Econometrica, Vol. 59, No. 1 (January, 1991), 241–250

STOCHASTIC PROCESS SWITCHING: SOME SIMPLE SOLUTIONS

BY KENNETH A. FROOT AND MAURICE OBSTFELD [1]

1. INTRODUCTION

ECONOMISTS HAVE RECENTLY begun to explore the effects of prospective regime changes on forward-looking variables. In an example inspired by Britain's return to the gold standard in 1925, Flood and Garber (1983) study the case in which an asset price (specifically, the exchange rate) floats freely until it reaches a pre-announced level, at which time the government intervenes to keep it fixed thereafter. They show how a first-stopping time methodology can be applied to their problem, but are unable to derive a closed-form solution using that mathematically cumbersome technique.

In this paper we apply techniques of regulated Brownian motion to derive easy solutions for a large class of regime-switching problems, including the one posed by Flood and Garber (1983).[2] The technical approach we use has several advantages over the alternatives. First, and most important, the method is both simple and helpful for understanding the intuition behind the mathematics. Second, it clarifies the economic similarities between Flood-Garber process switches and other types of stochastic regime change. Finally, the approach allows us to be precise about potential indeterminacies in asset prices that can crop up when policy makers do not fully specify how they intend to achieve their economic targets.[3]

The rest of the paper is structured as follows. In Section 2 below, we lay out a very general exchange-rate model. Section 3 contains solutions to several specific process-switching examples, and it also discusses how some formulations of future policy intentions may leave asset prices under-determined.

2. THE MODEL

To keep the analysis simple, we use the standard flexible-price monetary model of the exchange rate.[4] In this framework, the (log) spot exchange rate at time t, $x(t)$, is the sum of a scalar indicator of macroeconomic fundamentals, $k(t)$, plus a speculative term proportional to the expected percentage change in the exchange rate:

$$x(t) = k(t) + \alpha E(dx(t)|\phi(t))/dt. \tag{1}$$

[1] The authors are grateful to Bob Flood, an anonymous referee, and an editor of this journal for helpful comments, and to the John M. Olin, Henry Ford, and National Science Foundations for generous financial support.

[2] Smith (1991) has shown how to derive an explicit solution to the Flood-Garber problem using the methods they suggested. Naturally, our solution agrees with that found by Smith. Smith and Smith (1990) apply this result to the 1925 British episode.

[3] Froot and Obstfeld (1989) present a detailed discussion of the relationship between the process-switching examples in this paper and some others, including systems of exchange-rate "target zones." Under a target zone, the policy maker keeps an asset price from moving out of some predetermined range, but he does not intervene when the price is within the range. The original target-zone solution is due to Krugman (1990); see Miller and Weller (1988), Flood and Garber (1989), and Klein (1990) for other studies of exchange-rate zones.

[4] The single-equation formulation assumed below is derived from underlying behavioral relationships by Flood and Garber (1983). We assume continuous trading, but a discrete-time binomial setup yields comparable results.

Above, the parameter α can be interpreted as the semi-elasticity of money demand with respect to the interest rate, E is the expectations operator, and $\phi(t)$ is the time-t information set, which includes the current value of fundamentals, $k(t)$, as well as any explicit or implicit restrictions the authorities have placed on the future evolution of fundamentals. For example, if the authorities have announced that they will fix the exchange rate once it reaches a certain level, this information would be incorporated into $\phi(t)$. Included among the fundamental factors that affect $k(t)$ is a variable measuring relative national money supplies, which are assumed to be controlled directly by monetary authorities. Included as well are other, exogenous determinants of exchange rates that the authorities cannot influence.

The monetary authorities may intervene to influence exchange rates by altering the stochastic process governing (relative) money-supply growth. This in turn will alter the process driving the fundamentals, $k(t)$. A regime of freely floating exchange rates is said to be in effect when the authorities refrain from intervening to offset shocks to fundamentals. Under a free float, we assume that the fundamentals evolve according to the process:

$$dk(t) = \eta\, dt + \sigma\, dz(t), \tag{2}$$

where η is the (constant) predictable change in k, dz is a standard Wiener process, and σ is a constant.[5] As noted above, the authorities can control k through intervention, so k need not follow (2) under regimes other than a free float.

In a rational-expectations equilibrium with no speculative bubbles, there is a unique exchange-rate path that satisfies (1). This path has the integral representation:

$$x(t) = \alpha^{-1}\int_t^{\infty} e^{(t-s)/\alpha} E(k(s)|\phi(t))\, ds, \tag{3}$$

a representation valid under *any* policy regime or sequence of policy regimes. In words, (3) equates the current exchange rate to the present discounted value of expected future fundamentals (the discount rate is $1/\alpha$). We refer to the equilibrium exchange rate, given by the present-value formula (3), as the *saddlepath* exchange rate.[6]

Given (2) and the types of regime change we will consider, it is reasonable to suppose that the saddlepath exchange rate can be written as a twice continuously differentiable function of a single variable, the current fundamental:

$$x(t) = S(k(t)). \tag{4}$$

Naturally, the precise form of the function $S(k)$ depends (as is demonstrated below) on the nature of the regime shifts that the market thinks are possible.

A well-known special case is the one in which the authorities are committed to a *permanent* exchange-rate float, so that fundamentals are expected to follow process (2) forever. In this case, the conditional expectations in (3) are easy to evaluate, since they depend exclusively on current fundamentals, and not on possible future regime shifts. The saddlepath exchange rate $S(k(t))$ is given by:

$$\begin{aligned} &\alpha^{-1}\int_t^{\infty} e^{(t-s)/\alpha} E(k(s)|k(t))\, ds \\ &\qquad = \alpha^{-1}\int_t^{\infty} e^{(t-s)/\alpha}(k(t) + (s-t)\eta)\, ds = k(t) + \alpha\eta. \end{aligned} \tag{5}$$

[5] Propositions 1 and 2 below can be rederived using more complex forcing processes. See Section 3.3.

[6] This terminology is meant to differentiate the present-value exchange rate from other solutions to (1), which include extraneous bubble components driven by self-fulfilling expectations. We assume that such bubble solutions are ruled out by market forces.

If, however, the market expects the authorities to alter (2) in the future, the exchange rate need not satisfy (5), even while allowed to float. In such cases, direct computation of the sequence of conditional expectations in the present-value formula (3) is likely to be burdensome. We therefore follow an alternative, two-step approach to determine $S(k)$ when a regime switch from (2) to some other process is possible. First, we characterize the family of functions of form $x = G(k)$ that satisfy the equilibrium condition (1) so long as fundamentals evolve according to (2). Second, we find the member of this family that satisfies boundary conditions appropriate to the stochastic regime switch under consideration. This last function is the saddlepath solution, $S(k)$.

To implement step one of the procedure outlined above—finding the general solution $x = G(k)$—use Itô's Lemma and equation (1) to express expected depreciation during the float as:[7]

$$(6) \qquad E(dx|\phi)/dt = E(dG(k)|\phi)/dt = \eta G'(k) + \frac{\sigma^2}{2}G''(k),$$

where we have assumed $G(k)$ is twice continuously differentiable. Combining (1) and (6) yields a second-order differential equation that the exchange rate in (1) and (3) must satisfy:

$$(7) \qquad G(k) = k + \alpha\eta G'(k) + \frac{\alpha\sigma^2}{2}G''(k).$$

The general solution to (7) is:

$$(8) \qquad G(k) = k + \alpha\eta + A_1 e^{\lambda_1 k} + A_2 e^{\lambda_2 k},$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$ are the roots of the quadratic equation in λ,

$$(9) \qquad \lambda^2 \alpha\sigma^2/2 + \lambda\alpha\eta - 1 = 0,$$

and A_1, A_2 are constants of integration. Equation (8) forms the basis of our analysis below. As just discussed, a lone member of the family defined by (8) will turn out to be equivalent to the present-value formula for x in (3), and this function is just the saddlepath function, $S(k)$.

Notice that the general solution (8) consists of two components: one is linear, and the other is nonlinear, in k. The linear part, $k + \alpha\eta$, would be the standard linear saddlepath solution if no change in the fundamentals process (2) were possible, so that a free float were permanently in effect (see equation (5)). When there is a possibility of regime switching, however, fundamentals may *not* remain permanently a random walk with trend, and the present-value formula (3) therefore need *not* equal (5). Under prospective regime switches, the exchange rate's saddlepath value will generally depend on the nonlinear terms in (8). Just which initial conditions A_1 and A_2 are appropriate depends on the boundary conditions associated with the regime switch, conditions to be determined in step two of the two-step solution procedure outlined above.

3. STOCHASTIC PROCESS SWITCHING

This section carries out the second step of the solution method outlined in the last section. As an example of the simplicity of the technique, we solve the problem posed by

[7] Where it does not create confusion, we drop the time-dependence notation. It is worth noting that while we refer to $G(k)$ as a "general" solution, it is general only if attention is restricted to solutions that depend on current fundamentals alone. In fact, (1) has even more general solutions, for example, solutions that are functions not only of current fundamentals, but also of variables extraneous to the model. Such solutions are not considered in this paper. This exclusion is not restrictive given the economic problem we are considering.

Flood and Garber (1983). In terms of the mathematics, all that is involved is the appropriate choice of the two arbitrary constants in (8), A_1 and A_2.

3.1. *Deriving a Closed-Form Solution*

Suppose that the authorities wish to let the exchange rate float until it reaches a lower or an upper level $\underline{x}$ or $\bar{x}$, at which time they plan to fix x permanently. How does the rate behave prior to pegging? This class of problems is a generalization of that posed by Flood and Garber (1983), who are concerned with the behavior of a floating exchange rate when the authorities plan to switch to a fixed-rate regime at a single, predetermined level of the exchange rate, $\bar{x}$. We return to a more detailed discussion of their formulation at the end of this subsection. Our rationale for looking at a more general problem first is that its solution will clarify the economics of more specialized (and realistic) cases.

One way for the authorities to enforce their policy is to place lower and upper limits, $\underline{k}$ and $\bar{k}$, on the *fundamentals*, and to freeze fundamentals when one of these bounds is reached. This policy implies that $\underline{k}$ and $\bar{k}$ must be *absorbing* barriers on the fundamentals. Thus, if $S(k)$ is monotonically increasing in k (as turns out to be the case in equilibrium), then as long as k moves between the absorbing barriers $\underline{k}$ and $\bar{k}$, the exchange rate will float between the lower and upper values $\underline{x} = S(\underline{k})$ and $\bar{x} = S(\bar{k})$ until it reaches one of them.

Expectations about the way future regime shifts will be implemented are actually quite important in determining the exchange rate's saddlepath. For now we stay with the formulation of the previous paragraph, which is the same as the one used by Flood and Garber in their calculations. In particular, the assumption that the absorbing barriers are set as just described implies that the fundamentals will not jump at the moment of transition. As the next subsection shows, this restriction may be relaxed, but doing so will change the equilibrium exchange-rate path.

To determine exchange-rate behavior during the initial float, we solve for the exchange-rate path that satisfies (1), given that (2) holds for $k \in (\underline{k}, \bar{k})$ and that $\underline{k}$ and $\bar{k}$ are absorbing barriers. The saddlepath solution is a special case of (3):

$$x(t) = S(k(t)) = \alpha^{-1}\int_t^{\infty} e^{(t-s)/\alpha} E\big(k(s)|k(t), k(s) \in [\underline{k}_a, \bar{k}_a]\big)\, ds, \tag{10}$$

where the a subscript denotes that the barriers on fundamentals are absorbing. As noted in Section 2, direct evaluation of the conditional expectation in (10) is much more difficult than for the permanent free float (equation (5)): the bounds on fundamentals imply that the saddlepath exchange rate $S(k)$ generally is no longer a purely linear function of k.

We have already taken the first step toward solving the problem by deriving the general nonlinear solution $x = G(k)$ given by (8). Some member of this family of solutions must characterize exchange-rate behavior when k is *in the interior of* $[\underline{k}, \bar{k}]$, where (1) and (2) simultaneously hold. A nontrivial logical gap must be bridged, however, before concluding that equation (8) is also relevant *at the boundary* of this interval, that is, at the barriers $k = \underline{k}$ and $k = \bar{k}$. The needed bridge is supplied by the fact that the saddlepath solution $S(k)$ is *continuous* on the entire interval $[\underline{k}, \bar{k}]$. Continuity of $S(k)$ ensures that if that function coincides with a continuous function of form $G(k)$ on the interior of an interval, it coincides with the same function at the edges.[8]

All that remains, then, is to determine the boundary conditions on $G(k)$, which deliver unique values for the undetermined coefficients A_1 and A_2 in (8), and therefore tie

[8] Formally, taking limits as (say) k approaches $\bar{k}$ from below, continuity implies that $S(\bar{k}) = \lim_{k \to \bar{k}} S(k) = \lim_{k \to \bar{k}} G(k) = G(\bar{k})$.

down uniquely the member of the class $G(k)$ that coincides with $S(k)$. But the boundary values of integral (10) are easily found; by our choice of the barriers, they are:[9]

$$S(\underline{k}) = \alpha^{-1}\int_t^\infty e^{(t-s)/\alpha}E\big(k(s)|k(t)=\underline{k}_a\big)\,ds = \alpha^{-1}\int_t^\infty e^{(t-s)/\alpha}\underline{k}\,ds = \underline{k} = \underline{x}, \tag{11}$$

$$S(\bar{k}) = \alpha^{-1}\int_t^\infty e^{(t-s)/\alpha}E\big(k(s)|k(t)=\bar{k}_a\big)\,ds = \alpha^{-1}\int_t^\infty e^{(t-s)/\alpha}\bar{k}\,ds = \bar{k} = \bar{x}. \tag{12}$$

Combining (8) with (11) and (12) leads to the desired equations for A_1 and A_2:

$$\alpha\eta + A_1 e^{\lambda_1 \underline{k}} + A_2 e^{\lambda_2 \underline{k}} = 0, \tag{13}$$

$$\alpha\eta + A_1 e^{\lambda_1 \bar{k}} + A_2 e^{\lambda_2 \bar{k}} = 0. \tag{14}$$

These two expressions lead to the following proposition:

PROPOSITION 1: *When fundamentals follow* (2) *within the absorbing barriers* $\underline{k}$ *and* $\bar{k}$, *the saddlepath solution* (10) *is*:

$$x = S(k) = k + \alpha\eta\left(1 + \frac{e^{\lambda_2\bar{k}+\lambda_1 k} - e^{\lambda_2\underline{k}+\lambda_1 k} + e^{\lambda_1\underline{k}+\lambda_2 k} - e^{\lambda_1\bar{k}+\lambda_2 k}}{e^{\lambda_2\underline{k}+\lambda_1\bar{k}} - e^{\lambda_2\bar{k}+\lambda_1\underline{k}}}\right). \tag{15}$$

If we let the lower bound, $\underline{k}$ *go to minus infinity*, (15) *simplifies to*:

$$x = k + \alpha\eta\left(1 - e^{\lambda_1(k-\bar{k})}\right). \tag{16}$$

If in addition the upper bound, $\bar{k}$, *goes to infinity, we get the familiar linear solution* (5):

$$x = k + \alpha\eta. \tag{17}$$

When both boundaries are infinitely distant the exchange rate is linear in fundamentals.

The saddlepath solution given in the proposition is of the form hypothesized earlier: it is a function of the current state k and the two barriers. It is also straightforward to verify that $S(k)$ is monotonically increasing over its domain, as claimed earlier.

To understand the economics of the proposition, consider first the simple case in which fundamentals have no trend, $\eta = 0$. The exchange rate then is simply equal to current fundamentals: $x = k$. To understand why, notice that when $\eta = 0$, the expected growth of fundamentals always is zero—at as well as within the absorbing barriers. Thus, the best forecast of all future fundamentals is just today's value of k; and the conditional expectations in (3), as well as the exchange rate, all equal current k.

Matters are more complicated when $\eta > 0$, so we use Figure 1 to illustrate. The line labelled FF indicates the linear solution given by (5), which corresponds to the case $A_1 = A_2 = 0$. (FF is also the saddlepath under a permanent free-float regime.) The figure shows two exchange rate paths with the same upper bound, but different lower bounds. Path 1 depicts the behavior of x when the absorbing barriers are the points $\underline{k}$ and $\bar{k}$.

[9] The boundary conditions used here generalize easily to other types of regime change. For example, suppose that the authorities announce that at $k = \bar{k}$, the stochastic process driving fundamentals will switch permanently to $dk = \eta'\,dt + \sigma'\,dz$. Then condition (12) becomes $S(\bar{k}) = \bar{k} + \alpha\eta' = \bar{x}$. As another example, suppose that the authorities wish to implement a target-zone exchange-rate regime by keeping fundamentals within reflecting barriers $\underline{k}$ and $\bar{k}$. Then if policy interventions are infinitesimal, the boundary conditions (11) and (12) above would be replaced by $G'(\underline{k}) = G'(\bar{k}) = 0$. The saddlepath solution for the exchange rate combines these new boundary conditions with (8).

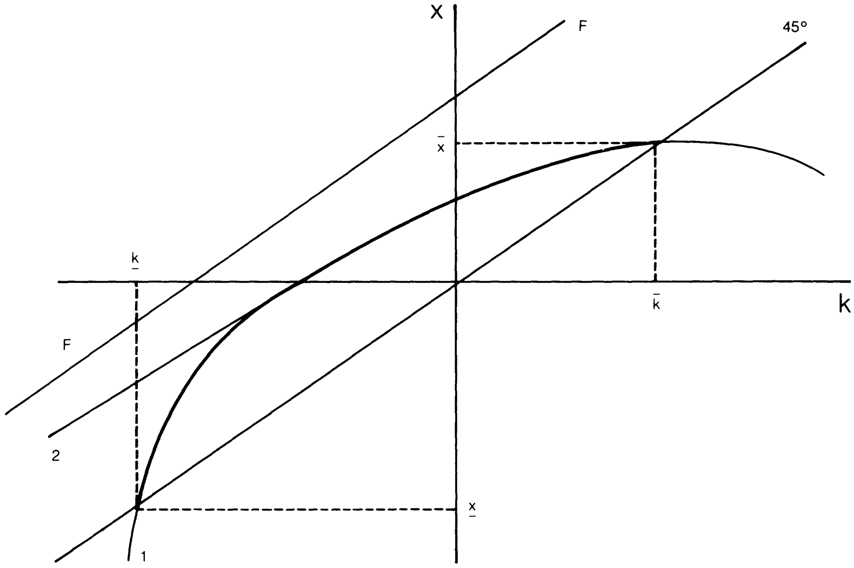

FIGURE 1

Path 2 is drawn to correspond to the extreme case in (16), where the lower bound is at minus infinity. It is clear from (11) and (12) that the exchange rate must lie on the 45-degree line through the origin at both absorbing barriers. The free-float saddlepath, *FF*, is relevant when both boundaries are infinitely distant.

The intuition behind the bent curves in Figure 1 is as follows. On the saddlepath, the exchange rate is the present discounted value of fundamentals, and the evolution of fundamentals is governed in part by their deterministic trend growth rate, which depends on η. Suppose that $\eta > 0$ (the case shown in the figure). As k approaches either $\bar{k}$ or $\underline{k}$, the probability that the exchange rate will still be floating on any given future date declines; and since η is set permanently to zero at the moment of pegging, the expected rate of fundamentals growth on any future date also declines as either absorbing barrier is approached. As a result, there is a progressive currency appreciation (fall in x) relative to *FF* as k moves towards a barrier. For $\eta < 0$, *FF* would lie below the 45-degree line and the saddlepath solution would be the mirror image of the one in Figure 1. As we have seen, when $\eta = 0$ the bending effects are absent because absorption of k has no effect on the expected change in fundamentals. Think of the saddlepath as being trapped between *FF* and the 45-degree line, which collapse into a single line when η shrinks to zero.

The path given by (16), whose derivation implies setting $A_2 = 0$ in (8), is the unique path for the stochastic-process-switching problem of Flood and Garber (1983). They attempt to solve directly the integral representation for the exchange rate when pegging occurs at $x = \bar{x}$ with no jump in fundamentals at the moment of pegging:

$$(18) \qquad x(t) = \alpha^{-1}\int_t^{\infty} e^{(t-s)/\alpha} E\big(k(s)|k(t), k(s) \leqslant \bar{k}_a = \bar{x}\big)\, ds.$$

Equation (16) is the unique solution to this integral.[10]

[10] See also Smith (1991).

3.2. *Policy Announcements and Exchange-Rate Indeterminacy*

In the foregoing discussion, we assumed that the authorities announced bounds on the fundamentals, and that their policy was a passive one until one of the bounds was reached. In practice, policy makers are unlikely to make future policy contingent on the level of a policy instrument such as the money supply. They are more likely to announce bounds on the exchange rate itself. Indeed, in their verbal description of the process-switching problem, Flood and Garber (1983) assumed that the authorities committed themselves simply to peg the exchange rate once it reached a target level $\bar{x}$. Only in their calculations did they add the supplementary—but crucial—assumption that fundamentals cannot jump at the instant of pegging. In the last section, we solved the Flood-Garber problem assuming explicitly that this additional information had been conveyed to the market.

If policy makers are less explicit in their announcements than we have assumed so far, however, the market may have too little information to calculate an equilibrium exchange rate. We now show that a unique equilibrium will exist if the authorities announce *both* a future exchange-rate peg *and* the size of the intervention they will undertake at the moment of transition. But prospective interventions of different size give rise to different exchange-rate paths. It follows that exchange-rate information alone does not generally suffice to determine an equilibrium. Our basic point is a familiar one: a determinate market outcome requires that different expectations not be validated by accommodating official behavior.[11]

We can see how a prospective transitional intervention affects the equilibrium by imagining that the authorities make the following announcement: "When the exchange rate reaches $\bar{x}$, we will peg it by means of an intervention that instantaneously raises the fundamental k by the amount $\bar{I}$. From then on, we will do whatever is necessary to hold the exchange rate at $\bar{x}$." To solve, it is best to work backwards from the moment after the intervention, when the exchange rate has already been fixed. At this time, and forever after, it must be the case that $k=\bar{k}=\bar{x}$. The intervention that brings k to $\bar{k}$ occurs at the moment x reaches $\bar{x}$, so at that moment, fundamentals move discontinuously from $\bar{k}-\bar{I}$ to $\bar{k}$. Note that investors fully anticipate this large change in fundamentals when x reaches its boundary level; but, if there are to be no riskless profit opportunities, the exchange rate must remain steady at $\bar{x}$ as the anticipated intervention is carried out. This condition is another application of asset-price continuity. In terms of our earlier notation, it can be written as:

$$S(\bar{k}-\bar{I})=\bar{k}=\bar{x}. \tag{19}$$

As before, the appropriate boundary conditions, applied to (8), yield a closed-form solution. Since the barrier is one-sided, one condition is $A_2=0$, which forces the saddlepath to approximate the linear solution for k far below $\bar{k}-\bar{I}$. Given that $A_2=0$, continuity condition (19) is:

$$\bar{k}-\bar{I}+\alpha\eta+A_1e^{\lambda_1(\bar{k}-\bar{I})}=\bar{k}=\bar{x}. \tag{20}$$

This second boundary condition leads to the following proposition.

PROPOSITION 2: *Assume that fundamentals are expected to rise discontinuously by the amount $\bar{I}$ at the moment the exchange rate is pegged at $\bar{x}$. While floating, the equilibrium exchange rate is given by*:

$$x=k+\alpha\eta+(\bar{I}-\alpha\eta)e^{\lambda_1(k-\bar{k}+\bar{I})}. \tag{21}$$

[11] Obstfeld (1984, p. 209), points out this indeterminacy. Flood and Garber (1989) use a boundary condition identical to the one invoked below to study target zones with discrete interventions at the margins.

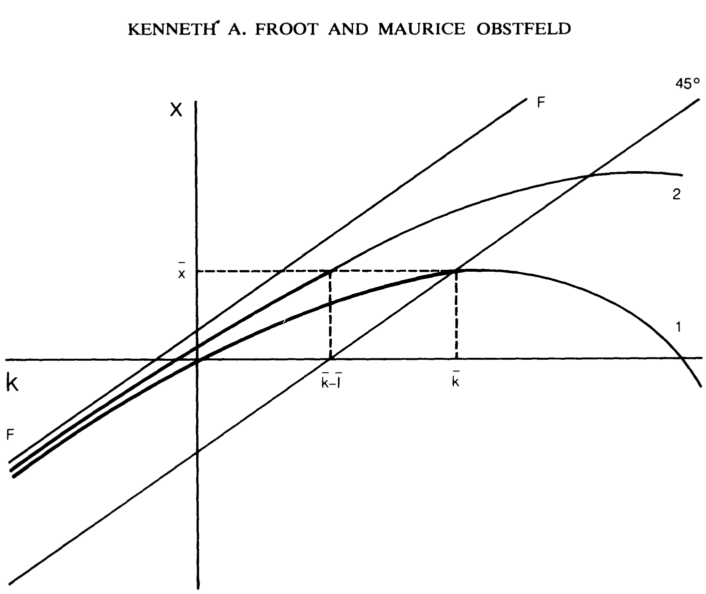

FIGURE 2

Figure 2 illustrates how the prospect of a discrete intervention alters the equilibrium. Path 1 depicts the equilibrium with $\bar{I}=0$ described by (16) and shown previously in Figure 1. Path 2, by contrast, graphs the relationship described by (21), where the authorities undertake an intervention of $\bar{I}>0$ at $x=\bar{x}$. Path 2 is steeper than path 1: near the upper bound, where a large positive intervention is imminent, the greater discounted value of expected future fundamentals increases the exchange rate.[12]

Equation (21) underscores the main point: when the authorities commit themselves *only* to peg the spot rate once it reaches $\bar{x}$, they are implicitly committing themselves to perform whatever intervention the market happens to conjecture will occur at that moment. Indeed, in the extreme case, suppose that when the policy is first announced at $t=0$, with $k(0)<\bar{k}=\bar{x}$, the market immediately pushes the exchange rate to $\bar{x}$. If the authorities are to fulfill their commitment, they would be forced to adjust fundamentals instantly by the amount $\bar{k}-k(0)$.

To rule out multiple equilibria, the authorities can announce both the size of the intervention they are willing to undertake as well as the exchange-rate peg. Alternatively, as was assumed in the previous subsection, they can announce that they will remain passive while fundamentals follow (2), until absorption occurs at the fundamental $\bar{k}=\bar{x}$.

3.3. *More Complex Forcing Processes*

The techniques above are practical only when the driving process in (2) is relatively simple; it is usually impossible to find closed-form general solutions to the analogues of (7) when k follows a more complicated forcing process. Nonetheless, some special cases do have solutions. Suppose, for example, that fundamentals are mean reverting, follow-

[12] If we think of the authorities as intervening by a sharp increase in the money supply when x reaches $\bar{x}$ along path 2, then equilibrium is maintained by a discontinuous fall in the nominal interest rate. The counterpart of this fall in the interest rate is a fall to zero in the expected rate of currency depreciation, which depends on the curvature of path 2. In general, the exchange-rate path (21) can be either convex or concave, depending on whether $\bar{I}$ is greater or less than $\alpha\eta$, respectively.

ing the Ornstein-Uhlenbeck process:

$$dk(t) = (\eta - \theta k(t))\,dt + \sigma\,dz(t), \tag{22}$$

where η, θ, and σ are known constants. Use of (1) and application of Itô's Lemma lead to the differential equation:

$$G(k) = k + \alpha(\eta - \theta k)G'(k) + \frac{\alpha\sigma^2}{2}G''(k). \tag{23}$$

The following proposition gives the general solution to (23):

PROPOSITION 3: *When fundamentals follow* (22), *any solution to equation* (1) *must satisfy*:

$$x = G(k) = \frac{k + \alpha\eta}{1 + \theta} + A_1 M\left(\frac{1}{2\theta\alpha}, \frac{1}{2}, \frac{2(\eta - \theta k)^2}{\theta\sigma^2}\right) + A_2 M\left(\frac{1 + \theta\alpha}{2\theta\alpha}, \frac{3}{2}, \frac{(\eta - \theta k)^2}{\theta\sigma^2}\right)\frac{(\eta - \theta k)}{\sqrt{\theta}\,\sigma}, \tag{24}$$

where A_1 and A_2 are arbitrary constants and $M(.,.,.)$ is the confluent hypergeometric function.[13]

Using the procedures discussed above, it is straightforward to rederive all the propositions when fundamentals evolve according to (22). Naturally, for values of k such that $\theta k \approx \eta$ the mean-reversion component of (22) is unimportant, so that the solutions appear qualitatively very similar to those shown in the graphs. For values of k where mean reversion is important, the mean reversion introduces a new source of bending (toward the unconditional mean of k, η/θ) into the paths above.

Graduate School of Business, Harvard University, Boston, MA 02163, U.S.A.
and
Dept. of Economics, University of California-Berkeley, Berkeley, CA 94720, U.S.A.

Manuscript received February, 1989; final revision received January, 1990.

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[13] See Slater (1965) for the properties of confluent hypergeometric functions.

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