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The Determinants of Optimal Currency Hedging

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This paper examines the important factors that determine an optimal currency hedging policy for a global portfolio. We show how the optimal policy depends on the composition of the investor's aggregate portfolio, the expected return to and risks of currency exposure, the transaction costs of currency hedging, the investor's time horizon, and the investor's risk tolerance. This is a synthesis of previous and ongoing work of the authors and of the findings of other academic and practitioner research in this area.[1]

We proceed first with a brief qualitative discussion of the critical considerations that enter into an analysis of currency hedging policy. Then we show more quantitatively how these ought to be combined to determine appropriate hedge ratios.

Currency exposure

Underlying any analysis of hedging policy is the notion of *currency exposure*. The exposure of an asset to a currency is the expected percent change in value of the asset per 1% change in the value of the currency.[2] For example, an exposure of 0.5 implies that a one-percent increase in the price of foreign exchange will increase the value of the portfolio (expressed in domestic currency) by one-half percent. The exposure of a portfolio of assets to a given currency is just the weighted average of the exposures of the individual assets that comprise the portfolio.

In the analysis that follows, we need to distinguish between the exposure of an asset or a portfolio before hedging versus the desired level of exposure once the hedge is in place. We will therefore refer to exposure prior to hedging as the *preexisting exposure*, and to the exposure after implementing the desired hedge as the *target exposure* of the portfolio.

Currency forward, futures, and options contracts are most often used to adjust currency exposure. These contracts increase exposure when held long, and decrease (hedge) exposure when held short; fowards and futures involve no initial investment.[3]

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[1]See, for example, Perold and Schulman (1989), Black (1989), and Froot (1993) for more on these topics and additional citations to the literature.

[2]For the purposes of hedging, one should use the percentage change in the value of the currency *in excess of* the interest differential (i.e., the return on the hedge itself) to determine the exposure. However, in most cases the variation over time in the interest rate differential is dwarfed by that of the exchange rate change. Thus, the difference in these two approaches is negligible.

[3]None, that is, other than collateral demanded by counterparties.

Expected returns to currency hedging

The return on a forward currency position is equal to the change in the exchange rate minus the interest rate differential between the two countries. The *expected return* on a forward position is thus the expected change in the currency minus the interest rate differential. Historically, average currency returns have fairly closely approximated the interest rate differential. Thus the average returns to currency forward positions, whether long or short, have been close to zero. This is the basis for the widely held assumption that the expected future return to a currency forward position will be small.

However, it is problematic to assume that the expected future return is literally zero. For if the expected return is zero when measured in home currency, it cannot be zero when measured in foreign currency. This is the so-called Siegel's Paradox which follows from the mathematical fact that the average value of a reciprocal is greater than the reciprocal of the average. Thus, for example, the average value of the yen/dollar rate is greater than the reciprocal of the average value of the dollar/yen rate. More specifically, the expected return on a forward position measured in home currency plus the expected return measured in foreign currency equals the variance of changes in the exchange rate.[4]

An assumption that is perhaps more reasonable than "zero" is that the expected return on a long currency forward position is split equally between the two currencies. This would imply that the expected return is equal to half the exchange rate variance. With a typical standard deviation of short-term exchange rate movements of 10% per annum, the variance evaluates to 100 basis points per annum. Accordingly, this methodology puts the expected return on a long forward position in foreign currency at 50 basis points per annum, and the expected return on a currency hedge at -50 basis points per annum. While these numbers are still relatively small, they are clearly non-zero.

Clearly, expected returns from holding foreign currency will positively affect the amount of currency exposure the investor should hold, i.e., the *target exposure.*

Transaction costs

The bid-ask spreads on near-term currency forward contracts are variously estimated in the range of 3 to 10 basis points. The contracts need to be rolled over as they expire. Thus, the annual transaction cost of hedging with three month contracts is in the range of 12 to 40 basis points. Longer maturity contracts need to be rolled less frequently, but usually have larger bid-ask spreads.

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[4]This is precisely true if the exchange rate follows a geometric random walk motion.

Positive transactions costs must be incurred whether the "hedge" is used to increase or decrease exposure. Such costs will tend to prevent "fine tuning" of exposures in the neighborhood of the target exposure. That is, as is shown below, if the initial and target exposures are near one another, transactions costs will eliminate the incentive to undertake any hedging at all. The result will be a band of inaction around the target exposure.

Investor risk tolerance

Risk tolerance reflects the investor's tradeoff between expected return and risk (expressed here in terms of the variance of returns). In the computations below, a risk tolerance of 0.5 indicates the investor is willing to accept an increased variance of 1 percent per annum in order to increase expected return by 100 basis points. Someone who invests 70/30 in stocks/bills when faced with an allocation between stocks and treasury bills has an implied risk tolerance of 0.34; a portfolio invested entirely in equities reflects a risk tolerance of 0.5; and an investor with risk tolerance of 0.25 holds a mix of 52/48 in stock/bills.[5]

Figure 1 helps gauge implied risk tolerance for unhedged, internationally diversified portfolios of stocks and bonds (not bills as above). The stock/bond mix is given by the horizontal axis, and applies to both the US and foreign components of the portfolio. For any given mix of US/foreign assets, implied risk tolerance increases as the fraction of stocks rises. However, as foreign stocks and bonds are added into an all-US portfolio (holding constant the mix of stocks and bonds), implied risk tolerance declines. While this may seem counterintuitive at first (US investors who add international assets are often considered adventurous), it is really just another way to demonstrate that globally diversified portfolios are safer than purely domestic portfolios.

The larger an investor's risk tolerance, the greater his or her propensity to hold riskier, high-return investments. An investor with greater risk tolerance will have a higher target exposure level. The reason is that, due to Siegel's paradox, long-positions in foreign exchange provide positive expected returns (in addition to higher risk).

Investor horizon

Investor horizon can also have an important impact on the optimal hedge ratio. At short horizons, the *preexisting exposure* of portfolios of international assets is quite high. That is, their returns (expressed in home-currency terms) covary strongly with exchange-rate surprises. However, preexisting exposures of foreign assets often diminish at long horizons, in many cases implying a lower optimal hedge ratio. Indeed, at long

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[5]These computations assume a risk premium of 6% per annum for stocks over treasury bills, and a standard deviation of stock returns of 17% per annum.

horizons, preexisting exposures may be so low that a high hedge ratio actually *raises* the portfolio's real return volatility.

To see why preexisting exposures may fall with horizon for stocks, note that at longer horizons, stocks can be thought of as "real" assets -- they are "naturally hedged" against a decline in the purchasing power of the currency in which they are denominated. When a currency falls in value, nominal prices of "real" goods in that currency tend to rise (relative to dollar prices) by an amount that offsets the currency depreciation. This reversion in the purchasing power of different currencies is what practitioners have in mind when they say that currency fluctuations eventually "wash out" of international asset returns.

Investor horizon can also impact the exposures of fixed-income investments. At short horizons, the cost of living in local currency is fairly stable, whereas the value of foreign exchange is not. At longer horizons, however, changes in the value of foreign exchange are largely reflected in inflation rates. Hedging can help insulate foreign bonds over short periods when consumption prices are sticky and currency values fluctuate widely. However, at longer horizons, hedging offers a bond portfolio little protection against domestic inflation risks. If, for example, declines (increases) in the value of the dollar are ultimately reflected one-for-one in higher (lower) *US* inflation, foreign bonds are "naturally hedged" against currency fluctuations. Thus, at long horizons, preexisting exposures of foreign bonds are lower than at short horizons. This effect is strengthened when domestic inflation shocks are relatively important in comparison with foreign inflation shocks.

The Determination of Target Exposures

a) no transactions costs of hedging

From standard portfolio theory (see the model in the appendix) the target exposure of a portfolio to any asset, including foreign exchange, is proportional to the asset's expected return as follows:

Target Exposure = Risk Tolerance x Reward-to-Risk Ratio

where the reward to risk ratio is the expected return of the asset in excess of the riskless rate divided by the variance of the asset's return:

Reward-to-Risk Ratio = Expected Excess Return ÷ Variance of the Asset's Returns.

This optimal exposure formula assumes **no transaction costs** for buying or holding the asset. In the case of exchange rate exposure, the reward/risk ratio is the expected return on a long forward position divided by the exchange rate variance. To illustrate, if the expected return from a long position in foreign exchange is zero, the formula implies that target exchange rate exposure is zero. Thus, any preexisting currency exposure in the underlying portfolio should be fully hedged.

As previously mentioned, the exposure of the underlying portfolio is the weighted average of the exposures of the assets comprising the portfolio. If US assets have no exchange rate exposure and foreign assets have exchange rate exposures of one, then the underlying exposure is the fraction of the portfolio invested abroad. Thus, if the expected return to currency hedging is zero, the optimal dollar amount to hedge is simply the face value of the amount invested abroad, regardless of risk tolerance.

On the other hand, if the expected return from being long foreign exchange is given by the Siegel's-Paradox method discussed above, then the target level of currency exposure increases. To continue with the example above, if the standard deviation of foreign exchange returns is 10% per annum (so that the annualized variance is 1%), then the expected return from foreign exchange exposure is 50 basis points. For an investor with risk tolerance equal to 0.4, this would imply a target exposure of 0.20 (from the equations above, target exposure is given by 0.4 x (0.5% / 1%)); for an investor with risk tolerance of 0.25, the target exposure falls to 13%. Thus, if the underlying portfolio has a currency exposure of, say, 30%, the optimal hedge ratio for the investor with risk tolerance of 0.25 is 17% (30%-13%), i.e., a hedge amount equal to 57% (17% / 30%) of the value of the underlying international assets.[6]

Figure 2 depicts the behavior of target exposure levels as an increasing function of risk tolerance. When the risk/reward ratio is lower, the target exposure level shifts downward proportionately. Note that target exposures remain at under 25% for reasonable levels of risk tolerance and foreign exchange reward/risk ratios.

b) target exposures in the presence of transaction costs of hedging

Transaction costs, and other costs associated with hedging such as management fees, reduce the amount one should hedge.[7] In the presence of such costs, investors should not hedge so much as to fully equate the actual exposure with the target exposure defined above. Instead, the optimal adjusted hedge will bring the exposures to within a "band" around the original target. The Adjusted Target Exposure Range is then a band around the original target:

$$\text{Adjusted Target Exposure Range} = \text{Target Exposure} \pm \text{Band}$$

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[6]In this framework, the target exposure depends only on risk tolerance and the risk-to-reward ratio, and not on the preexisting exposure of the portfolio. Thus, in the above example, if the underlying portfolio had no preexisting currency exposure, the optimal decision would be to *increase* the currency exposure to 17%, for example by taking on long positions in currency forward contracts.

[7]If, instead of hedging, one would ordinarily increase the portfolio's exposure to currency risk, transactions costs again would lessen this increase in exposure.

Band = Risk Tolerance x
(Annualized Transaction Costs / Variance of the Asset's Returns)

To illustrate, suppose that the transaction costs of hedging are 20 basis points per annum, that the standard deviation of exchange rate movements is 10% per annum, and that the investor's risk tolerance is 0.3. The band then evaluates to ± 6%.[8] The optimal hedging policy is therefore to hedge to the adjusted target exposure, which is within plus or minus 6% of the original target exposure.

Clearly, transactions costs can create a relatively large band of inaction: for transactions costs of 40 basis points and risk tolerance of 0.5, the band evaluates to ± 0.20.[9] Recall that for such an investor, the original target exposure was 0.25. This implies that with preexisting exposures of 0.5 or greater, the optimal adjusted target exposure is 0.45 (0.25 + 0.20). If the preexisting exposure lies within the band -- that is, between 0.45 and 0.05 (0.25 ± 0.20) -- then the optimal adjusted target exposure is *equal* to the preexisting exposure. The optimal hedge is not to hedge at all. Finally, if the preexisting exposure is actually less than 0.05 (even negative), the investor should take on additional foreign exchange risk until reaching an adjusted target exposure of 0.05.

For a more conservative example, suppose that the investor's risk tolerance is 0.25 and that hedge transactions costs are 30 basis points per annum. This investor would have a band of 0.08. Moreover this particular investor has an original target exposure of 0.13, so that the adjusted target exposure is:

i. 0.05 for preexisting exposures smaller than 0.05;
ii. equal to the preexisting exposure for preexisting exposures between 0.05 and 0.21;
iii. and 0.21 for preexisting exposures greater than 0.21.

c) comments

Note the implications of the analysis so far. If the portfolio contains even a relatively large fraction of international assets, say 30 percent, preexisting exposure will be in the neighborhood of only 0.30. And with such low levels of initial exposure, only investors with the least risk tolerance or with the lowest-cost access to hedging services, will hedge in large amounts. Specifically, a preexisting exposure of 0.30, relatively low risk tolerance of 0.25, and transactions-cost level of 30 basis points per annum, result in an adjusted target exposure of 0.21 (0.13+0.08). This, in turn implies hedging an amount equal to 9% (0.30-0.21) of the overall value of the portfolio. Relative to the value of the international assets, this is implies a hedge ratio of 30% (0.09 / 0.30).

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[8]The band is the risk tolerance x (transactions costs / variance) = ± 0.2 x (0.4% / 1.0%)= ± 0.08.
[9]The band is now ± 0.4 x (0.5% / 1.0%)= ± 0.2.

Moreover, this hedge ratio is very sensitive to risk tolerance. For example, if risk tolerance is increased from 0.25 to 0.4, the adjusted target exposure increases to 0.32 (0.20+0.12), implying that no hedging should be done at all!

Finally, the hedge ratio is also sensitive to transactions costs. If transactions costs in the above example (with risk tolerance equal to 0.25), are reduced by one-half to15 basis points, the adjusted target exposure becomes (0.13 ± 0.04). This in turn implies hedging an amount that is equal to 43%=(0.30-0.17)/0.30 of the value of the international assets. (See Tables 1 and 2 below for a complete analysis of the sensitivity of hedge ratios.) The implication here is that transactions costs can impose bands of inaction whose boundaries are important to be aware of. They may approach, or even engulf, the preexisting exposures of many global portfolios.

Preexisting Exposures

Now that we have analyzed target exposures for different investors in the presence of transactions costs, we turn to the computation of actual, preexisting exposures. Exposures are best measured empirically over relatively short return horizons (such as a day, week or month) because there is plenty of such short-horizon data available. In the next subsection, we therefore estimate actual short-horizon exposures of several major asset classes. Long-horizon exposures, on the other hand, are very difficult to measure empirically, because typical time-series samples do not contain much information about them. In subsection b, we therefore provide a model which helps determine how preexisting exposures are affected by increases in horizon.[10] We also comment on how the results from this model square with long-horizon empirical estimates that have been reported elsewhere.

a) preexisting exposures over short horizons: estimation

Estimation of preexisting short-horizon exposures for different asset classes is relatively straightforward. There are a large number estimates in the literature on this topic. While the numbers differ by sample period and particular asset class, there is reasonable qualitative agreement on exposure sizes. To gain a sense for the magnitude of these exposures and how they evolve over time, Figures 4-6 display rolling regressions of preexisting exposures measured using monthly data. Several conclusions emerge from the results.

First, even the short-horizon exposures may be imprecisely measured because they vary over time. This is not too surprising, as different factors drive short-horizon exposures at different times. Take for example the exposure of US bonds to the

[10]In the model (which is presented formally in the appendix), a change in horizon affects only the preexisting exposures, and has no affect on target exposures. However, this neat separation holds only for objective functions which are linear in the mean and variance of terminal wealth.

exchange rate. If the market is worried about high inflation, then an increase in the US money supply will result in low bond returns (as long rates rise) and a concurrent depreciation of the dollar -- i.e., in a *negative* exposure to foreign exchange. However, if the market is concerned that the Fed will tighten, then a current increase in the money supply may still lower bond returns, but can be associated with a concurrent *appreciation* of the dollar -- i.e., in a positive exposure to foreign exchange. The actual exposure of US bonds will therefore be a combination of these two effects, and will vary over time.

Second, the average short-term exposures which emerge from the data are roughly consistent with earlier estimates. Historically, the short-term currency exposures of US stocks and bonds have been time-varying but small, with the exposure of US stocks being on average positive (about 0.1, although measured with little precision), and the exposure of US bonds being on average even more positive (about 0.25, although again measured with little precision). The short-term currency exposures of foreign stocks and bonds (with returns measured in US dollars) have been large, varying around one, with averages slightly greater than 1.0 for stocks, and approximately 1.1 for bonds. The long-term real currency exposures of foreign stocks are smaller over longer time periods.

b) the effect of horizon on preexisting exposures

The analysis of the optimal adjusted target exposure remains valid regardless of investor horizon.[11] However, investor horizon can and *does* appear to affect the preexisting exposure of the portfolio -- i.e., longer horizons typically imply lower levels of preexisting exposure.

Before continuing, it is important to clarify what we mean by "horizon." Investors whose estimates of risk and expected return for major asset classes (and currencies) do not depend on time horizon are implicitly assuming that asset returns follow random walks. When returns do not follow random walks, the horizon over which the investor plans will matter. For example, in the presence of mean-reverting stock returns, an investor planning to liquidate his or her portfolio after five years will perceive much lower *annualized* risk to holding stocks than an investor planning to liquidate after one year. For comparable asset allocation mixes, the longer-horizon investor will have a lower preexisting exposure to stocks.

Currency returns exhibit a similar mean-reverting feature. Nominal exchange rate changes have a large short-run, but no long-run, effect on the cost of living in one country compared with another. This reversion in the "real" exchange rate is called the doctrine of purchasing power parity (PPP), and there is strong evidence that PPP holds over long periods.[12] Deviations from PPP put pressure on domestic and foreign inflation rates, and have a substantial correlation with them over long periods of time. That is, a

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[11]Note, however, the caveat in the previous footnote.

[12]For recent empirical work on this topic, see, for example, Frankel (1986) or Diebold, Husted, and Rush (1991) and the citations therein.

large depreciation of the dollar is associated with higher US inflation and lower foreign inflation than originally interpreted. Assets' exposures to exchange rate fluctuations will generally be affected by horizon, as exchange rate changes slowly get reflected in domestic and foreign price levels.

Consider stocks first. Over longer time periods stocks behave like "real" assets -- that is, they retain their value in the presence of inflation.[13] For example, while fluctuations in the dollar/pound exchange rate affect the price of UK stocks relative to US stocks in the short run, in the long run UK stocks are priced at "international" rates. The evidence that is available supports this view that short-term currency fluctuations wash out of the real price of both foreign and domestic stocks. (For a more detailed discussion see Froot, 1993.) The implication is that the very long-horizon exposure of (foreign and domestic) stocks to exchange rate fluctuations is very low, perhaps even zero.

The long-horizon exposure of a bond portfolio is more complex, as it depends both on its currency-denomination as well as the magnitude of domestic and foreign inflation shocks. The long-horizon exposure of foreign bonds will depend on the relative size of foreign inflation shocks. If foreign prices move to fully offset exchange rate changes, then foreign bonds have a long-horizon exposure equal to 1.0. Alternatively, if foreign prices do not respond at all to exchange rate movements, foreign bonds will retain their real purchasing power (in foreign currency) regardless of what happens. Since PPP holds over long periods, foreign bonds will also retain their purchasing power in the *domestic* currency. Thus, in this case foreign bonds have an exposure of 0.0 at long horizons. (Note that, over short horizons, foreign bonds have exposures of about 1.1 on average -- see Figure 6.)

Domestic bond exposures will be analogously influenced by horizon. That is, if domestic prices ultimately move to fully offset exchange rate changes (implying, of course, that foreign prices do not respond at all), the long horizon exposure of domestic bonds will be -1.0. That is, real bond returns will be highly negatively exposed to exchange rates, because exchange-rate changes result in equiproportional changes in the purchasing power of the bond payments. Alternatively, if domestic prices do not respond at all to exchange-rate changes, domestic bonds will have a long-horizon exposure of 0.0.

c) model results

Now we have a sense for the magnitudes of very long -- indeed, infinite -- horizon exposures of different asset classes. We next need to know how long is long. That is, how different from short-horizon exposures are the exposures over more normal investor horizons? The answer to this comes from the above-mentioned model, in which exposures at each horizon are shown to be a weighted average of short-term and very long-term exposures, with the weights depending on the overall speed with which

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[13]This is not true of stocks over short time periods. For a study of the stock-return / inflation relationship at short as well as long horizons, see Boudoukh and Richardson (1993) and the citations therein.

deviations from PPP die away. Empirical estimates suggest that the half-life of real exchange rate fluctuations is about 4 years.[14] This implies that approximately 16 percent of a PPP deviation on a given date can be expected to die away over the next year.

Specifically, the model calculates preexisting exposures at each time horizon as:

Preexisting Exposure =
Instantaneous Preexisting Exposure x HAW + Infinite Horizon Exposure x (1 - HAW)

where "instantaneous" implies that the preexisting exposure is measured on high-frequency return data (for example, daily, weekly, or monthly). The infinite-horizon exposure, which is derived in the context of the model, follows the intuitions above. The weighting term HAW -- for "horizon adjustment weight" -- in the above equation is equal to 1.0 at short horizons and 0.0 at infinitely long horizons. As shown in the appendix, HAW is given by:

$$\text{Horizon Adjustment Weight} = \left(1 - (1-0.16)^{T+1}\right) / \left((T+1) \times 0.16\right)$$

T = Horizon in Years
0.16 = estimated annual decay rate of PPP deviations

Finally, to implement the model, we need to know the relative sensitivity of domestic and foreign inflation rates to PPP deviations. (These two sensitivities are assumed to be equal in the estimates below.) Actual estimates of short-term exposures were also used, in order to calibrate the model to specific asset classes.

Using this expression and parameter choices, preexisting exposures are computed as:

Preexisting Exposures at Different Horizons

	foreign		**domestic**	
Horizon (years)	**stocks**	**bonds**	**stocks**	**bonds**
0	0.95	1.10	0.10	0.24
1	0.88	1.04	0.09	0.19
2	0.81	1.00	0.09	0.15
3	0.75	0.95	0.08	0.11
4	0.70	0.91	0.07	0.07
5	0.65	0.87	0.07	0.04
10	0.47	0.74	0.05	-0.08

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[14]See Frankel (1986).

20	0.28	0.60	0.03	-0.20
30	0.20	0.53	0.02	-0.26
50	0.12	0.48	0.01	-0.31
infinite	0.00	0.39	0.00	-0.39

When the horizon is 0 years, the exposures are exactly the instantaneous exposures discussed above. When the horizon is infinite, the exposures follow from the above discussion of very long-horizon exposures. The infinite-horizon exposures of foreign and domestic bonds are equal but opposite in the table because we assumed that foreign and domestic consumer prices are equally sensitive to deviations of the exchange rate from PPP. Clearly, this result can be changed by assuming asymmetric sensitivity to PPP deviations.

To illustrate how to apply the number in the table, consider a portfolio that is equally weighted across foreign and domestic bonds and stocks (i.e., 25 percent in each asset class). The instantaneous preexisting exposure of the portfolio is the average of the actual instantaneous exposures in the first line of the table: 0.25 x (0.95+1.10+0.10+0.24) = 0.60. At a ten-year horizon, the preexisting exposure drops to 0.30. At an infinite horizon the exposure falls to exactly zero.

These numbers in the table show a somewhat less rapid decline with horizon than actual estimates of long-horizon exposures. Froot (1993) estimates that the preexisting exposure on a portfolio of international equities falls from about 0.92 at short horizons to 0.37 at eight-year horizons. Interestingly, the actual estimates of stock exposures decline much more rapidly than the constant geometric rates of decay assumed by the model: at three-year horizons stock exposures in Froot's data fall to 0.39, and level off after that.

For foreign bonds, Froot's estimates show a much lower short-horizon exposure, of only about 0.82 at one-year horizons. The same data have an exposure of 0.39 at eight-year horizons. Here, however, the decline occurs much more slowly than for stocks: at five-year horizons foreign bond exposures decline to only 0.72. This rate of decline appears somewhat slower than that show in the table. For domestic bonds, Froot finds an initial exposure of about 0.0, which declines to about -0.8 at eight year horizons. If we were to try to replicated the pattern of these coefficients using the above model we could do so most closely by assuming that domestic prices were more sensitive to PPP deviations than foreign prices. This has the effect of reducing the infinite-horizon exposures of both foreign and domestic bonds from the estimates in last line of the table above.

The optimal hedge ratio

We can now put together all of the pieces to determine the optimal amount to hedge as a fraction of the value of the international assets in the portfolio:

Optimal Hedge Ratio =
Preexisting Exposure − Adjusted Target Exposure

Optimal Hedge Ratio =
Instantaneous Preexisting Exposure x HAW + Infinite Horizon Exposure x (1 - HAW)
− Risk Tolerance
x (Expected Excess Return ± Annualized Transactions Costs) / Return Variance

Overall Results

Tables 1 and 2 lay out the implications of all of our discussion for hedging demands. Table 1 focuses on short-term horizons (monthly), whereas Table 2 performs analogous computations for a long-term horizon of five years.

Table 1 makes it clear that for investors with relative high risk tolerance, and for no transactions costs, hedging demands are extremely sensitive to the preexisting exposure levels. -- hedging demands range from 105% of the value of the international assets to -15% (i.e., going long foreign exchange through futures in an amount equal to 15 percent of the value of the international assets). Once transactions costs are added, the optimal hedge ratios are somewhat less volatile -- ranging from 0% to 77% of the value of international investments. However, optimal hedge ratios are equal to zero in a number of cases. This reflects the transactions-cost drag on returns as well as the band of inaction associated with such hedging costs. Only the least risk tolerant investors hedge, and among those, those with relatively substantial international components are the most aggressive hedgers.

Table 2 features exactly the same layout, only now the horizon is increased to five years. Optimal hedge ratios in the absence of transactions costs are still very sensitive to the assumptions -- ranging from 49% to -75% -- but they involve less hedging on average than comparable figures in Table 1. The most important impact of the horizon change falls on the hedge ratios with transactions costs. Now there is relatively little hedging by anyone, as the longer horizon attenuates the preexisting exposures by about 30%. The investor who is most disposed toward hedging -- one who displays little risk tolerance, yet has a relatively large fraction of his portfolio in foreign assets -- hedges only 30% of the value of the international assets. There is even some evidence that risk tolerant investors with relatively little preexisting foreign asset exposure take on additional currency exposure by going long foreign exchange (indicated by a negative value in column 14 of Table 2).

The sensitivity of the results in these two tables bears emphasis. Slightly higher transactions costs will strongly reduce hedging demands. Similarly, changing the assumptions on risk tolerance and expected excess returns on holdings of foreign exchange will have large effects on optimal hedge ratios.

Conclusions

This paper has examined the determinants of optimal currency hedge ratios and calibrated a simple model to gauge how much hedging should be undertaken. Our results show a wide range of hedge ratios across a number of parameter values. However, if one allows for a reasonable level of transactions costs, none of these hedge ratios approach 1.0. Our work is particularly relevant for investors with a relatively long planning horizon. The long-horizon results suggest that optimal hedge ratios are quite low (never above 30%), and are usually zero in the presence of normal transactions costs.

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1. Appendix

In this appendix, we present the formal model that underlies the analysis in the main text.

1.1. Basic setup

Consider an investor who holds a portfolio of assets, P, and who must choose what fraction of wealth, w, to hedge. This investor is assumed to have preferences which are linear in the mean and variance of terminal wealth (which is T periods from the present). To keep this analysis simple, we also assume that the investor keeps w fixed over the entire investment period, and that other asset allocation decisions are predetermined with respect to movements in exchange rates.[1] Finally, we assume that the investor uses short-term forward contracts to adjust the degree of exchange-rate exposure in the portfolio.[2]

The investor wishes to choose w to maximize the utility of terminal wealth, which is here assumed to be a linear function of mean and variance:

$$\max_{w} E[r_P] - \frac{\gamma}{2}\text{Var}[r_P], \tag{1}$$

where r_P is the return on the total portfolio over the horizon, and γ is a measure of investor risk aversion. The return r_P, can be thought of as the return on the initial portfolio, call it r_I, minus the return on a currency hedge position, weighted by the size of the hedge:

$$r_P = r_I - w\epsilon, \tag{3}$$

where ϵ is the return on the hedge position (expressed as a percent of the notional value of the forward contract). Thus, we can write:

$$E[r_P] = \mu_I - w\mu_\epsilon - |w|\,c \tag{4}$$

$$\text{Var} = \sigma_I^2 - 2w\text{cov}[\epsilon, r_I] + w^2\sigma_\epsilon^2,$$

[1] The assumption that w remains fixed over time is not particularly applicable to an individual investor, who might wish to adjust w as his or her horizon changes. (However, such an assumption might be more appropriate for an long-lived institution, such as a pension fund.) Froot (1993) and Froot and Perold (1993) develop more complex models which allow long-horizon investors to choose hedge ratios frequently. Qualitatively, the results of these models are similar to the results above, provided that investor preferences are more risk averse than logrithmic.

[2] Short-term forward contract returns are equal to those on futures, provided that there are no costs of maintaining margins.

where μ_I and μ_ϵ are the expected return on the initial portfolio and hedge contract, respectively; σ_I^2 and σ_s^2 are the corresponding variances; and $|w|\,c$ represents the transactions costs involved in using hedges.

The first-order condition for the problem in (2), using (3) and (4), can be written:

$$w^* = \frac{-\mu_\epsilon - c}{\gamma\sigma_\epsilon^2} + E \quad \text{if} E > \frac{\mu_\epsilon + c}{\gamma\sigma_\epsilon^2} \tag{5a}$$

$$w^* = \frac{-\mu_\epsilon + c}{\gamma\sigma_\epsilon^2} + E \quad \text{if} E < \frac{\mu_\epsilon - c}{\gamma\sigma_\epsilon^2} \tag{5b}$$

$$w^* = 0 \quad \text{if} E \in \left[\frac{\mu_\epsilon - c}{\gamma\sigma_\epsilon^2}, \frac{\mu_\epsilon + c}{\gamma\sigma_\epsilon^2}\right], \tag{5c}$$

where w^* is the optimal hedge ratio, and $E \equiv \frac{\text{cov}[\epsilon, r_I]}{\sigma_\epsilon^2}$ is the exposure of the initial portfolio – i.e., the expected percentage change in the domestic-currency value of the initial portfolio for a one-percentage-point increase in the value of foreign exchange. In the language of the main text, E is the *preexisting exposure* of the portfolio and $\frac{\mu_\epsilon \pm c}{\gamma\sigma_\epsilon^2}$ is *target exposure*. Thus, equation (5) says that the optimal hedge ratio is just the difference between the preexisting and target exposures.

There are several noteworthy aspects of (5). First, it holds regardless of the composition and optimality of the other holdings in the portfolio. Second, the presence of positive expected returns on currencies ($\mu_\epsilon > 0$) tends to decrease the magnitude of the hedge, i.e., it increases the desired exposure to foreign exchange. Third, transactions costs push the optimal hedge toward zero. They also create a band of inaction in which no hedging occurs – for small disparities between the preexisting and target exposures, transactions costs make exposure adjustments too costly to undertake. Fourth, the smaller is the preexisting exposoure, E, the lower is the optimal hedge.

In order to examine the effect of horizon in (5), we need to make the model more precise. Let us define P_t and P_t^* as the log of the time-t domestic and foreign consumer price levels, respectively, and S_t as the log of the time-t nominal exchange rate (the domestic price of foreign currency). Let $\pi_t = P_t - P_{t-1}$ and $\pi_t^* = P_t^* - P_{t-1}^*$ be the corresponding domestic and foreign inflation rates. We then can define the real exchange rate as:

$$R_t \equiv S_t + P_t^* - P_t, \tag{6}$$

and the change in the real exchange rate as:

$$\Delta R_t \equiv \Delta S_t + \pi_t^* - \pi_t. \tag{6a}$$

The hedge return at time t is that of a short position in foreign exchange:

$$\epsilon_t = i_t - i_t^* - \Delta S_t, \tag{7}$$

where i_t and i_t^* are the domestic and foreign short-term interest rates, respectively. We assume that the hedge return, ϵ_t, is white noise around some mean, μ_ϵ. Thus, overtime the cumulative return on a currency position follows a random walk with drift. In this model, therefore, there are no opportunities to earn high risk-adjusted returns by trading foreign exchange.

Using equations (6) and (7), the change in the real exchange rate can be written as the *ex post* real interest differential less the return on the hedge:

$$\Delta R_t = (i_t - \pi_t) - (i_t^* - \pi_t^*) - \epsilon_t. \tag{8}$$

We also assume that the real exchange rate is mean reverting, i.e., that purchasing power parity (PPP) holds over long horizons. Over short horizons, however, we assume that changes in the nominal and real exchange rates are perfectly correlated. These two assumptions seem consistent with actual behavior: because at short horizons goods prices are "sticky," the relative cost of living is perfectly correlated with the change rate change, but, at long horizons when PPP holds, it becomes uncorrelated. Together these two assumptions suggest that the real exhange rate follows something like the mean-reverting process:

$$\Delta R_t = -\alpha(R_{t-1} - R^*) - \epsilon_t, \tag{9}$$

where R^* is a constant, and represents the PPP level of the real exchange rate.

What factors cause the real exchange rate to revert? From equation (8), it is clear that since the hedge return follows a random walk, the components of the real interest differential – interest rates and prices – must account for the reversion. A positive shock to ϵ_t tends to make the domestic real exchange rate overvalued (i.e., R falls below its long run value). There is then pressure on domestic consumer prices to grow more slowly and on foreign prices to grow more quickly. In addition, there is pressure on domestic interest rates to rise and on foreign interest rates to fall in order to compensate investors for the expected real depreciation of the currency. These pressures can be summarized by making future inflation and interest rates sensitive to the current deviation of the real exchange rate from PPP:[3]

$$\pi_t = \alpha_\pi(R_{t-1} - R^*) - \epsilon_t + \bar{\pi} \tag{10a}$$

$$\pi_t^* = \alpha_{\pi^*}(R_{t-1} - R^*) - \epsilon_t + \bar{\pi}^* \tag{10b}$$

$$i_t = \alpha_i(R_{t-1} - R^*) - \epsilon_t + \bar{i} \tag{10c}$$

$$i_t^* = \alpha_{i^*}(R_{t-1} - R^*) - \epsilon_t + \bar{i}^*, \tag{10d}$$

where $\bar{\pi}, \bar{\pi}^*, \bar{i}, \bar{i}^*$, are long-run inflation and interest rate levels, satisfying the restriction that long-run real interest rates are equal, i.e., that $\bar{i} - \bar{\pi} = \bar{i}^* - \bar{\pi}^*$. Combining equations (10) and (11) implies that $\alpha = \alpha_\pi + \alpha_{\pi^*} + \alpha_i + \alpha_{i^*}$ – the decay in the real exchange rate can be traced back to the decay rates of foreign and domestic prices and interest rates.

1.2. Multiperiod returns

Next we need to specify the real returns to domestic investors from different asset classes. We first specify the one-period (short-horizon) returns, and from these returns we then derive the implied many-period (long-horizon) returns.

1.2.1. domestic stocks

Consider domestic stocks first. Their return is composed of one component which is *iid* and another which is proportional to the mean-reverting real exchange rate change:

$$\begin{aligned} \Delta Y_t^{ds} &= \delta_t^{ds} + \beta \Delta R_t \\ &= \delta_t^{ds} - \beta\epsilon_t - \alpha\beta(R_{t-1} - R^*), \end{aligned} \tag{11}$$

where we have used equation (9) to get the last expression. If $\beta = 0$, equation (11) says that the real domestic return on stock is unaffected by changes in the real exchange rate. With $\beta > 0$, equation (11) says that a current one-percent

[3] The formulation in equation (10), which states that interest-rate and price differentials are linearly relative to the deviation of the real exchange rate from PPP, is derived rigorously in a celebrated paper by Dornbusch (1976).

appreciation of the domestic real exchange rate (i.e., a fall in R_t) leads to a β-percent fall in the contemporanous value of domestic stocks, but that their subsequent real value rises over time as the real currency overvaluation decays away. That is, over long periods, the real value of domestic stocks is unaffected by real exchange rate changes. In the short run, however, stock returns may be correlated with the contemporaneous exchange-rate change. (From the data presented in the main text, US stocks have $\beta \approx 0.1$.)

1.2.2. foreign stocks

Next consider foreign stocks. Once again, their return is comprised of an *iid* portion plus a portion related to the real exchange rate change:

$$\Delta Y_t^{fs} = \delta_t^{fs} + (1-\beta^*)\Delta R_t$$

$$= \delta_t^{fs} - (1-\beta^*)\epsilon_t - \alpha(1-\beta^*)(R_{t-1} - R^*). \tag{12}$$

With $\beta^* = 0$, (12) says that the local currecy value of foreign stocks is unrelated to current exchange rate shocks (i.e., realization of ϵ_t), so that the dollar value changes by the same percentage as the exchange rate change. So, for example, a current one-pecent real appreciation of the domestic currency makes foreign stocks on average on one-percent cheaper in the short run. In the long run, however, foreign stocks do not remain cheap. As above, we assume that the discount dies away with the deviation of the real exchange rate from PPP.

With $\beta^* > 0$, foreign stocks do not become instantly cheaper by the full amount of the depreciation of the foreign currency. Instead, foreign stocks *rise* in local-currency terms by β^* percent in response to a one-percent depreciation of the foreign currency. Thus in dollar terms, foreign stocks are $1-\beta^*$ percent cheaper in the short run. As before, in the long run, this underreaction in the real value of the stocks dies away with the deviation from PPP.

1.2.3. domestic bonds

Third, we consider domestic bonds. Their real returns are driven by interest rate changes less inflation:

$$\Delta Y_t^{db} = \delta_t^{db} - D(i_{t+1} - i_t) - \pi_t$$

$$= \delta_t^{db} - (1 - D\alpha_i)\epsilon_t - (\alpha\alpha_i D + \alpha_\pi)(R_{t-1} - R^*) - \bar{\pi}, \tag{13}$$

where D is a measure of bond duration relative to the investment horizon, and where equations (9) and (10) have been used to derive the last equation. Equation (13) says that real bond returns are negatively affected one-for-one by higher domestic inflation, as well as by increases in current interest rates.

1.2.4. foreign bonds

Finally, foreign bonds are analogous. Their real domestic returns are driven by foreign interest-rate changes and exchange-rate changes less domestic inflation:

$$\Delta Y_t^{fb} = \delta_t^{fb} - D(i_{t+1}^* - i_t^*) + \Delta S_t - \pi_t$$

$$= \delta_t^{db} - (1 - D\alpha_{i^*})\epsilon_t + (\alpha\alpha_{i^*} D + \alpha_{\pi^*} - \alpha)(R_{t-1} - R^*) - \bar{\pi}^*, \tag{14}$$

where equations (9) and (10) have been used to derive the last equality.

1.3. Derivation of asset exposures at different horizons

Equations (11)-(14) all share the same basic form for single-period returns:

$$\Delta Y_t = \delta_t - A\epsilon_t + B(R_{t-1} - R^*), \tag{15}$$

where only the parameters A and B differ. If we hedge an amount w against currency fluctuations, this return becomes:

$$\Delta Y_t^H = \delta_t - (A - w)\epsilon_t + B(R_{t-1} - R^*). \tag{16}$$

The T-period return can then be shown to be:

$$\Delta^T Y_0^H = \sum_{t=1}^{T} \Delta Y_t^H$$

$$= \sum_{t=1}^{T} \delta_t - \sum_{t=1}^{T} \Big(A - w + B\Big(\frac{1 - \lambda^{T-t}}{1 - \lambda}\Big)\Big)\epsilon_t + B\Big(\frac{1 - \lambda^T}{1 - \lambda}\Big)(R_0 - R^*), \tag{17}$$

where $\lambda = 1 - \alpha$.

Equation (17) allows computation of the exposure of different asset classes at different horizons. We denote these exposures by

$$E_{a,T} = \frac{\text{cov}[\sum_{t=1}^{T} \epsilon_t, \sum_{t=1}^{T} r_{a,t}]}{T\sigma_\epsilon^2}, \tag{18}$$

where $r_{a,t}$ is the time-t return on any of the above asset classes. Equations (17) and (18) yield the result that

$$E_{a,T} = A + \frac{B}{\alpha}\Big(1 - \frac{1-\lambda^T}{\alpha T}\Big), \tag{19}$$

where the term $\left(1 - \frac{1-\lambda^T}{\alpha T}\right)$ is the horizon adjustment factor discussed in the main text.

The optimal hedge ratio at long horizons thus uses equation (19) in place of E in equation (5). That ratio is used to derive Tables 1 and 2 in the main text.[4]

[4] To compute the long-horizon exposures reported in the text, we used the following parameter values: $\alpha = 0.16$ (this corresponds to a half-life of PPP deviations of 4 years), $\alpha_i = 0.06$, $\alpha_{i^*} = -0.025$ (these reflect average responses of interest rates to PPP deviations), $\alpha_\pi = 0.06$, $\alpha_{\pi^*} = 0.06$, $D = 4$, $\beta = 0.1$, $\beta^* = 0.05$ (these latter two are estimates from high-frequency data of US and foreign stock returns).

Figure 1
Implied Risk Tolerance*

*Assumes historical volatility and correlation (1984-1993)
Assumes expected excess returns on all stocks = 6% p.a.;
expected excess returns on all bonds = 2% p.a.

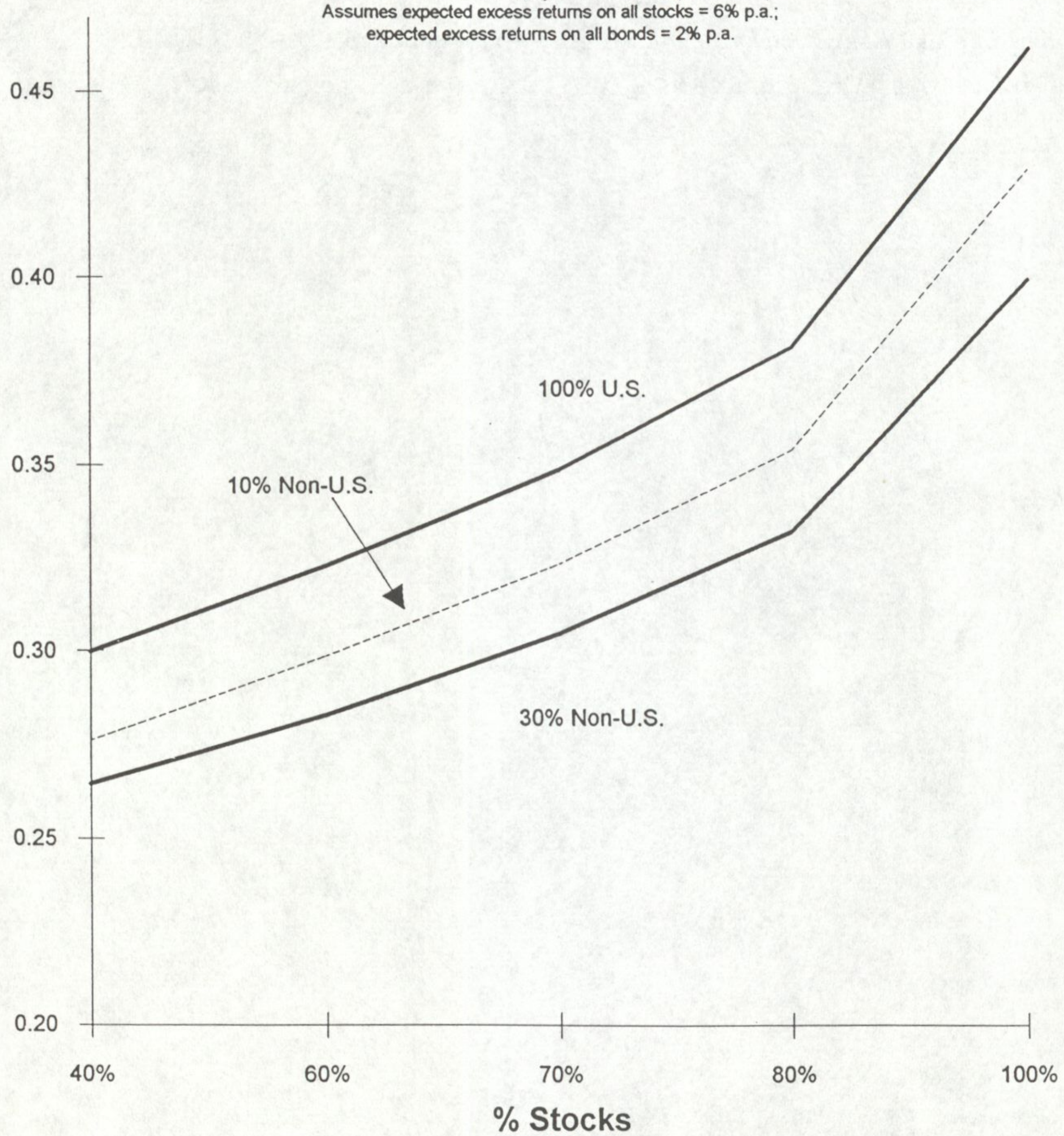

Figure 2
Target Exposure

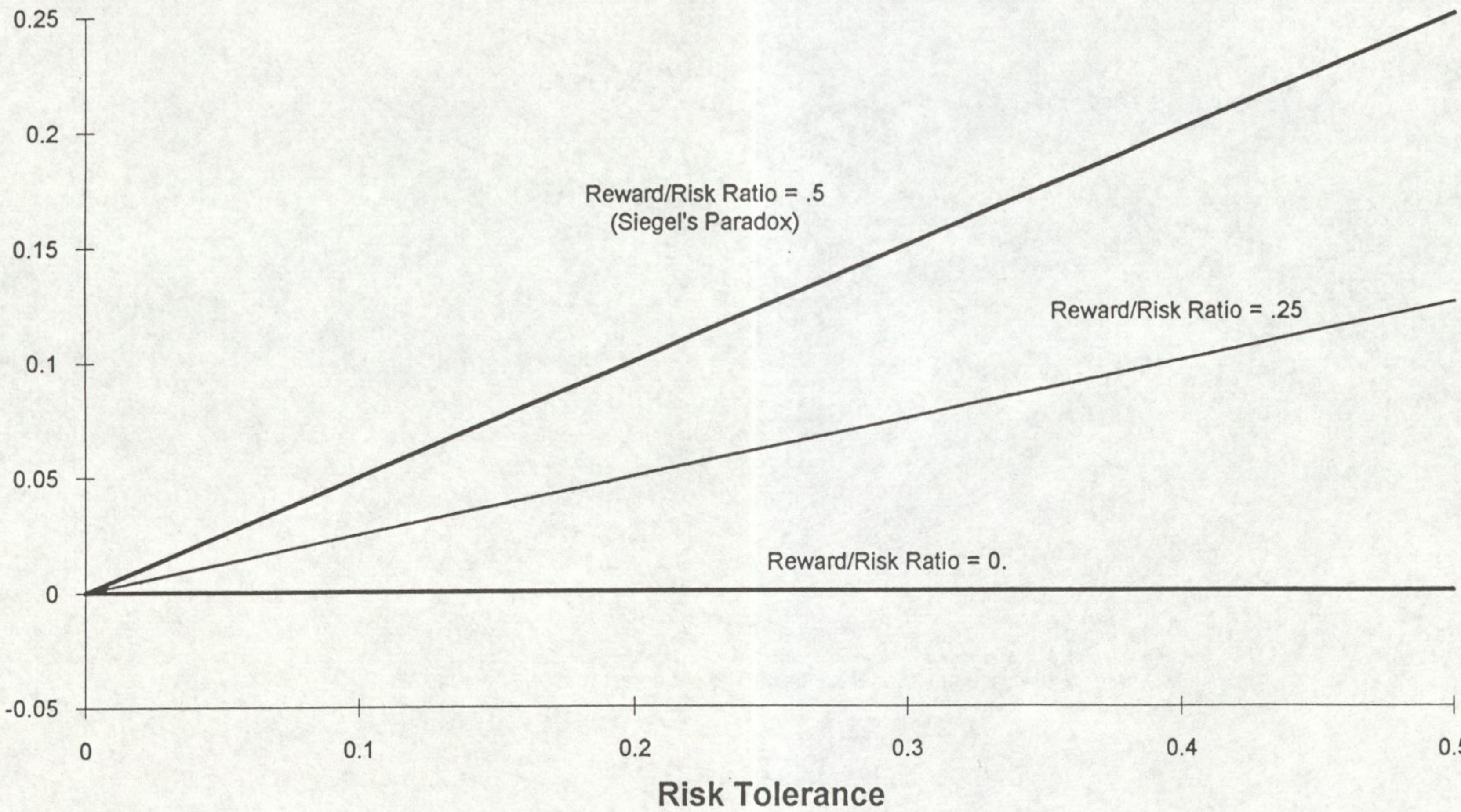

Figure 3
Transactions Cost Band

(Assumes Standard Deviation of Currency Movements = 10% p.a.)

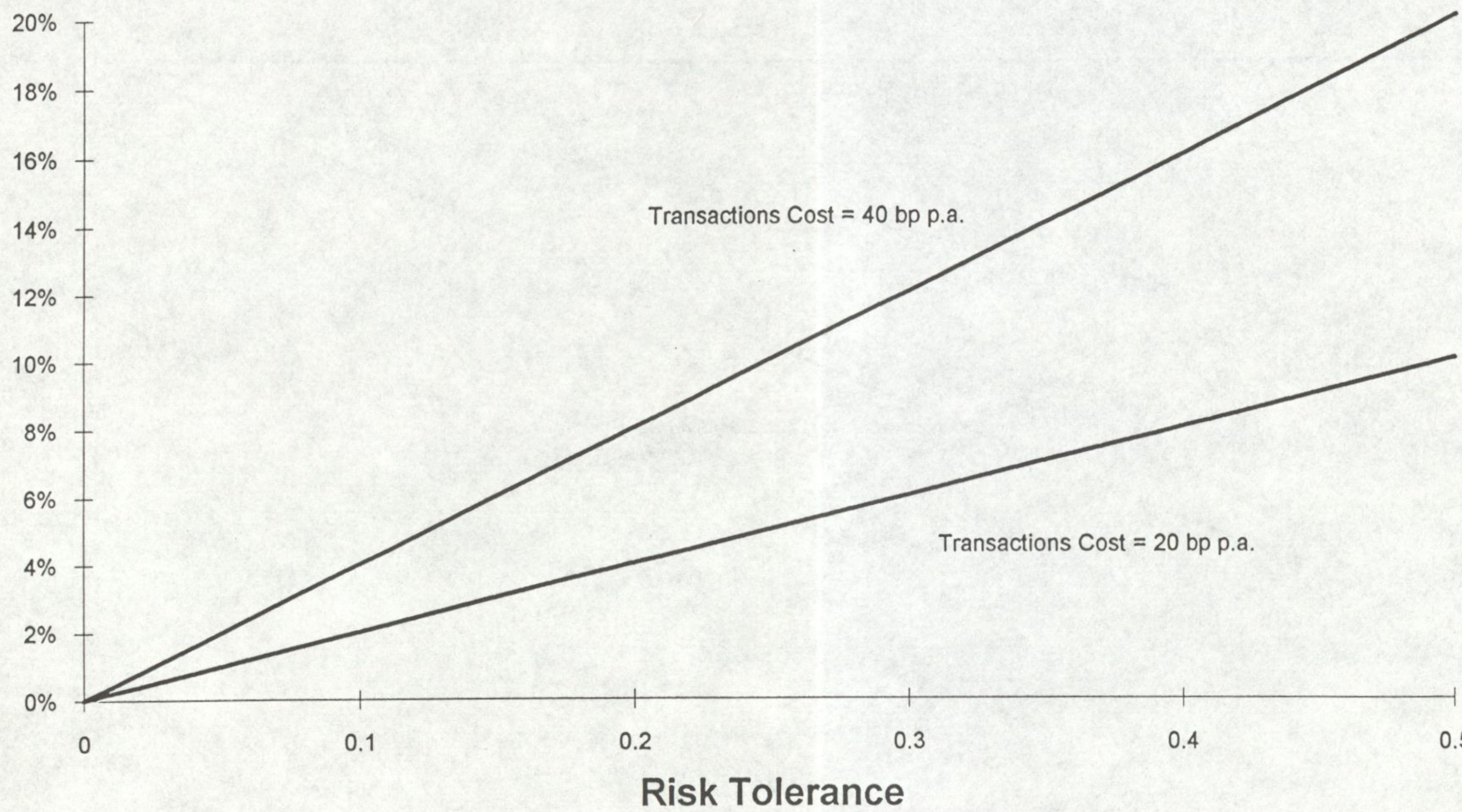

Figure 4
Currency exposure of US stocks and bonds

(Vs equal weighted basket of DM, Yen, and Pound;
Trailing 36 months exposure)

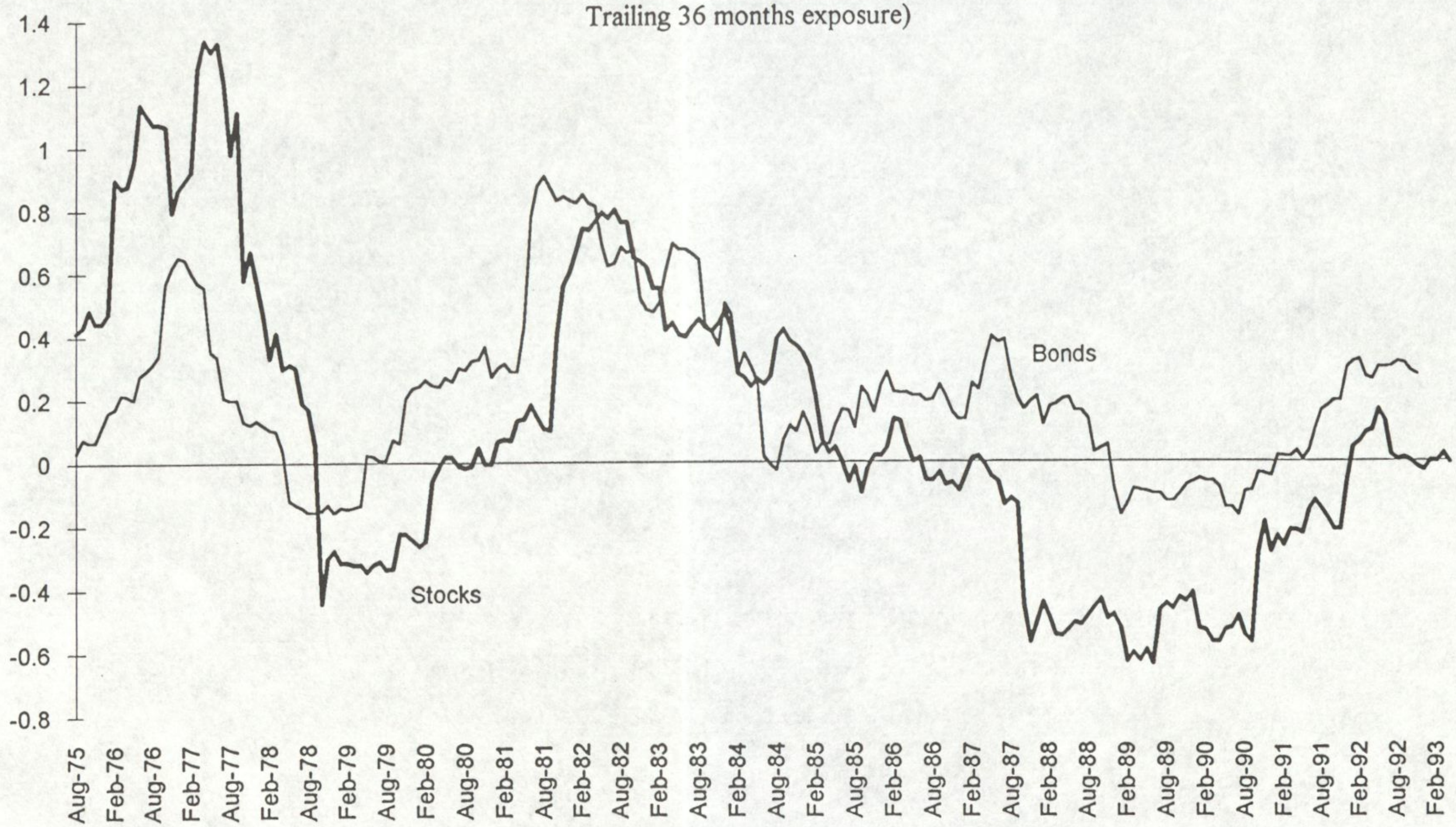

Figure 5
Foreign Stock Market Exposures

(Exposure to own currency; trailing 36 months exposures)

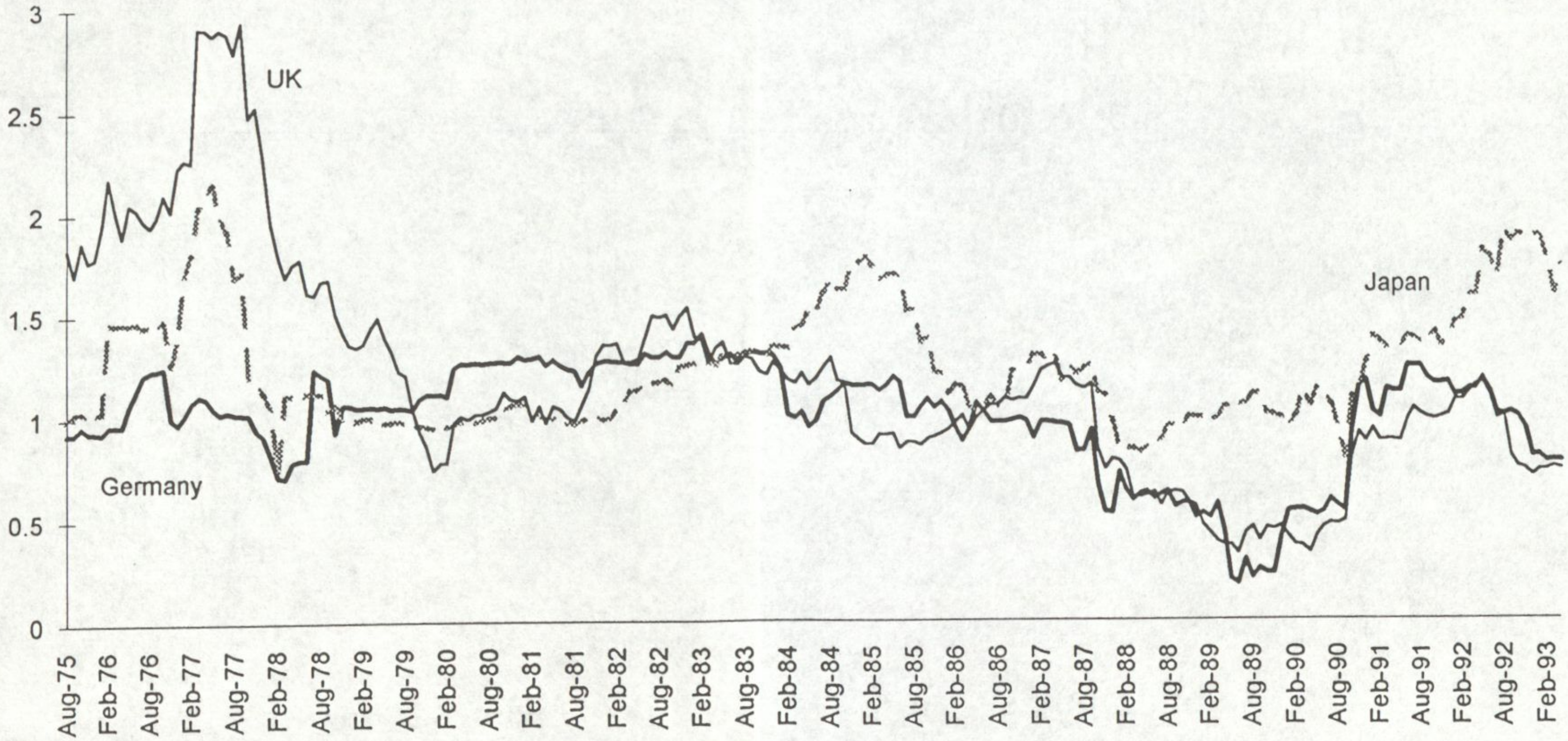

Figure 6
Foreign Bond Market Exposures

(Exposure to own currency; trailing 36 months exposures)

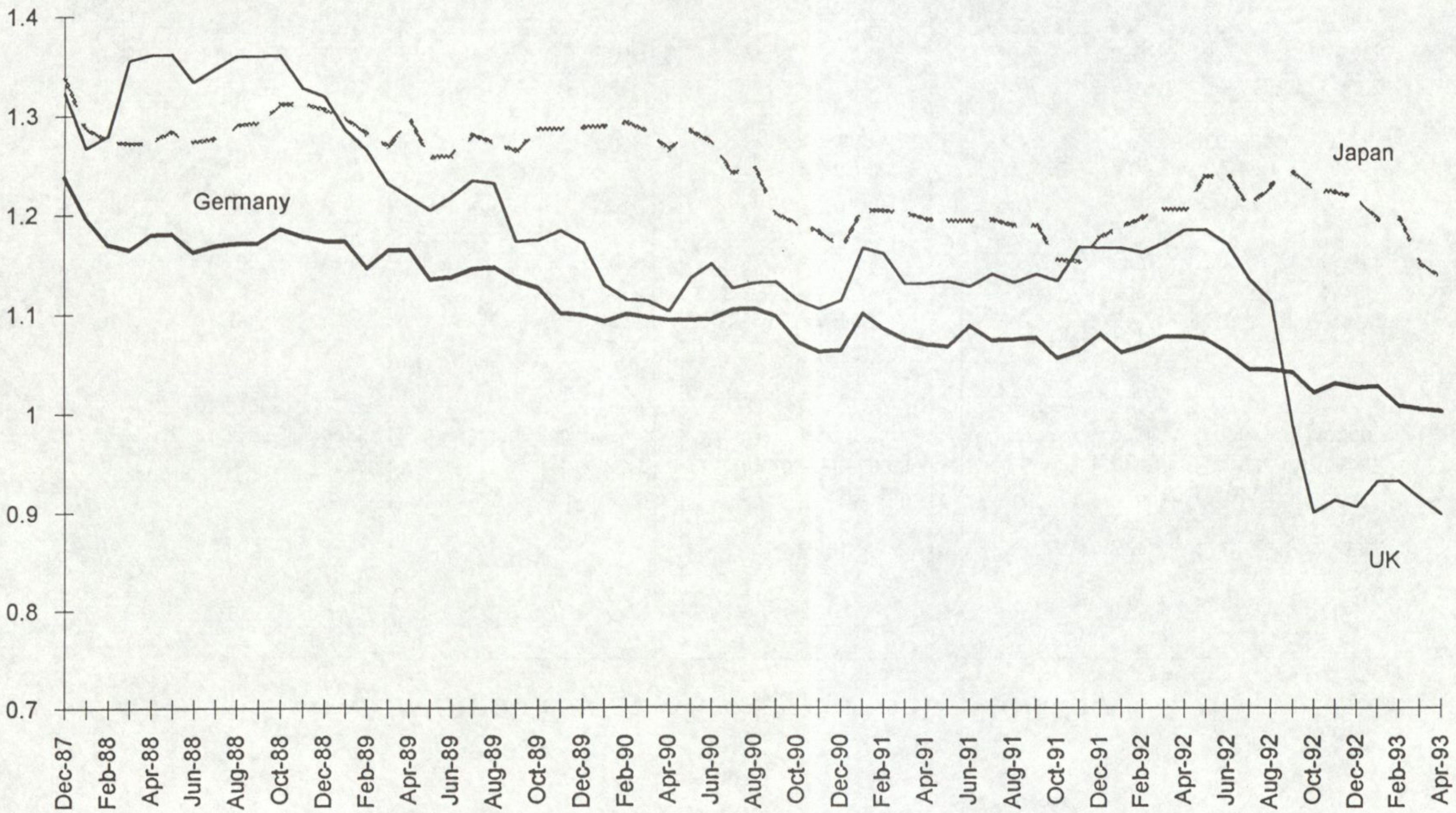

Table 1
Optimal Hedge Ratios: 1 Month Horizon

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
							No Transaction Costs		30 bp Transactions Costs				
										Adjusted Target Exposure Range			
% Stocks	% Foreign	Risk Tolerance	Target Exposure .5 * (3)	Preexisting Exposure U.S.	Int'l	Total	Amount to Hedge (7) - (4)	% of Int'l Holdings to hedge (7) / (2)	Trans Cost Band	Lower (4) - (10)	Upper (4) + (10)	Amount to Hedge 7) v (11) or (12	% of Int'l Holdings to hedge (13) / (2)
40%	30%	0.25	13%	13%	31%	44%	32%	105%	8%	5%	20%	24%	77%
70%	30%	0.25	13%	10%	30%	40%	27%	91%	8%	5%	20%	20%	66%
100%	30%	0.25	13%	7%	29%	36%	23%	77%	8%	5%	20%	16%	54%
40%	10%	0.25	13%	17%	10%	27%	14%	145%	8%	5%	20%	7%	67%
70%	10%	0.25	13%	13%	10%	23%	10%	102%	8%	5%	20%	3%	27%
100%	10%	0.25	13%	9%	10%	19%	6%	60%	8%	5%	20%	0%	0%
40%	30%	0.40	20%	13%	31%	44%	24%	80%	12%	8%	32%	12%	39%
70%	30%	0.40	20%	10%	30%	40%	20%	66%	12%	8%	32%	8%	26%
100%	30%	0.40	20%	7%	29%	36%	16%	52%	12%	8%	32%	4%	12%
40%	10%	0.40	20%	17%	10%	27%	7%	70%	12%	8%	32%	0%	0%
70%	10%	0.40	20%	13%	10%	23%	3%	27%	12%	8%	32%	0%	0%
100%	10%	0.40	20%	9%	10%	19%	-2%	-15%	12%	8%	32%	0%	0%

Table 2
Optimal Hedge Ratios: 5 Year Horizon

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
							No Transaction Costs		30 bp Transactions Costs				
										Adjusted Target Exposure Range			
% Stocks	% Foreign	Risk Tolerance	Target Exposure .5 * (3)	Preexisting Exposure U.S.	Int'l	Total	Amount to Hedge (7) - (4)	% of Int'l Holdings to hedge (7) / (2)	Trans Cost Band	Lower (4) - (10)	Upper (4) + (10)	Amount to Hedge 7) v (11) or (12	% of Int'l Holdings to hedge (13) / (2)
40%	30%	0.25	13%	4%	23%	27%	15%	49%	8%	5%	20%	7%	30%
70%	30%	0.25	13%	4%	21%	26%	13%	44%	8%	5%	20%	6%	27%
100%	30%	0.25	13%	5%	20%	24%	12%	40%	8%	5%	20%	4%	23%
40%	10%	0.25	13%	5%	8%	13%	0%	0%	8%	5%	20%	0%	0%
70%	10%	0.25	13%	5%	7%	13%	0%	2%	8%	5%	20%	0%	0%
100%	10%	0.25	13%	6%	7%	13%	0%	3%	8%	5%	20%	0%	0%
40%	30%	0.40	20%	4%	23%	27%	7%	24%	12%	8%	32%	0%	0%
70%	30%	0.40	20%	4%	21%	26%	6%	19%	12%	8%	32%	0%	0%
100%	30%	0.40	20%	5%	20%	24%	4%	15%	12%	8%	32%	0%	0%
40%	10%	0.40	20%	5%	8%	13%	-8%	-75%	12%	8%	32%	0%	0%
70%	10%	0.40	20%	5%	7%	13%	-7%	-74%	12%	8%	32%	0%	0%
100%	10%	0.40	20%	6%	7%	13%	-7%	-72%	12%	8%	32%	0%	0%