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The Law of One Price Over 700 Years

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This paper examines annual commodity price data from England and Holland over a span of seven centuries. Our data incorporates transaction prices on seven commodities: barley, butter, cheese, oats, peas, silver, and wheat, as well as pound/shilling nominal exchange rates going back, in some cases, to 1273. We find that the magnitude, volatility, and persistence of deviations from the law of one price have not declined by as much as one might expect. We find this despite lower transport costs, reduced trade protection, and fewer wars and plagues in the modern era. Our analysis is consistent with growing evidence that goods-market arbitrage remains highly imperfect, even today.

Key Words: Purchasing power parity; Exchange rates.
JEL Classification Numbers: F3, F31.

1. INTRODUCTION

One of the most striking empirical regularities in international finance is the volatility and persistence of deviations from the law of one price across relatively homogeneous classes of goods. Whereas goods-market arbitrage may force virtually instantaneous international price equalization for precious metals such as gold and platinum, price adjustment for most goods is relatively slow, with half lives for price deviations typically exceeding one year. Though there is some debate, the half-life of purchasing power parity

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deviations appears more on the order of two to four years for PPI and CPI deviations.[1]

The general presumption among most international economists is that the volatility, if not the persistence, of international price and real exchange rate deviations is a relatively modern phenomenon, reflecting a combination of domestic price rigidities and high nominal exchange rate volatility. We show here that this is not the case; between the fourteenth and twentieth centuries, the fall in the volatility and persistence of deviations from the law of one price has been remarkably small. Moreover, because our data also reveal a large common component in deviations from the law of one price across goods, we conclude that a secular decline in the importance of measurement error is not driving our results.[2]

The extent of goods-market segmentation is at the absolute core of many policy debates in international economics, including exchange rate policy, capital controls, coordination of financial regulation, and trade policies. Although the weight of the recent literature has moved sharply toward the view that international segmentation is large, our findings here provide a unique historical benchmark for what 'large' is.

Our data set, which we describe in Section II, consists of annual price observations on a variety of agricultural commodities for England and Holland, going back in some cases to the thirteenth century. The sheer length of the data set is, of course, interesting in its own right. With seven centuries of data, one can potentially say much more about low frequency characteristics of the data than is normally the case. Indeed, we are able to examine the behavior of relative price movements over five year intervals, while retaining over 100 degrees of freedom. Even during the Middle Ages, movements in cross-country relative prices of the same good constituted the major source of variation in real exchange rates. We show that this occurs despite the fact that nominal exchange rates were far less volatile and local-currency pricing (sticky prices) apparently far less important. This supports the view in the literature that even today goods-market arbitrage appears weak except over a relatively narrow range of goods, at least until price deviations exceed 25 percent or 30 percent.[3]

[1]Isard (1977) finds large persistent deviations from the law of one price in seven-digit SITC categories; Giovannini (1988) finds similar results even for extremely homogenous categories of goods such as screws. Other papers documenting the size of law of one price deviations include Engel (1993), Rogers and Jenkins (1995) and Engel and Rogers (1994). For surveys of this literature and the broader literature on purchasing power parity, see Froot and Rogoff (1995) and Taylor (2000).

[2]Engel (1993) and Engel and Rogers (1995) have emphasized that post 1973 within-country relative price movements for different goods are small relative to cross-country relative price movements for the same good.

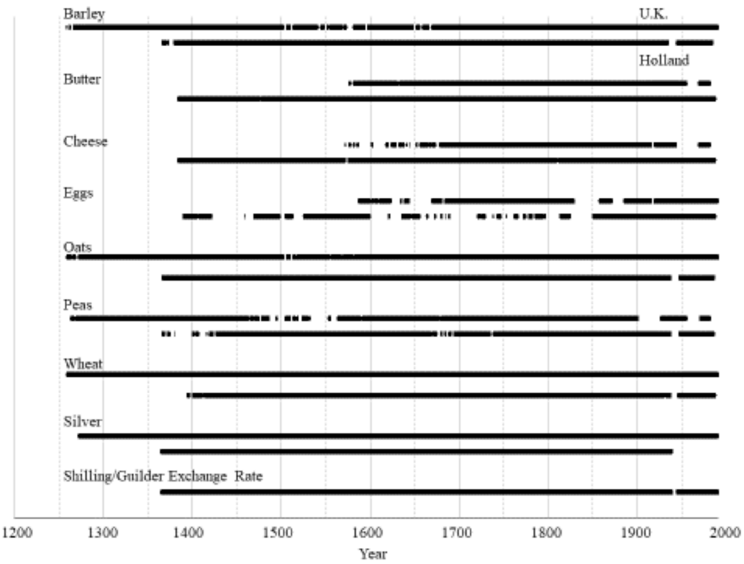
[3]See Obstfeld and Rogoff (2000) and Engel (1999).

We proceed as follows. We first discuss the data, look at some basic visual characteristics, and then present out estimates of volatility and persistence. The Section 4 concludes.

2. DATA DESCRIPTION

Our data set consists of annual wholesale prices for England and Holland for seven commodities, spanning the late thirteenth century to the present day. The commodities include three grains (wheat, oats and barley), two dairy products (butter and cheese), peas, and silver. Coverage varies somewhat over time and across commodities, as Figure 1 illustrates. The grain and market silver price series are quite solid with very few missing observations. In contrast, the dairy price data for England generally begin only in the mid-sixteenth century (Dutch data begin earlier), with a number of missing observations thereafter.

FIG. 1. English and Dutch Commodity Prices, 1273-1991 Data Coverage



2.1. Data Sources

The core references for the pre-nineteenth century data are two studies which grew out of the International Commission on Price History, a project headed by Lord William Beveridge (of Beveridge curve fame) that began in the 1930s. Beveridge's (1939) book on England mainly covers the mid-sixteenth century through the eighteenth century, whereas Posthumus's two books (1946, 1964) on Holland cover mainly the late fourteenth century through the early nineteenth century. Thanks in part to the coordinating efforts of the Commission, there is a significant correspondence in methodology across the two studies.

Although the Beveridge project volumes fill in several centuries of data, they still leave the earliest and most recent centuries of our sample period uncovered. The two main sources for early Middle Ages data are Thorgold Rogers (1866) for England, and Herman Van der Wee (1963) for Holland. As Appendix I details, for many commodities including especially the grains and silver, one can find multiple data sources even for the early years. The availability of multiple sources provides, of course, a helpful check on the data. The Beveridge and Posthumus books themselves provide duplicate price quotes for some commodities.

How do economic historians construct price data for the Middle Ages? In some cases, the prices are based on records from town markets or, for later periods, from organized exchanges. But by far the most important sources are the purchasing records of various institutions such as hospitals, colleges, orphanages, and the military. Though such data are not posted market prices, they are actual transactions prices paid by bulk purchasers. Appendix I lists many of the primary sources underlying the data, though of course the interested reader should turn to our various secondary sources for a more detailed data description; see also Kim (1996).[4]

The data for the past two hundred years come from a variety of sources, including back issues of The Economist, annual tables in the Journal of the Royal Statistical Society, Mitchell and Deane (1962), Mitchell and Jones (1971), and government statistical abstracts. Curiously, price data on nineteenth-century Holland has, until recently, been extremely sparse. Fortunately, recent work by Knibbe (1993) and van Reil (1995) has filled in some of the major gaps. For the nineteenth and twentieth century, most of the data are wholesale market prices, though for nineteenth century Holland, some still comes from institutional transactions records.

2.2. Caveats: Location, Averaging Procedures, Homogeneity of Goods

Although the data seem reasonably reliable, the reader must be aware of a host of caveats when trying to interpret our later time series results. First, prices come from a variety of different locales within each country. For England, all prices are either from London or from institutions in nearby southeastern England. For Holland, the distances are greater. Amsterdam, of course, was not a major trading center until the 1500s. Before that, Utrecht was the commercial capital of Holland and some of our early price

[4]See Kim (1996) also for a discussion of some of the many issues of data construction, e.g., keeping track of subtle changes to the official definition of a bushed over time.

observations come from there. Dutch prices for the early Middle Ages are from Flanders and Brabant, which were economically integrated with Holland from the time of the Holy Roman Empire until Holland gained its independence from Spain at the end of the sixteenth century. After the 1500s, virtually all of the data is from Amsterdam. Fortunately, the significant amount of overlapping data we have from the various markets suggests that price variation across markets within the same country are small compared to price variation we will later observe across commodities and countries (where differentials in excess of 20 percent are the norm).

Second, the annual data points for each commodity are actually averages of prices recorded throughout the year, with the method of averaging differing somewhat across studies. Not surprisingly, there is considerable controversy among historians on this issue. Beveridge criticizes Rogers for placing the same weight on individual transactions data as on price observations from town markets and large regular institutional purchasers. Rogers, in contrast, argues that any price that represents an arm's length transaction is a valuable piece of information.

In some cases, we have multiple sources of data for the same commodity in a single country. Our general procedure for handling multiple data sources and commodity types was to take a simple average of the available data for any given commodity in any given year. (Sources for the raw data are listed in the Appendix I.) Clearly, price variations due to multiple data sources for the same commodity within a country can be treated as a source of measurement error. However, this source of error does not appear important regardless of how we average across commodities.

Third, in addition to the problem of having shifting locales, one must recognize that none of the commodities we study are perfectly homogeneous. There are, for example, many different varieties of wheat. Posthumus provides prices for Konigsberg wheat, Polish wheat, red wheat, Warder wheat, Frisian wheat, and Zealand wheat. 'Barley' includes summer barley, winter barley, brewing barley and fudder barley. Beveridge notes that during bad harvest periods, the average quality of grains sold in town markets generally tends to drop.

In spite of this lack of homogeneity, the splicing across different types of a commodity does not drive our results. Like other sources of measurement error splicing might be a source of jumps and variation across all relative prices within a country. However, we show that the within-country variation of prices of different goods is not large compared to across-country variation of prices of the same good. Put differently, all the different series yield very similar results, across goods and across centuries. This suggests not only that splicing across data series does not dominate our findings, but that measurement error—which intuitively would be much larger in the earlier part of the sample—generally cannot be the dominant factor.

2.3. Other Issues in Data Construction

It should be noted that direct trade between England and Holland in the commodities listed here was quite limited over most of the period. Holland, for example, imported wheat from Austria, Germany, and Poland; see Posthumus (1946). Imports from England included mainly tin and lead, and exports to England included linens and spices. Thus any arbitrage to enforce the law of one price came mainly through third parties. Technology diffusion, of course, can also help equate relative prices, though over much longer periods.

Generally speaking, our tests for the law of one price involve converting nominal prices to silver prices within each country, and then comparing silver prices across countries. This approach is necessary as data on exchange rates is quite limited before the 1500s, whereas local-currency prices for silver are relatively easily obtained. For the post-1500 period where guilder-shilling exchange rates are available, deviations from the law of one price in silver for our data set appear to be extremely small (typically less than one percent).[5] One could, of course, also use gold as the numeraire. In choosing silver, we follow the lead of the Beveridge Commission.

The prices we use are generally producer (wholesale) prices, inclusive of taxes. Beveridge and Posthumus provide enough information to remove taxes for some years for some series, but since the law of one price is generally tested inclusive of taxes we leave them in. There are other caveats. Beveridge, for example, notes that military purchasers were notoriously delinquent in making payments and no inflation or interest adjustment is made in the data to allow for this. It was not uncommon during the Middle Ages for the families of hospital patients to pay bills in kind with grains, so some (small) percentage of the prices drawn from early hospital records may not really represent arm's-length transactions, and the historians may not always have been successful in weeding out such cases.

Finally, we should mention that our choice of countries and commodities was largely dictated by our goal of putting together the longest possible time series for testing the law of one price. After silver and gold data, grain data is by far the deepest and most complete. Our decision to include dairy commodities such as butter, where data become available only later, was dictated by a desire to have a spectrum of tradability across the different goods considered. Trade in butter was presumably far more difficult than trade in wheat, at least prior to modern refrigeration and packing techniques. If one is willing to start from the late 1600s, it would be possible to test the law of one price across a much broader range of goods than we

[5]Even silver is not quite homogenous. Dutch prices tend to be quoted for fine silver versus standard silver for British prices. The relative price within each country between standard and fine silver was quite stable over the period, however, at 0.925 (see Jastram (1981)), so all prices were converted to standard silver.

do here. Our view was that it was especially interesting to focus first on commodities for which really long time series are available.

In sum, a time series of this length must be spliced together from a variety of sources encompassing a range of market locales and subtle variations in commodity types. We will argue, though, that these imperfections are generally second-order compared to the huge swings one observes over time for relative prices of distinct commodities, and for price differentials across different countries for the same commodity.

2.4. Wars and Plagues

Over such a long sample period, there are a plethora of major events one might want to control for in forming inferences about law of one price deviations. England and Holland fought countless wars over the sample period, sometimes independently, sometimes as allies, sometimes as enemies. Clearly wars are special events that might disrupt integration; this is certainly the case in the modern era. One also has to consider the effects of the scores of plagues that ravaged Europe during the Middle Ages, more than once eliminating sizable fractions of England and Holland's populations. Again, sometimes plagues occurred concurrently in the two countries, sometimes not. It is, of course, not obvious how plagues would affect price deviations. Did they have a greater effect on demand or supply? Our approach to dealing with plagues and wars is agnostic. We use dummy variables to control for war/plague effects, and to test the robustness of our results. (The notes to Tables 5 and 6 at the end of the paper list the various major war and plague episodes.)

We do not separately treat peacetime changes in trade regimes or commercial policy. For example, we do not adjust for Britain's Corn Laws of the eighteenth and nineteenth centuries. We also do not separate out the period 1950-1973 when the Netherlands was included in the EEC's agricultural policies, but Britain was not. Nor do we segregate periods of extraordinary innovation in transportation technologies and costs. Certainly, at the level of individual markets and cities, there were undoubtedly many exogenous shocks, including the weather, that we do not control for. This makes it all the more striking how stable the law of one price deviations turn out to be.

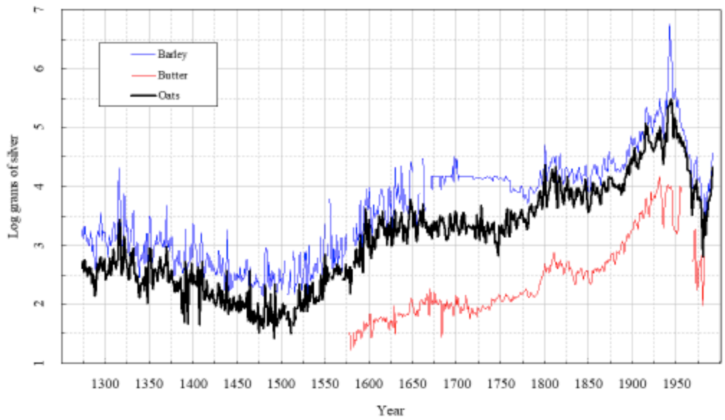
3. TESTING AND RESULTS

Because our data set is so interesting and unusual, we focus on very simple graphs and descriptive statistics before attempting to estimate some of the basic parameters of volatility and persistence.

3.1. Graphical Results

Before looking at relative prices across countries, it is useful to look at broad trends in within-country price levels for individual commodities. Figures 2 and 3 show the (log) prices of barley, oats, and butter relative to silver in England and Holland, respectively. In the graphs, an upward movement denotes a rise in the value of the good relative to silver. Not surprisingly, the two figures reveal clear evidence of a large common low-frequency component, both across goods and countries. For example, the logs of goods prices relative to silver rose by about 1.50 (or about 450 percent) from 1525 to about 1600. After that, there is no clear trend until the eighteenth century.

FIG. 2. English Barley, Butter and Oats, 1273-1991 Log Price in Grams of Silver Per Unit

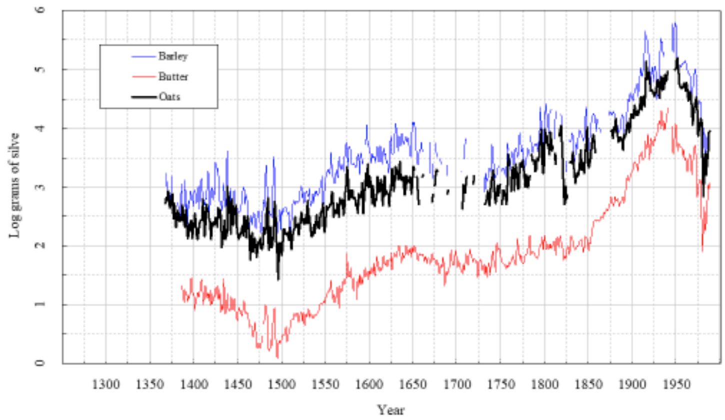


Historians have articulated two primary causes for this century-long surge in prices relative to silver.[6] The first main cause was the discovery of massive silver deposits in the Americas, including especially the 1545 Potosi discovery in modern-day Peru. Combined with improved mining techniques, these new lodes produced a sharp increase in European silver stocks, with growth peaking during the 1590s. The second cause was the rapidity of population growth after the Black Plague of the mid-fifteenth century. As additional lower quality lands had to be farmed to meet increased demand, prices of agricultural products rose.[7]

[6]For good overviews of European history during much of this period, see Garraty and Gay (1972), Palmer and Colton (1978), and Cameron (1993).

[7]One problem with this hypothesis is that, holding silver stocks constant, the per capita supply of silver falls with increases in population. This effect would tend to lower the prices of grains relative to silver, and it is unclear whether the supply of grains was sufficiently inelastic to dominate it.

FIG. 3. Dutch Barley, Butter and Oats, 1273-1991 Log Price in Grams of Silver Per Unit



Prices in terms of silver grew by a similar amount in the late nineteenth and early twentieth centuries as private silver stocks grew again, this time due largely to the discovery of the Comstock lode in Nevada in 1859 and the progressive demonetization of silver during the latter part of the century.

Thus, while it is true that the price of silver has risen sharply since World War II, this change comes on the heels of a much longer period of falling price trends. In 1990, barley, oats, and wheat were worth on average about 4.5 times as much silver as they were in 1273. Of course, over seven hundred years, this amounts to only 21 basis points a year. Long-term relative price movements among the various agricultural goods is even smaller. Parenthetically, one might conclude from the constancy of relative prices over very long horizons that there is a substantial degree of convergence in productivity across different commodities. If so, this would provide support for the view that technological innovation responds endogenously to price differentials, if only sporadically and only over very long time periods.

We next use the data to examine deviations from the law of one price (henceforth LOP) over the sample. Figures 4 and 5 show the disparity between British and Dutch prices (in logs) for barley and butter, respectively. (Oats is visually quite similar to barley.) In the graphs, an upward movement denotes a rise in British relative to Dutch prices (after converting both sets of prices to a common silver price numeraire.) As we have already discussed, our interpretation of these prices assumes that the law of one price holds for silver; this appears to be a very good approximation over the period from the 1500s on where exchange rate data is available.

FIG. 4. Relative English to Dutch Barley Prices, 1273-1991

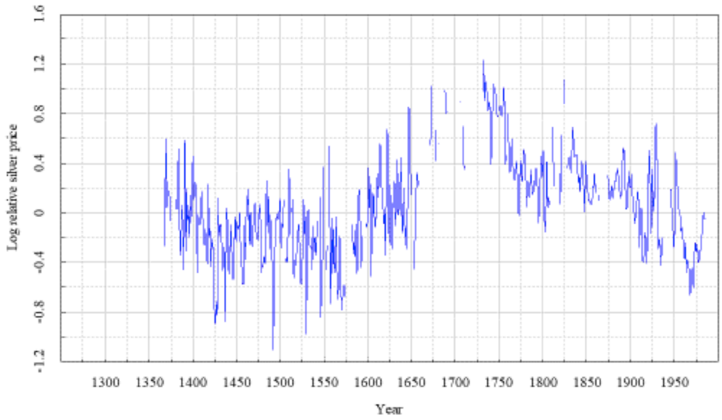
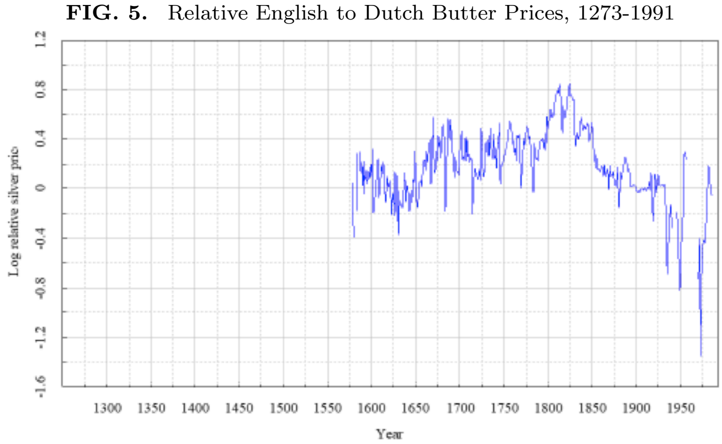


FIG. 5. Relative English to Dutch Butter Prices, 1273-1991



The most remarkable characteristic of Figures 4 and 5 is the volatility of LOP deviations. This volatility is very large—a simple year-to-year standard deviation of 0.25 in logs is not unusual in the earlier part of the sample. Casual inspection of the figures suggests that there are no large trends in LOP deviations over the full period. However, there do appear to be low-frequency movements at one- to two-century time intervals.

A comparison of the two figures suggests that the relative price movements are highly correlated across the two goods, at least at frequencies of one or two centuries. As we shall see, similar high corrections hold broadly across all the goods in our sample. When the relative price of barley is persistently high in England, English prices of other goods tend to be persistently high as well. Though low frequency correlations are not our main focus here, it nevertheless interesting to pause to reflect on what kind of model might explain these very long-term trend comovements in the relative prices of English and Dutch goods.

The most obvious explanation for the relatively high level of English prices during eighteenth and early nineteenth centuries are the Corn Laws in England, that placed significant tariffs on the import of grains. High relative Dutch prices from 1950 to 1973 might be due to the Netherlands participation in the protectionist agricultural policy of the EEC, while Britain stayed out until 1973. But one can only push the commercial policy explanation so far. Note that the secular rise in British prices (relative to Dutch) appears to begin already in the first half of the seventeenth century. Yet the Corn Laws only came into being only in 1698 (they were repealed in 1846).

Therefore, it is also interesting to consider the Balassa-Samuelson hypothesis, that the relative prices of nontraded goods will tend to rise in countries with faster income growth (see Froot and Rogoff (1995)). If one views silver as being far more easily traded than agricultural commodities then, indeed, the fluctuations evident in Figures 4 and 5 accord well with relative GNP growth movements across the two countries. Specifically, the rise of Amsterdam as a major commercial center culminates around 1609, when under the 12 Year Truce between Holland and Spain, the port of Antwerp was effectively cut-off. This rise might explain the (slight) negative relative price trends to 1609. After that, Holland's fortunes fell relative to those of the English. By 1713, Holland had basically exhausted herself fighting wars against France, allowing Great Britain to build herself into the world's main naval power. Holland was still a major financial and trade center and an important source of foreign capital but its relative position declined as Great Britain developed rapidly. The industrial revolution in England started somewhat later, circa 1760 (e.g., James Watt invented his steam engine in 1769). Interestingly, our data show that the industrial revolution had less of an effect on deviations from the LOP than did England's earlier commercial and political ascendancy. Figures 4 and 5 also suggest that English-to-Dutch prices fell from about 1825 on. At that time, Britain had become the richest country in the world, but its growth began to be eclipsed by other European countries. It is interesting to note in Figures 4 and 5 that low-frequency deviations from the law of one price appear on a one-or-two century basis, but that these deviations appear to revert over the long run.[8] [9] The issue of what drove the long-term price trends is quite an interesting one, but not central here. Essentially, while we do need to control for the overlay of long-term trends in interpreting our volatility data, over most periods these trends are swamped by short-term fluctuations.

FIG. 6. Normal Shilling/Guilder Exchange Rate and Relative English/Dutch Wheat Prices, 1273-1991

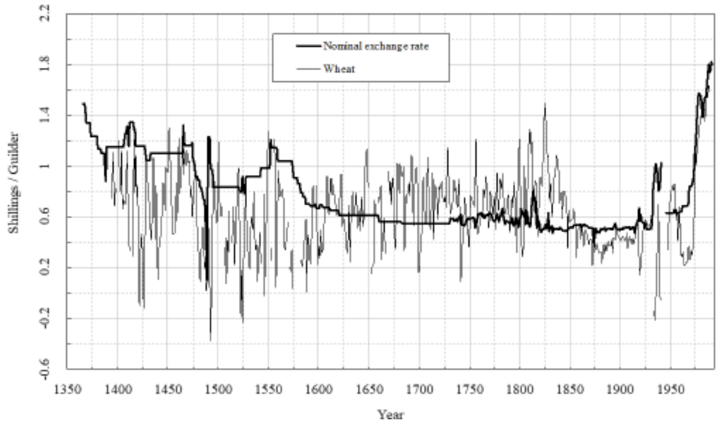


Figure 6, which decomposes law of one price changes into domestic currency price movements versus exchange rate movements, gives a very different perspective on the data. A central issue in the modern debate about international monetary transmission mechanisms is the importance of "local-currency pricing" in driving deviations from the law of one price. Under local currency pricing, firms set prices in advance in local currency terms; it is assumed that arbitrageurs are unable to equalize prices, perhaps due to monopoly power at the wholesale level. Though at one time, most trade economists viewed local currency pricing as characterizing only

[8]Trend long-run law of one price deviations are possible across goods containing a nontraded component, but only if there are trend productivity differentials between the two countries (see Froot and Rogoff (1995)). Given that England and Holland had roughly similar cumulative growth rates over the full sample, the long-run price convergence is perhaps not surprising.

[9]Our data also provide some perspective on the Baumol-Bowen (1966) effect, which relates the evolution of relative prices to differences in productivity growth (i.e., more rapid productivity growth results in secular price declines). Baumol and Bowen argued that as a rule, productivity growth is slower in more labor intensive goods. In the NBER working paper version, we depict the behavior of the log price of wheat relative to butter, cheese, and eggs, and demonstrate that these log relative prices have risen by a factor 1.2 during the sample, an increase of 330 percent in the relative price levels since the fifteenth century. Assuming that dairy production is more labor intensive than grain production, this price rise is broadly consistent with the Baumol-Bowen view.

a small number of goods which are difficult to arbitrage (e.g., automobiles), many recent articles have argued that local-currency pricing is by far the predominant mode of international price setting.[10]

In the context of this debate, Figure 6 is truly striking. It shows the extent to which deviations from the law of one price are associated with nominal exchange rate fluctuations as opposed to fluctuations in local currency prices. Specifically, Figure 6 compares the nominal pound-shilling exchange rate with the ratio of English-to-Dutch local prices of wheat.

The Figure makes the point that the nominal exchange rate prior to about 1600 was quite turbulent, that the period between approximately 1600, and 1930, was relatively quiescent, and that the post-1930 period became turbulent again, with the post-Bretton-Woods period being extremely so. Interestingly, from 1600 until the late twentieth century, the variability in local-currency grain prices was very large, accounting for the majority of deviations in the law of one price. Beginning in the mid-nineteenth century, and carrying over to today (and especially through the 1973-91 floating-rate period), the variability of local-currency prices has fallen dramatically, and the variability of the nominal exchange rate has increased dramatically. This suggests that the importance of local currency pricing in driving deviations from the law of one price is a relatively recent phenomenon. To the extent that local-currency prices are sticky, such stickiness appears to have emerged only in the last century or so. The implication is that while the volatility of deviations from the law of one price has not changed conspicuously from century to century over the last 700 years, the composition of these deviations is now completely different. Granted, some of the early fluctuations in price may be due to measurement error but, as we have mentioned repeatedly, this can only be a part of the story since nominal price changes across good will turn out to be highly correlated.

This is as far as we go here using visual metrics. Section IV attempts to characterize some basic statistical properties of the data.

3.2. Econometric Methodology

We next examine simple measures of trends, volatilities, and persistence. Obviously, a data set of this length allows one to contemplate implementing the most extravagant time series techniques, but it seemed to us that in a first pass at the data, it would be useful to focus first on relatively transparent statistics. The first difficult issue is that there are missing observations (especially for cheese, peas and butter), which makes estimation of persistence complicated. The second difficult issue is how to control for the myriad of special circumstances and events (e.g., the Black Death

[10]See, for example, Devereaux and Engel (2000), Engel (1993), and Engel and Rogers (1994).

which eliminated 1/3 of Europe's population in the fourteenth century) which take place over a time span of this magnitude.

To deal with missing observations, we employ a Kalman filter methodology in addition to standard time-series techniques in order to simultaneously estimate missing data points and coefficients of persistence. By working with state-space representations, we can estimate ARIMA specifications while keeping track of unobservable state variables. The specific algorithm we use is similar to that developed by Gardner, Harvey, and Phillips (1992).[11] This algorithm allows us to compute an exact likelihood function which can be maximized iteratively using numerical optimization techniques. Appendix III for a discussion of our approach. The usual disadvantages of this technique due to poor small sample properties is not a problem in this context.

Out of the large universe of special circumstances and events one might try to control for, it seemed natural to us (in this first pass at the data) to focus on plagues and wars, which are the most straightforward and obvious departures from normal circumstances. With little in the way of theory to guide us, we tried a number of specifications, averaging across them. As the individual specifications yielded by and large similar results, we report only averaged results across the specifications. (These specifications are detailed in the footnotes to Table 1.) The plague and war dummies are detailed in Tables 5 and 6. Here we note two unusual properties of these dummies. First, we label as 'transitory' those dummies that are nonzero only during a plague or war event period. 'Permanent' dummies become nonzero when a plague or war commences, but then remain nonzero for the rest of the sample. The permanent dummies better pick up persistent effects that major plagues and war are likely to exhibit. Second, to avoid identification problems (many plagues and war overlap one another), we combined transitory dummies into single 'plague' and 'war' dummies. Each of these takes on the value of 1 or -1 during the occurrence. The sign for each plague or war was determined in a prior stage in which we regressed prices on transitory dummies for individual wars and plagues and observed the sign of the estimated dummy coefficient.

Fundamentally, plagues do not appear to have any obvious systematic effect on deviations from the law of one price, and wars only appear to have a striking effect beginning with the Napoleonic wars in the early nineteenth century.[12] Our results leave open the question of whether controlling for other special circumstances, most notably commercial policy, might have

[11]On state-space procedures for estimating ARIMA models with missing observations, see also Jones (1980) and Harvey and Pierce (1984).

[12]Rogers (1994) presents evidence that in more modern times, wars do affect the time series behavior of relative prices. Specifically, he finds that PPP deviations die out at different rates during war and nonwar years.

made a fundamental difference. (We have no doubt some such events would prove statistically significant, if only by due to type II error.)

3.3. Trends in Relative Prices

In Table 1, we look at trends both over the entire sweep of the data sample, and over individual centuries. Trend increases over different subsamples are important because they reveal something about low frequency fluctuations in the data. Of course, being able to detect low-frequency movements in the data is one of the great benefits to having 700 annual observations.

For the purposes of estimating trends, our universe of specifications consists of regressions of relative prices on: i) a constant and time trend; ii) a constant, transitory war and plague dummies, and time trend; and iii) a constant, transitory war and plague dummies, permanent war and plague dummies, and time trend.

Table 1 reports trend estimates obtained from the full sample (1273-1991), as well as century-long subperiods (1273-1399, 1400-1499, etc.). We also report estimates from the 1973-1991 subperiod.

The variable $\ln(P_i^{UK}/P_i^{Hol})$ is the (log) relative price of the ith British good relative to the same Dutch good. This is the category of relative prices we use to examine the performance of LOP. Obviously, data coverage is less than for either of the corresponding individual-country series, since to test LOP one must have prices for both countries. The category marked $\ln(P_i^{UK}/P_{Wheat}^{UK})$ denotes the price of various goods in England relative to wheat; the category marked $\ln(P_i^{Hol}/P_{Wheat}^{Hol})$ is defined analogously. Finally, the categories marked $\ln(P_i^{UK})$ and $\ln(P_i^{Hol})$ denote the simple average prices of goods relative to silver in Britain and Holland, respectively.

Table 1 gives the average estimated trend coefficient across our full universe of specifications. The first point made by Table 1 is that the trend estimates are reasonably large economically. Average trend movements of half a percent a year are not uncommon over many century-long intervals, even for LOP deviations. For example, the point estimates show the relative price of English-to-Dutch barley falling until 1600, at which point it began to grow at 1.40 percent a year during 1600-99, only to decline again in the following century at an average annual rate of 1.10 percent.

To save space we do not report averages of the estimated standard errors of these trend coefficients. However, we generally find that these trends, at least on a century-by-century basis, are typically not statistically significant, regardless of the precise specification considered. Basically, the deviations from the law of one price are so volatile as to swamp the trends, even over a century.

TABLE 1.

English and Dutch Commodity Prices 1273-1991: Average Trend Times 100

Category	Index	1273-1991	1273-1399	1400-1499	1500-1599	1600-1699	1700-1799	1800-1899	1900-1991	1973-1991
$\ln(P_i^{UK}/P_i^{Hol})$	Barley	−0.11	−1.44	−0.32	0.25	1.40	−1.10	−0.17	−0.67	3.39
	Butter	−0.17	—	—	1.15	0.26	0.30	−0.83	−0.67	9.85
	Cheese	−0.21	—	—	3.79	−0.93	0.22	−0.54	−0.69	7.04
	Oats	−0.07	−1.11	−0.70	0.49	0.39	−0.26	−0.39	−0.21	0.67
	Peas	0.14	−3.16	0.37	−1.48	0.67	0.18	0.23	−0.68	0.60
	Wheat	−0.07	−8.98	−0.33	−1.21	0.29	−0.01	−0.50	0.14	0.87
	Mean*	[(0.12)]	(−3.67)	(−0.23)	(0.19)	(0.46)	(−0.13)	(−0.34)	(−0.54)	(3.48)
S.D.*	[(0.34)]	(2.11)	(0.19)	(0.76)	(0.31)	(0.20)	(0.14)	(0.15)	(1.48)	
$\ln(P_i^{UK}/P_{Wheat}^{UK})$	Barley	0.08	−0.13	0.11	0.79	0.64	−0.85	0.72	−0.44	−0.08
	Butter	0.41	—	—	−2.21	0.13	−0.07	1.02	−0.07	8.27
	Cheese	0.45	—	—	−2.16	−0.96	0.10	0.86	0.44	5.30
	Oats	0.15	−0.04	−0.08	1.27	0.14	−0.14	0.72	−0.18	0.48
	Peas	0.28	−0.20	0.39	−0.26	0.66	0.02	1.28	−0.65	−3.81
	Mean*	[(0.31)]	(−0.12)	(0.14)	(−0.51)	(0.12)	(−0.19)	(0.92)	(−0.18)	(2.03)
	S.D.*	[(0.32)]	(0.06)	(0.17)	(0.81)	(0.33)	(0.19)	(0.12)	(0.21)	(2.38)
	Silver	−0.14	0.17	0.46	−0.40	−0.18	−0.63	0.44	1.79	−4.27
$\ln(P_i^{Hol}/P_{Wheat}^{Hol})$	Barley	0.15	−11.13	0.13	−0.33	0.71	−0.10	0.42	−0.18	−1.44
	Butter	0.36	−7.47	−0.28	−0.55	0.16	−0.36	1.38	0.29	1.23
	Cheese	0.41	−11.83	−0.03	−0.12	0.35	−0.10	0.92	1.10	3.46
	Oats	0.22	−20.68	0.22	−0.74	0.74	−0.02	0.76	0.00	0.28
	Peas	0.23	—	0.12	−0.23	0.31	−0.18	0.53	−0.12	1.70
	Mean*	[(−1.05)]	(−12.78)	(0.03)	(−0.39)	(0.45)	(−0.15)	(0.80)	(0.22)	(1.05)
	S.D.	[(0.71)]	(3.24)	(0.10)	(0.13)	(0.13)	(0.06)	(0.19)	(0.26)	(0.90)
	Silver	−0.25	−14.75	0.19	−1.47	−0.01	−0.59	−0.03	1.74	0.08
	Mean*	[(−0.03)]	(−0.28)	(−0.27)	(0.92)	(0.37)	(0.46)	(0.34)	(−2.39)	(0.45)
$\ln(P_i^{UK})$	S.D.*	[(0.35)]	(0.06)	(0.14)	(0.30)	(0.24)	(0.16)	(0.20)	(0.37)	(2.66)
	Mean*	[(0.40)]	(2.36)	(−0.06)	(1.14)	(0.14)	(0.56)	(0.76)	(−1.57)	(−0.16)
$\ln(P_i^{Hol})$	S.D.*	[(0.39)]	(2.75)	(0.09)	(0.11)	(0.06)	(0.10)	(0.22)	(0.21)	(1.84)

Average trend across three regression specifications: (a) constant and time trend; (b) constant, transitory war dummy, transitory plague dummy and time trend; and (c) constant, transitory war dummy, transitory plague dummy, permanent war and plague dummies, and time trend. These estimates are GLS estimates obtained using a single Cochrane-Orcutt iteration. The first-order autocorrelation coefficient of the OLS residuals was used to make this adjustment.

* Single-bracketed figures in the rows labeled "Mean" and "S.D." give the sample mean and accompanying standard error of trend estimates for relative prices in that category and century (i.e., barley, butter, cheese, oats, peas and wheat in the category $\ln(P_i^{UK}/P_i^{Hol})$. The double-bracketed figures given for Mean and S.D. in the column labeled 1273-1991 represent the mean and associated standard error of trend estimates across all seven century-long periods and commodities in a given category.

Over the entire sample, the point estimates of the trends are very small, and there is little evidence of any statistical trend in cross-country relative prices. Table 1 shows that the six commodity prices fell on average in

England relative to Holland, but by an average of less than 10 basis point a year. However, in spite of having over 700 years of data (ignoring missing observations), the average time-series standard error for this point estimate is almost 11 basis points, so the long-term trend does not appear strongly significant.

The second point made by Table 1 is that century-long trends in deviations from the LOP are correlated across commodities. That is, within the P_i^{UK}/P_i^{Hol} category, the trend estimates tend to cluster around similar values for each century, yet they differ considerably across centuries, not only in magnitude, but in sign. The last line in the P_i^{UK}/P_i^{Hol} group shows this, recording the mean trend estimate for each century.

In addition to the means, we report the standard deviation of the means across commodities, in order to gauge how different average trends were from zero and from each other. Several of the mean trends are different from zero, and a number are different from one another. Indeed it appears that the dispersion of trends across goods within each century is small relative to the dispersion of trends over the broader sample, suggesting that there is correlation in low frequency movements across deviations from LOP, as suggested by the graphs above.

TABLE 2.

English and Dutch Commodity Prices 1273-1991: Volatility Around “Theoretical” Value

Category	Index	1273-1991	1273-1399	1400-1499	1500-1599	1600-1699	1700-1799	1800-1899	1900-1991	1973-1991
$\ln(P_i^{UK}/P_i^{Hol})$	Barley	0.42	0.25	0.34	0.39	0.46	0.64	0.35	0.32	0.27
	Butter	0.33	—	—	0.19	0.24	0.33	0.41	0.36	0.41
	Cheese	0.54	—	—	1.30	0.62	0.37	0.60	0.29	0.22
	Oats	0.35	0.32	0.41	0.35	0.32	0.34	0.44	0.21	0.20
	Peas	0.65	0.76	1.00	0.70	0.76	0.46	0.31	0.34	0.23
	Wheat	0.37	0.37	0.52	0.51	0.24	0.21	0.27	0.34	0.19
	Mean Corr.*	[(0.53]	(0.52)	(0.63)	(0.62)	(0.43)	(0.36)	(0.57)	(0.60)	(0.70)
	S.E. Corr.*	[(0.08)]	(0.17)	(0.11)	(0.07)	(0.10)	(0.10)	(0.08)	(0.07)	(0.07)

Simple standard deviation of each price series about its theoretical value of 0.

* Figures in the row labeled “Mean Corr.” give the average of the pairwise correlation coefficients between the individual commodity price series in that category. Figures in the row labeled “S.E. Corr.” are approximate standard errors for the “Mean Corr.” estimates, calculated on the assumption that the correlations are distributed normally. Single-bracketed estimates correspond to a single century-long period, while the double-bracketed figures are calculated across al1 eight century-long periods.

3.4. Volatility of Relative Prices

In this subsection we focus on higher-frequency price movements, looking at volatility much as we did conditional trends above. Table 2 gives the simplest measure of volatility, the standard deviation of the log of British prices (in terms of silver) minus the log of Dutch prices, i.e., the standard deviation of absolute deviations from LOP. The series are extremely volatile by this measure, with standard deviations for annual movements generally in excess of 30 percent.

TABLE 3.

English and Dutch Commodity Prices 1273-1991: Average Conditional Volatility

Category	Index	1273-1991	1273-1399	1400-1499	1500-1599	1600-1699	1700-1799	1800-1899	1900-1991	1973-1991
$\ln(P_i^{UK}/P_i^{Hol})$	Barley	0.34	0.23	0.26	0.27	0.30	0.25	0.20	0.28	0.11
	Butter	0.25	—	—	0.17	0.17	0.14	0.16	0.29	0.19
	Cheese	0.30	—	—	0.47	0.35	0.19	0.17	0.22	0.17
	Oats	0.29	0.26	0.27	0.23	0.20	0.19	0.26	0.17	0.13
	Peas	0.36	0.34	0.36	0.32	0.35	0.31	0.24	0.29	0.21
	Wheat	0.30	0.16	0.30	0.30	0.20	0.18	0.23	0.28	0.11
	Mean Corr.*	[(0.50]	(0.57)	(0.63)	(0.58)	(0.43)	(0.38)	(0.55)	(0.58)	(0.64)
	S.E. Corr.*	[(0.03)]	(0.08)	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)	(0.06)
$\ln(P_i^{UK}/P_{Wheat}^{UK})$	Barley	0.26	0.15	0.18	0.29	0.25	0.27	0.17	0.21	0.03
	Butter	0.31	—	—	0.30	0.23	0.22	0.25	0.28	0.19
	Cheese	0.39	—	—	0.49	0.35	0.20	0.23	0.20	0.13
	Oats	0.24	0.20	0.22	0.24	0.18	0.20	0.18	0.13	0.07
	Peas	0.37	0.21	0.25	0.31	0.24	0.17	0.35	0.29	0.10
	Silver	0.49	0.30	0.23	0.32	0.21	0.24	0.21	0.44	0.37
	Mean Corr.*	[(0.68)]	(0.77)	(0.78)	(0.78)	(0.61)	(0.71)	(0.69)	(0.58)	(0.59)
	S.E. Corr.*	[(0.02)]	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.09)
$\ln(P_i^{Hol}/P_{Wheat}^{Hol})$	Barley	0.20	0.13	0.20	0.14	0.14	0.13	0.15	0.21	0.09
	Butter	0.34	0.09	0.25	0.19	0.20	0.22	0.32	0.16	0.06
	Cheese	0.37	0.12	0.26	0.20	0.23	0.22	0.29	0.30	0.12
	Oats	0.24	0.26	0.22	0.15	0.17	0.20	0.21	0.17	0.05
	Peas	0.31	—	0.30	0.18	0.28	0.31	0.28	0.26	0.23
	Silver	0.42	0.19	0.30	0.23	0.24	0.24	0.25	0.43	0.38
	Mean Corr.*	[(0.68)]	(0.72)	(0.67)	(0.66)	(0.61)	(0.60)	(0.67)	(0.64)	(0.57)
	S.E. Corr.*	[(0.02)]	(0.08)	(0.02)	(0.02)	(0.04)	(0.03)	(0.03)	(0.04)	(0.07)

Average regression standard error across six OLS regression specifications: (a) constant; (b) constant, transitory war dummy and transitory plague dummy; (c) constant, transitory war dummy, transitory plague dummy, and permanent war and plague dummies; and (d) specifications (a) - (c) run with a time trend.

* Figures in the rows labeled "Mean Con." give the average of the pairwise correlation coefficients between the individual commodity price series in that category. Figures in the rows labeled "S.E. Corr." are approximate standard errors for the "Mean Corr." estimates, calculated on the assumption that the correlations are distributed normally. Single-bracketed estimates correspond to a single century-long period, while the double-bracketed figures are calculated across all eight century-long periods.

Even more remarkably, none of the commodity price series show any obvious trend in volatility over the centuries; volatility of absolute deviations from LOP are actually lower under the post-1973 float despite the high volatility of nominal exchange rates.

Table 3 reports average conditional volatility across our specifications (discussed above) allowing for constants, trends, and both temporary and permanent plague/war dummies. While somewhat lower than the volatility estimates given in Table 2, the standard deviations for LOP deviations in Table 3 remain quite large, generally in excess of 20 percent a year.[13] As in Table 2, there has been little, if any, decline in the volatility of LOP deviations over time. The volatilities in the twentieth century are roughly the same as in the fourteenth century.

Table 3 also presents measures of volatility of within-country prices relative to wheat. Note that the volatility of within-country prices of different goods is generally of the same order of magnitude, if not larger, than the volatility of price differentials for the same good in different countries. Also, as in the case of LOP deviations, within-country relative price volatilities are generally quite stable across time.

The final point to take from Tables 2 and 3 is that deviations from LOP appear to exhibit strong common country components. The final two lines under the P_i^{UK}/P_i^{Hol} category in these two tables show: a) the average pairwise off-diagonal correlations across LOP deviations, and b) the standard deviation of this average. The century-by-century correlations are all strongly positive, averaging about 0.55. Including diagonal elements, the average correlation is $\frac{0.55(N-1)+1}{N} = 0.625$.

These high correlations across the goods help assure us that measurement error is not the dominant force behind our volatility results. Highly volatile measurement errors early in the sample would lead to a lower correlation across commodities during that time relative to later in the sample. However, we do not observe this—the correlations remain extremely stable throughout. This is a stunning result in view of the many changes affecting competition, the organization of markets, and the collection of data that have occurred over the 700 years.

The large magnitude of real exchange rate fluctuations during the float has occupied many international economists (some of whom argue that investor irrationality is to blame). What is striking here is that, over the longer time frame, the recent variability of commodity baskets appears normal. Thus, the puzzle of why real exchange rates are so volatile applies to pre-1850 prices as well as to those of the recent floating-rate period.

3.5. Relative Price Dynamics

Until now, we have restricted ourselves to examining very straightforward characteristics of the data, with the only complication being how to deal with plague and war dummies. Now we turn to looking at some of the dynamic properties of the data, using simple ARIMA specifications.

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[13]By comparison, the annual standard deviation of the S&P 500 stock index since the mid-1920s has only been roughly 19 percent a year.

In addition to the tables reported below, we also formally tested to see whether the data could reject the presence of unit roots in LOP deviations. Not surprisingly, given our earlier graphical analysis, we found that deviations from the LOP appear strongly stationary. Using data at both 1-year and 5-year intervals, we reject strongly the hypothesis that the LOP price deviations contain unit roots. In spite of the rejections, the point estimates for the autocorrelation coefficients are generally quite high, suggesting a relatively long half life for LOP deviations.[14] For relative within country prices (relative to wheat), one can also reject nonstationarity. However, as one might suspect from Figure 2, within-country prices measured in silver do not reject nonstationarity (see Froot, Kim and Rogoff (1995) further details).

Chow Tests on ARIMA Specifications

In Table 4, we present the results for a simple AR(1) specification. We exploit the enormous length of our sample by using specifications involving 5-year intervals in addition to the more standard 1-year intervals. By looking at the longer intervals we hope to be less reliant on left-out lags in our specifications.

The first column represents estimates for the full sample taken at 5-year intervals, whereas the second column gives the full sample estimates for the annual data. The third column gives the estimates for a dummy slope coefficient for the twentieth century, and the fourth column gives estimates for a dummy slope coefficient for the post-1973 period.[15] T-tests of these dummy slope coefficients against zero are equivalent to Chow tests of the hypothesis that there is no change in persistence across the subsamples.

Viewing the results for the full sample, the annual AR coefficients (ρ_1 in Table 4) for barley, butter and wheat are 0.84, 0.89 and 0.78 respectively. These estimates imply half lives for LOP deviations of 3.9, 6.2 and 2.8 years. These estimates are remarkably similar in magnitude to typical estimates on modern floating data for the half lives of deviations from purchasing power parity.[16] Deviations from LOP, even in these relatively homogenous and highly traded commodities, is remarkably slow. The rate of convergence during the twentieth century does not appear to be signifi-

[14]Since the augmented Dickey-Fuller tests allow for richer dynamics than simply an AR(1) specification, one cannot read the half-lives directly off the AR coefficients; we will turn to discussing half lives in more detail below.

[15]We chose this subperiod partly because Britain joined the EEC in 1973 (and the Netherlands did in 1950), and partly because exchange rates more broadly began to float at that time.

[16]As Taylor (2001) points out these estimates may be biased upwards because they do not account for nonlinear convergence and/or time-averaging of the data. Nevertheless, using similar methods to other studies, with our 700 years of data, we get similar estimates.

TABLE 4.

AR(l) Estimates for English and Dutch Commodity Prices, 1273-1991

Category	Good	5-Year	Annual		
		p_1	p_1	p_2	p_3
$\ln(P_i^{UK}/P_i^{Hol})$	Barley	0.672	0.839	−0.097	−0.035
		(0.065)	(0.022)	(0.091)	(0.221)
	Butter	0.761	0.894	0.032	−0.136
		(0.068)	(0.024)	(0.060)	(0.081)
	Wheat	0.628	0.784	0.048	−0.118
		(0.071)	(0.027)	(0.075)	(0.228)
$\ln(P_i^{UK}/P_{Wheat}^{UK})$	Barley	0.508	0.630	0.290	0.250
		(0.072)	(0.030)	(0.070)	(0.373)
	Butter	0.341	0.693	0.111	0.088
		(0.105)	(0.039)	(0.087)	(0.195)
$\ln(P_i^{Hol}/P_{Wheat}^{Hol})$	Bartley	0.399	0.455	0.419	0.499
		(0.085)	(0.040)	(0.059)	(0.096)
	Butter	0.636	0.745	0.231	0.267
		(0.073)	(0.029)	(0.044)	(0.070)
$\ln(P_i^{UK})$	Bartley	0.784	0.881	0.081	0.020
		(0.053)	(0.021)	(0.037)	(0.098)
	Butter	0.811	0.793	0.169	0.115
		(0.065)	(0.038)	(0.043)	(0.069)
	Wheat	0.700	0.851	0.019	0.084
		(0.061)	(0.021)	(0.066)	(0.064)
$\ln(P_i^{Hol})$	Bartley	0.625	0.823	0.158	−0.177
		(0.072)	(0.026)	(0.032)	(0.098)
	Butter	0.884	0.951	0.045	−0.341
		(0.043)	(0.023)	(0.027)	(0.070)
	Wheat	0.578	0.829	0.123	0.067
		(0.076)	(0.025)	(0.051)	(0.082)

Column 1: Estimated coefficient ρ_1 from the specification $y_1 = \delta_0 + \delta_1 t + \rho_1 y_{t-1} + e_1$ where y_t is every fifth observation. Columns 2-4: Estimated coefficients p_1, p_2 and p_3 from the specification $y_1 = \delta_0 + \delta_1 t + \rho_1 y_{t-1} + \rho_2 D_{1900-1972} y_{t-1} + \rho_3 D_{1973-1991} y_{t-1} + e_t$ where $D_{s,t}$ is a dummy variable taking the value 1 for the years s through t, and 0 otherwise. Thus ρ_2 and ρ_3, capture the increments over ρ_1 of the first-order autoregressive coefficient in each of two subsamples, 1900-1991, and 1973-1991. Standard errors in parentheses. For $\ln(P_i^{UK}/P_i^{Hol})$ the mean and trend coefficients are constrained to equal 0, as the price indices in this category represent real exchange rates. Estimation is carried out using the Kalman filter to cater to the presence of missing observations.

cantly different from that in earlier centuries: ρ_2 is small and insignificant for all three commodities. Nor is there any evidence of change in serial correlation after the advent of modern floating exchange rates; ρ_3 is also insignificant for all three commodities. This last result accords with the results of Taylor (2000) and Lothian and Taylor (1995) who use up to 200 years of consumer price index data and find no significant evidence that PPP deviations die out any faster or slower since the advent of floating. Taylor (2000) shows also that there has been no change in persistence over the last century.

Interestingly, the estimates of ρ_1 for prices of barley and butter relative to wheat (P_i^{UK}/P_{Wheat}^{UK} and $P_i^{Hol}/P_{Wheat}^{Hol}$) suggest that shocks to relative prices of different goods within the same country die out faster than deviations from LOP; estimates range from 0.46 to 0.74. The rates of convergence appear to have slowed during the twentieth century as the estimate for ρ_1 is positive in four cases, and significantly different from zero in three.

In addition to the AR(1) models reported above, we estimated a number of simple alternative ARIMA specifications. In most instances, adding higher order autoregressive and moving average terms had little effect on the first-order coefficients and led to only a marginal improvement in overall R^2s. Thus, we do not report other ARIMA specifications here.

Finally, if one examines the five-year estimates, half-lives of the LOP deviations appear even longer than they do from the one-year estimates. This suggests that the AR(1) specifications popular in the literature may be too simple, leaving out important sources of additional persistence. This merits further exploration.

4. CONCLUSIONS

It is remarkable that, despite the steady decline in transportation costs over the past 700 years, the repeated intrusion of wars and disease, and the changing fashions of commercial policy, the volatility and persistence of deviations in the law of one price have remained quite stable, even as the composition of volatility has moved strongly from nominal prices to nominal currency changes. To us, the most natural explanation of these results are that, despite superficial evidence of extensive two-way trade flows, the markets for Dutch and British agricultural commodities are not, and have never been, close to fully integrated. Perhaps this is not so surprising, given that we have learned from the recent literature that complete goods market integration is rarely, if ever, achieved. Still, our comparison of the fourteenth through twentieth centuries provides a unique perspective on just how segregated markets remain, even today. Our conjecture is that further refinements of the data set here, which are likely to reduce the volatility of early period prices more than later-period ones, will only serve to reinforce the conclusion that market segmentation is extremely important in the modern era even compared to the Middle Ages.

Obviously, despite our best efforts to construct the data carefully, one cannot reject the hypothesis that our results are explained by various offsetting forces. Given, however, that one would expect both greater measurement error and more impediments to trade for the early period, it is surprising that measured volatility for the Middle Ages is not higher. To the extent that measurement error is more important in the earlier data, and to the extent that shocks to relative prices have become more persistent since the sixteenth century, then volatility has actually increased since these earlier times.

It is difficult to do more than scratch the surface of potential possibilities for the data set developed here. With the exception of our Kalman filter estimates, we have generally restricted our analysis to simple descriptions of the data. Clearly, the data set admits a host of possibilities for further research, including allowing for richer time series dynamics, and a more complete investigation of the effects of wars and plagues, and, certainly, commercial and tax policies.

APPENDIX: AN ANNOTATED BIBLIOGRAPHY OF DATA SOURCES

England

Beveridge, Sir William. Prices and Wages in England from the Twelfth to the Nineteenth Century, Vol. I. London: Longmans, Green and Co., 1939. (BEV) During the 1930s, Sir William (later Lord) Beveridge headed the International Scientific Committee on Price History, which produced two of the key references used in our study (the Beveridge book for England and the Posthumus book for Holland). The Beveridge book contains data on a variety of commodity prices for Southeastern England (London and nearby vicinity), covering the early sixteenth century to the late eighteenth century. The data were largely constructed from the purchasing records of various institutions, including colleges and hospitals, and the Royal Navy ("Navy Victualling"). In addition, some prices are based on records kept by "the Lord Steward's Department" which (under the monarchy) purchased the provisions for the various Royal Palaces around London.

Board of Trade. Report of Wholesale and Retail Prices, London: 1903. (BOT) The Report, published at the request of Parliament, contains commodity prices encompassing the nineteenth century.

The Economist. (ECO) During the nineteenth and early twentieth century, The Economist published weekly statistics on wholesale prices of various commodities.

Jastram, Roy W. Silver: The Restless Metal. New York: John Wiley & Sons, 1981. (JAS) The book examines the development of silver in the

United Kingdom and the United States. Tables at the end of the book provide a long (1273-1977) time series on the price of silver in England.
Journal of the Royal Statistical Society. (JRSS) Over the period 1895-1938, the Journal annually published a table of wholesale commodity prices.
Mingay, G.E. The Agrarian History of England and Wales: Volume VI: 1750-1850. Cambridge: Cambridge University Press, 1989. (MIN) Tables at the end of this book contain the prices of various agricultural products in towns in England. The primary source for these data are price quotations from area newspapers.
Ministry of Agriculture and Fisheries. Index Numbers of Agricultural Prices, 1930. (IND) A report providing wholesale prices of agricultural products for the 1920s.
Mitchell, B.R. and Deane, P. Abstract of British Historical Statistics. Cambridge: Cambridge University Press, 1962. (MITI) Both Mitchell and Dean, and Mitchell and Jones (below) contain a wide variety of time series encompassing a wide range of socio-economic issues.
Mitchell, B.R., and H. G. Jones. British Historical Statistics. Cambridge: Cambridge University Press, 1988. (MITII).
Rogers, J.E. Thorgold. A History of Agriculture and Prices in England. Vol. I, IV, and V. Oxford: Clarendon Press, 1866. (ROG) Includes annual average tables, constructed by taking the average of all the prices available for a given year. Most of the disaggregated series comes from various institutions, large estates and town markets.
Statistical Abstract of Great Britain. (SAGB) Statistical Office of the European Communities. Agricultural Price Statistics. (APS)
Stratton, J.M. and Brown, J.H. Agricultural Records. London: John Baker Limited, 1978. (AR) This book contains price tables in the appendix, drawn largely from secondary sources, including the Rogers book (above) or various government statistics.

Holland

Bieleman, Jan, Boeren Op Het Drentie Zand 1600-1910. Utrecht: Hes Vitgevers, 1987. (BIE) Contains a table of annual wheat and rye prices.
Central Bureau of Statistics. Negentig Jaren In Tydreeksevl 1899-1988. 1989. (NEG) Commodity price tables for the twentieth century, based on government sources.
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the most important source of data for pre-nineteenth century Holland. Volume I contains wholesale prices from the Amsterdam Exchange covering the seventeenth through nineteenth centuries. This volume also contains extensive time series on silver prices in Holland and market foreign exchange rates.

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Van Der Wee, Herman. The Growth of the Antwerp Market and the European Economy, Vol. I. The Hague: Martinus Nijhoff, 1963. (WEEI) Contains various price tables for commodities in various cities in Flanders and Brabant. (These areas were economically integrated with Holland from the time of the Holy Roman Empire until the late sixteenth century when Holland gained its independence.) The prices here are from institutional sources, mainly hospitals, cited in the Flemish silver groat; which equalled 1/40 of a Dutch guilder.

Van Reil, Arthur. Prices and Economic Development in the Netherlands, 1800-1913. (University of Utrecht, 1995). (VRI) Contains annual prices on basic commodities drawn from provincial annual reports and archive records of market prices for various cities in Holland, including Amsterdam and Utrecht.

DATA COVERAGE BY COMMODITY/YEAR/COUNTRY

For listing of data on each commodity, including years covered by each source, see Froot, Rogoff, and Kim (1995); the most up-to-date version is posted at http://post.economics.harvard.edu/faculty/rogoff/rogoff.html.

APPENDIX: PLAGUE AND WAR DUMMIES

See Tables 5 and 6 for definitions and estimates of plague and war dummies.

TABLE 5.

Relative English/Dutch Commodity Prices in Log Grams of Silver, 1273-1991: Average Coefficient on War Dummies (continued)

War	Period	Barley	Butter	Cheese	Oats	Peas	Wheat
Wars	1273-1991	0.10	0.02	−0.06	0.12	0.15	0.09
		(0.14)	(0.13)	(0.15)	(0.12)	(0.12)	(0.12)
	1273-1399	—	—	—	—	—	—
		—	—	—	—	—	—
	1400-99	−0.04	—	—	−0.15	−0.07	−0.18
		(0.34)	—	—	(0.34)	(0.41)	(0.36)
	1500-99	−0.06	—	—	0.07	−0.26	−0.10
		(0.20)	—	—	(0.20)	(0.26)	(0.26)
	1600-99	—	−0.02	−0.01	—	0.17	0.13
		—	(0.15)	(0.25)	—	(0.22)	(0.17)
	1700-99	−0.30	0.10	−0.03	0.01	0.21	0.06
		(0.36)	(0.20)	(0.27)	(0.28)	(0.29)	(0.19)
	1800-99	−0.10	−0.09	−0.05	0.16	0.11	−0.07
		(0.28)	(0.23)	(0.25)	(0.33)	(0.28)	(0.28)
	1900-91	—	—	—	—	—	—
		—	—	—	—	—	—
Roses	1273-1991	−0.13	—	—	0.08	0.14	−0.25
		(0.47)	—	—	(0.43)	(0.50)	(0.45)
	1400-99	−0.26	—	—	−0.31	−0.22	−0.61
		(0.65)	—	—	(0.68)	(0.78)	(0.69)
DutInd	1273-1991	−0.02	—	—	−0.26	0.28	−0.00
		(0.46)	—	—	(0.42)	(0.47)	(0.44)
	1500-99	−0.14	—	—	−0.50	0.12	−0.02
		(0.40)	—	—	(0.33)	(0.42)	(0.42)
EngCiv	1273-1991	0.08	0.03	−0.12	0.12	−0.20	0.20
		(0.33)	(0.32)	(0.40)	(0.26)	(0.28)	(0.31)
	1600-99	−0.02	0.08	−0.22	0.12	−0.34	0.08
		(0.27)	(0.23)	(0.41)	(0.25)	(0.37)	(0.27)
AngDutl	1273-1991	−0.37	0.04	−0.10	−0.15	−0.03	−0.44
		(0.43)	(0.35)	(0.46)	(0.41)	(0.41)	(0.39)
	1600-99	−0.50	−0.04	0.19	−0.26	−0.01	−0.44
		(0.38)	(0.30)	(0.49)	(0.38)	(0.47)	(0.34)
AngDut2	1273-1991	0.39	0.14	−0.03	0.06	0.12	0.18
		(0.43)	(0.32)	(0.34)	(0.42)	(0.37)	(0.34)
	1600-99	0.33	0.10	−0.09	−0.02	0.06	0.12
		(0.43)	(0.23)	(0.26)	(0.40)	(0.41)	(0.26)
AngDut3	1273-1991	0.06	0.10	−0.10	−0.06	0.15	0.00
		(0.39)	(0.30)	(0.31)	(0.35)	(0.34)	(0.30)
	1600-99	−0.18	0.04	−0.10	−0.14	0.02	−0.01
		(0.38)	(0.22)	(0.27)	(0.34)	(0.41)	(0.25)

TABLE 5.

Relative English/Dutch Commodity Prices in Log Grams of Silver, 1273-1991: Average Coefficient on War Dummies (concluded)

War	Period	Barley	Butter	Cheese	Oats	Peas	Wheat
SpanSucc	1273-1991	0.02	0.06	0.29	0.14	0.05	0.03
		(0.24)	(0.21)	(0.18)	(0.20)	(0.19)	(0.19)
	1700-99	1.05	−0.42	−0.02	0.08	−0.27	0.14
		(1.00)	(0.51)	(0.61)	(1.70)	(1.95)	(0.54)
AngDut4	1273-1991	−0.31	0.06	0.09	−0.00	0.24	0.11
		(0.23)	(0.25)	(0.22)	(0.19)	(0.22)	(0.24)
	1700-99	−0.13	−0.11	−0.06	−0.02	0.28	0.03
		(0.30)	(0.19)	(0.27)	(0.21)	(0.25)	(0.19)
FrenRev	1273-1991	−0.02	0.15	0.11	0.01	0.19	0.03
		(0.29)	(0.30)	(0.28)	(0.25)	(0.29)	(0.30)
	1700-99	−0.00	0.12	0.01	0.01	0.30	0.11
		(0.34)	(0.24)	(0.32)	(0.26)	(0.32)	(0.25)
NapWar	1273-1991	0.18	0.03	0.28	0.21	−0.24	0.04
		(0.25)	(0.28)	(0.25)	(0.21)	(0.25)	(0.27)
	1800-99	0.51	0.17	0.52	0.25	0.84	0.49
		(0.52)	(0.46)	(0.48)	(0.62)	(0.68)	(0.57)
WWI	1273-1991	−0.09	−0.22	−0.08	−0.15	0.43	−0.14
		(0.19)	(0.22)	(0.19)	(0.16)	(0.23)	(0.20)
	1990-91	0.02	−0.02	0.17	−0.09	0.35	−0.07
		(0.38)	(0.37)	(0.30)	(0.28)	(0.51)	(0.38)
WWII	1273-1991	−0.12	−0.11	−0.12	−0.12	−0.16	0.04
		(0.20)	(0.24)	(0.24)	(0.16)	(0.24)	(0.21)
	1990-91	0.11	−0.03	−0.02	−0.20	−0.03	−0.19
		(0.39)	(0.36)	(0.33)	(0.27)	(0.29)	(0.36)

Average regression coefficient across the six specifications run for each series and time period: (a) constant; (b) constant, transitory war dummy and transitory plague dummy; (c) constant, transitory war dummy, transitory plague dummy, and permanent war and plague dummies; and (d) specifications (a) through (c) run with a time trend. The variable "Wars" is a minus one-zero-one dummy as described in the accompanying text. The remaining variables are zero-one dummies that take the value 1 in all years subsequent to the outbreak of that war. The war years are as follows: Roses: 1455-85; Dutind: 1572-1609; EngCiv: 1642-49; AngDut: 1652-54; AngDut2: 1665-67; AngDut3: 1672-74; SpanSucc: 1702-13; AngDu4: 1780-84; FrenRev: 1793-1802; NapWar: 1803-15; WWl: 1914-18; WW2: 1939-45.

TABLE 6.

Relative English/Dutch Commodity Prices in Log Grams of Silver, 1273-1991: Average Coefficient on Plague Dummies (continued)

Plague	Period	Barley	Butter	Cheese	Oats	Peas	Wheat
Plagues	1273-1991	0.12	0.04	−0.05	−0.04	−0.09	0.30
		(0.19)	(0.50)	(0.79)	(0.18)	(0.24)	(0.18)
	1273-1399	—	—	—	—	—	—
		—	—	—	—	—	—
	1400-99	0.34	—	—	−0.26	0.14	0.38
		(0.33)	—	—	(0.37)	(0.60)	(0.34)
	1500-99	0.04	—	—	0.16	−0.04	0.31
		(0.22)	—	—	(0.19)	(0.27)	(0.23)
	1600-99	—	—	—	—	—	—
		—	—	—	—	—	—
	1700-99	—	—	—	—	—	—
		—	—	—	—	—	—
	1800-99	—	—	—	—	—	—
		—	—	—	—	—	—
	1900-91	—	—	—	—	—	—
		—	—	—	—	—	—
ENGPL1	1273-1991	—	—	—	—	—	—
		—	—	—	—	—	—
	1273-1399	—	—	—	—	—	—
		—	—	—	—	—	—
ENGPL2	1273-1991	—	—	—	—	—	—
		—	—	—	—	—	—
	1273-1399	—	—	—	—	—	—
		—	—	—	—	—	—
ENPL3	1273-1991	0.15	—	—	−0.00	−3.65	—
		(1.42)	—	—	(0.59)	(2.82)	—
	1273-1399	1.40	—	—	0.02	9.18	—
		(2.61)	—	—	(0.59)	(3.64)	—
ENPL4	1273-1991	−0.22	—	—	−0.03	−0.34	—
		(0.37)	—	—	(0.27)	(0.46)	—
	1273-1399	−0.10	—	—	0.09	−0.16	—
		(0.29)	—	—	(0.32)	(0.31)	—
ENPL5	1273-1991	—	—	—	—	—	—
		—	—	—	—	—	—
	1273-1399	—	—	—	—	—	—
		—	—	—	—	—	—
ENPL6	1273-1991	−0.09	—	—	−0.22	0.35	0.03
		(0.27)	—	—	(0.23)	(0.55)	(0.41)
	1400-99	—	—	—	—	—	—
		—	—	—	—	—	—

TABLE 6.

Relative English/Dutch Commodity Prices in Log Grams of Silver, 1273-1991: Average Coefficient on Plague Dummies (continued)

Plague	Period	Barley	Butter	Cheese	Oats	Peas	Wheat
ENPL7	1273-1991	−0.42	—	—	−0.20	−0.58	−0.38
		(0.29)	—	—	(0.25)	(0.41)	(0.31)
	1400-99	−0.37	—	—	−0.11	−0.65	−0.32
		(0.28)	—	—	(0.26)	(0.41)	(0.34)
ENPL8	1273-1991	0.22	—	—	0.18	0.54	0.55
		(0.32)	—	—	(0.28)	(0.40)	(0.33)
	1400-99	0.33	—	—	0.22	0.58	0.65
		(0.29)	—	—	(0.27)	(0.37)	(0.35)
DTPL1	1273-1991	0.02	—	—	−0.32	−0.24	−0.17
		(0.29)	—	—	(0.25)	(0.27)	(0.30)
	1400-99	0.01	—	—	−0.22	−0.27	−0.11
		(0.27)	—	—	(0.25)	(0.26)	(0.32)
DTPL2	1273-1991	0.16	—	—	0.30	−0.16	0.36
		(0.35)	—	—	(0.30)	(0.34)	(0.36)
	1400-99	0.19	—	—	0.37	−0.23	0.42
		(0.31)	—	—	(0.29)	(0.30)	(0.36)
ENPL9	1273-1991	0.22	—	—	0.07	0.42	0.23
		(0.43)	—	—	(0.39)	(0.46)	(0.41)
	1400-99	0.30	—	—	0.17	0.54	0.31
		(0.40)	—	—	(0.40)	(0.43)	(0.44)
DTPL3	1273-1991	−0.02	—	—	−0.12	−0.05	−0.10
		(0.27)	—	—	(0.23)	(0.27)	(0.29)
	1400-99	0.02	—	—	−0.03	−0.12	−0.01
		(0.27)	—	—	(0.25)	(0.28)	(0.32)
ENPL10	1273-1991	−0.22	—	—	−0.28	−0.15	−0.29
		(0.31)	—	—	(0.27)	(0.34)	(0.32)
	1400-99	−0.11	—	—	0.13	0.07	0.01
		(0.47)	—	—	(0.51)	(0.59)	(0.49)
DTPL4	1273-1991	0.35	—	—	0.59	0.19	0.40
		(0.35)	—	—	(0.31)	(0.44)	(0.36)
	1400-99	0.43	—	—	0.57	0.11	0.55
		(0.33)	—	—	(0.31)	(0.42)	(0.39)
DTPL5	1273-1991	−0.02	—	—	−0.27	0.24	−0.07
		(0.33)	—	—	(0.29)	(0.42)	(0.34)
	1400-99	0.11	—	—	0.01	—	0.13
		(0.50)	—	—	(0.53)	—	(0.52)
ENDTPLl	1273-1991	−0.21	—	—	−0.17	−0.15	−0.09
		(0.28)	—	—	(0.24)	(0.32)	(0.29)
	1500-99	−0.12	—	—	−0.14	0.10	0.21
		(0.27)	—	—	(0.23)	(0.33)	(0.29)

TABLE 6.

Relative English/Dutch Commodity Prices in Log Grams of Silver, 1273-1991: Average Coefficient on Plague Dummies (continued)

Plague	Period	Barley	Butter	Cheese	Oats	Peas	Wheat
ENPL11	1273-1991	0.24	—	—	0.28	−0.02	0.34
		(0.32)	—	—	(0.28)	(0.37)	(0.33)
	1500-99	0.12	—	—	0.37	0.18	0.54
		(0.33)	—	—	(0.27)	(0.43)	(0.36)
ENPLl2	1273-1991	−0.16	—	—	−0.02	−0.32	−0.48
		(0.35)	—	—	(0.31)	(0.58)	(0.35)
	1500-99	−0.04	—	—	−0.22	−0.13	−0.33
		(0.30)	—	—	(0.24)	(0.61)	(0.33)
ENPL13	1273-1991	−0.03	—	—	−0.06	0.46	0.04
		(0.40)	—	—	(0.32)	(0.67)	(0.37)
	1500-99	−0.17	—	—	−0.06	0.50	0.14
		(0.35)	—	—	(0.25)	(0.75)	(0.34)
ENPL14	1273-1991	0.30	—	—	0.28	−0.41	0.47
		(0.41)	—	—	(0.32)	(0.60)	(0.36)
	1500-99	0.40	—	—	0.24	−0.14	0.55
		(0.36)	—	—	(0.24)	(0.62)	(0.33)
ENDTPL2	1273-1991	−0.36	—	—	−0.07	−0.20	−0.33
		(0.37)	—	—	(0.31)	(0.54)	(0.36)
	1500-99	−0.29	—	—	0.00	−0.14	−0.04
		(0.32)	—	—	(0.25)	(0.50)	(0.34)
ENPL15	1273-1991	0.00	—	—	−0.03	0.16	−0.05
		(0.35)	—	—	(0.31)	(0.47)	(0.36)
	1500-99	−0.09	—	—	−0.15	0.32	−0.03
		(0.28)	—	—	(0.23)	(0.44)	(0.32)
DTPL6	1273-1991	0.19	0.53	1.87	0.18	−0.00	0.20
		(0.45)	(1.39)	(3.35)	(0.40)	(0.47)	(0.42)
	1500-99	0.29	—	—	0.34	0.19	0.37
		(0.42)	—	—	(0.33)	(0.43)	(0.43)
DTPL7	1273-1991	0.11	0.45	0.17	0.45	0.48	0.18
		(0.35)	(0.41)	(0.46)	(0.28)	(0.35)	(0.33)
	1500-99	0.16	—	—	0.34	0.63	0.35
		(0.29)	—	—	(0.22)	(0.32)	(0.30)
ENPL16	1273-1991	0.27	−0.06	0.51	0.05	0.18	0.38
		(0.43)	(0.35)	(0.49)	(0.33)	(0.43)	(0.38)
	1500-99	0.15	—	—	0.05	0.31	0.52
		(0.40)	—	—	(0.26)	(0.39)	(0.35)
ENDTPL3	1273-1991	0.07	−0.00	−0.l9	0.06	−0.61	−0.18
		(0.41)	(0.33)	(0.46)	(0.31)	(0.38)	(0.35)
	1500-99	−0.01	—	—	−0.17	−0.55	−0.28
		(0.45)	—	—	(0.31)	(0.42)	(0.41)

TABLE 6.

Relative English/Dutch commodity prices in log grams of silver, 1273-1991: Average Coefficient on Plague Dummies (concluded)

Plague	Period	Barley	Butter	Cheese	Oats	Peas	Wheat
DTPL8	1273-1991	0.04	0.03	−0.38	0.11	−0.39	0.06
		(0.31)	(0.31)	(0.37)	(0.28)	(0.30)	(0.32)
	1600-99	−0.09	−0.05	−0.21	0.04	−0.47	−0.04
		(0.28)	(0.23)	(0.36)	(0.27)	(0.37)	(0.27)
ENDTPL4	1273-1991	−0.04	−0.04	−0.69	−0.19	0.22	−0.06
		(0.30)	(0.31)	(0.34)	(0.27)	(0.29)	(0.31)
	1600-99	−0.18	−0.05	−1.00	−0.26	0.13	−0.14
		(0.28)	(0.21)	(0.28)	(0.27)	(0.34)	(0.25)
DTPL9	1273-1991	0.66	0.14	0.31	0.25	0.14	0.38
		(0.43)	(0.35)	(0.41)	(0.44)	(0.40)	(0.39)
	1600-99	0.65	0.21	0.25	0.21	0.11	0.37
		(0.41)	(0.28)	(0.37)	(0.43)	(0.44)	(0.32)
ENPL5	1273-1991	0.20	—	—	0.40	−0.61	—
		(0.29)	—	—	(0.24)	(0.57)	—
	1600-99	—	—	—	—	—	—
		—	—	—	—	—	—

Average regression coefficient across the six specifications run for each series and time period: (a) constant; (b) constant, transitory war dummy and transitory plague dummy; (c) constant, transitory war dummy, transitory plague dummy, and permanent war and plague dummies; and (d) specifications (a) through (c) run with a time trend. The variable "Plagues" is a minus one-zero-one dummy as described in the accompanying text. The remaining variables are zero-one dummies that take the value 1 in all years subsequent to the outbreak of that plague. The plague years and their associated names are as follows (ENPL denotes English plague, DTPL denotes Dutch plague, and ENDTPL denotes a plague that occurred in both countries): ENPL1: 1348,1349; ENPL2: 1361,1362; ENPL3: 1368,1369; ENPL4: 1375; ENPL5: 1390,1391; ENPL6: 1400; ENPL7: 1420; ENPL8: 1427; DTPL1: 1439; DTPL2: 1450; ENPL9: 1457,1458; DTPL3: 1467-1774; ENPL10: 1485; DTPL4: 1493; DTPL5: 1499; ENDTPL1: 1511-1521; ENPL11: 1526-1532; ENPL12: 1536,1537; ENPL13: 1543; ENPL14: 1548; ENDTPL2: 1557, 1558; ENPLl5: 1563; DTPL6: 1574, 1575; DTPL7: 1585, 1586, 1587, 1588; ENPL16: 1593; ENDTPL3: 1597-1605; DTPL8: 1617; ENDTPL4: 1624-1632; DTPL9: 1655, 1656; ENDTPL5: 1664, 1665, 1666.

APPENDIX: TIME-SERIES METHODOLOGY

We wish to accomplish two objectives using time series methods: to test for stationarity, and to estimate ARIMA specifications for relative prices. Because of the presence of missing observations, straightforward application of standard tests using linear estimation techniques is precluded. To handle missing observations properly in this context, it is necessary to employ a full-information method of estimation, one which involves simultaneously estimating missing observations and computing parameters. We employ the Kalman filter as a way of doing this.

Consider first the Dickey-Fuller specification. We wish to test the null hypothesis that the data are generated by a unit root autoregression with drift, using the augmented Dickey-Fuller regression

$$y_t = \xi_1 \Delta y_{t-1} + \xi_2 \Delta y_{t-2} + \rho y_{t-1} + \delta_0 + \delta_1 t + \varepsilon_t \tag{A.1}$$

Under the null hypothesis, $\rho = 1$. With no missing observations, the t-statistic from OLS estimation of equation (A.1) can be compared directly to the critical values tabulated in Fuller (1976).[18]

Let $\{\alpha_t\}_{t=1}^T$ be the full series of observed data — if we had no missings then $\{\alpha_t\}_{t=1}^T \equiv \{y_t\}_{t=1}^T$. Whenever we observe y_t, α_t is observed with certainty. However, whenever y_t is not observed, then an estimate of α_t can be obtained using the Kalman filter.

Replacing y_t in equation (A.1) with α_t and rearranging, we have

$$\alpha_t - \mu = \xi_1 \Delta\alpha_{t-1} + \xi_2 \Delta\alpha_{t-2} + \rho(\alpha_{t-1} - \mu) + \delta_1 t + \varepsilon_t, \tag{A.2}$$

where $\mu = \frac{\delta_0}{1-\rho}$. Under the hypothesis that $\rho = 1$, $(\alpha_t - \mu)$ is a zero-drift unit root autoregression. Equation (A.2) can be cast in the following state space form:

State Equation

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{A.3}$$

Observation Equation

$$y_t = A'x_t + H'\xi_t. \tag{A.4}$$

where

$$\xi_t = \begin{bmatrix} \alpha_1 - \mu \\ \Delta\alpha_t \\ \Delta\alpha_{t-1} \end{bmatrix}$$

$$F_t = \begin{bmatrix} \rho & \xi_1 & \xi_2 \\ \rho - 1 & \xi_1 & \xi_2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$v_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \end{bmatrix}$$

[18]Clearly, equation (A.1) can be used to estimate a simple AR(1) specification by setting ξ_1, ξ_2 and δ_1 to be zero. In what follows, we present our Kalman filter methodology for equation (A.1); the results for the AR(1) estimation can be seen easily as a special case.

$$A' = [\mu \quad \delta_1]$$

$$x_t = \begin{bmatrix} 1 \\ t \end{bmatrix}$$

$$H' = [1 \quad 0 \quad 0]$$

where ε_t is assumed to be Gaussian white noise, with $E(\varepsilon_t, \varepsilon_t) = \sigma^2$.

Under the null hypothesis, the process for the state vector is nonstationary. The Kalman filter is therefore initialized with a diffuse prior. Accordingly, $\hat{\xi}_{1|0}$, the conditional forecast of the state vector in period 1 conditional on no observations of y, is chosen to be

$$\hat{\xi}_{1|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{A.5}$$

while $P_{1|0}$, the mean squared error associated with $\hat{\xi}$, is chosen to be

$$P_{1|0} = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{A.6}$$

where λ is some large number. These starting values are then updated on the basis of observations on y using the formula for updating a linear projection, yielding

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} + P_{t|t-1}H(H'P^{-1}_{t|t-1}H)(y_t - A'x_t - H'P_{t|t-1}), \tag{A.7}$$

and

$$P_{t|t}P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H)^{-1}H'P_{t|t-1}, \tag{A.8}$$

The state equation is then used to predict the first and second moments of the state in period $t+1$, conditional on all observations up to and including t:

$$\begin{aligned} \hat{\xi}_{t+t|t} &= F\hat{\xi}_{t|t} \\ P_{t+1|t} &= FP_{t|t}F' + Q, \end{aligned}$$

where

$$Q = \begin{bmatrix} \alpha^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{A.9}$$

On the assumption that both ε_t and the initial state are Gaussian, the forecast of $\hat{\xi}_{t+1|t}$ and hence $\hat{y}_{t+1|t}$ are optimal among any functions of past observations of y. In particular, we have

$$y_t|x_t, y_1, \ldots, y_{t-1} \sim N(A'x_t + H'\hat{\xi}_{t|t-1}), (H'P_{t|t-1}H), \tag{A.10}$$

Comparing each forecast with the observed value of y_{t+1} generates a series of prediction errors and associated mean squared errors which can be used to construct the likelihood function from the prediction error decomposition. If y_t is not observed in a given period, we omit the prediction error for that period from the likelihood function. The updating equations are skipped for this observation, but the prediction equations are still used to generate an optimal forecast and associated mean squared error for the subsequent period.

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