, given by $1/\theta^*$, is the average market value of the debt:*

$$\frac{1}{\theta^*(x)} = \frac{V^*(x)}{D}. \tag{8}$$

Part (i) of the proposition shows that debtor welfare is an increasing function of the amount forgiven: Pure debt relief always makes debtors better off. Part (ii) shows that creditors are better off only when an increase in foregiveness increases expected payments, that is, when $dV^*/dx > 0$.[12] (Notice we assumed that creditors are risk neutral and that they know the optimal investment schedule of the country.) V^* is increasing when (7) is dominated by the first term, which represents the increase in expected payments due to a higher level of investment. The second term in (7), which is negative, measures the loss in full-repayment states as the face value of the debt falls. When the probability of full repayment is small, (7) is positive. Creditors gain from a reduction in contracted payments, with the

[12] The debt sells at a discount in the secondary market as long as the probability of full repayment is less than one, $G < 1$. In the neighborhood of $dV^*/dx = 0$, G is necessarily less than one.

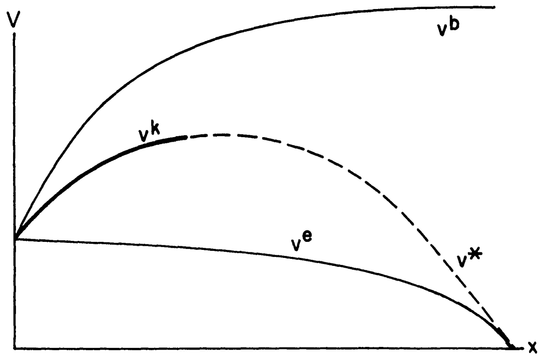

FIGURE 1
TOTAL VALUE OF CREDITOR CLAIMS

size of the gain proportional to the impact on investment of the change in incentives. On the other hand, when the probability of complete repayment is high, (7) is negative. $V^*(x)$ is therefore hump-shaped, as drawn in Figure 1. This, of course, is Krugman's (1989) debt-relief Laffer curve. The value of creditors' claims is maximized at the top of the curve, where $dV^*/dx = 0$. Pure debt relief is in the interest of *both* the debtor and its creditors when the country is on the wrong side of the Laffer curve.

As long as the country is on the right side of the Laffer curve, pure debt relief is not in creditors' collective interest. But, even worse, it is *never* in an individual's interest. A given creditor's claims can have only a small impact on V^*. Thus, conditional on other creditors ripping up their claims, an individual creditor would prefer to hold on to his. The free-rider problem therefore blocks pure debt relief. Even a country on the wrong side of the Laffer curve should not expect individual creditors of their own volition to set x such that $dV^*/dx = 0$.

The difficulty of getting creditors to act as a collective entity has spawned the market-based proposals we consider next. There are two important features that distinguish the equilibria envisaged in these proposals from that in pure debt relief. First, the country, and not the creditors' collective, acts by making a take-it-or-leave it buyback offer. Second, individual creditors must voluntarily participate in a market-based scheme. To be successful such schemes must therefore circumvent the free rider problem.

3.2. *Buybacks Out of Aid from Foreign Governments.* Suppose that another country donates resources earmarked exclusively for debt repurchase.[13] In period zero, the country informs creditors that in period one it will auction off these

[13] A country's ability to buy back debt on the secondary market is a matter of some *controversy*, although Bolivia recently completed a buyback of more than 1/2 of its debt. Syndication agreements

(*Continued on next page*)

resources in return for old debt. We assume that the debtor and its creditors rationally anticipate the optimal period-one investment response, $I = I^*(x)$, and that the auction is competitive. Let b represent the donated funds, given exogenously, and let $x = x(b) = \theta^b(b)b$ denote the face amount of old debt repurchased in the auction, i.e., the *effective* amount of relief generated by the buyback. The buyback equilibrium is summarized in the following proposition (and proven in the Appendix).

PROPOSITION 2. *When resources for a competitive buyback are donated, the equilibrium is characterized by:*[14]

(i) *Debtor welfare is the same as under pure debt relief:*

$$W^b(x) = W^*(x), \tag{9}$$

and is strictly increasing in amount of effective relief, $dW/dx > 0$.

(ii) *Creditors' collective welfare is greater than under pure debt relief by the amount of aid:*

$$V^b(x, b) = V^*(x) + b, \tag{10}$$

which is increasing in the amount of the buyback, $dV^b/db \geq 0$.

(iii) *The buyback takes place at a price where a marginal unit of debt after the buyback,* $1/\theta^b$, *is equal to the average market value of the debt remaining:*

$$\frac{1}{\theta^b(x)} = \frac{V^*(x)}{D - x}. \tag{11}$$

Effective relief is strictly increasing in b: $dx/db = d(\theta^b b)/db > 0$.

Notice the similarity between parts (i) and (ii) of Propositions 1 and 2. Buybacks funded by a third party reduce future debt payments. Debtors clearly gain. Ignoring the transfer, b, *these buybacks are equivalent to pure debt relief, for any given level of effective relief.* It is as if the aid goes directly to creditors in return for a write down of size x. The auction merely serves to translate a fixed amount of buyback resources, b, into effective debt relief, x. The larger the transfer, the more creditors gain. From an efficiency point of view, nothing is different from Proposition 1: Once the transfer is netted out, Pareto improvements are possible only if the country is on the wrong side of the debt-relief Laffer curve. Figure 1 graphs expected creditor payments under pure debt relief and a buyback out of

(Continued)
in commercial bank loans make buybacks problematic. These agreements contain a mandatory repayment clause, which stipulates that any prepayment must be distributed among creditors according to exposure, and a sharing clause which requires that any payment received by a creditor in excess of its exposure must be shared among all banks according to exposure. See the discussion in Section 3.5 below.

[14] Dooley (1988) discusses some of these results.

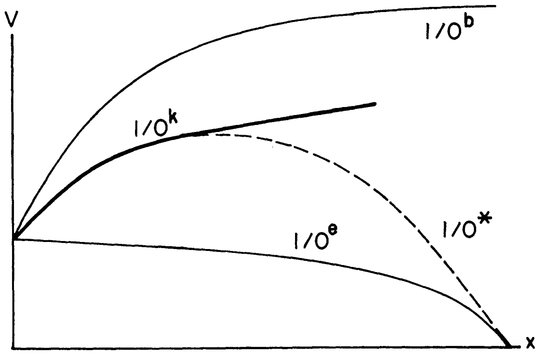

FIGURE 2
PRICE OF REMAINING DEBT

aid, V^* and V^b, respectively. While V^* is hump-shaped, V^b is concave and increasing everywhere, which reflects the added value of the transfer, b.

Part (iii) of the proposition says that the buyback price is the inverse of the expected value of the last unit of old debt repurchased. If the auction is competitive the price, $1/\theta^b$, must be such that individual creditors are indifferent between holding onto their old debt, and trading it in for cash. Thus in equilibrium, the expected payoff from holding θ^b units of old debt must be one: $\theta^b[V^*(x)/(D-x)] = 1$, which is just Equation (11).

Figure 2 shows a graph of the unit price of remaining debt, $1/\theta^b(x)$. Before the buyback announcement, the price is as in Proposition 1, $1/\theta^b(0) = 1/\theta^*(0)$. The price of remaining debt rises with the size of the buyback for two reasons. First, as debt is bought back, the quality of the remaining obligations improves. Second, the country invests more as it gains surplus in better states of nature, further improving the remaining debt's quality. The interaction of these two factors determines the concavity (or convexity) of the curve. The frequency distribution, $g(\varepsilon)$, determines how quickly increases in b raise the probability of complete repayment. When $g(\varepsilon)$ is increasing, the price curve tends to be convex. On the other hand, the country's investment response, $f'(dI^*/dx)$, is decreasing (due to the concavity of f), which tends to make the price curve concave. If ε is uniformly distributed and f is concave, then the path of the price will resemble the concave curve in Figure 2.[15] Finally, when the buyback is large enough to retire completely the outstanding debt, $\lim_{x \to D} \theta^b(x) = 1$, the entire debt can be repurchased only at its full face value.[16]

Finally, part (iii) shows that the amount of effective relief increases with the size

[15] See Dooley (1988), who discusses in detail the impact of alternative frequency distributions on buyback pricing.

[16] This will be the case as long as the last unit of debt is riskless, i.e., if output is positive in all states, $f(I^*) + \underline{\varepsilon} > 0$.

of the buyback, even though the rate at which old debt is exchanged, θ^b, falls. An increase in the size of the buyback, b, therefore must raise the welfare of both the debtor and its creditors. This is true whether or not the country is on the back side of the Laffer curve.

3.3. *Buybacks Out of Future Cash Flows, or Exit Bonds.* Next we consider the case in which old debt is repurchased by issuing senior claims to future cash flows. We call these claims exit bonds. If these bonds are to be senior to the existing debt, every creditor must agree to honor their seniority before the auction takes place.[17] For the moment we assume the seniority of these bonds, but we return to whether creditors would in fact grant it. We do not require, however, that the exit bonds are riskless.

In period zero, the country announces the face amount of exit bonds it plans to issue, given by k, and simultaneously asks creditors to make the bonds senior. As in the previous section, we assume that everyone rationally anticipates the optimal period-one investment response, $I = I^*(x)$, and that the auction is competitive. The period-one auction retires $\theta^k k$ in face value of old debt. The amount of effective debt relief—the reduction in the total face value of old debt less the face value of the exit bonds—is then $x = x(k) = (\theta^k(k) - 1)k$. Because they are senior, exit bonds are more valuable than the same face amount of old debt. Thus $\theta^k(k) > 1$, and exit bonds generate effective relief, $x(k) > 0$, $\forall\ k > 0$.

The exit-bond equilibrium is summarized in the following proposition, with proofs in the Appendix.

PROPOSITION 3. *When resources for a competitive buyback come from future cash flows, the equilibrium must satisfy:*

(i) *Debtor welfare is the same as under pure debt relief:*

$$W^k(x) = W^*(x), \qquad \forall\ x. \tag{12}$$

(ii) *Creditors' collective welfare is the same as under pure debt relief:*

$$V^k(x) = V^*(x), \qquad \forall\ x. \tag{13}$$

(iii) *If the buyback is small enough to be riskless—that is, if $f(I^*) + \underline{\varepsilon} > k$—then the buyback price, θ^k, is equal to the price under pure debt relief:*

$$\theta^k(x) = \theta^*(x), \qquad \forall\ x. \tag{14}$$

The greater the exit bond offering, the greater the level of effective relief, $dx/dk > 0$.

(iv) *If the buyback is not riskless, then the equilibrium price solves:*

[17] Under the sharing clause, mentioned in footnote 13, individual creditors can sue for their share of payments. New securities can therefore be effectively senior only if creditors agree unanimously to waive the sharing clause. A single "holdout" creditor will undermine the assurance of other creditors that they will be able to keep their exit-bond repayments.

$$(15)\quad \theta^k(k) = \left(\frac{D - \theta^k k}{E(\max(0 \min(f(I^*) + \varepsilon, D - \theta^k k)))}\right)\left(\frac{E(\min(f(I^*) + \varepsilon, k))}{k}\right),$$

where $\theta^k(k) < \theta^*(x(k)), \forall k.$

Note that in parts (i) and (ii) of the proposition welfare is a function of x only, regardless of the size of the exit bond offering. The auction generates effective debt relief by distinguishing between senior and subordinate claims, but has no other effects on the debtor or its creditors. After all, exit bonds do not represent new funds that have become available for debt service. They merely repackage the existing claims in a way that is advantageous to the debtor. Thus, *for any given amount of effective relief, an issue of exit bonds is equivalent to pure debt relief.*[18]

Part (iii) of the proposition shows that the repurchase price, $1/\theta^k$, is purely a function of effective relief. Indeed, the price is exactly equal to that which would prevail if creditors granted an equivalent amount of pure debt relief. Part (iii) applies only to riskless exit bonds, however. The price of the larger, risky bond issues in part (iv) is not generally a function of effective relief alone. The riskier the bond issue, the *relatively* less risky is original debt. *Ceteris paribus*, the repurchase price must rise above what it would have been were the exit bonds riskless [$1/\theta^*(x(k))$]. Figure 2 graphs the repurchase price. The $1/\theta^*(x)$ and $1/\theta^k(x)$ curves separate at the point where the bond offering becomes risky.[19]

It is worth dwelling for a moment on how the swap rate, $\theta^k(k)$ evolves. Consider the impact on the value of creditors' claims of an increase in the size of the bond offering. Using (7) and (13):

$$(16)\qquad \frac{d\theta^k}{dk} = \left(\frac{-D}{(V^k)^2}\right)\left((1 - G)f'\left(\frac{dI^*}{dx}\right) - G\right)\frac{dx}{dk}.$$

Suppose for a moment that investment is fixed, $dI^*/dx = 0$, so that the first term in (16) is zero. Then larger exit bond offerings *lower* total expected payments. How is it that a strictly positive exit bond offering reduces expected payments without any change in the total resources available for debt service? Because creditors are competitive, subordination of old debt creates an externality: As creditors swap in old debt for senior exit bonds, they degrade the value of remaining old debt. At the price of the first increment of the buyback, $1/\theta^k(0) = 1/\theta^*(0)$, each creditor strictly prefers to swap in old debt: Conditional on no other creditors swapping, each creditor is indifferent between swapping and not swapping, and conditional on all other creditors swapping, each creditor is strictly better off by swapping in old debt. The resulting excess supply drives up the price

[18] Krugman (1988) discusses this equivalence and presents results for small buybacks.

[19] A loose intuition for this result is as follows. When exit bonds are risky, a marginal increase in the offering may increase the riskiness of the exit bonds substantially, without changing much the riskiness of the old debt. (In terms of the model, this means the density function—evaluated at the point where there is just enough output to service the outstanding exit bonds, $g(\varepsilon')$, where $\varepsilon' = k - f(I^*)$—is large.) The relative riskiness of the old debt would then *improve* rapidly, so that $1/\theta^k$ would rise sharply.

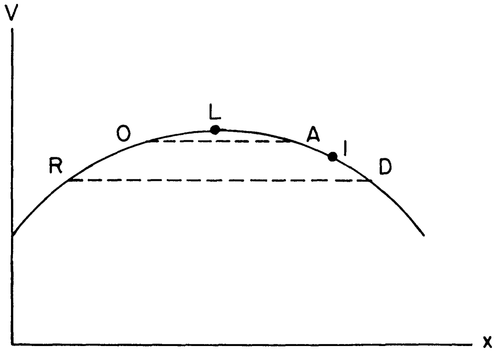

FIGURE 3
EXIT BOND OFFERINGS

of the exit bond in terms of old debt (θ^k rises). Now if we allow investment to respond to effective relief, the first term in (16) becomes positive. The excess supply of old debt at $\theta^k(0)$ is smaller. When the country is on the wrong side of the Laffer curve, the investment response dominates the subordination effect, and θ^k actually falls with k.

Exit bond equilibria. While it is clear from Proposition 3 that exit bonds and pure debt relief have many similarities, it is their differences that explain exit bonds' popularity. First, there are differences between the free-rider and seniority problems. Consider an individual creditor's decision about whether to grant seniority when a country is at a point like O in Figure 3. Suppose the country announces a small issue of exit bonds, and other creditors agree to treat the bonds as senior. If the individual creditor refuses to grant seniority, then the exit bonds will be perfect substitutes for old debt. In that case, the equilibrium generates no relief, $\theta^k = 1$ and $x = 0$, and the individual creditor's claims do not change in value. If, on the other hand, this individual creditor agrees to the subordination of old debt, the value of its claims will rise marginally as the country moves up the Laffer curve. As long as the country is on the wrong side of the Laffer curve, each creditor will find that granting seniority is a dominant strategy. (When the country is on the right side of the Laffer curve—point A in Figure 3—the argument runs in reverse; refusing seniority is the dominant strategy.) Because individual creditors are not small with respect to seniority, exit bonds break the free-rider barrier to debt relief.

A second difference between pure debt relief and an exit bond offering is the amount of effective relief generated in equilibrium. Consider again a country at point O in Figure 3. If creditors were to coordinate and write down their claims, then they would choose x such that $dV^*/dx = 0$. The country moves to point L—the top of the Laffer curve. Under exit bonds, however, the country chooses the amount of effective relief, and would like to set x as high as possible.[20] But there are limits. If the country announces an offering so large as to lower the value of creditors' claims, moving, say, from point L to I in Figure 3, creditors will not grant seniority, and the exit bond offering will generate no effective relief. The country will abide by the individual creditor's rationality constraint, so it sets k such that $V^k(x(k)) \geq V^k(0)$. At a point like O, the debtor chooses x such that $V^k(x^*) = V^k(0)$—across the Laffer curve at point A. Assuming the exit bond offering is small enough to be riskless, Equation (14) implies that the equilibrium price of the remaining debt will be $1/\theta^k(x^*) = 1/\theta^k(0) = V^*(0)/D$, the pre-existing price of a unit of old debt. The face value of the optimal offering follows directly: $k^* = x^*/(\theta^k(0) - 1)$. Thus in equilibrium *exit bonds can generate more effective relief than pure debt relief.*

It would appear that an exit bond equilibrium provides at least as much effective relief as pure debt relief, and sometimes strictly more. Do exit bonds dominate pure debt relief from the country's point of view? In general the answer is no. While the exit bond equilibrium yields greater effective relief than a pure debt relief equilibrium in the neighborhood to the left of point L, the neighborhood may be small. A country that starts out at point R will *not* be able to generate enough relief to reach point D.

To see this note that the exit bond issue can, at most, retire the entire outstanding debt.[21] The precise level of k that exhausts the old debt has no closed form solution. But the important point is that the further to the left of the Laffer-curve peak the country starts out, the more likely it is to run out of old debt *before* reaching the other side. Indeed, the country may run out of old debt before reaching the top.[22]

3.4. *Buybacks Out of the Country's Endowment.* Propositions 1 through 3 have stressed the similarities between buybacks and pure debt relief. The schemes we have considered—pure debt relief, buybacks out of aid, and exit bonds—are all ways of releasing resources to the country in the second period. Their common feature is that the relief funds become available in the same period in which they are used. In this section we turn to a different source of funds for buybacks: the country's current resources. We will see that these buybacks have an intertemporal effect on investment incentives, and they therefore fail to be equivalent to pure debt relief.

We now assume that the country must finance the debt repurchase using its period-one endowment, E. This pertains to a country that purchases the debt with reserves (savings), or raises taxes on current consumption. Of course, in a maximizing model, such a distinction is irrelevant. The country will spread any

[20] The upper bound on the amount of effective relief a country can obtain by offering riskless exit bonds is given by the point at which all of the old debt is retired: $x_{max} = (\theta^k - 1)k_{max}$, and k_{max} is such that $\theta^k k_{max} = D$.

[21] See footnote 20 above.

[22] Note that in Figure 2, the $1/\theta^k$ curve stops when all of the old debt is retired, before reaching $x = D$.

reduction in period-one resources optimally across consumption, saving, and investment.[23]

In period zero the country announces its buyback, e, out of the initial endowment, E. Let the buyback price be given by θ^e and effective relief by $x = x(e) = \theta^e e$. Once the resources for the buyback are fixed at e, the country's investment problem is

$$\max_I U_1(\mathrm{E} - e - I) + \beta E(\max(0, y - D + x)), \tag{17}$$

with the first-order condition again given by Equation (4). Inspection of (17) and (4) shows that optimal investment is no longer completely summarized by effective relief. We now denote optimal investment by $I^{**} = I^{**}(x(e), e)$. The following proposition is proven in the Appendix.

PROPOSITION 4. *For any given level of effective relief, x, the investment incentives associated with a buyback out of current resources are smaller than under pure debt relief:*

$$I^{**}(0, 0) = I^*(0), \tag{18}$$

$$I^{**}(x(e), e) < I^*(x), \qquad \forall\, x, e > 0, \tag{19}$$

$$\frac{\partial I^{**}(x, e)}{\partial e} < 0, \qquad \forall\, x. \tag{20}$$

The intuition for Proposition 4 is straightforward. A buyback out of current resources must lower the available endowment (by e) in order to generate positive effective relief. When $\mathrm{E} - e$ falls, the marginal utility of period-one consumption must rise. The marginal return on investment then rises above what it otherwise would have been. Investiment is lower than if the buyback resources came from elsewhere. Indeed, *these intertemporal considerations can dominate the investment-incentive effects, so investment falls with an increase in the size of the buyback.*[24]

The buyback is characterized in the following proposition.

PROPOSITION 5. *For a given level of effective relief, a buyback out of the period-one endowment implies:*

[23] If the period-one consumption decision is made *before* reserves are used for the buyback, then the buyback will have no effect on period-one consumption. But this timing would also imply that the buyback cannot have an effect on investment either.

[24] The results in Proposition 4 are fairly general. Even though it is doubtful that a country would finance the entire buyback out of period-one resources, the proposition holds as long as a *portion* of the buyback resources comes from the period-one endowment and the remainder comes from one of the sources discussed in Sections 3.1 through 3.2. Investment falls as the size of the buyback increases if the subutility, U_1, is sufficiently concave. For small buybacks (i.e., $e = 0$) the condition for this is:

$$\frac{-U_1''}{U_1'} > \frac{\theta^*(0)g(\varepsilon^*)}{G},$$

which can be thought of as a condition on the coefficient of absolute risk aversion.

(i) *Debtor welfare is lower than under pure debt relief:*

(21) $$W^e(x, e) = U_1(\mathrm{E} - e - I^{**}(x, e)) + U_2(I^{**}(x, e), x) < W^*(x), \qquad \forall x, e > 0,$$

where $\partial W^e/\partial e < 0$.

(ii) *Creditors' collective welfare is lower than under an equivalent buyback out of aid:*

(22) $$V^e(x, e) < V^b(x, e) = V^*(x) + e, \qquad \forall e > 0.$$

(iii) *The rate at which old debt is exchanged,* θ^e, *is greater than the corresponding rate for a buyback out of aid:*

(23) $$\theta^e(x, e) = \frac{D - x}{V^e(x, e)} > \theta^b(x), \qquad \forall x, c > 0.$$

The equivalence in Propositions 1 through 3 between pure debt relief, buybacks out of aid, and buybacks out of future cash flows does not carry over to buybacks out of current resources. In these earlier propositions, the debtor was always better off after the buyback. There are now two opposing effects. Debt relief still raises debtor welfare, $\partial W^e/\partial x > 0$, and improves investment incentives, $\partial I^{**}/\partial x > 0$, but in the buyback process current resources are sacrificed, $\partial W^*/\partial e < 0$. Either of these terms may dominate, so that the overall effect on debtor welfare is ambiguous. Effective debt relief can easily come at too high a cost: *The country's optimal buyback may be zero.*[25] The country is always worse off under a buyback out of current resources than under a buyback out of future receipts.

Creditors' interest in such a buyback is similarly ambiguous. On the positive side, creditors receive the transfer e from the country's endowment. Because these resources would not have otherwise been used for debt service—that is, they are "additional"—creditors gain. On the negative side, the sacrifice of current resources implies less investment, $\partial I^{**}(x, e)/\partial e < 0$, and this reduces the value of remaining debt. For countries with a relatively large endowment, the additionality of the resources outweighs the negative liquidity effect, so that creditors gain from a buyback out of current resources. This can be true even when the country loses. *For countries that have small initial endowments, however, the* sacrifice *of current resources may reduce investment so much that the value of claims falls,* $dV^e/de < 0$. Even creditors may not be able to gain from a buyback out of current resources.[26]

[25] The condition for a small buyback out of current resources to lower country welfare is $U_1' > \beta G\theta^*(0)$, which from the first-order condition (4) is equivalent to $f' > \theta^*(0)$. If f satisfies the Inada conditions, then the above condition will be met for sufficiently low E. Even if investment is zero, the debt will have value as a claim to the random variable ε. Thus while $\lim_{I \to 0} f' = \infty$, the price remains bounded, $\lim_{I \to 0} \theta(0) = M < \infty$.

[26] This requires a condition stronger than that given in footnote 24. Intuitively, period-one subutility must be even more concave: Investment must not only fall with e_1 it must fall rapidly enough for the value of the remaining debt to decline.

In terms of Figures 1 and 2, $V^e(x, e)$ and $1/\theta^e$ must lie below $V^b(x, b)$ and $1/\theta^b$, respectively. (Note that we have drawn the pessimistic case in which the curves do not even rise.) Because investment may actually fall with e, there is no guarantee the price will be increasing in the amount of the buyback, even when the country is on the back side of the Laffer curve. *In spite of potent investment-incentive contraints, the "Laffer curve" for a buyback out of current resources can be flat, or even declining everywhere.*

In sum, the effect on both creditors and debtors of a buyback out of current resources is ambiguous. This is true even if the country is initially on the wrong side of the debt-relief Laffer curve. But note that since the buyback resources are additional, creditors are more likely to benefit than are debtors.

3.5. *Assessing Buybacks Versus Pure Debt Relief.* Our analysis has highlighted both the similarities and differences between market-based schemes and pure debt relief. The differences are not only a result of the mechanics of each scheme, but also of the conditions needed to make the scheme workable.

Clearly, the free-rider problem will be a substantial barrier to pure debt relief, even when the country is on the wrong side of the debt-relief Laffer curve. The foregoing buyback proposals could be alternatives when creditors fail to coordinate. Nevertheless, they may be unworkable in practice. Buybacks out of aid will make both creditors and debtors better off, but at the expense of the donor. This makes large-scale buybacks for the major debtors a remote possibility.[27] While none of the buyback proposals is subject to the free-rider problem, all nevertheless require a measure of coordination among creditors. Creditors would have to design and then agree unanimously to a wavier of the sharing and mandatory prepayment clauses. This would necessitate negotiation among creditors and the input of legal resources. Because the syndicates include banks from all over the world, it is not clear who would enforce the waiver, or whether it would be enforceable at all.

Assuming buybacks are workable, market-based schemes may be best for some countries, even in the absence of a large donor. We saw that a successful exit bond offering could conceivably take a country beyond the top of its debt-relief Laffer curve, where it is better off than under pure debt relief. Under other circumstances, however, exit bonds would not allow the country to reach the top. The informational requirements in determining the optimal size of a bond offering and how far along the Laffer curve it would take the country are formidable. As Krugman (1988) has stressed, the investment incentive effects behind the Laffer curve's upward slope are inherently hard to measure.

In practice, the chance is small that market-based schemes would be preferred to pure debt relief from the countries point of view. Almost inevitably, an exit bond offering would use some current reserves as collateral—as in the recent Mexican case. Then the results of Section 3.4 apply, so the buyback may hurt the

[27] Note from Figure 1 that creditors do best under a buyback out of aid. Bulow and Rogoff (1988) point out that as long as there is a chance of such a buyout, creditors have an incentive to block other types of debt-reduction schemes.

debtor. It is important to note that reserves should be thought of as current, and not future resources, even if they are unavailable for current consumption. (In other words, a buyback out of reserves is not equivalent to a buyback out of future cash flows.) When a credit-constrained country holds reserves, the shadow return on foreign exchange is likely to be higher than the return on physical investment. Given that an increase in effective debt relief implies a lower probability that the reserves will be needed for future debt service, a marginal increase in debt relief does *not* generate more investment, it merely increases desired holdings of reserves. Buybacks out of reserves will reduce investment-incentive effects, as will buybacks out of other current resources.[28]

4. INCENTIVE VERSUS LIQUIDITY EFFECTS ON INVESTMENT

We have so far seen two problems with market-based debt relief schemes. First, relief cannot be Pareto improving unless the country is on the wrong side of the debt-relief Laffer curve. Second, if the country sells current resources in return for forgiveness, Pareto improvements may not be possible regardless of where on the Laffer curve the country is.

We have also seen that future incentives are not the only factor determining investment. In Section 3.4, the usual investment response to debt relief is distorted by the use of current resources. Countries may be liquidity constrained in addition to being incentive constrained. This suggests that optimizing creditors would adjust the level of both debt and current liquidity.

It is not new to argue that creditors have an interest in providing sufficient liquidity to problem debtors. Sachs (1984) and Krugman (1985) study the role of liquidity in averting default. If a debtor is about to declare default, it makes sense to lend at a loss today in order to retain the chance of full repayment tomorrow. The incentive-constraint argument for promoting sufficient liquidity is, however, different: By taking advantage of high-return projects that otherwise would have been foregone, additional lending stimulates investment and allows countries to pay more in the future. In this case, there is no choice between either financing or forgiving; there is an optimal combination of the two.

4.1. *Optimal Liquidity and Debt Relief.* In this section we study a simple optimal liquidity-and-debt contract from creditors' point of view. We then compare the results of this optimal contract with creditors welfare under pure debt relief.

[28] We can easily make this point in the model above. Consider a case in which the endowment is large enough for the country to hold reserves which earn the world interest rate. Then the choice of reserves and investment is given jointly by Equation (4) and the first-order condition that reserves earn the world rate of interest:

$$1 = \frac{U_1'}{\beta G}.$$

As long as reserves are strictly positive, then the first-order conditions together imply $f' = 1$. The optimal level of investment is constant and, therefore, debt relief has no impact on investment.

We employ a variant of the model in Section 3, with only two changes. First, we leave out the uncertainty in production since it is no longer essential. Output is simply $y = f(I)$. Second, the creditors will make a take-it-or-leave-it offer which consists of a period-two repayment, D, and an injection of liquidity, L, in period one. The initial contractual debt is given by $D_0 \geq D$. In this simple framework, the country must first decide whether to invest. If it invests, the optimal level of investment, $I^* = I^*(L)$, is given by the first-order condition

$$(24) \qquad f'(I^*) = \frac{U_1'(\mathrm{E} + L - I^*)}{\beta},$$

where, as before, we assume that the country is credit constrained, $f' > 1$. By the implicit function theorem, the country invests only a portion of any additional liquidity, the rest is consumed:

$$(25) \qquad \frac{dI^*}{dL} = \frac{U_1''}{\beta f'' + U_1''} < 1.$$

Notice that creditors have no control over how the country divides the new liquidity between investment and consumption. If "conditionality" were applied, forcing the country to invest a larger-than-desired share of L, then the argument for liquidity relief would be even stronger.

The fact that (25) is positive implies that the most severely liquidity-constrained debtors will have the lowest chosen levels of investment. Debt relief increases investment from 0 to $I^*(L)$; liquidity-constrained countries will therefore gain less from pure debt relief than countries with more liquidity.

The country will invest only if it gains from doing so. Its rationality constraint requires that welfare with investment is greater than welfare with no investment:

$$(26) \qquad U_1(\mathrm{E} + L - I^*) + \beta(f(I^*) - D) \geq U_1(\mathrm{E} + L),$$

where we again assume that the period-two repayment is min (y, D). Equation (26) implies that for any given amount of liquidity, creditors will maximize the value of their claims by lowering the debt payment to

$$(27) \qquad D(L) = \frac{U_1(\mathrm{E} + L - I^*)}{\beta} - \frac{U_1(\mathrm{E} + L)}{\beta} + f(I^*).$$

Equation (27) says that if creditors write down the debt, they will do so to be at the top of the debt-relief Laffer curve. Given L, lower values of D imply a one-for-one reduction in expected payments, while higher values imply expected payments fall to zero. The function $D(L)$ defines a family of debt-relief Laffer curves, one for each L.

It is easy to show that the debt payment is an increasing function of liquidity, $D'(L) > 0$.[29] Greater liquidity raises the optimal level of investment and, there-

[29] The envelope theorem implies that

$$\frac{dD(I^*(L), L)}{dL} = \frac{\partial D(I^*, L)}{\partial L} = \frac{U_1'(\mathrm{E} + L - I^*)}{\beta} - \frac{U_1'(\mathrm{E} + L)}{\beta} > 0,$$

because marginal utility is higher when investment crowds out current consumption.

fore, increases the payment creditors can extract. It follows that *countries that are more liquidity constrained are more likely to be on the wrong side of the debt-relief Laffer curve.* Figure 4 demonstrates, showing three Laffer curves with different underlying levels of liquidity, $L_2 > L_1 > L_0$. As the country is more illiquid, the Laffer curve shifts down (since from (25), $dI^*/dL > 0$), and the peak shifts toward the left (since $D' > 0$).[30] Suppose the debt is initially at D_0. Then it is clear that if the country has liquidity equal to L_2, pure debt relief will not be in creditors' interest. On the other hand, if the country is severely liquidity constrained, at $L = L_0$, then there is scope for pure debt relief. The irony is that countries with weak investment-incentive effects are also the most likely recipients of pure debt relief.

Fortunately for all, creditors may gain by adjusting the level of liquidity. They will not, however, choose the L that gives the highest Laffer curve. They will instead set the pair $\{D, L\}$ to maximize the discounted value of cash flows, $D - L$. Since creditors can collectively choose to set $L = 0$ and still receive a period-two repayment (by setting $D = D(0)$), any new lending must be profitable in itself. Notice, however, that as long as the initial debt, D_0, is high enough, the free-rider problem remains: An individual creditor would prefer not to write down his portion of the debt in the first place, even when others do.[31] We then have the following proposition, which is proven in the Appendix.

PROPOSITION 6. *The optimal contract, $\{D^*, L^*\}$, solves:*[32]

$$f'(I^*) = 1 + \frac{U'(\mathrm{E} + L^*)}{\beta}, \tag{28}$$

$$D^*(L^*) = \frac{U_1(\mathrm{E} + L^* - I^*)}{\beta} - \frac{U_1(\mathrm{E} + L^*)}{\beta} + f(I^*), \tag{29}$$

where I^* is given by Equation (24).

The intuition for this contract can be seen in Figure 4. Suppose the country has an initial obligation D_0 and liquidity $L_0 = 0$. The expected value of the debt payment is shown by point A. Pure debt relief (or one of the buyback scenarios discussed in Section 3) can move the country to the top of the L_0 Laffer curve, point B. The improvement in incentives raises debtor welfare and investment and

[30] Note that the horizontal axis in Figure 4 is D.

[31] Some debt relief is required before profitable lending can be undertaken. If debt relief were not needed, then there would be no free-rider problem; individual creditors would find it in their interest to lend, regardless of the behavior of others.

[32] We assume that the country is sufficiently liquidity constrained to satisfy the second-order condition for this problem,

$$\frac{f''U_1''(\mathrm{E} + L^* - I^*)}{\beta f'' + U_1''(\mathrm{E} + L^* - I^*)} - U_1''(\mathrm{E} + L^*) < 0.$$

This condition holds, for example, for isoelastic utility and production functions at sufficiently low levels of the endowment, E.

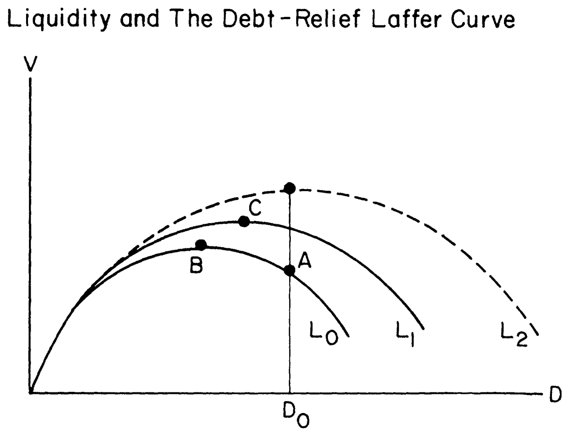

FIGURE 4
LIQUIDITY AND THE DEBT-RELIEF LAFFER CURVE

reduces current consumption. But since marginal utility rises (see equation 4), the return on investment will not fall as much as the improved incentives merit. The country will therefore be unwilling to undertake all of the investment projects that become profitable at world interest rates. For the liquidity constrained country, we would have such a high marginal return on investment that $f'(I^*) > 1 + U'(E + L_0)$.[33] Creditors can capture a surplus above the world interest rate on additional investment by providing liquidity while *reducing* (by more) the amount of debt relief. This shifts the value of the claims from point B to C. Note that creditors would be strictly worse off if debt relief and new lending were negotiated separately, because then the new lending would be competitive. Creditors obtain the surplus by offering to provide simultaneously new lending and debt relief. Provided the second-order conditions above hold we have:

PROPOSITION 7. *The more liquidity-constrained the country is, the more creditors sacrifice with simple debt-reduction schemes in comparison with the optimal liquidity and debt relief given in Proposition 6.*

5. CONCLUSIONS

Our four main conclusions can be stated as follows:

1) Market-based debt relief schemes are similar to pure debt relief in the sense

[33] This condition is equivalent to:

$$\frac{U_1'(E + L^* - I^*)}{\beta} - \frac{U_1'(E + L^*)}{\beta} > 1,$$

which will be satisfied for low enough E and, for example, isoelastic production and utility functions.

that they reduce the debt overhang. These plans can therefore be Pareto improving only if investment-incentive effects are sufficiently important.

2) Market-based plans differ from pure debt relief, and from one another, according to the source of resources used to retire old debt. Creditors' preferred market-based scheme is a buyback out of aid, followed either a buyback out of current resources or (if the country is on the back side of the Laffer curve) a buyback out of future receipts. Debtor's preferred scheme is a tie between a buyback out aid and a buyback out of future receipts.

3) If investment-incentive effects are important enough to make debt reduction profitable for creditors, then debt reduction alone will not generally be optimal from the creditor's perspective. Thus neither market-based schemes nor pure debt relief will generally maximize the value of creditor claims.

4) In general, countries that are liquidity-constrained are the best candidates for an optimal relief package which includes new lending as well as partial debt forgiveness.

These conclusions are relatively general, and are likely to come out of more realistic, and more complicated models of the investment process. We have abstracted from such issues as capital flight, the debtor's internal financing constraints, and how creditors impose penalties in instances of default. Nevertheless, we believe that our general conclusions will remain when these issues are considered explicitly. We have also ignored the adverse selection problems which naturally arise once debt relief is on the table (Froot, Scharfstein and Stein, 1988, study these problems).

Finally, our analysis takes as given the presence of investment-incentive effects. The specification of these incentives should not be taken too literally. To be fair to reality, these effects should be interpreted in the broadest possible sense. They include concerns about debtor-government taxation as well as penalties imposed by creditors.[34] They might also include the uncertainty about (not just the expectation of) future creditor and/or debtor-government policies. These uncertainties may discourage physical investments which are costly to reverse and encourage capital flight and investment in other nonproductive liquid assets.[35] To the extent lower levels of debt reduce these uncertainties, the above analysis applies. Nevertheless, there is thus far no empirical evidence that suggests incentive effects are important, or present at all.

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APPENDIX

Proof of Proposition 2.

(i) Recall that effective relief is equal to the amount of old debt retired in the buyback, $x = \theta^b b$. The debtor welfare is given by:

$$U_1(\mathrm{E} - I^*(x)) + U_2(I^*(x), x) = W^*(x).$$

[34] See Rotemberg (1988).

[35] See Froot and van Wijnbergen (1986).

By the envelope theorem,

$$\frac{dW^*(I^*(x), x)}{dx} = \frac{\partial W^*(I^*(x), x)}{\partial x} = \frac{\partial U_2(I^*(x), x)}{\partial x} = \beta \int_{\varepsilon^*}^{\bar{\varepsilon}} g(\varepsilon)\, d\varepsilon = \beta G > 0.$$

Thus debtor welfare increases with the amount of effective relief. Next we show that the amount of effective relief increases with the size of the buyback, $dx/db > 0$:

$$\frac{dx}{db} = \theta^b + \frac{d\theta^b}{dx}\frac{dx}{db} b = \frac{\theta^b}{1 - b(d\theta^b/dx)}.$$

From Proposition 1 (ii) and Proposition 2 (iii):

$$\frac{d\theta^b}{dx} = \frac{-\theta^b\left(\dfrac{dV^*}{dx}\right) - 1}{V^*}.$$

Since for all x,

$$\theta^b = \frac{D - x}{E(\min(D - x, f(I^*) + \varepsilon))} \geq 1, \quad \text{and} \quad \frac{dV^*}{dx} \in [-1, \infty]$$

then $d\theta^b/dx < 0$, and it follows that $dx/db \geq 1$.

(ii). To see that creditor welfare increases in the size of the buyback, note that:

$$\frac{dV^b}{db} = \left(\frac{dV^*}{dx}\right)\left(\frac{dx}{db}\right) + 1.$$

Since $dx/db > 1$ and $dV^*/dx \in [-1, \infty]$, it follows that $dV^b/db > 0$.

Proof of Proposition 3.

(i). Debtor welfare under an exit bond offering is given by:

$$W^k(x) = U_1 + \beta E(\max(0, f(I^*) + \varepsilon - D + (\theta^k - 1)k) = W^*(x).$$

(ii). The value of creditors' claims under an exit bond offering is given by:

$$\begin{aligned} V^k &= E(\max(0, \min(f(I^*) + \varepsilon - k, D - \theta^k k)) + k) \\ &= E(\min(f(I^*) + \varepsilon, D - x)) = V^*, \end{aligned}$$

where the first equality follows from the definitions of exit bonds and limited liability, and the second equality is by algebra.

(iii) and (iv). The competitive auction requires:

$$\theta^k\left(\frac{E(\max(0, \min(f(I^*) + \varepsilon, D - \theta^k k)))}{D - \theta^k k}\right) = \frac{E(\min(f(I^*) + \varepsilon, k))}{k},$$

where the right-hand side is the expected return on a one-dollar exit bond. If the exit bond is riskless, then this expected return is one, and Proposition 3 (iii) follows after a little algebra.

PROOF OF PROPOSITION 4. Follows from the debtor's first-order condition, and application of the implicit function theorem.

PROOF OF PROPOSITION 5. Follows directly from Proposition 4.

PROOF OF PROPOSITION 6. The creditors' collective maximizes $D(L) - L$. Taking the first-order condition and using Equation (24) yields (28). Equation (29) follows from (27) directly.

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