

Intrinsic Bubbles: The Case of Stock Prices

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Several puzzling aspects of the behavior of United States stock prices may be explained by the presence of a specific type of rational bubble that depends exclusively on aggregate dividends. We call bubbles of this type "intrinsic" bubbles because they derive all of their variability from exogenous economic fundamentals and none from extraneous factors. Intrinsic bubbles provide a more plausible empirical account of deviations from present-value pricing than do the traditional examples of rational bubbles. Their explanatory potential comes partly from their ability to generate persistent deviations that appear to be relatively stable over long periods. (JEL G12)

After a decade of research, financial economists remain unsatisfied with simple accounts of stock-price fluctuations. The initial rejections by Stephen LeRoy and Richard Porter (1981) and Robert Shiller (1981) of a simple present-value model based on constant discount rates and rational expectations have not been reversed by subsequent work. Although departures from present-value prices appear to be large and persistent, it has nevertheless proved difficult to find empirical support for parsimonious alternatives to the simple present-value model.¹

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¹Marjorie Flavin (1983), N. Gregory Mankiw et al. (1985), Allan Kleidon (1986), Terry Marsh and Robert Merton (1986), John Campbell and Shiller (1987), Kenneth West (1987, 1988a) and Froot (1987) address

At one time, rational bubbles were viewed as one such alternative. Interest in bubbles has waned, however, in part because econometric tests have not produced persuasive evidence that rational bubbles can help explain stock prices. That is, no one has produced a specific bubble parameterization that is both parsimonious and capable of explaining the data.

In this paper, we propose and test empirically a new rational-bubble specification with both of these properties. Our formulation is parsimonious because it introduces no extraneous sources of variability. Instead, the bubbles we examine are driven exclusively—albeit nonlinearly—by the exogenous fundamental determinants of asset prices. For this reason, we refer to these bubbles as “intrinsic.” One striking property of an intrinsic bubble is that, for a given level of

econometric shortcomings of the original studies. John Cochrane (1989) argues that, in principle, time-varying discount factors could explain failures of the simple present-value model. There is little positive empirical evidence, however, that discount-factor variation alone can explain these failures; see, for example, Shiller (1981), Robert Flood et al. (1986), and Campbell and Shiller (1988a). Robert Pindyck (1984) suggests that low-frequency price fluctuations may be a result of time-varying risk premia driven by changing stock-price volatility. However, James Poterba and Lawrence Summers (1986) argue that volatility is not sufficiently persistent to explain a large portion of low-frequency price movements.

exogenous fundamentals, the bubble will remain constant over time: intrinsic bubbles are deterministic functions of fundamentals alone. Thus, this class of bubbles predicts that stable and highly persistent fundamentals lead to stable and highly persistent over- or undervaluations. In addition, these bubbles can cause asset prices to "overreact" to changes in fundamentals.

Surprisingly, our parametric example of an intrinsic bubble also appears to be capable of explaining long-term movements in stock prices. It turns out that the component of prices not explained by the simple present-value model is highly positively correlated with dividends, as an intrinsic bubble would predict. As a result, an intrinsic bubble fits well both the bull market of the 1960's, a period of high and rising real dividends, and the market decline of the early 1970's. We use our estimated model to separate out the present-value and bubble components of stock prices and find that the former implies a realized annual real return on stocks of about 9.1 percent—very close to the 9.0 percent average for this century.

Of course, there are nonbubble hypotheses that could in principle explain our results. It is often argued that stationary fads or noise trading lie behind departures from present-value prices.² Both fads and intrinsic bubbles can generate departures that are highly persistent; but an important theoretical distinction between the two is that the former entail short-term speculative profit opportunities, whereas bubbles alone do not. Because stock-market returns appear to have a predictable component, our empirical tests are designed to separate the bubble from possible sources of predictable returns, such as fads and variable discount rates. While this predictability ultimately should be useful in explaining certain features of the data, our results suggest that it

is not the main explanation for the simple present-value model's failure.

A second alternative hypothesis involves possible future changes in regime. It is well known that any bubble path is observationally equivalent to a present-value path for which the process generating fundamentals may change in the future.³ Our results therefore could be interpreted as evidence of such prospective changes. Indeed, present-value pricing formulas similar in form to the bubble formulas derived below arise in asset-pricing models that assume stochastic regime shifts. In this paper, we posit no specific regime-switch model to explain the apparently nonlinear relationship between stock prices and dividends.⁴

Notwithstanding our empirical results, we find the notion of rational bubbles to be problematic. It is difficult to believe that the market is literally stuck for all time on a path along which price:dividend ratios eventually explode. If the market began on such a path, surely investors would at some point attempt the kind of infinite-horizon arbitrage that rules bubbles out in theoretical models; and since *fully* rational agents would anticipate such attempts, bubbles could never get started. It seems to us an empirical question, however, whether this much foresight should be ascribed to the

³Flood and Peter Garber (1980), James Hamilton and Charles Whiteman (1985), and Flood and Robert Hodrick (1986) discuss this observational equivalence.

⁴See Paul Krugman (1987) on regime switches in the stock market; see Krugman (1987) and Froot and Obstfeld (1991) on the foreign-exchange market. Krugman suggests that trigger-price sell strategies can make the price-dividend relation nonlinear. Results like those we report below could be rationalized within a fad model (without bubbles) in which investors perceive "psychological barriers" to upward movements in stock prices. When a price barrier is reached, a group of noise traders might enter the market with some probability, in which case a new barrier is established, or exit ("profit taking"). Speculation by rational investors would, in this environment, create a nonlinear relationship between prices and dividends. An alternative model predicting bubble-like behavior is based on market learning about managerial competence; see Nobuhiro Kiyotaki (1990). Stephen Cecchetti et al. (1990) study the empirical implications of particular nonlinear fundamentals processes.

²For examples of models with fads or noise, see Shiller (1984), Kyle (1985), Fischer Black (1986), Jeffrey Frankel and Froot (1986), Summers (1986), Campbell and Albert Kyle (1988), J. Bradford De Long et al. (1990), and Froot et al. (1990).

market. Perhaps agents do not really have as clear a picture of the distant future as the simplest rational-expectations models suggest. Stock prices and dividends might follow a nonlinear relation such as the one we estimate for some time before market participants catch on to the unreasonable implications of very high dividend realizations.

The paper is structured as follows. Section I shows how intrinsic bubbles arise in a standard present-value model. In Section II, we compare some properties of intrinsic bubbles and a more conventional extraneous bubble whose explosive dynamics are driven by calendar time. Section III then turns to the data. We examine the univariate and bivariate time-series properties of United States stock prices and dividends, and we argue that an intrinsic bubble is broadly consistent with the results. In the second part of Section III, we estimate our model directly and test it against several alternatives. Section IV concludes and offers our interpretations of the results.

I. Intrinsic Bubbles in a Present-Value Model

Stochastic linear rational-expectations models can have a multiplicity of solutions that depend on exogenous fundamentals but do not depend on extraneous variables such as time.⁵ In this section, we describe how such rational bubbles arise as nonlinear solutions to a linear asset-pricing model. Although our choice of a specific model is guided by the empirical application we have in mind, solutions similar to those derived below arise in a broader class of models.

⁵Included in the category of extraneous variables are irrelevant fundamentals, such as lagged fundamentals that play no economic role apart from their self-fulfilling effect on expectations. The excessive variability of an asset-price solution containing an intrinsic bubble comes entirely from its functional form, not from the introduction of extraneous state variables. An intrinsic-bubble solution is a reduced-form expression that depends only on the exogenous factors objectively affecting the economy, not on extraneous noise. In other words, every intrinsic-bubble solution is a "minimal-state-variable" solution in the sense of Bennett McCallum (1983).

The model is based on a simple condition that links the time-series of real stock prices to the time-series of real dividend payments when the expected rate of return is constant. Let P_t be the real price of a share at the beginning of period t , let D_t be real dividends per share paid out over period t , and let r be the constant, instantaneous real rate of interest. The condition we focus on is

$$(1) \quad P_t = e^{-r} E_t(D_t + P_{t+1})$$

where $E_t(\cdot)$ is the market's expectation conditional on information known at the start of period t .⁶

The present-value solution for P_t , denoted by P_t^{pv} , is

$$(2) \quad P_t^{\text{pv}} = \sum_{s=t}^{\infty} e^{-r(s-t+1)} E_t(D_s).$$

Equation (2) is a particular solution to the stochastic difference equation (1). It equates a stock's price to the present discounted value of expected future dividend payments. We assume that the present value (2) always exists, that is, that the continuously compounded growth rate of expected dividends is less than r .

The present-value formula is the solution to (1) usually singled out by the relevant economic theory as a unique equilibrium price. It can be derived by applying the transversality condition,

$$(3) \quad \lim_{s \rightarrow \infty} e^{-rs} E_t(P_s) = 0$$

and then observing that successive forward substitutions into (1) converge to (2).

Equation (1) has solutions other than (2). By construction, these alternative price paths satisfy the requirement of period-by-period efficiency, but they do not satisfy (3). Let $\{B_t\}_{t=0}^{\infty}$ be any sequence of random vari-

⁶In our empirical implementation of the model below we allow for errors in this equation, which does not hold exactly for United States data (Flood et al., 1986).

ables such that

$$(4) \quad B_t = e^{-r} E_t(B_{t+1}).$$

Then, $P_t = P_t^{pv} + B_t$ is a solution to (1), which can be thought of as the sum of the present-value solution and a rational bubble. Clearly, property (4) implies that P_t violates the transversality condition (3) if $B_t \neq 0$.

Rational bubbles are sometimes viewed as being driven by variables extraneous to the valuation problem. However, some bubbles may depend only on the exogenous fundamental determinants of asset value. We call such bubbles "intrinsic" because their dynamics are inherited entirely from those of the fundamentals. An intrinsic bubble is constructed by finding a nonlinear function of fundamentals that satisfies (4). In the above stock-price model with only one stochastic fundamental factor (the dividend process), intrinsic rational bubbles depend on dividends alone.

To see how an intrinsic stock-price bubble might look, suppose that log dividends are generated by the geometric martingale,

$$(5) \quad d_{t+1} = \mu + d_t + \xi_{t+1}$$

where μ is the trend growth in dividends, d_t is the log of dividends at time t , and ξ_{t+1} is a normal random variable with conditional mean zero and variance σ^2 . Using (5) and assuming that period- t dividends are known when P_t is set, we see that the present-value stock price in (2) is directly proportional to dividends:

$$(6) \quad P_t^{pv} = \kappa D_t$$

where $\kappa = (e^r - e^{\mu + \sigma^2/2})^{-1}$. Equation (6) is essentially a stochastic version of Myron Gordon's (1962) model of stock prices, which predicts that $P_t^{pv} = (e^r - e^{\mu})^{-1} D_t$ under certainty. The assumption that the sum in (2) converges implies that $r > \mu + \sigma^2/2$.

Now define the function $B(D_t)$ as

$$(7) \quad B(D_t) = cD_t^\lambda$$

where λ is the positive root of the quadratic

equation

$$(8) \quad \lambda^2 \sigma^2 / 2 + \lambda \mu - r = 0$$

and c is an arbitrary constant. It is easy to verify that (7) satisfies (4):

$$\begin{aligned} (9) \quad e^{-r} E_t(B(D_{t+1})) &= e^{-r} E_t(cD_t^\lambda e^{\lambda(\mu + \xi_{t+1})}) \\ &= e^{-r} (cD_t^\lambda e^{\lambda\mu + \lambda^2\sigma^2/2}) \\ &= e^{-r} (cD_t^\lambda e^r) = B(D_t). \end{aligned}$$

By summing the present-value price and the bubble in (7), we get our basic stock-price equation:

$$(10) \quad P(D_t) = P_t^{pv} + B(D_t) = \kappa D_t + cD_t^\lambda.$$

Even though (10) contains a bubble (for $c \neq 0$) and thus violates (3), it is driven exclusively by fundamentals: $P(D_t)$ is a function of dividends only and does not depend on time or any other extraneous variable. $B(D_t)$ is therefore an example of an intrinsic bubble.⁷

The inequality $r > \mu + \sigma^2/2$ can be used to show that λ must always exceed 1. It is this explosive nonlinearity that permits $B(D_t)$ to grow in expectation at rate r . We will assume from now on that $c > 0$, so that stock prices cannot be negative. Negative stock prices would violate free disposability.⁸

⁷Thomas Sargent (1987 pp. 348–9) characterizes a rational bubble as a function $\tilde{B}(t, X_t) = e^{r't} X_t$ of time and a variable X_t that obeys $E_t(X_{t+1}) = X_t$. However, his definition does not imply that bubbles have to contain deterministic time components. To write the bubble $B(D_t)$ defined by (7) in Sargent's form, simply let $X_t = e^{-r't} cD_t^\lambda$.

⁸Let λ' be the negative root of equation (8). Then the general solution to (1) [within the class of functions $P = P(D_t)$] is

$$P(D_t) = P_t^{pv} + c_1 D_t^\lambda + c_2 D_t^{\lambda'}.$$

We have imposed $c_2 = 0$ in (10) on the grounds that the stock price P_1 should go to zero (not to infinity) as

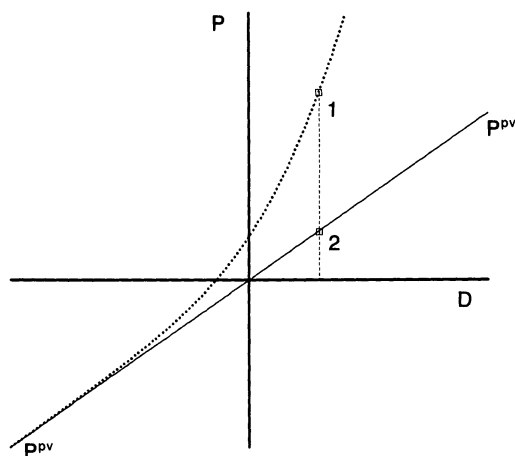


FIGURE 1. INTRINSIC-BUBBLE PRICE PATHS

It might seem paradoxical that movements in a bubble can be completely attributed to movements in fundamentals. Economists are accustomed to an almost instinctive decomposition of asset prices into two components, one dependent on market fundamentals and a second reflecting self-fulfilling beliefs and driven, at least in part, by extraneous factors. In the context of linear models, for example, McCallum (1983) argues that bubble solutions can be avoided by restricting attention to "minimal-state-variable" solutions that depend only on fundamentals. The possibility of intrinsic bubbles reveals that McCallum's approach does not rule out multiple solutions unless some additional requirement (e.g., linearity of the price function) is imposed.

Like all rational bubbles, intrinsic bubbles rely on self-fulfilling expectations. Instead of being driven by extraneous variables, however, these expectations are driven by the nonlinear form of the price solution itself. Figure 1 shows the family of solutions

dividends D_t go to zero. The argument in the text shows that any variable Y_t whose logarithm follows a martingale with drift μ and variance σ^2 leads to a bubble solution to (1), $P(D_t, Y_t) = P_t^{PV} + B(Y_t)$. Thus, a formula like (7) can be used to construct extraneous as well as intrinsic bubbles.

given by (10). The straight line $P^{PV}P^{PV}$ indicates the present-value solution (6); this solution implies that $E_t(P_{t+1}/P_t) = e^{\mu + \sigma^2/2} < e^r$. A point like 1 on the bubble path satisfies rate-of-return condition (1) because of Jensen's inequality. At point 1, the next innovation in log dividends is distributed symmetrically around zero, but the market's belief that the relevant price function has the shape shown means that the expected rise in the stock price and, hence, the current stock price itself are higher at point 1 than at the corresponding point 2 on $P^{PV}P^{PV}$.⁹

II. Alternative Bubble Specifications: A Partial Comparison

Why might intrinsic bubbles succeed in characterizing stock prices when other bubble formulations have failed? In this section, we argue that intrinsic bubbles have several empirically appealing properties that the bubble parameterizations used in previous applied studies lack.

To begin, we need to know why bubble explanations of stock prices have fared so poorly. A first reason might be a belief that prices simply do not diverge from their present-value levels.¹⁰ There are strong theoretical arguments behind this view. However, while short-horizon excess-profit opportunities are plausibly quite small, the theoretical conditions required to rule out rational bubbles assume substantial, per-

⁹It is easy to check that various theorems used to identify unique solutions of the form $P(D_t)$ to equations like (1) do not apply under this section's assumptions. For example, (10) is not within any of the classes of solutions considered by Robert Lucas (1978), Rusdu Saracoglu and Sargent (1978), Christian Gourieroux et al. (1982), or Whiteman (1983). The problem is not that the process in (5) is nonstationary. Multiple solutions analogous to (10) exist when (5) is a mean-reverting Ornstein-Uhlenbeck process; see Froot and Obstfeld (1991). Rather, the problem is that standard uniqueness theorems impose additional restrictions, such as linearity of the solution or the assumption that all state variables are restricted to assume values in compact sets. These assumptions rule out solutions such as (10).

¹⁰Flood and Hodrick (1990) survey the empirical literature on bubbles from this perspective.

haps unrealistic, infinite-horizon foresight on the part of economic agents.

A second reason for the poor empirical track record of bubbles is that the specific parameterizations that have been tested have failed. These parameterizations generally assume that bubbles and, hence, stock prices contain deterministic exponential time trends.¹¹ However, there is little evidence of such behavior in U.S. stock-market data.

Some general specification tests have been employed in the hope that bubbles can be detected without the need to take a stand on a specific bubble form. Even though these tests may have low power, they nevertheless reject the no-bubble null hypothesis frequently. However, they cannot reveal the precise source of rejection, so they yield no hard evidence that bubbles really are the culprits.¹² The tendency to ascribe these rejections to sources other than bubbles has been strengthened both by the theoretical arguments against bubbles and by the failure of the specific parameterizations mentioned above. However, consideration of stochastic bubbles that look quite different from the usual time-driven examples may throw a different light on the specification-test results.

How then do intrinsic bubbles behave, and why might they do a better job of tracking stock prices? First, intrinsic bubbles capture well the idea that stock prices overreact to news about dividends, as argued by Shiller (1984), among others. Equation (10) implies that $dP_t/dD_t = \kappa + \lambda c D_t^{\lambda-1} > \kappa$, so prices move more when div-

idends change than the present-value formula (6) would predict.

In addition, intrinsic bubbles are not obviously inconsistent with the apparent time-series properties of stock prices. Even though the bubbles are expected to grow at the rate of interest, specific realizations may fluctuate within some limited range for rather long periods. A given dividend realization corresponds to a unique stock price regardless of the date on which the dividend is announced. Because dividends are persistent, deviations from present-value prices may also be persistent. An implication of this property is that, even with a very long data series, the fundamentally explosive nature of an intrinsic bubble might be impossible to detect through diagnostic time-series tests.

Some simulations illustrate these points by comparing the intrinsic bubble in (10) with a particular alternative bubble specification. Each simulation experiment involves three solutions to the difference equation in (1). The first of these is the present-value price P_t^{pv} given by (6); the second is a purely stochastic, nonlinear intrinsic bubble of the form (10), denoted by \tilde{P}_t ; and the third is a bubble that depends on time as well as on dividends:

$$(11) \quad \tilde{P}_t = P_t^{pv} + b D_t e^{(r - \mu - \sigma^2/2)t}.$$

The precise formulation in (11) is chosen for two reasons. First, it makes the bubble a function of dividends and thus allows stock prices to overreact to dividend news, just as the bubble in (10) does. Second, (11) follows the majority of parametric bubble tests in adopting a specification in which the extraneous variable t affects prices.

Dividends are assumed to follow (5), and in each experiment successive innovations ξ_t are drawn independently from a normal distribution. P_t^{pv} is calculated using estimates of r , μ , and σ^2 implied by U.S. stock-price and dividend data, and the values of the parameters κ , c , and b are those estimated below in Section III. The simulations are run over 200 years. However, it is important to note that there is little importance to these specific choices of parame-

¹¹See Flood and Garber (1980) and Olivier Blanchard and Mark Watson (1982) for specific examples.

¹²The general specification test for bubbles used by West (1987) can alternatively be interpreted as a test of model specification, the purpose for which it was originally proposed by Robert Cumby et al. (1983). A second type of specification test for bubbles compares the time-series properties of prices and dividends, which should differ if condition (1) holds but stock prices contain a rational bubble (see Hamilton and Whiteman, 1985; Behzad Diba and Herschel Grossman, 1988a).

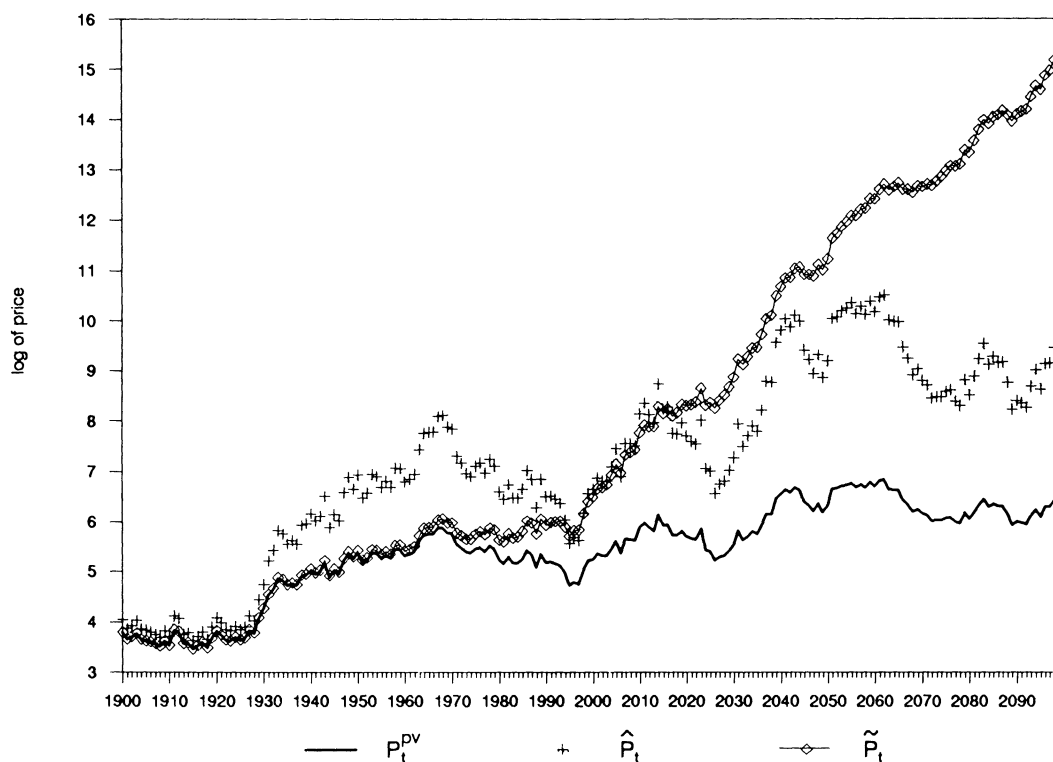


FIGURE 2. SIMULATED STOCK-PRICE PATHS

Notes: P_t^{PV} is the simulated present-value stock-price path assuming that log dividends follow a random walk with trend. \hat{P}_t gives simulated stock prices under the intrinsic bubble in equation (10). \tilde{P}_t gives simulated stock prices under the bubble example in equation (11).

ters and sample size: the qualitative patterns displayed in the following figures are quite general.

Figure 2 shows a first run in which the simulated intrinsic bubble, \hat{P}_t , does not produce noticeable explosive behavior within the simulation sample. The percentage overvaluation of stocks is not very different at the end of the sample (the year 2100) than it is around 1970 or 2015. In contrast, the partially deterministic bubble \tilde{P}_t explodes decisively.

The behavior of the time-driven bubble is similar in Figure 3, but the underlying dividend realization makes the explosive expected growth of the intrinsic bubble more apparent. Figures 2 and 3 highlight the sharply different paths for intrinsic bubbles that different paths of fundamentals can

produce. (Of course, paths qualitatively similar to the intrinsic-bubble paths could be generated by purely random bubbles that depend on extraneous variables.)

Diba and Grossman (1988b) have argued on theoretical grounds that stochastic rational bubbles cannot “pop” and subsequently start up again. This feature, they assert, makes rational bubbles empirically implausible. Figure 4, however, shows an intrinsic-bubble realization that falls over time to a level quite close to fundamentals. Indeed, if dividends follow a process like (5) but without drift, the logarithm of dividends reaches any given lower bound with probability 1; we can therefore be sure that the bubble term in (10) gets arbitrarily close to zero in finite time. For practical purposes, this is the same as periodically popping and

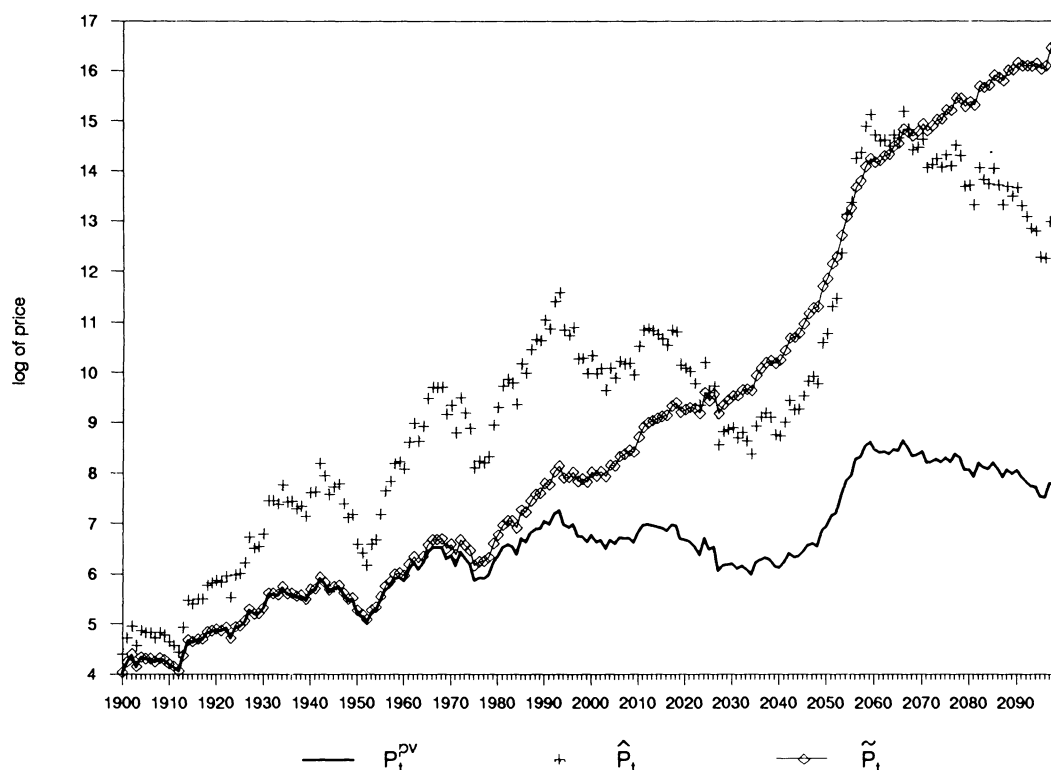


FIGURE 3. SIMULATED STOCK-PRICE PATHS

Notes: P_t^{pv} is the simulated present-value stock-price path assuming that log dividends follow a random walk with trend. \hat{P}_t gives simulated stock prices under the intrinsic bubble in equation (10). \tilde{P}_t gives simulated stock prices under the bubble example in equation (11).

restarting with probability 1. Intrinsic bubbles allow stock prices to get very close to present-value levels and then diverge. (They also, however, allow arbitrarily large divergences.)¹³

¹³To gain a sense of the likelihood with which an intrinsic bubble such as the one in (10) is likely to recede dramatically, we ran Monte Carlo experiments on the future evolution of stock prices using stock prices in 1987 as the initial condition. (These experiments use the following parameters, estimated from the data described in the following section: $\sigma = 0.122$, $\mu = 0.011$, $r = 0.086$, $c = 0.34$). We found that the probability that the bubble falls to a level one-half of its size in 1987 in the next 100 years is 81 percent, and the probability that it falls to a level one-quarter of its size in 1987 is 53 percent. These results suggest that, with moderate dividend growth rates, the bubble is likely to appear to shrink substantially over longer time-series samples.

Notice that all three simulations share the feature that the intrinsic-bubble path lies above the time-driven bubble in the early part of the sample, but below it by the sample's end. This pattern in the early part of the sample is merely a result of initial conditions and is therefore purely arbitrary.¹⁴ By contrast, the feature that the time bubble eventually exceeds the intrinsic bubble is more general. It is easy to show that, as the sample size T grows, the probability that $\hat{P}_T > \tilde{P}_T$ goes to zero for any set

¹⁴It turns out that, if model (11) is to have any hope of fitting the data, the estimate of b must be very close to zero, implying that \hat{P}_t is very close to P_t^{pv} for the first part of the sample; see Section III-B and Figure 7.

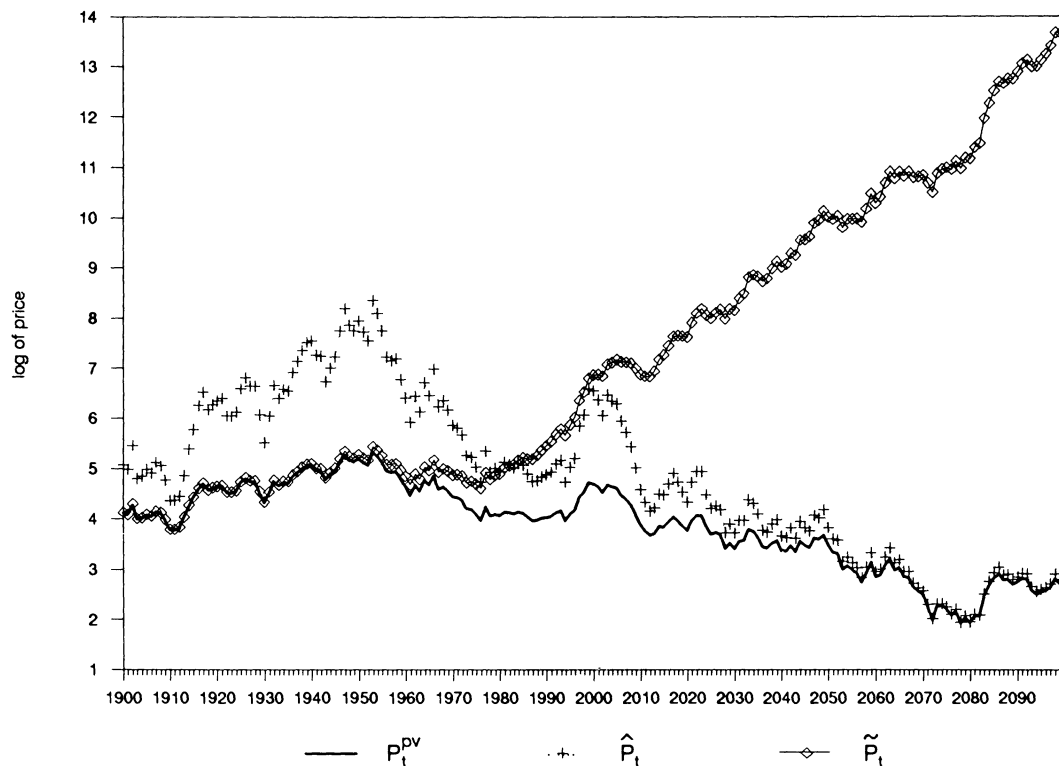


FIGURE 4. SIMULATED STOCK-PRICE PATHS

Notes: P_t^{PV} is the simulated present-value stock-price path assuming that log dividends follow a random walk with trend. \hat{P}_t gives simulated stock prices under the intrinsic bubble in equation (10). \tilde{P}_t gives simulated stock prices under the bubble example in equation (11).

of initial conditions.¹⁵ The intrinsic bubble in \hat{P}_T ultimately exceeds the time-driven bubble in \tilde{P}_T very rarely in large samples, but when it does, it does so by an amount

¹⁵PROOF: Define $\psi = r - \mu - \sigma^2/2$ and assume, without loss of generality, that the bubbles are equal at $t = 0$: $bD_0 = cD_0^\lambda$. Then,

$$\begin{aligned} \Pr[\tilde{P}_T < \hat{P}_T] &= \Pr[bD_T e^{\psi T} < cD_T^\lambda] \\ &= \Pr[\psi T < (\lambda - 1)(\mu T + \sum_{s=1}^T \xi_s)] \\ &= \Pr[r - \lambda\mu - \sigma^2/2 < (\lambda - 1)(\sum_{s=1}^T \xi_s)/T]. \end{aligned}$$

Equation (8) implies, however, that $r - \lambda\mu - \sigma^2/2 = \sigma^2(\lambda^2 - 1)/2 > 0$ (recall that $\lambda > 1$). Since $\text{plim}(\sum_{s=1}^T \xi_s)/T = 0$, the proof is complete.

large enough to equalize the two bubbles' expected growth rates.

This latter property is important empirically. It implies that it would be unusual to draw a long dividend series which yields an intrinsic bubble that appears as explosive as a comparable time-driven bubble. Even though intrinsic and time-driven bubbles are expected to grow at the same rate on average, a long intrinsic-bubble sample path is very likely to appear less explosive than the path a time-driven bubble such as (11) generates.

III. Application to the U.S. Stock Market

This section applies the model developed above to U.S. stock-market data. The model's specification is generalized, however, to allow for errors in the difference

equation describing stock-price movements, equation (1). Now, time- t prices are given by

$$(1') \quad P_t = e^{-r} E_t(D_t + P_{t+1}) + e^{-r} u_t$$

where u_t is a predictable single-period excess return. Equation (1') implies that (10) is replaced by the statistical model,

$$(12) \quad P_t = c_0 D_t + c D_t^\lambda + \varepsilon_t$$

in which $c_0 = \kappa = (e^r - e^{\mu + \sigma^2/2})^{-1}$ and ε_t is the present value of the errors in (1'), $\varepsilon_t \equiv \sum_{s=t}^{\infty} e^{-r(s-t+1)} E_t(u_s)$. Estimation of (12) is complicated by collinearity among the regressors, but dividing by D_t mitigates the problem and leads to

$$(13) \quad \frac{P_t}{D_t} = c_0 + c D_t^{\lambda-1} + \eta_t$$

where $\eta_t \equiv \varepsilon_t / D_t$. The null hypothesis of no bubble implies that $c_0 = \kappa$ and $c = 0$, whereas the bubble alternative in (10) predicts that $c_0 = \kappa$ and $c > 0$.

The new error term,

$$\eta_t = D_t^{-1} \sum_{s=t}^{\infty} e^{-r(s-t+1)} E_t(u_s)$$

is assumed to be statistically independent of dividends at all leads and lags and to have unconditional mean zero. This assumption is critical in the tests below.¹⁶ The error in (13) could be interpreted, for example, as

¹⁶Independence is an unnecessarily strong assumption for some purposes. The tests carried out in the following subsections will produce consistent parameter estimates provided only that $E_t(\eta_t | D_t) = 0$. As a partial check on this weaker assumption, we formed an estimate of u_t / D_t from equation (1'): $\theta_{t+1} \equiv u_t / D_t + \omega_{t+1} = (e^r P_t / D_t) - 1 - (P_{t+1} / D_t)$, where ω_{t+1} is unforecastable given time- t information. We then regressed θ_{t+1} on actual log-dividend changes. The results show that Δd_t has no statistically significant explanatory power for θ_{t+1} .

However, for correct statistical inferences, we must make the stronger assumption that dividends and η_t are independent at all leads and lags. Again, we examined this assumption by regressing θ_{t+1} on changes in log dividends (current, past, and future) and found no significant explanatory power.

the result of time-varying effective income-tax rates or time-varying discount factors. One could also think of η_t as partly reflecting a fad—a shock to the demand for stocks which is unrelated to efficient forecasts of future dividends. For the latter interpretation, specification (13) allows separate identification of bubble and fad components in stock prices.

Estimation is based on the Standard and Poor's stock price and dividend indexes from the *Securities Price Index Record*, as extended backwards in time by Alfred Cowles and Associates (1939). Following Robert Barsky and De Long (1989), we examine the period 1900–1988, using nominal stock prices recorded in January of each year and deflated by the January producer price index (PPI). Dividends are annual averages of nominal data for the calendar year, deflated by the year-average PPI.¹⁷ We would have preferred data on beginning-of-period- t dividends to match the beginning-of-period- t stock price, P_t . Because these are not available, we use the average of period- t dividends as our measure of D_t .^{18,19}

¹⁷Although the price and dividend series extend back to 1871, we chose to begin our sample at 1900 because the composition of the market portfolio becomes increasingly restrictive as one goes back in time. In 1871, the portfolio comprises only 47 stocks, of which 31 are railroads. Because many other authors (e.g., Campbell and Shiller, 1987) have used the longer series, we also ran our statistical tests on the 1871–1986 sample. The results were qualitatively unaffected.

¹⁸A potential problem with this choice is that D_t may not be completely known at the beginning of period t . Nevertheless, we see two reasons why D_t is likely to be a better measure of the dividend information contained in beginning-of-period- t price, P_t , than is the average period- $(t-1)$ dividend, D_{t-1} . First, P_t is not recorded on January 1, but is itself an average over the period- t month of January. Second, to mitigate the effects of any time lapse between the determination and actual distribution of dividends, it is better to use average period- t dividends than those from period $t-1$. In any case, unless otherwise mentioned, the results below are not importantly different when average period- $(t-1)$ dividends are used to proxy for beginning-of-period- t dividends.

¹⁹Applying the notion of intrinsic bubbles to aggregate stock price and dividend data raises a question of interpretation. One possibility is that each firm's share price equals the present value of its own dividends, plus an intrinsic bubble on aggregate dividends. Such a

TABLE 1—COINTEGRATING REGRESSIONS OF ANNUAL REAL STOCK PRICES AND DIVIDENDS

Row	Regression equation	Cointegrating coefficient (β)	R^2	DW	d.f.
1	$P_t = \alpha + \beta D_t + \nu_t$	36.65	0.85	0.57	87
2	$D_t = \alpha + \beta P_t + \nu_t$	0.023	0.85	0.69	87
3	$p_t = \alpha + \beta d_t + \nu_t$	1.591	0.88	0.69	87
4	$d_t = \alpha + \beta p_t + \nu_t$	0.556	0.88	0.70	87

Notes: Cointegrating regressions are estimated using OLS. The sample period for all regressions was 1900–1988.

A. The Price–Dividend Relation

In deriving (13) we assume that the log-dividend process follows a martingale with trend. As an empirical matter, it seems unreasonable to suppose that market participants use only the information in past dividends to forecast future dividends. In Appendix A, we argue, however, that the stochastic process in (5) is a plausible approximation to the mechanism the market uses to forecast aggregate dividends. Appendix A describes several univariate and bivariate tests of the log-dividend specification in (5). We find little evidence against the martingale hypothesis: log-dividend changes are essentially unpredictable when conditioning on the lags of log dividends and/or log price:dividend ratios.²⁰ The data estimate the parameters in (5) as $\mu = 0.011$ and $\sigma = 0.122$.

A general implication of (13) is that stock prices may appear to overreact to changes in dividends. Also, (13) predicts that price:dividend ratios are nonstationary and

positively correlated with dividends. This subsection presents a brief empirical examination of these basic implications of intrinsic bubbles.

First, what does the simple present-value model predict for the sensitivity of prices to changes in dividends? From (6), a one-dollar change in dividends should raise prices by κ dollars. Using the fact that the sample-average gross real return on stocks is $e^r = 1.090$ per annum, we have that $\kappa = (e^r - e^{\mu + \sigma^2/2})^{-1} = (1.090 - e^{0.011 + 0.122^2/2})^{-1} = 14.0$. In general, if P_t and D_t are cointegrated of order (1,1), then under the present-value model the cointegrating coefficient should be approximately κ . Equation (6) also implies that the elasticity of prices with respect to dividends is 1. If log stock price p_t and d_t are cointegrated, it is also with a coefficient of 1.

The first row of Table 1 presents estimates of κ , obtained by regressing prices on dividends. The coefficient is estimated to be 36.7—much larger than the value of 14.0 predicted by the simple present-value model.²¹ If P_t and D_t are cointegrated then the ordinary least-squares (OLS) estimate of the cointegrating factor, while consistent, is biased in small samples. In order to bound the cointegrating coefficient, we run the reverse regression (projecting D_t on P_t) in the second row of Table 1. This produces a larger estimate of κ : $1/0.0233 = 42.9$. Even

formulation would remove the incentive for managers to influence the market price of their firms' shares by altering the timing of dividend payments. However, there is a semantic issue of whether the bubble is extraneous to individual companies' share prices.

²⁰Some of this evidence may be controversial. We have placed our discussion in Appendix A because the controversial aspects are somewhat tangential to our main argument. Provided one is prepared to accept (5) as a reasonable approximation to the forecasting model investors use, equation (13) will still approximate the relation between the price:dividend ratio and dividends.

²¹Similar estimates of the cointegrating factor are obtained by Campbell and Shiller (1987), Diba and Grossman (1988a), and West (1987), among others.

TABLE 2—UNIT-ROOT TESTS FOR ANNUAL PRICE:DIVIDEND RATIOS

Variable	β_1	
	With time trend	Without time trend
Spread, $P_t - 14D_t$	-0.1355 (-2.08)	-0.0702 (-1.14)
Price:dividend ratio, P_t/D_t	-0.2157 (-2.99)	-0.1343 (-2.11)
Log price:dividend ratio, $p_t - d_t$	-0.2122 (-3.55*)	-0.1315 (-2.55)

Notes: Values reported are the coefficients β_1 in the following regressions: with trend, $\Delta x_{t+1} = \beta_0 + \beta_1 x_t + \beta_2 t + \nu_{t+1}$; without trend, $\Delta x_{t+1} = \beta_0 + \beta_1 x_t + \nu_{t+1}$. Standard errors are constructed allowing for an MA(4) process in the residual. The t statistics, reported in parentheses beneath the point estimates, are for the test $\beta_1 = 0$.

*Statistically significant at the 5-percent level, using confidence intervals proposed by Phillips and Perron (1988) and Phillips (1987).

the lower of the two estimates would imply that the required rate of return on stocks less the expected growth rate of dividends is an implausibly low $1/36.7 = 2.7$ percent per annum. (The actual value over our sample period is 7.1 percent.) The third and fourth rows of Table 1 perform analogous regressions in logs instead of levels. Here, the cointegrating coefficient predicted by the present-value model is 1, but the estimates are again much higher—bounded between 1.59 and $1/0.556 = 1.80$. These estimates suggest that simple present-value models cannot explain why price:dividend ratios are so high given historical stock returns or, equivalently, why returns have been so high given price:dividend ratios.

To test whether these estimates are statistically incompatible with the simple present-value model, we examine various measures of the price:dividend ratio, including Campbell and Shiller's (1987) spread, for nonstationarity. Table 2 reports unit-root tests for the theoretically warranted spread ($P_t - 14D_t$) as well as the price:dividend ratio in levels (P_t/D_t) and in logs ($p_t - d_t$) (see Peter Phillips and Pierre Perron, 1988). Results of tests with and without time trends are reported. Under the present-value model, we should reject nonstationarity in each of these regressions; yet in five of six

cases we cannot reject the unit-root hypothesis.²²

We question whether these tests can be decisive, however, because of acknowledged problems with both their size and power.²³ One approach, exemplified by Campbell and Shiller (1987, 1988a, b) and Campbell (1990), is to assume at the outset that the price:dividend ratio is stationary. This assumption is important for their results; for example, Campbell's (1990) attribution of substantial price volatility to predictable excess returns relies crucially on the near-nonstationarity of the price:dividend ratio. Our view is that the ambiguous evidence on stationarity

²²Some of our results may be sensitive to the timing of dividends. Diba and Grossman (1988a), for example, use lagged dividends and deflate by the wholesale price index (WPI). They find that the log price:dividend ratio, $p_t - d_{t-1}$, is stationary. Using lagged dividends, but deflating by the PPI, Campbell and Shiller (1988a) also reject nonstationarity. Campbell and Shiller (1987) find results similar to those reported above for the spread, $P_t - \kappa D_t$, using data from 1871 to 1986.

²³For some Monte Carlo evidence on the size of these tests, see G. William Schwert (1988). Tests using lagged dividends (mentioned in footnote 22) may reject too frequently under the assumption that $p_t - d_t$ actually contains a unit root. On the power of unit-root tests applied to price:dividend ratios, see Cochrane (1989).

TABLE 3—ESTIMATES OF EQUATION (13), $P_t/D_t = c_0 + cD_t^{\lambda-1} + \eta_t$

Row	Regression method	c_0	c	$\lambda - 1$	F test ($\hat{c} = 0$)	R^2	DW	d.f.
1	OLS	12.24 (1.14)	0.34** (0.05)			0.57	0.71	87
2	Maximum likelihood	14.18 (1.77)	0.26** (0.06)			0.75	1.91	86
3	OLS	14.63 (2.28)	0.04 (0.12)	2.61* (1.15)	128.0**	0.57	0.71	86
4	Maximum likelihood	16.55 (2.02)	0.01 (0.02)	3.29* (1.45)	9.62**	0.75	1.91	85

Notes: Standard errors are reported in parentheses; OLS regressions report Newey–West standard errors allowing for fourth-order serial correlation and conditional heteroscedasticity. (Higher orders of serial correlation did not yield larger standard errors.) Maximum-likelihood estimates specify the error term as an AR(1) process. The sample period for all regressions was 1900–1988.

* Statistically significant at the 5-percent level; ** statistically significant at the 1-percent level.

makes it worthwhile to move beyond simple time-series diagnostics.

In sum, the evidence presented in this subsection has three important implications for our argument. First, prices are too sensitive to current dividends to be consistent with a simple present-value model. The implication, of course, is that the portion of stock prices unexplained by such a model must be highly correlated with dividends.²⁴ Second this overreaction apparently cannot be explained by other variables which are incorporated into stock prices and help forecast future dividends. If, for example, when dividends are high investors tend to get other reliable information that dividends will grow more quickly than previ-

ously expected, then this information is likely to be incorporated in stock prices, which therefore should Granger-cause dividends. The results in Appendix A suggest, however, that this is not the case. Finally, a specification such as (13) has at least the potential to explain these failures of the present-value model.

B. A Direct Test for Intrinsic Bubbles

To see whether this potential is at all realized, we turn in Table 3 to estimates of (13) and several related expressions. Before interpreting the estimates, however, some discussion of econometric issues is in order.

The regressor in (13), $D_t^{\lambda-1}$, presents difficulties because it is explosive. Two assumptions are necessary for valid statistical inferences. If the t statistic from testing $\hat{c} = 0$ is to have a known distribution under the null hypothesis, we require that: (i) the residuals, η_t , are distributed normally and identically—but not necessarily independently—with unconditional mean zero; and (ii) the dividend innovations, ξ_t , are distributed independently of the residuals η_t at all leads and lags. Appendix B provides a proof that the standard t statistic does indeed approximate a normal distribution un-

²⁴This result is essentially a restatement of Shiller's (1981) volatility findings. West's (1987) general specification test and Campbell and Kyle's (1988) noise-trading model also exploit the excess sensitivity of prices to dividend changes. Stephen Durlauf and Robert Hall (1988) find noise in prices that is more highly correlated with prices themselves than with dividends. Their definition of noise, however, is not the difference between prices and a multiple of current dividends, but the difference between prices and an *ex post* measure of the present value of future dividends.

der these assumptions, despite the presence of the exploding regressor.²⁵

A second aspect of estimation requiring discussion is the effect of serial correlation on the estimated standard errors of coefficients. Because theory offers no guide to η_t 's serial correlation, the usual standard errors may be incorrect. We try to account for this possibility in two ways. First, we estimate (13) by OLS, but correct the residuals using Whitney Newey and West's (1987) covariance-matrix estimator for serial correlation of unknown form. This estimator allows for conditional heteroscedasticity.²⁶ Second, since the residuals appear to be well described by a first-order autoregressive process, we compute maximum-likelihood estimates of the parameters under the assumption that the residuals are AR(1).

Finally, there is the issue of how to estimate the exponent, λ , and the present-value multiplier, κ . In some of the regressions below, we do not estimate λ concurrently with the other parameters. Instead, we use the point estimates from the log-dividend process obtained earlier, together with the mean return on stocks over the period, to compute $\lambda = 2.74$.²⁷ In other regressions, we estimate all parameters simultaneously, without imposing additional restrictions. The restriction that $c_0 = \kappa = 14.0$ is not imposed on the constant term in (13), even though it holds under both the null and alternative hypotheses. Instead, we use the unrestricted estimate of c_0 as a kind of sensibility check on our model.

²⁵The assumption that η_t in (13) is normal must be approximate because negative values of the price:dividend ratio are excluded. However, the approximation is likely to be accurate because the average value of the price:dividend ratio is equal to more than five times the estimated standard deviation of η_t .

²⁶It is plausible to think of the residual in (12), ε_t , as growing at a rate similar to that of dividends. In such a case, we would not expect η_t to exhibit much conditional heteroscedasticity. Indeed, in our estimates the heteroscedasticity-corrected standard errors were similar to the uncorrected standard errors.

²⁷We tried a variety of parameter estimates for r , μ , and σ^2 . These do have a minor effect on the exponent but are unimportant for the general regression results reported below.

The first two rows of Table 3 report estimates of (13) using OLS and maximum likelihood. These two regressions constrain λ to equal 2.74. In both cases, \hat{c} is statistically very significant. The estimates are comparable in magnitude and significance for the two estimation methods.²⁸ In the third and fourth rows, we estimate all of the parameters of the model simultaneously. The point estimates of c_0 are similar to those above, although λ is estimated to be larger and c correspondingly lower.²⁹ The larger standard error for c is expected here because the derivatives of the likelihood function with respect to the parameters c and λ are highly positively correlated [specifically, these derivatives include the terms $D_t^{\lambda-1}$ and $c_0(\lambda-1)D_t^{\lambda-2}$, with $\lambda > 2$]. Rather than using a t test to judge the importance of the nonlinear term, it is therefore more appropriate to compute an F test of the no-bubble hypothesis, $c = 0$, $\lambda = \hat{\lambda}$, where $\hat{\lambda}$ is the unrestricted estimate of λ . This hypothesis is rejected strongly at any reasonable level of significance.^{30,31}

²⁸We also tried estimating an extended form of (13):

$$\frac{P_t}{D_t} = c_0 + c_1 D_t^{\lambda'-1} + c_2 D_t^{\lambda'-1} + \eta_t$$

where λ' is the negative root from equation (8). Our estimates of r , μ , and σ^2 suggest that $\lambda' = -4.22$. Because $\lambda' < 0$ and dividends have a positive trend, $D_t^{\lambda'-1}$ will be of vanishing importance in explaining prices. Indeed, when we included $D_t^{\lambda'-1}$ in the regression, it had no effect on the estimate of c_1 . Furthermore, c_2 was imprecisely estimated and varied widely across different estimation techniques. As we expected, there seemed to be no evidence that the second nonlinear term helped in explaining stock prices. We therefore do not report these results.

²⁹Despite these differences in point estimates, there is virtually no improvement in R^2 . A likelihood-ratio test cannot reject the hypothesis that row 3 is no improvement over row 1 of Table 3.

³⁰We used the Newey-West covariance-matrix estimator for this test. In nonlinear models, t tests and F tests are not equivalent, as the t test is a Wald test (i.e., it is based entirely on the unrestricted model) while the F test is based on the likelihood-ratio principle (i.e., it explicitly compares the unrestricted model with the restricted model in which dividends are unable to explain any movements in the price:dividend

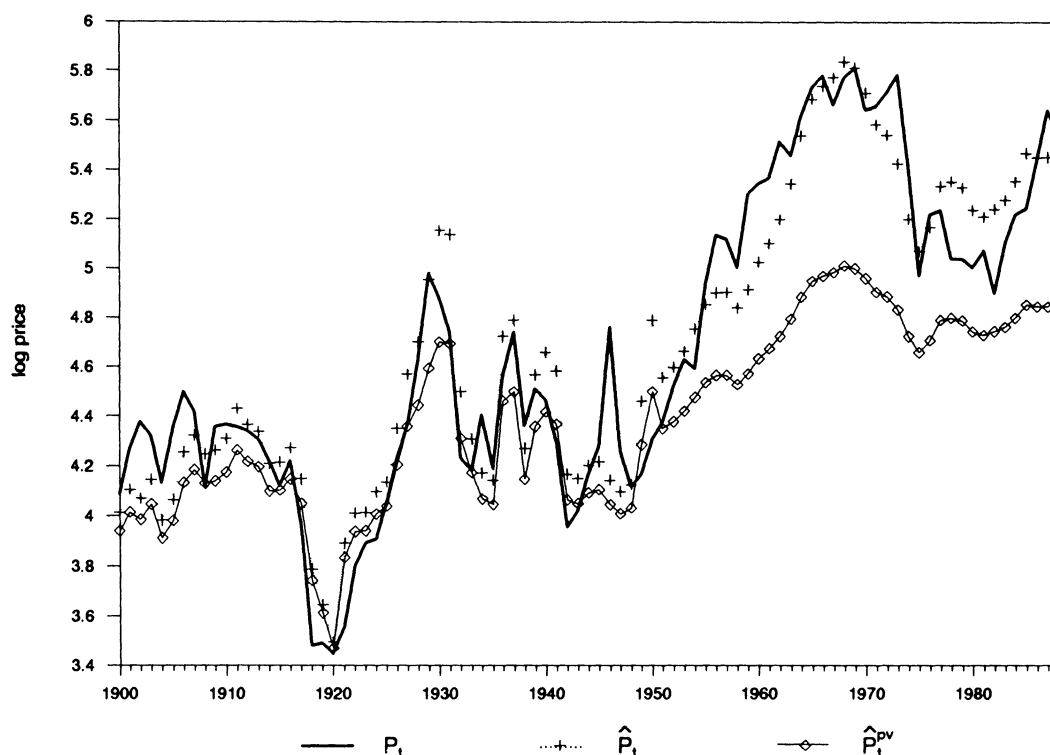


FIGURE 5. ACTUAL AND PREDICTED STOCK PRICES

Notes: P_t is the actual real stock price; $\hat{P}_t = D_t(\hat{c}_0 + \hat{c}D_t^{\lambda-1})$ is the predicted stock price under the intrinsic bubble; and $\hat{P}_t^{pv} = D_t\hat{c}_0$ is the model's predicted present-value stock price.

The finding that c is statistically positive suggests that prices become increasingly overvalued relative to the nonbubble price, P_t^{pv} , as dividends rise. Similarly, when dividends are low, the bubble component of price shrinks: P_t approaches P_t^{pv} . (Recall the dotted curve in Fig. 1, which graphs the relationship between fundamentals and prices implied by $c > 0$.) The size of the bubble (the distance between P_t and P_t^{pv})

explodes as the dividend becomes large. Of course, if realized dividends do not reach a high enough level, the bubble component will remain small.

Note also that the model's estimates of c_0 are sensible. All four estimates from Table 3 imply that P_t^{pv} is measured on average to be close to 14 times current dividends; indeed, each estimate is statistically indistinguishable from $\kappa = 14.0$, the value predicted by the simple present-value model set out above. In our estimates of (13), $\hat{P}_t^{pv} = \hat{c}_0 D_t$ turns out to be consistent with the long-run average return on stocks, because the non-linear dividend term soaks up a reasonable amount of the excessive sensitivity of actual prices to dividends.

The economic significance of the bubble is, of course, another matter. How large is the bubble component in prices, and how

ratio, $c = 0$). We thank one of the referees for pointing this out.

³¹One way of checking the assumption that ξ_t and η_t are distributed independently is to regress the estimated residuals obtained from (13) directly on current, past, and future changes in the log of dividends. In doing so, we could not reject the hypothesis that leads and lags of Δd_t have no explanatory power for η_t .

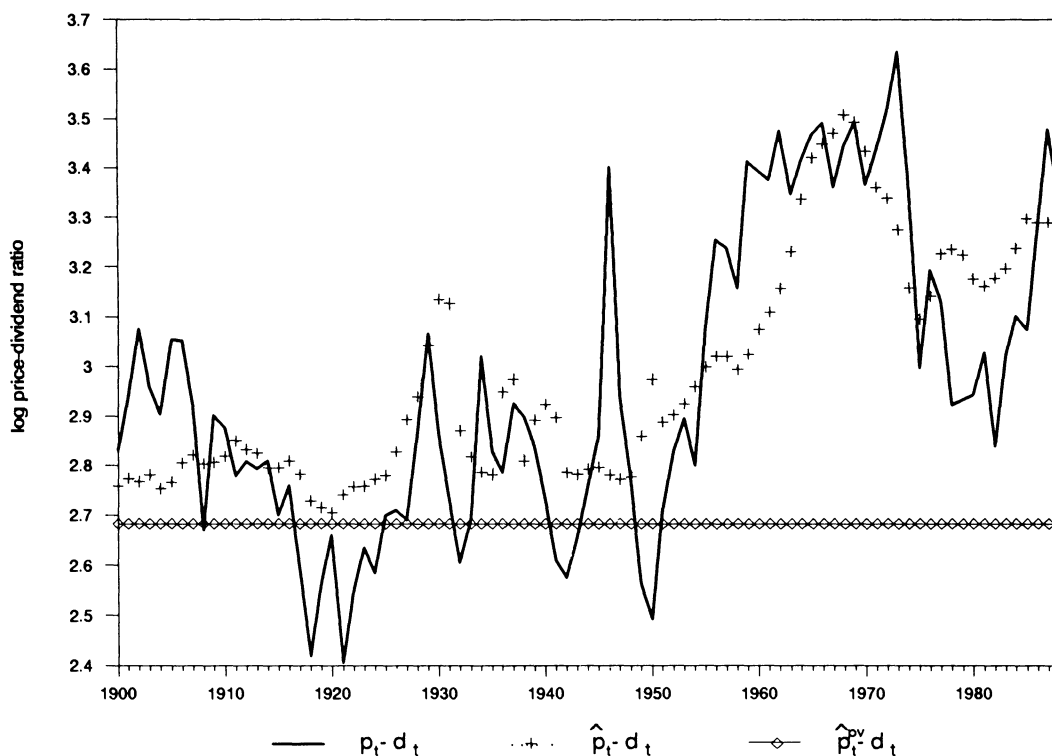


FIGURE 6. ACTUAL AND PREDICTED LOG PRICE:DIVIDEND RATIOS

Notes: $p_t - d_t$ is the actual log price:dividend ratio; $\hat{p}_t - d_t$ is the predicted log price:dividend ratio under the intrinsic bubble; and $\hat{p}_t^{pv} - d_t$ is the model's predicted present-value price:dividend ratio.

well does the model track actual price movements? Figure 5 helps explore these issues. It compares actual stock prices, P_t , with both \hat{P}_t^{pv} (the model's estimate of the nonbubble component of prices) and \hat{P}_t (the model's estimated price inclusive of the bubble term). Figure 6 presents comparable graphs of log price:dividend ratios.³² The figures are striking in two respects.

First there is the sheer size of the bubble itself (the distance between \hat{P}_t and \hat{P}_t^{pv}). It has grown over time and has been particularly large during the post-World War II

period. Indeed, the estimates suggest that at this writing the nonbubble level of the S&P 500 is less than 50 percent of its current value! The difference $\hat{P}_t - \hat{P}_t^{pv}$ is estimated to be this large recently because the levels of both dividends and price:dividend ratios are historically high.

Second, Figures 5 and 6 indicate that \hat{P}_t explains a good deal of actual stock-price movements. The sustained run-up in prices from 1950 to 1968 appears to be captured by the model, as does the post-World War II tendency for stocks to sell at historically large multiples of dividends.³³ The model

³² Figures 5 and 6 use the estimated coefficients from the third row of Table 3. However, this choice is immaterial to the results: it is almost impossible to distinguish visually among all the models estimated in Table 3.

³³ The model does a better job of explaining movements in the price:dividend ratio in the postwar period than in the earlier part of the century. This is evident in Figures 5 and 6, which show that prices and divi-

also does a plausible job of explaining the year-to-year variability of stock prices. Note from Figure 5 that the variance of dividends appears to have fallen relative to the variance of prices over the sample. Stock-price variability has been puzzling not only because it is so high, but also because it has not declined over time as rapidly as has the variability of dividends. Figure 5 and equation (13) together suggest a possible resolution of this paradox: stock-price volatility has not fallen with that of dividends because the *level* of dividends (and therefore the scope for volatility due to an intrinsic bubble) has been historically high.³⁴

Of course, the “fit” of \tilde{P}_t in Figures 5 and 6 cannot be judged without a standard of comparison. Because there are infinitely many bubble specifications that depend on time or other extraneous variables, sufficient excavation would allow us in principle to fit perfectly the actual price path.

One way of judging the model’s fit is to try alternative specifications. Table 4 helps to compare (13) to specifications in which additional terms are included in the regression, sometimes instead of and sometimes alongside $D_t^{\lambda-1}$. We start by examining the effects of two alternatives, the time-driven bubble term in (11) (divided by D_t) and a linear time trend. In isolation, either of these regressors appears to be a statistically significant determinant of the price:di-

vidend ratio (see rows 1 and 3 of Table 4, either estimation method). However, neither regressor remains statistically significant when the nonlinear term in (13), $D_t^{\lambda-1}$, is added to the regression (rows 2 and 4; as in Table 3, we have set λ at its estimated theoretical value of 2.74). Note that in the OLS estimates, the coefficients on the additional regressors become negative once $D_t^{\lambda-1}$ is added. The coefficients on the nonlinear term, however, remain statistically significant and of the same basic magnitude as in Table 3s.

To see whether the nonlinearity of the dividend term in (13) is important, rows 5 and 6 of Table 4 add a *linear* dividend term, D_t , to the regression. As the earlier results might suggest, D_t is positive and statistically significant on its own. However, once $D_t^{\lambda-1}$ is also included, the estimated coefficient on D_t becomes statistically insignificant, and in the maximum-likelihood estimate its sign is reversed. The signs and magnitudes of the estimates of c appear to be consistent with the results of Table 3, but multicollinearity raises the standard errors of the coefficients.

A second way of judging the model’s fit is to compare visually the dividend bubble in (13) with the time-driven bubble \tilde{P}_t defined in (11). Figure 7 graphs the predicted values of the present-value price, \tilde{P}_t^{pv} , and the bubble-inclusive price, \tilde{P}_t , from OLS estimates (row 1 for OLS estimates in Table 4). In comparing Figures 5 and 7, it is evident that the time-driven bubble, $\tilde{P}_t - \tilde{P}_t^{pv}$, captures little of the post-World War II variability of the stock market. Correlation with dividends, per se, is not enough to enable this bubble to explain stock prices. This result is not surprising: the presence of a deterministic time component forces the time-driven bubble to be essentially zero for most of the sample period.

C. Tests of an Alternative Hypothesis: Present-Value Prices under Variable Dividend Growth Processes

In testing our bubble specification, we assumed that ξ_t in (5) and η_t in (13) are independent at all leads and lags. While

dividends moved strongly together prior to 1950. Tests for subsample stability tend to reject the hypothesis: the OLS estimate of c in the postwar sample remains statistically significant at about 0.34 but becomes indistinguishable from zero prior to 1951.

³⁴To see how much the estimated sensitivity of prices to dividends has changed over time, recall that $dP_t/dD_t = \kappa + c\lambda D_t^{\lambda-1}$. Using the estimates from Table 3, we can compute rough estimates of dP_t/dD_t , which can be interpreted as the model’s prediction of the coefficient in a “cointegrating” regression of prices on dividends. Using average dividends over the period 1951–1988, we find (using row 2 of Table 3) $dP_t/dD_t \approx 14.2 + (0.26)(2.74)(7.86^{1.74}) = 39.9$. Similarly, over the period 1900–1950, $dP_t/dD_t \approx 14.2 + (0.26)(2.74) \times (4.31^{1.74}) = 23.2$. The estimated sensitivity of prices to dividends has therefore nearly doubled over the post-World War II period.

TABLE 4—ESTIMATES OF ALTERNATIVE MODELS, $P_t/D_t = c_0 + cD_t^{\lambda-1} + gX_t + \eta_t$

Row	X_t	c_0	c	g	R^2	DW	d.f.
<i>Estimation Method = OLS:</i>							
1	Time bubble, $X_t = e^{(r-\mu-\sigma^2/2)t}$	18.28 (1.47)		0.030* (0.013)	0.21	0.35	87
2	Time bubble, $X_t = e^{(r-\mu-\sigma^2/2)t}$	11.85 (1.09)	0.377** (0.060)	-0.008 (0.010)	0.57	0.75	86
3	Linear trend, $X_t = t$	13.68 (2.22)		0.15** (0.05)	0.35	0.43	87
4	Linear trend, $X_t = t$	12.43 (1.39)	0.364* (0.066)	-0.019 (0.052)	0.57	0.75	86
5	Linear dividends, $X_t = D_t$	6.88 (2.02)		2.273** (0.39)	0.55	0.70	87
6	Linear dividends, $X_t = D_t$	18.09 (8.62)	0.684 (0.448)	2.397 (3.32)	0.57	0.70	86
<i>Estimation Method = Maximum Likelihood:</i>							
1	Time bubble, $X_t = e^{(r-\mu-\sigma^2/2)t}$	18.34 (2.16)		0.027 ^a (0.014)	0.75	2.04	86
2	Time bubble, $X_t = e^{(r-\mu-\sigma^2/2)t}$	14.48 (1.86)	0.223** (0.075)	0.008 (0.012)	0.75	1.94	85
3	Linear trend, $X_t = t$	14.02 (2.87)		0.145** (0.055)	0.75	1.99	86
4	Linear trend, $X_t = t$	13.19 (2.16)	0.190* (0.084)	0.060 (0.056)	0.75	1.93	85
5	Linear dividends, $X_t = D_t$	11.39 (2.76)		1.530** (0.420)	0.75	1.93	86
6	Linear dividends, $X_t = D_t$	24.54 (6.64)	0.904* (0.404)	-4.372 (2.710)	0.76	1.91	85

Notes: Standard errors are reported in parentheses; OLS regressions report Newey-West standard errors, allowing for fourth-order serial correlation and heteroscedasticity. (Higher orders of serial correlation did not yield larger standard errors.) Maximum-likelihood estimates specify the error term as an AR(1) process. The sample period for all regressions was 1900–1988. The time-bubble specification (lines 1 and 2) is derived by dividing equation (11) by D_t .

^aStatistically significant at the 10-percent level; *statistically significant at the 5-percent level; **statistically significant at the 1-percent level.

we have attempted to test this assumption directly, our failure to reject it does not mean that it is true. For example, if dividends follow a more complex stochastic process than (5), persistent variation in the growth rate of dividends could lead to persistent movements in the price:dividend ra-

tio.³⁵ Such variation in dividend growth rates could invalidate our independence assumption.

³⁵See Barsky and De Long (1989) for one such model of time-varying dividend growth rates.

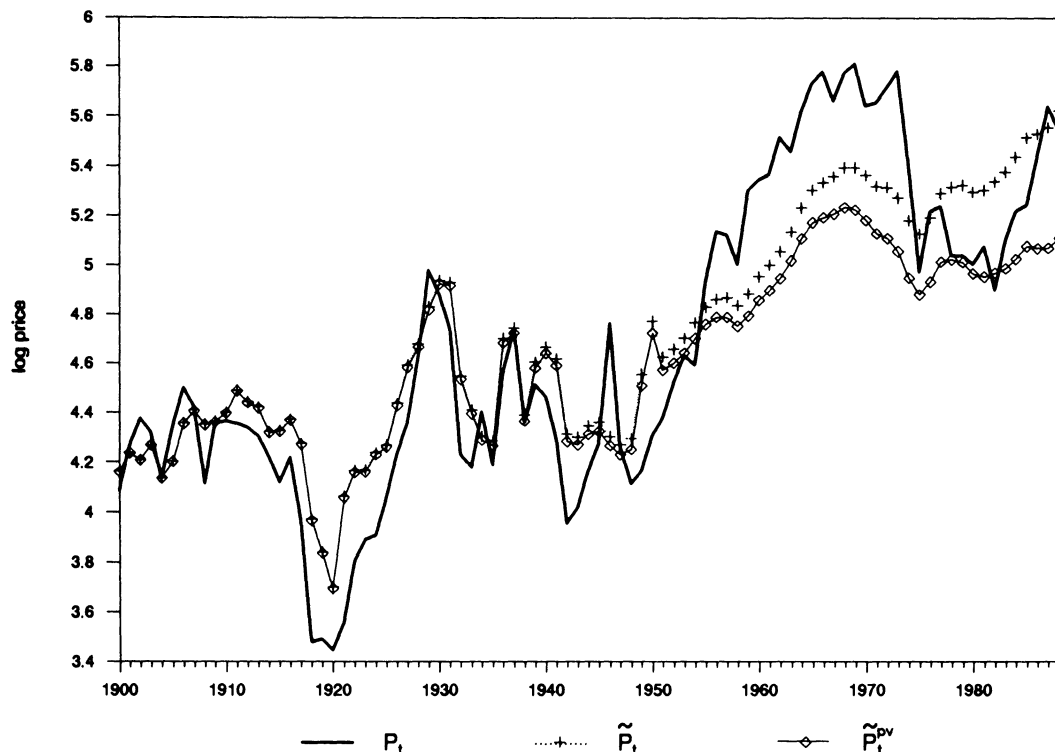


FIGURE 7. ACTUAL AND PREDICTED STOCK PRICES

Note: P_t is the actual real stock price; \tilde{P}_t is the predicted stock price under the time bubble in equation (11); and \tilde{P}_t^{pv} is that model's predicted present-value stock price.

tion by creating a correlation between price:dividend ratios and the level of dividends, even in the absence of a bubble.

In order to explore this possibility in more detail, suppose that the stock price does not contain a bubble but that the growth rate of dividends follows the autoregressive process:

$$(14) \quad \Delta d_{t+1} = \gamma_1 + \gamma_2 \Delta d_t + \xi_{t+1}.$$

If $\gamma_2 > 0$, a positive shock to the dividend growth rate tends to increase both dividends and the price:dividend ratio, tending to give a positive sample correlation between D_t and η_t in regressions such as (13).

To ascertain the importance of this effect, we ran a set of Monte Carlo experiments. We first estimated equation (14) in the data, finding that $\gamma_1 = 0.008$, $\gamma_2 = 0.1755$, and $\sigma_\xi = 0.122$. Our Monte Carlo procedure was then to draw 88-year paths of dividends, generated randomly according to (14), with ξ_t independently and identically distributed normal. We then estimated P_t^{pv} , the mathematically expected present value of dividends in (2). We then defined $P_t/D_t \equiv P_t^{pv}/D_t + \nu_t$, where ν_t is independently and identically distributed normal and calibrated such that simulated and actual price:dividend ratios have comparable average variability. Finally, we regressed the resulting price:dividend-ratio path on the associated path of $D_t^{\lambda-1}$, as in equation (13), and computed the t statistic for the test of

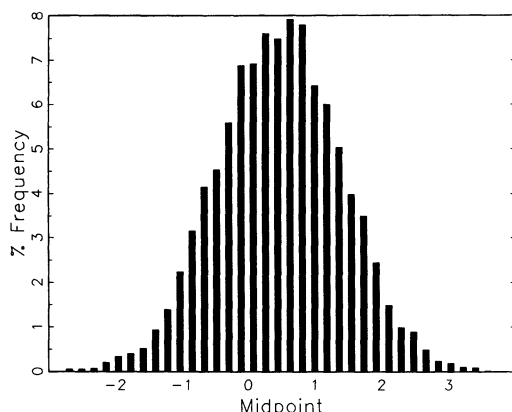


FIGURE 8. MONTE CARLO ESTIMATE OF THE DISTRIBUTION OF THE t STATISTIC FOR $c = 0$ IN EQUATION (10)

Note: The estimates were made under the assumption that dividends follow (14) and that stock prices do not contain a bubble.

$c = 0$.³⁶ This procedure was repeated 5,000 times.

Figure 8 shows the frequency distribution of the t statistics obtained from this procedure. The distribution is essentially normal, although upward-biased, with a mean of 0.47 and a standard deviation of 0.93. Slightly fewer than 5 percent of the statistics were greater than 2.0, and fewer than 0.3 percent were greater than 3.0. These results suggest that time-varying dividend growth rates could make our test of $c = 0$ in (13) reject too often in favor of $c > 0$, but the bias appears to be too small to explain the large t statistic in the actual data.

IV. Summary and Concluding Remarks

This paper has proposed a class of rational bubbles that depend exclusively on exogenous fundamentals. The resulting class of asset-price solutions has intuitive appeal because it avoids the introduction of extraneous driving variables and captures the

idea that prices can overreact to changes in fundamentals.

We applied a version of this model to United States stock-market data. The estimates reveal a strong nonlinear relationship between prices and dividends, which can be interpreted as a rejection of the hypothesis that there is no bubble. The estimates also help to reconcile the historical return on stocks with the level of the price:dividend ratio (and with its correlation with dividends), something that simple present-value models appear to be unable to do. In addition, the estimates imply that the bubble component in today's stock prices is very large. Even if one is reluctant to accept the bubble interpretation, the apparent nonlinearity of the price:dividend relation requires attention.

The hypothesis tests reported above have some desirable statistical properties. Unlike general specification tests, for example, the tests in this paper use estimates that are consistent under both the null and alternative hypotheses (given our identifying assumptions). The tight parametric form of intrinsic bubbles allows us to offer an interpretation of earlier specification-test results, which often did not pinpoint the factors causing model failure.

Our formulation allows variables such as the price:dividend ratio to predict excess returns. To carry out statistical inference, we do require that dividends themselves cannot be used to forecast returns, but in any case there is little direct evidence to the contrary. By relaxing the present-value assumption, the tests allow the data to allocate deviations from the simple present-value model across a bubble term and predictable excess returns. Our interpretation of Section III's results is that, once intrinsic bubbles are admitted as an empirical possibility, the predictability of excess returns no longer appears to be the only cause of the simple present-value model's failure.

It is hard not to be skeptical about the long-run implications of any kind of rational bubble. The fact that our identifying assumptions (that log dividends follow a martingale and that dividend innovations are unrelated to nonbubble components of

³⁶As in row 1 of Table 3, we used OLS and set $\lambda = 2.74$.

TABLE A1—UNIT-ROOT TESTS FOR ANNUAL REAL DIVIDENDS

Variable	β_1	
	With time trend	Without time trend
Log dividends, d_t	-0.1644 (-2.56)	-0.0545 (-1.40)

Notes: Figures reported are the coefficients β_1 in the following regressions: with trend, $\Delta x_{t+1} = \beta_0 + \beta_1 x_t + \beta_2 t + \nu_{t+1}$; without trend, $\Delta x_{t+1} = \beta_0 + \beta_1 x_t + \nu_{t+1}$. Standard errors are constructed allowing for an MA(4) process in the residuals. The t statistics reported in parentheses beneath the point estimates are for the test $\beta_1 = 0$.

price:dividend ratios) have not been rejected does not mean that they are true. In fact, we suspect that the class of assumptions that cannot be rejected is sufficiently large that, on the basis of currently available data, it may be impossible to determine conclusively whether deviations from present-value prices are nonstationary (i.e., rational bubbles) or stationary (i.e., fads) or even whether such deviations exist at all (i.e., time-varying discount factors or dividend growth rates). Perhaps the results above merely show that there is a coherent case to be made for bubbles alongside these alternative possibilities. If that is so, then we should not feel too comfortable about how well we really understand stock prices.

APPENDIX A

Time-Series Properties of Dividends

In deriving equation (13), we assumed that the log-dividend process follows a martingale with trend. In this appendix we briefly examine the time-series evidence on the dividend-generating process to see whether it is consistent with our assumption.

Table A1 reports tests of the null hypothesis that the log-dividend process, d_t , contains a unit root.³⁷ We perform the unit-root

tests allowing for alternative assumptions about the presence of a time trend. Neither version produces significant evidence against the unit-root hypothesis at the 10-percent level of statistical significance. As noted in the text, the data indicate that $\mu = 0.011$ and $\sigma = 0.122$.³⁸

Of course, if the stock price in (10) is to be a correct solution to (1), investors' conditional expectation of d_{t+1} must equal $\mu + d_t$. It follows that the disturbance ξ_{t+1} in (5) must not only be unpredictable given the past history of dividends, it must also be unpredictable given any broader time- t information set that investors use. In particular, because investors' forecasts of future dividends must depend on current dividends only, stock prices (which presumably reflect investor information beyond that in dividends) should not improve the accuracy of dividend forecasts that are based on current dividends alone. This is a strong assumption, so we check to see how well it fares in the data.

Table A2 reports tests for Granger-causality from prices to dividends. In the first row, we regress log-dividend changes on a constant and the lags of both log-dividend changes and log price:dividend ratios. Because the price:dividend ratio should in principle include all information relevant for forecasting future dividends, it should pick up any forecastable nontrend compo-

order serial correlation in the residuals, as suggested by those authors. For similar tests, see Kleidon (1986), Campbell and Shiller (1987) (who examine the level, rather than the log, of real dividends), and Campbell and Shiller (1988a).

³⁸There is some evidence that the residuals in this regression are not white, indicating that a more complex ARIMA process might perform better. The Durbin-Watson statistic was 1.65, which is inconclusive, but a $Q(27)$ test rejects the hypothesis of no serial correlation in the residuals at a 3.8-percent level of significance. Using the 1871–1986 sample of S&P data, Campbell and Shiller (1988a) reject (at the 5-percent level) the hypothesis that the d_t process contains a unit root. This finding could in principle be due to structural instability over the sample. There is also some evidence of kurtosis in the estimated residuals from equation (5). This could be evidence of time-varying volatility of log-dividend innovations.

³⁷The tests are those proposed by Phillips (1987) and Phillips and Perron (1988). We allow for fourth-

TABLE A2—TESTS FOR WHETHER PRICES GRANGER-CAUSE DIVIDENDS

Row	Regression equation	<i>F</i> test (<i>P</i> value)	<i>R</i> ²	DW	<i>d.f.</i>	Lag length
1	$\Delta d_{t+1} = \alpha(L)\Delta d_t + \beta(L)(p_t - d_t) + \nu_{t+1}$	0.812 (0.52)	0.13	1.96	75	4
2	$d_{t+1} = \alpha(L)d_t + \beta(L)p_t + \nu_{t+1}$	1.868 (0.12)	0.91	1.98	75	4

Notes: Granger-causality tests are based on OLS estimates and Newey-West standard errors. The sum of the coefficients on the log price:dividend ratio and on the log of price are reported in rows 1 and 2, respectively. The numbers in parentheses are probability values from *F* tests of the hypothesis that $\beta_i = 0 \forall i$. Alternative lag lengths were also tried for these regressions, but they did not change the results. The sample period for all regressions is 1900–1988. Constant terms were included in both regressions.

nent of dividend changes. The table reports the sum of the coefficients on $p_t - d_t$ and its lags, as well as an *F* test of the hypothesis that these coefficients are jointly zero. This test shows that we cannot reject the hypothesis that $p_t - d_t$ has no incremental power for forecasting future dividend changes.³⁹ The test's formulation is, however, unnecessarily restrictive. If log prices and log dividends are cointegrated of order (1,1) but with a coefficient other than 1, our inferences may not be valid. In the second row of Table A2, we therefore run a less restrictive regression, which asks directly whether log prices Granger-cause log dividends. Once again, the data provide no strong evidence that log prices have incremental predictive power for future log dividends.⁴⁰

In their tests of the present-value model, Campbell and Shiller (1987) report evidence to the contrary: that the spread does Granger-cause future dividend changes. However, these rejections appear to depend

on a different convention for dating prices and dividends: Campbell and Shiller use the beginning-of-period price, P_{t+1} , and the average of the previous period's dividend, D_t , to predict average period-(*t* + 1) dividends, D_{t+1} (Campbell and Shiller, 1987 p. 1074).⁴¹ If P_{t+1} contains cleaner, more up-to-the-minute information about the beginning-of-period-(*t* + 1) dividend than does the time-averaged variable D_t , then one would expect to find Granger-causality using the Campbell-Shiller dating convention, even when stock prices contain no information beyond that in the past history of dividends. Furthermore, as we have argued (see footnote 18), substantial information about the current year's dividends could become available during the month of January. We therefore see little basis at present for rejecting the hypothesis that prices do not Granger-cause dividends. While the view that prices contain information beyond that in current dividends is plausible, there just is not much evidence in its favor in these data.⁴² We conclude that (5) is a reasonable

³⁹We also ran this test in levels rather than logs, using the Campbell-Shiller spread, $P_t - \kappa D_t$, in place of the log price:dividend ratio and using ΔD_t in place of Δd_t . The results, using various measures of κ , are not importantly different from those reported above.

⁴⁰Christopher Sims et al. (1990) and West (1988b) give the asymptotic justification for this procedure. In both regression tests, we used a lag length of 4. Similar tests on alternative lag lengths yielded the same results. We also duplicated these tests on the 1871–1986 data set used by Campbell and Shiller (1987), with no change in the results.

⁴¹Following the dating convention described at the beginning of Section III, we instead use the beginning-of-period-*t* price, P_t , along with D_t to predict D_{t+1} . Robert Engle and Watson (1985) also use this convention and obtain Granger-causality results similar to ours.

⁴²We ran the regressions in Table A2 using Campbell and Shiller's (1987) dating convention and found results similar to theirs. If there is substantial additional information about future dividends in stock prices, then one might nevertheless expect to find that

empirical approximation to the true process investors use to forecast dividends.

APPENDIX B

Derivation of the Finite-Sample Distribution of the Test for $\hat{c} = 0$ in (13)

Consider the model $y_t = cx_t + \eta_t$, where $t = 1, \dots, T$, $y_t = P_t/D_t$, c is a parameter to be estimated, $x_t = D_t^{\lambda-1}$, and the log of D_t evolves according to (5):

$$(B1) \quad d_t = \mu t + d_0 + \sum_{s=1}^t \xi_s.$$

For simplicity, we assume that the constant term in (13), c_0 , is known and has been removed. Let \tilde{x} represent the random sequence of regressors from time 1 to T , a particular realization of which is given by x . We wish to derive the distribution of the test $\hat{c} = c$, where \hat{c} is the OLS estimate of c . To do this, we require the following assumptions.

prices Granger-cause dividends even when our dating convention is used. The results in Table A2, however, suggest that this is not the case. Campbell and Shiller (1988b) present evidence that a 30-year moving average of past earnings, deflated by the time- t real stock price, helps forecast Δd_t when added to a regression of that variable on its lags and on current and lagged values of $p_t - d_{t-1}$. Shiller (pers. comm.) reports that the explanatory power of this equation declines (but still remains highly significant) when the earnings and spread variables are redefined using the average real stock price for period $t-1$. What is puzzling, if rational expectations are assumed, is that the information content of past earnings apparently is not reflected in past stock prices. If it were, one would expect the lagged stock-price variables in the Table A2 regressions to pick up some of the ability of earnings to forecast future dividends.

We tried regressing the change in log dividends on lagged dividend changes and lagged values of $p_t - d_t$, where p_t is the year average price. We found that lagged log price:dividend ratios had no predictive power for changes in log dividends. However, if the level of log dividends is regressed on lagged log dividends and lagged log average prices, prices have predictive power at the 5-percent (but not the 1-percent) level over 1900–1986.

ASSUMPTION 1: *The residuals, η_t , are normally and identically, but not necessarily independently, distributed with unconditional mean 0 and autocorrelation function $\delta(k)$.*

ASSUMPTION 2: *The dividend innovations, ξ_t , are independently distributed of the residuals, η_t , at all leads and lags and have mean 0 and variance σ^2 .*

To proceed, note that the OLS estimate of c is

$$(B2) \quad \hat{c}(\tilde{x}) - c = \frac{\sum_{t=1}^T \tilde{x}_t \eta_t}{\sum_{t=1}^T \tilde{x}_t^2} = \sum_{t=1}^T \left(\frac{\tilde{x}_t}{\sum_{s=1}^T \tilde{x}_s^2} \right) \eta_t \\ \equiv \sum_{t=1}^T \tilde{w}_t \eta_t$$

where the \tilde{w}_t are a random set of weights, which by Assumption 2 are independently distributed of the η_t 's. By Assumptions 1 and 2, the linear combination in (B2), for a given sample path of the regressors, x , is a weighted average of normals and is therefore normally distributed:

$$(B3) \quad \hat{c}(x) - c \sim \mathcal{N}(0, (x'x)^{-1} x' \Omega x (x'x)^{-1})$$

where $\Omega_{i,j} = \delta(i-j)$. Notice that since the distribution of \hat{c} depends on the particular realization, x , the unconditional distribution of \hat{c} will be a mixture of normals and will therefore have fat tails. Nevertheless, under both the null and alternative hypotheses, c is estimated consistently.

Even though the unconditional distribution of \hat{c} is not normal, the usual t statistic for $\hat{c}(x) = c$ is distributed $\mathcal{N}(0,1)$, even in finite samples, provided that Ω is known. To see this, note that from (B3) the t statistic is given by

$$(B4) \quad \frac{\hat{c}(x) - c}{\sqrt{(x'x)^{-1} x' \Omega x (x'x)^{-1}}} \sim \mathcal{N}(0,1).$$

Because this distribution does not depend on the sample realization, x , it holds uncon-

ditionally. This is true under both the null and alternative hypotheses.

Of course, (B4) assumes that Ω is known. If Ω must be estimated and if η_t is serially uncorrelated, then the expression on the left-hand side of (B4) has an exact t distribution in finite samples. If Ω must be estimated and η_t is serially correlated, then the expression on the left-hand side of (B4) does not have a t distribution in finite samples but will converge to $\mathcal{N}(0,1)$ in distribution as T approaches infinity.

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