Understanding the US Dollar in the Eighties: The Expectations of Chartists and Fundamentalists*

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I Introduction

There is general agreement that the large appreciation of the dollar from 1980 to February 1985 was attributable to an increase in the demand for dollar assets relative to other currencies. The only alternative hypothesis would be a decrease in the supply of dollar assets relative to other currencies. Given the rapid rate of increase in the supply of US assets as measured by the Federal government deficit, and the related current account deficit, it does not seem that asset supplies moved the right way to explain the rise in the value of the dollar.

There is less agreement about why investors suddenly found dollar assets so much more attractive after 1980. One set of explanations based on fundamentals is that there was an increase in the expected rate of return on dollar assets — a decline in the US expected inflation rate, increase in the interest rate, or combination of the two — relative to other countries' assets.

A second set of explanations would attribute the increased demand for dollar assets to a self-confirming increase in expected future appreciation of the dollar (or decrease in expected future

depreciation). By self-confirming we mean that the change in expectations is not driven by fundamentals; the dollar may have been on a rational speculative bubble path.

A third set of explanations is that there was an increase in the perceived safety of US assets relative to other countries. This is the so-called safe-haven explanation.

We have concluded elsewhere (for example, Frankel and Froot, 1986b) that the increase in the real interest differentials between the US and its trading partners readily explains most of the 1981–84 appreciation of the dollar, but that this standard explanation misses some of the dynamics, particularly the last 20 per cent or so of the appreciation up to February 1985.[1] We have also argued that the alternatives which have been proposed, the safe-haven and rational speculative bubble hypotheses, are even less capable of explaining how the appreciation could have persisted for four years. It thus seems that a new theory is called for.

In this paper we propose the outlines of a model of a speculative bubble that is not constrained by the assumption of rational expectations. The model features three classes of actors: fundamentalists, chartists and portfolio managers. None of the three acts utterly irrationally, in the sense that each performs the specific task assigned him in a reasonable, realistic way. Fundamentalists think of the exchange rate according to a model — say, the Dornbusch overshooting model for the sake of concreteness — that would be exactly correct if there were no chartists in the world. Chartists do not

[1] Calculated using the Federal Reserve's multilateral index of the value of the dollar, from the end of the first quarter, 1984, until February 1985.

have fundamentals such as the long-run equilibrium rate in their information set; instead they use autoregressive models — say, simple extrapolation for the sake of concreteness — that have only the time series of the exchange rate itself in the information set. Finally portfolio managers, the actors who actually buy and sell foreign assets, form their expectations as a weighted average of the predictions of the fundamentalists and chartists. The portfolio managers update the weights over time in a rational Bayesian manner, according to whether the fundamentalists or the chartists have recently been doing a better job of forecasting. Thus each of the three is acting rationally subject to certain constraints. Yet the model departs from the reigning orthodoxy in that the agents could do better, in expected value terms, if they knew the complete model. When the bubble takes off, agents are irrational in the sense that they learn about the model more slowly than they change it. Furthermore, the model may be unstable in the neighbourhood of the fundamentals equilibrium, but stable around a value for the dollar that is far from that equilibrium.

II Fundamentalists and Chartists

In Frankel and Froot (1986b) we presented evidence supporting the following five propositions, each with elements of paradox.

(1) The dollar continued to rise even after all fundamentals (the interest differential, current account, etc.) apparently began moving the wrong way. The only explanation left would seem to be, almost tautologically, that investors were responding to a rising expected rate of change in the value of the dollar. In other words, the dollar was on a bubble path.

(2) Evidence suggests that the investor-expected rate of depreciation reflected in the forward discount is not equal to the rationally expected rate of depreciation. The failure of a fall in the dollar to materialize in four years implies that the rationally expected rate of depreciation was less than the forward discount.

(3) On the other hand, current account calculations in the spirit of Krugman (1985) and Marris (1985) suggested that the rationally expected rate of depreciation was greater than the current forward discount.

(4) Data from two surveys conducted by the *Economist*'s Financial Report and the *American Express Bank Review* show that the respondents have since 1981 indeed held an expected rate of

depreciation substantially greater than the forward discount.[2] But interpreting their responses as true investor expectations, and interpreting the excess over the forward premium as a negative risk premium, raises several problems. First, if investors seriously expected the dollar to depreciate so fast, why did they buy dollars? Second, the theory of exchange risk says that the risk premium should generally be small and, for the dollar in the 1980s, that it probably has moved in the positive direction.

(5) In the safe-haven theory, a perceived shift in country risk rather than exchange risk might seem to explain many of the foregoing paradoxes. However, the covered differential between European and US interest rates actually fell after 1982 suggesting that perceptions of country risk, if anything, shifted against the United States.

The model of fundamentalists and chartists that we are proposing has been designed to reconcile these conflicting conclusions. To begin with, we hypothesize that the views represented in the American Express and *Economist* six-month surveys are primarily fundamentalist, like the views of Krugman and Marris (and most other economists). But it may be wrong to assume that investors' expectations are necessarily the ones reported in the six-month surveys or that they are even homogeneous (as most of our models do). Expectations are heterogeneous. Our model suggests that the market gives heavy weight to the chartists, whose expected rate of change in the value of the dollar has been on average much closer to zero, perhaps even positive. Paradox (4) is answered if fundamentalists' expectations are not the only ones determining positions that investors take in the market.

The increasing dollar over-valuation after the interest differential peaked in 1982 or 1984 would be explained by a falling market-expected rate of future depreciation (or rising expected rate of appreciation), with no necessary basis in fundamentals. The market-expected rate of depreciation declined over time, not necessarily because of any change in the expectations held by chartists or fundamentalists, but rather because of a shift in the weights assigned to the two by the

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[2] The Money Market Services survey has been conducted weekly or bi-weekly since 1983. The *Economist* survey covers 13 leading international banks and has been conducted six times a year since 1981. The American Express survey covers 250 to 300 central bankers, private bankers, corporate treasurers and economists, and has been conducted more irregularly since 1976.

The Term Structure of Expected Depreciation and Prediction Errors
(in per cent per annum)

Forecast Horizon	Survey Source	Dates	N	Expected Depreciation $E[s(t+1)]-s(t)$			Survey Prediction Error $E[s(t+1)]-s(t+1)$			Forward Discount $f(t)-s(t)$			Forward Rate Prediction Error $f(t)-s(t+1)$		
				Mean	SD of Mean	t stat	Mean	SD of Mean	t stat	Mean	SD of Mean	t stat	Mean	SD of Mean	t stat
1 week															
Total	MMS	10/84–9/85	183	−8.13	4.23	−1.92	−23.36	10.55	−2.21	NA			NA		
UK			46	−24.26	9.08	−2.67	−41.13	25.70	−1.60						
WG			46	−3.81	7.58	−0.50	−19.40	20.23	−0.96						
SW			45	−0.75	9.96	−0.07	−13.88	23.58	−0.59						
JA			46	−3.53	6.74	−0.52	−18.81	13.41	−1.40						
2 Weeks															
Total	MMS	1/83–10/84	187	4.22	1.26	3.36	16.57	3.38	4.91	NA			NA		
UK			47	−2.66	2.48	−1.07	13.49	6.77	1.99						
WG			47	5.08	2.59	1.96	20.28	7.51	2.70						
SW			46	6.09	2.54	2.40	19.95	6.49	3.07						
JA			47	8.40	2.21	3.79	12.63	6.32	2.00						
1 Month															
Total	MMS	10/84–9/85	48	−10.25	2.88	−3.56	−23.80	7.76	−3.07	−0.16	0.87	−0.18	−13.71	7.49	−1.83
UK			12	−25.06	6.07	−4.13	−38.95	20.29	−1.92	−5.63	2.09	−2.69	−19.52	19.52	−1.00
WG			12	−6.91	6.10	−1.13	−19.91	14.82	−1.34	1.56	1.47	1.06	−11.44	13.93	−.082
SW			12	−6.38	5.15	−1.24	−19.59	17.09	−1.15	1.99	1.52	1.31	−11.22	16.76	−0.67
JA			12	−2.66	3.65	−0.73	−16.76	8.84	−1.90	1.45	0.77	1.87	−12.65	9.95	−1.27
3 Months															
Total	MMS	1/83–10/84	187	7.76	0.38	20.53	18.53	2.89	15.52	3.75	0.17	21.52	14.51	2.87	5.07
UK			47	4.46	0.55	8.08	18.38	5.98	3.07	0.37	0.19	1.96	14.29	5.97	2.40
WG			47	8.33	0.62	13.45	22.01	5.96	3.69	4.68	0.14	34.37	18.36	6.05	3.03
SW			47	9.62	1.01	9.51	22.23	5.26	4.23	6.13	0.14	43.69	18.74	4.91	3.82
JA			47	8.68	0.54	15.95	11.58	5.20	2.23	3.85	0.16	23.77	6.75	5.02	1.35
Total	*Economist*	6/81–8/85	165	10.03	0.63	15.80	15.54	2.75	5.64	2.47	0.36	6.82	7.98	2.59	3.09
UK			33	5.16	1.34	3.84	14.38	6.80	2.11	0.47	0.36	1.34	9.69	6.07	1.60
FR			33	5.57	1.08	5.17	15.88	5.80	2.74	−4.23	0.70	−6.07	6.08	5.42	1.12
WG			33	12.79	1.17	10.91	17.19	5.86	2.93	4.54	0.21	21.26	8.94	5.41	1.65
SW			33	12.96	1.11	11.68	16.16	6.95	2.33	6.39	0.33	19.30	9.58	6.63	1.45
JA			33	13.67	1.51	9.04	14.10	5.16	2.73	5.16	0.46	11.25	5.59	5.17	1.08
6 Months															
Total	*Economist*	6/81–8/85	165	9.88	0.45	21.76	18.07	2.27	7.30	2.47	0.35	7.16	10.24	2.44	4.20
UK			33	5.03	0.74	6.81	16.82	5.52	3.05	0.64	0.29	2.20	11.99	5.38	2.23
FR			33	4.77	0.80	5.95	17.98	4.97	3.61	−4.30	0.57	−7.54	8.45	4.84	1.75
WG			33	12.97	0.68	19.17	20.33	5.20	3.91	4.52	0.19	24.22	11.35	5.06	2.24
SW			33	12.77	0.70	18.12	19.35	6.58	2.94	6.27	0.28	22.65	12.47	6.26	1.99
JA			33	13.86	0.69	19.98	15.86	5.07	3.13	5.21	0.42	12.51	6.93	5.32	1.30
Total	AMEX	7/81–8/84	20	8.03	0.90	8.88	13.01	3.34	3.89	3.89	0.93	4.19	8.88	3.13	2.84

Forecast Horizon	Survey Source	Dates	N	Expected Depreciation $E[s(t+1)] - s(t)$			Survey Prediction Error $E[s(t+1)] - s(t+1)$			Forward Discount $f(t) - s(t)$			Forward Rate Prediction Error $f(t) - s(t+1)$		
				Mean	SD of Mean	t stat	Mean	SD of Mean	t stat	Mean	SD of Mean	t stat	Mean	SD of Mean	t stat
12 Months															
Total	*Economist* 6/81–8/85		165	8.29	0.35	23.57	17.76	1.76	10.07	2.54	0.31	8.08	12.12	1.92	6.33
UK			33	4.05	0.56	7.22	16.72	4.08	4.10	0.78	0.22	3.49	13.81	4.21	3.28
FR			33	3.92	0.53	7.34	18.41	2.29	8.04	−3.83	0.43	−8.86	10.66	2.98	3.57
WG			33	11.25	0.39	29.03	19.90	3.29	6.04	4.38	0.15	28.93	13.02	3.32	3.92
SW			33	10.91	0.43	25.49	19.67	4.07	4.84	6.27	0.22	29.05	15.41	4.13	3.73
JA			33	11.31	0.53	21.18	14.10	4.59	3.07	5.08	0.34	15.05	7.70	5.04	1.53

Note: Expectations are for four currencies against the dollar: UK = British Pound; WG = German Mark; SW = Swiss Franc; JA = Japanese Yen.

portfolio managers, who are the agents who take positions in the market and determine the exchange rate. They gradually put less and less weight on the big-depreciation forecasts of the fundamentalists, as these forecasts continue to be proven false, and more and more weight on the chartists.

Before we proceed to show how such a model works, we offer one piece of evidence that there is not a single homogeneous expected rate of depreciation reflected in the survey data: the very short-term expectations (one-week and two-week) reported in a third survey of market participants, by Money Market Services, Inc., behave very differently from the medium-term expectations (three, six or 12 month) reported in any of the three surveys.[3]

Table 1 shows expected depreciation (from all three surveys) at a variety of time horizons. Perhaps most striking is a large fall in the standard deviation of the mean as the forecast horizon increases. At the short end of the spectrum, none of the means from the one-week forecasts is significantly different from zero at the 1 per cent level, and the standard deviations are large, ranging from 4.2 per cent to 9.1 per cent.[4] At the other extreme, the one-year forecast horizon, all of the means are highly significant with t statistics approaching 30, and the standard deviations are below 0.6 per cent. The intermediate horizons conform to this pattern of decline.

A second striking fact is that the one-week and one-month surveys, which were conducted only for 10/84 to 9/85, indicate that respondents on average expected the dollar to appreciate, often at a rapid annual rate. During the comparable period for which 12-month forecasts are available (1/85 to 4/84), expected depreciation was still large and positive at 7.32 per cent as well as significant ($t = 8.29$). For more on the different behaviour of long-term and short-term expectations in the survey data, see Frankel and Froot (1986c).

These two facts suggest that there are far more consistent views about the value of the dollar in the longer run than in the shorter run; while short-

[3] For more extensive analyses of the Money Market Services survey data set, see Dominguez (1986), Frankel and Froot (1986a), and Froot and Frankel (1986).

[4] For all currencies combined, the standard deviation of the means treat the value of each currency against the dollar as independent. To the extent that all the forecasts contain a common dollar component, these aggregate standard deviations are biased downward, so that the corresponding t statistics are overstated.

run expectations may predict appreciation or depreciation at different times, longer run forecasts consistently call for substantial depreciation. It is as if there are actually two models of the dollar operating, one at each end of the spectrum, and a blend in between. The fundamentalist model, for which we specify a Dornbusch overshooting model, can be identified with the longer run expectations. The chartist model, a simple ARIMA forecasting equation such as a random walk, might be identified with the shorter run. Under this view, respondents use some weighted average of the two models in formulating their expectations for the value of the dollar at a given future date, with the weights depending on how far off that date is.

These results suggest an alternative interpretation of how chartist and fundamentalist views are aggregated in the marketplace, an aggregation that takes place without the benefit of portfolio managers. It is possible that the chartists are simply people who tend to think short term and the fundamentalists are people who tend to think long term. For example, the former may by profession be 'traders', people who buy and sell foreign exchange on a short-term basis and have evolved different ways of thinking than the latter, who may by profession buy and hold longer term securities.[5]

In any case, one could interpret the two groups as taking positions in the market directly, rather than merely issuing forecasts for the portfolio managers to read. The market price of foreign exchange would then be determined by demand coming from both groups. But the weights that the market gives to the two change over time, according to the groups' respective wealths.[6] If the fundamentalists sell the dollar short and keep losing money, while the chartists go long and keep gaining, in the long run the fundamentalists will go bankrupt and there will only be chartists in the marketplace. The model that we develop in the next section pursues the portfolio managers' decision-making problem instead of the marketplace-aggregation idea, but the two are similar in spirit.

Yet another possible interpretation of the survey data is that the two ways of thinking represent conflicting forces within the mind of a single

[5] It sounds strange to describe three to six months as 'long-term', but such descriptions are common in the foreign exchange markets.

[6] Figlewski (1978, 1982) considers an economy in which private information, weighted by traders' relative wealth, is revealed in the market price.

representative agent. When respondents answer the longer term surveys, they give the views that their economic logic tells them are correct. When they get into the trading room, they give greater weight to their instincts, especially if past bets based on their economic logic have been followed by ruinous 'negative reinforcement'. A respondent may think that when the dollar begins its plunge, he or she will be able to get out before everyone else does. This opposing instinctual force comes out in the survey only when the question pertains to the very short term — one or two weeks; it would be too big a contradiction for his conscience if a respondent were to report a one-week expectation of dollar depreciation that was (proportionately) just as big as the answer to the six-month question, at the same time that he or she was taking a long position in dollars. Again, we prefer the interpretation where the survey reflects the true expectations of the respondent, and the market trading is done by some higher authority; but others may prefer the more complex psychological interpretation.

The fragments of empirical evidence in Table 1 are the only ones we will offer by way of testing our approach. The aim in what follows is to construct a model that reconciles the apparent contradictions discussed above. There will be no hypothesis testing.

We think of the value of the dollar as being driven by the decisions of portfolio managers who use a weighted average of the expectations of fundamentalists and chartists. Specifically.

$$\Delta s_{t+1}^m = \omega_t \, \Delta s_{t+1}^f + (1 - \omega_t) \, \Delta s_{t+1}^c \qquad (1)$$

where Δs_{t+1}^m is the rate of change in the spot rate expected by the portfolio managers, Δs_{t+1}^f and Δs_{t+1}^c are defined similarly for the fundamentalists and chartists, and ω_t is the weight given to fundamentalist views. For simplicity we assume $\Delta s_{t+1}^c = 0$. Thus equation (1) becomes

$$\Delta s_{t+1}^m = \omega_t \Delta s_{t+1}^f \qquad (2)$$

or

$$\omega_t = \Delta s_{t+1}^m / \Delta s_{t+1}^f.$$

If we take the six-month forward discount to be representative of portfolio managers' expectations and the six-month survey to be representative of fundamentalists' expectations we can get a rough idea of how the weight, ω_t, varies over time.

Table 2 contains estimates of ω_t from the late 1970s to 1985. (There are, unfortunately, no survey data for 1980.) The table indicates a preponderance of fundamentalism in the late seventies; portfolio

TABLE 2

*Estimated Weight Given to Fundamentalists
by Portfolio Managers*

	Year					
	1976–79	1981	1982	1983	1984	1985
Forward Discount (1)	1.06	3.74	3.01	1.10	3.07	− 0.16
Survey expected Depreciation (2)	1.20	8.90	10.31	10.42	11.66	4.00
ω (1)/(2)	0.88	0.42	0.29	0.11	0.26	− 0.04

Note: Forward discount 1976–85 is at six months and includes data through September 1985 for the average of five currencies, the pound, French franc, mark, Swiss franc and yen. Survey expected depreciation 1981–85 is from the *Economist* six-month survey data, and for 1976–79 is from the AMEX survey data for the same five currencies.

managers gave almost complete weight to this view. But beginning in 1981, as the dollar began to rise, the forward discount increased less rapidly than fundamentalists' expected depreciation, indicating that the market (or the portfolio managers in our story) was beginning to pay less attention to the fundamentalists' view. By 1985 the market's expected depreciation had fallen to about zero. According to these computations, fundamentalists were being completely ignored.

While the above scenario solves the paradox posed in proposition (4), it leaves unanswered the question of how the weight ω_t, which appears to have fallen dramatically since the late 1970s, is determined by portfolio managers. Furthermore, if portfolio managers have small risk premia, and thus expect depreciation at a rate close to that predicted by the forward discount, we still must account for the spectacular rise of the dollar (proposition (1)), and resolve how the rationally expected depreciation differs from the forward discount (propositions (2) and (3)).

III Portfolio Managers and Exchange Rate Dynamics

Up to this point we have characterized the chartist and fundamentalist views of the world, and hinted at the approximate mix that portfolio managers would need to use if the market risk premium is to be near zero. We now turn to an examination of the behaviour of portfolio managers, and of the determination of the equilibrium spot rate. In particular, we first focus exclusively on the dynamics of the spot rate which are generated by the changing expectations of portfolio managers. We then extend the framework

to include the evolution of fundamentals which eventually must bring the dollar back down.

A general model of exchange rate determination can be written

$$s_t = c \, \Delta s_{t+1}^m + z_t \qquad (3)$$

where $s(t)$ is the log of the spot rate, Δs_{t+1}^m is the rate of depreciation expected by 'the market' (portfolio managers) and z_t represents other contemporaneous determinants. This very general formulation, in which the first term can be thought of as speculative factors and the second as fundamentals, has been used by Mussa (1976) and Kohlhagen (1979). An easy way to interpret equation (3) is in terms of the monetary model of Mussa (1976), Frenkel (1976) and Bilson (1978). Then c would be interpreted as the semi-elasticity of money demand with respect to the alternative rate of return (which could be the interest differential, expected depreciation or expected inflation differential; the three are equal if uncovered interest parity and purchasing power parity hold), and z_t would be interpreted as the log of the domestic money supply relative to the foreign (minus the log of relative income, or any other determinants of real money demand). An interpretation of equation (3) in terms of the portfolio-balance approach is slightly more awkward because of nonlinearity. But we could define

$$z_t = d_t - f_t - c(i_t - i_t^*) \qquad (4)$$

where d_t is the log of the supply of domestic assets (including not only money but also bonds and other assets), f_t is the log of the supply of foreign assets, and $i_t - i_t^*$ is the nominal interest differential. Then equation (3) can be derived as a linear approximation to the solution for the spot rate in

a system where the share of the portfolio allocated to foreign assets depends on the expected return differential or risk premium, $i_t - i_t^* - \Delta s_{t+1}^e$. If investors diversify their portfolios optimally, c can be seen to depend inversely on the variance of the exchange rate and the coefficient of relative risk-aversion.[7] In any case, the key point behind equation (3), common throughout the asset-market view of exchange rates, is that an increase in the expected rate of future depreciation will reduce demand for the currency today, and therefore will cause it to depreciate today.

The present paper imbeds in the otherwise standard asset pricing model given by equation (3) a form of market expectations that follows equation (1). That is, we assume that portfolio managers' expectations are a weighted average of the expectations of fundamentalists, who think the spot rate regresses to long-run equilibrium, and the expectations of chartists who use time-series methods. We define $\bar{s}$ to be the logarithm of the long-run equilibrium rate and v to be the speed of regression of s_t to $\bar{s}$. In the view of fundamentalists

$$\Delta s_{t+1}^f = v(\bar{s} - s_t). \tag{5}$$

In the context of some standard versions of equation (3) — the monetary model of Dornbusch (1976) in which goods prices adjust slowly over time or the portfolio-balance models in which the stock of foreign assets adjusts slowly over time — it can be shown that equation (5) might be precisely the rational form for expectations to take if there were no chartists in the market, $\omega_t = 1$. (Unfortunately for the fundamentalists, the distinction is crucial; equation (5) will not be rational given the complete model.)

For example, if we define z_t in equation (3) as the interest differential we have

$$s_t = \alpha + cv(\bar{s} - s_t) - b(i_t - i_t^*). \tag{6}$$

Uncovered interest parity, $i_{(t)} - i^* = v(\bar{s} - s_{(t)})$, implies that $v = 1/(b - c)$ and $\alpha = \bar{s}$. It is then straightforward to show that v can be rational within the Dornbusch (1976) overshooting model.[8]

[7] See, for example, Frankel (1986).

[8] Assume that prices evolve slowly according to $\dot{p} = \pi(\gamma(s - p) - \sigma(i - i^*))$ (where γ and σ are elasticities of goods demand with respect to the real exchange rate and the interest rate, respectively), that the interest rate differential is proportional to the gap between the current and long-run price levels, $\lambda(i - i^*) = p - \bar{p}$ (where λ is the semi-elasticity of money demand with respect to the

In the second group of models (Kouri, 1976, and Rodriguez, 1980, are references), overshooting occurs because the stock of net foreign assets adjusts slowly through current account surpluses or deficits. A monetary expansion creates an imbalance in investors' portfolios which can be resolved only by an initial increase in the value of net foreign assets. This sudden depreciation of the domestic currency sets in motion an adjustment process in which the level of net foreign assets increases and the currency appreciates to its new steady state level. In such a model (which is similar to the simulation model below), the rate of adjustment of the spot rate, v, may also be rational, if there are no chartists. Repeating equation (6) but using the log of the stock of net foreign assets instead of the interest differential as the important fundamental, we have in continuous time

$$s(t) = \alpha + cv(\bar{s} - s(t)) - df(t). \tag{7}$$

Suppose the actual rate of depreciation is $\dot{s}(t) = u(\bar{s} - s(t))$. Equation (7) then can be rewritten in terms of deviations from the steady state levels of the exchange rate and net foreign assets, $\bar{s}$ and $\bar{f}$,

$$\dot{s}(t) = (-u/cv)(\bar{s} - s(t)) - (du/cv)(\bar{f} - f(t)) \tag{8}$$

where rationality implies that $u = v$. Following Rodriguez (1980), the normalized current account surplus may also be expressed in deviations from steady state equilibrium:

$$\dot{f} = -q(\bar{s} - s(t)) + \gamma(\bar{f} - f(t)) \tag{9}$$

where q and γ are the elasticities of the current account with respect to the exchange rate and the level of net foreign assets, respectively. The system of equations (8) and (9) then has the rational expectations solution:

$$v = [c\gamma - 1 + ((1 - c\gamma)^2 + 4c(\gamma + dq))^{1/2}]/2c. \tag{10}$$

IV The Model with Exogenous Fundamentals

We now turn to describe the model, assuming for the time being that important fundamentals remain fixed. Regardless of which specification we use for the fundamentals, the existence of chartists whose views are given time-varying weights by the

8 cont.

interest rate) and that the long-run equilibrium exchange rate is given by long-run purchasing parity, $s = p$. Then it can be shown that rationality implies:

$$v = 1/(b - c) = (\pi/2\lambda)(\gamma\lambda + \sigma + (\gamma^2\lambda^2 + 2\lambda\gamma\sigma + \sigma^2 + 4)^{1/2}).$$

portfolio managers complicates the model. For simplicity we study the case in which the chartists believe the exchange rate follows a random walk, $\Delta s_{t+1}^c = 0$. Thus equation (1) becomes

$$\Delta s_{t+1}^m = \omega_t v(\bar{s} - s_t). \tag{1a}$$

Since the changing weights by themselves generate self-sustaining dynamics, the expectations of fundamentalists will no longer be rational, except for the trivial case in which fundamentalist and chartist expectations are the same, $v = 0$.

The 'bubble' path of the exhange rate will be driven by the dynamics of portfolio managers' expected depreciation. We assume that the weight given to fundamentalist views by portfolio managers, ω_t, evolves according to

$$\Delta \omega_t = \delta(\hat{\omega}_{t-1} - \omega_{t-1}). \tag{11}$$

$\hat{\omega}_{t-1}$ is in turn defined as the weight, computed *ex post*, that would have accurately predicted the contemporaneous change in the spot rate, defined by the equation

$$\Delta s_t = \hat{\omega}_{t-1} v(\bar{s} - s_{t-1}). \tag{12}$$

Equations (11) and (12) give

$$\Delta \omega_t = \delta \, \Delta s_t / [v(\bar{s} - s_{t-1})] - \delta \omega_{t-1}. \tag{13}$$

The coefficient δ in equation (13) controls the adaptiveness of ω_t.

One interpretation for δ is that it is chosen by portfolio managers who use the principles of Bayesian inference to combine prior information with actual realizations of the spot process. This leads to an expression for δ which changes over time. To simplify the following anaylsis we assume that δ is constant; in the first appendix we explore more precisely the problem that portfolio managers face in choosing δ. The results that emerge there are qualitatively similar to those that follow here.

Taking the limit to continuous time, we can rewrite equation (13) as

$$\dot{\omega}(t) = \delta[\dot{s}(t)/[v(\bar{s} - s(t))] - \omega(t)]$$
$$\text{if } 0 < \omega(t) < 1; \tag{14}$$

if $\omega(t) = 0$ then

$$\begin{bmatrix} \dot{\omega}(t) = 0 & \text{if } \dot{s}(t) \leq 0 \\ \dot{\omega}(t) = (\delta \dot{s}(t)/[v(\bar{s} - s)] & \text{if } \dot{s}(t) > 0; \end{bmatrix} \tag{14a}$$

if $\omega(t) = 1$ then

$$\begin{bmatrix} \dot{\omega}(t) = 0 & \text{if } \dot{s}(t) \geq v(\bar{s} - s(t)) \\ \dot{\omega}(t) = \delta \dot{s}(t)/[v(\bar{s} - s(t)] - \delta & \text{if } \dot{s}(t) < v(\bar{s} - s(t)) \end{bmatrix} \tag{14b}$$

where a dot over a variable indicates the total derivative with respect to time. The restrictions that are imposed when $\omega(t) = 0$ and $\omega(t) = 1$ are to keep $\omega(t)$ from moving outside the interval $[0,1]$. These restrictions are in the spirit of the portfolio managers choice set: the portfolio manager can at most take one view or the other exclusively.

The evolution of the spot rate can be expressed by taking the derivative of equation (3) (for now holding z and the long-run equilibrium, $\bar{s}$, constant

$$\dot{s}(t) = \dot{\omega}(t)(cv/[1 + cv\omega(t)]) (\bar{s} - s(t)). \tag{15}$$

Equations (14) and (15) can be solved simultaneously and rewritten, for interior values of ω, as

$$\dot{\omega}(t) = -[\delta \omega(t)(1 + cv\omega(t))]/[1 + cv\omega(t) - \delta c];$$
$$\text{if } 0 < \omega(t) < 1 \tag{16}$$

$$\dot{s}(t) = [-\delta \omega(t)cv]/[1 + cv\omega(t) - \delta c](\bar{s} - s(t)). \tag{17}$$

In principle, an analytic solution to the differential equation (16) could be substituted into (17), and then (17) could be integrated directly. For our purposes it is more desirable to use a finite difference method to simulate the motion of the system. In doing so we must pick values for the coefficients, c, v and δ, and starting values for $\omega(t)$ and $s(t)$.

To exclude any unreasonable time paths implied by equations (16) and (17), we impose the obvious sign restrictions on the coefficients. The parameter v must be positive and less than one if expectations are to be regressive, that is, if they are to predict a return to the long-run equilibrium at a finite rate. By definition, δ and $\omega(t)$ lie in the interval $[0,1]$ since they are weights. The coefficient c measures the responsiveness of the spot rate to changes in expected depreciation and must be positive to be sensible.

These restrictions, however, are not enough to determine unambiguously the sign of the denominator of equations (16) and (17). The three possibilities are that: $1 + cv\omega(t) - \delta c < 0$ for all ω; $1 + cv\omega(t) - \delta c < 0$ for all ω; and

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[9] In this case, however, $\omega(t)$ does not have a close analytic form.

$1 + cv\omega(t) - \delta c \underset{>}{\overset{<}{=}} 0$ as $\omega(t) \underset{>}{\overset{<}{=}} \omega^*$, where $0 <$ $\omega^* < 1$.[10]

If $1 + cv\omega(t) - \delta c < 0$, the system will be stable and will tend to return to the long-run equilibrium from any initial level of the spot rate. This might be the case if portfolio managers use only the most recent realization of the spot rate to choose $\omega(t)$, that is, if $\delta \approx 1$. If, on the other hand, portfolio managers give substantial weight to prior information so that δ is small, the expression $1 + cv\omega(t) - \delta c$ will be positive. In this case the spot rate will tend to move away from the long-run equilibrium if it is perturbed.

Let us assume that portfolio managers are slow learners.[11] What does this assumption imply about the path of the dollar? If we take as a starting point the late 1970s, when $s(t) \approx \bar{s}$ and when $\omega_t \approx 1$ (as the calculations presented in Table 2 suggest), equation (17) says that the spot rate is in equilibrium, that $\dot{s}(t) = 0$. From equation (14b), we see that $\dot{\omega}(t) = 0$ as well. Thus the system is in a steady state equilibrium, with market expectations exclusively reflecting the views of fundamentalists.

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[10] We do not consider the third case, because equations (16) and (17) are not defined at $1 + cv\omega(t) - \delta c = 0$.

[11] The following intuition may help see why the system is stable when portfolio managers are 'fast' learners and unstable when they are 'slow' learners. Suppose the value of the dollar is above $\bar{s}$, so that the portfolio managers are predicting depreciation at the rate $\omega v(\bar{s} - s(t))$. If the spot rate were to start depreciating at a rate slightly faster than this, portfolio managers would then shift $\omega(t)$ upwards, in favour of the fundamentalists. Under what circumstances would these hypothesized dynamics be an equilibrium? Recall from equations (14) and (15) that if δ is big, portfolio managers place substantial weight on new information. The larger is δ, the more quickly the spot rate changes. It is easy to show that if portfolio managers are fast learners (i.e. if $\delta > 1/c + v\omega$), they update ω so rapidly that the resulting rate of depreciation must in fact be greater than $\omega v(\bar{s} - s(t))$. Thus the system is stable. Alternatively, if portfolio managers are 'slow' learners, $\delta < 1/c + v\omega$, they heavily discount new information and therefore change $\omega(t)$ too slowly to generate a rate of depreciation greater than $\omega v(\bar{s} - s(t))$. If we instead hypothesize an initial rate of depreciation which is less than $\omega v(\bar{s} - s(t))$, portfolio managers would tend to shift ω downwards, more towards the chartists. From equation (15), a negative $\dot{\omega}(t)$ causes the spot rate to appreciate. Thus slow learning will tend to drive the spot rate further away from the long-run equilibrium (given $0 < \omega < 1$), making the system unstable.

But given that $1 + cv\omega(t) - \delta c > 0$, this equilibrium is unstable, and any shock starts things in motion. Suppose that there is an unanticipated appreciation (the unexpected persistence of high long-term US interest rates in the early 1980s, for example). The sign restrictions imply that $\omega(t)$ is unambiguously falling over time. Equation (16) says that the chartists are gaining prominence, since $\dot{\omega}(t) < 0$. The exchange rate begins to trace out a bubble path, moving away from long-run equilibrium; equation (17) shows that $\dot{s}(t) < 0$ when $\bar{s} > s(t)$. This process cannot, however, go on forever, because market expectations are eventually determined only by chartist views. At this point the bubble dynamics die out since both $\omega(t)$ and $\dot{\omega}(t)$ fall to zero. From equation (17) $\dot{s}(t)$ then stops moving away from long-run equilibrium, as it approaches a new, lower equilibrium level where $\dot{s}(t) = 0$. In the words of Dornbusch (1983), the dollar is both high and stuck.

Figures 1 and 2 trace out a 'base-case' simulation of the line profile of the spot rate and ω. They are intended only to suggest that the model can potentially account for a large and sustained dollar appreciation. The figures assume that the dollar is perturbed out of a steady state equilibrium where $\bar{s} = s(t)$ and $\omega(0) = 1$ in October 1980. The dollar rises at an decreasing rate until sometime in 1985, when, as can be seen in Figure 2, the simulated weight placed on fundamentalist expectations becomes negligible. A steady state obtains at a new higher level, about 31 per cent above the long-run equilibrium implied by purchasing power parity. Although we tried to choose reasonable values for the parameters used in this example, the precise
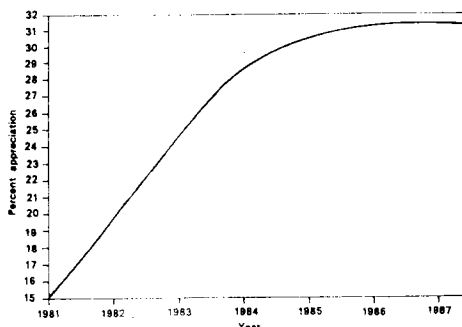

FIGURE 1

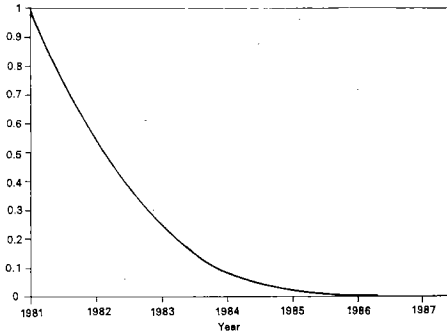*Simulated Value of the Dollar Above its Long-Run Equilibrium*

FIGURE 2

Simulated Weight Placed on Fundamentalist
Expectations by Portfolio Managers

level of the plateau and the rate at which the currency approaches it are sensitive to different choices of parameters. In the second appendix we give more detail on values used in the simulation.

It is worth emphasizing that the equilibrium spot rate appreciates along its bubble path even though none of the actors expects appreciation. This result is due to the implicit stock adjustment taking place. As portfolio managers reject their fundamentalist roots, they reshuffle their portfolios to hold a greater share in dollar assets. For fixed relative asset supplies, a greater dollar share can be obtained in equilibrium only by additional appreciation. This unexpected appreciation, in turn, further convinces portfolio managers to embrace chartism. The rising dollar becomes self-sustaining. In the end, when the spiral finally levels off at $\omega(t) = 0$, the level at which the currency becomes stuck represents a fully rational equilibrium: portfolio managers expect zero depreciation and the rate of change of the exchange rate is indeed zero.

What we term the irrationality of the model can be seen by inspecting equation (17). Recall that market-expected depreciation, that of portfolio managers, is a weighted average of chartist and fundamentalist expectations, $\omega(t)v(\bar{s} - s(t))$. But the actual, or rational, expected rate of depreciation is given by $[-\delta c/[1 + cv\omega(t) - \delta c]] \omega(t)v(\bar{s} - s(t))$. The two are not equal, unless $\omega = 0$.[12] The problem we gave portfolio managers was to pick

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[12] There is a second root, $\omega = -1/(vc)$, which we rule out since it is less than zero.

$\omega(t)$ in a way that best describes the spot process they observe (together with the prior confidence they had in fundamentalist predictions). But theirs is an impossible task, since the spot process is more complicated.

V The Model with Endogenous Fundamentals

The results so far offer an explanation for the paradox of proposition (1), that sustained dollar appreciation occurs even though all agents expect depreciation. But a spot rate that is stuck at a disequilibrium level is an unlikely end for any reasonable story. The next step is to specify the mechanism by which the unsustainability of the dollar is manifest in the model.

The most obvious fundamental which must eventually force the dollar down is the stock of net foreign assets. Reductions in this stock, through large current account deficits, cannot take place indefinitely. Sustained borrowing would, in the long run, raise the level of debt above the present discounted value of income. But long before this point of insolvency is reached, the gains from a US policy aimed at reducing the outstanding liabilities (either through direct taxes or penalties on capital, or through monetization) would increase in comparison to the costs. If foreigners associate large current account deficits with the potential for moral hazard, they would treat US securities as increasingly risky and would force a decline in the level of the dollar.

To incorporate the effects of current account imbalances, we consider the model, similar to Rodriguez (1980), given in equation (7):

$$s_t = \alpha + c\Delta s^m_{t+1} - df \tag{18}$$

where Δs^m_{t+1} is defined in equation (1a) and where f represents the log of cumulated US current account balances. The coefficient, d, is the semi-elasticity of the spot rate with respect to transfers of wealth, and must be positive to be sensible. The differential equations (16) and (17) now become

$$\dot{\omega}(t) = [\delta/[1 + cv\omega(t) - \delta c]] [-\omega(t)(1 + cv\omega(t)) - (d\dot{f})/(v(\bar{s} - s(t)))] \quad \text{if } 0 < \omega(t) < 1 \tag{19}$$

$$\dot{s}(t) = [-\delta\omega(t)cv(\bar{s} - s(t)) + d\dot{f}]/ [1 + c\omega(t)v - \delta c]. \tag{20}$$

If we were to follow the route of trying to solve analytically the system of differential equations, we would add a third equation giving the 'normalized' current account $\dot{f}$, as a function of $s(t)$. (See, for example, equation (9) above.) But we here instead pursue the simulation approach.

TABLE 3

Sensitivity Analysis for the Simulation of the Dollar

	Parameter	Values		Maximum appreciation of the dollar above the initial shock	# of months until peak
delta	c	theta	d	(in per cent)	
0.04	25	0.045	− 0.005	11.4	41
0.06	25	0.045	− 0.005	26.9	27
0.02	25	0.045	− 0.005	5.8	44
0.04	15	0.045	− 0.005	6.4	38
0.04	35	0.045	− 0.005	18.1	40
0.04	25	0.03	− 0.005	8.8	36
0.04	25	0.06	− 0.005	13.5	44
0.04	25	0.045	0	16.4	80
0.04	25	0.045	− 0.0025	11.6	45
0.04	25	0.045	− 0.0075	11.4	38

Notes: These estimates correspond to the simulation depicted in Figure 8 in Frankel and Froot (1986b). The parameter delta falls over time according to equation (A3) in Appendix 1.

In the simulation we use actual current account data for f, the change in the stock of net foreign assets. Figures 3 and 4 trace out paths for the differential equations (19) and (20). During the initial phases of the dollar appreciation, the current account, which responds to the appreciation with a lag, does not noticeably affect the rise of the dollar. But as ω becomes small, the spot rate becomes more sensitive to changes in the level of the current account, and the external deficits of 1983–85 quickly turn the trend. When ω is small and portfolio managers observe an incipient depreciation of the dollar, they begin to place more weight on the forecasts of fundamentalists, thus accelerating the depreciation initiated by the current account deficits. There is a 'fundamentalist revival'. Ironically, fundamentalists are initially driven out of the market as the dollar appreciates, even though they are ultimately right about its turn to $\bar{s}$.

Naturally, all of our results are sensitive to the precise parameters chosen. To gain an idea of the various sensitivities, we report in Table 3 results using alternative sets of parameter values in the simulation corresponding to Appendix 1. While there is some variation, the qualitative pattern of bubble appreciation, followed by a slow turn-around and bubble depreciation, remains evident in all cases.

Recall that one of the main aims of the model

is to account for the two seemingly contradictory facts given by propositions (2) and (3): first that market efficiency tests results imply that the rationally expected rate of dollar depreciation has been less than the forward discount, and second that the calculations based on fundamentals, such as those by Krugman and Marris, imply that the rationally expected rate of depreciation, by 1985, became greater than the forward discount.
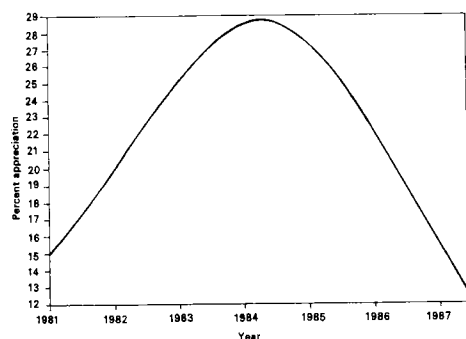

FIGURE 3

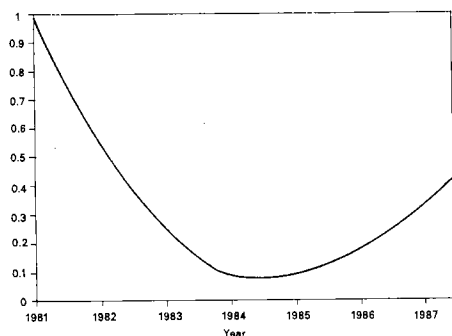*Simulated Value of the Dollar Above its Long-Run Equilibrium*

FIGURE 4

Similated Weight Placed on Fundamentalist
Expectations by Portfolio Managers

Table 4 clarifies how the model resolves this paradox. The first two lines show the expectations of our two forecasters, the chartists and fundamentalists. The third line repeats the six-month survey expectations to demonstrate that they may in fact be fairly well described by the simple regressive formulation we use to represent fundamentalist expectations in line two. The fourth line contains the expected depreciation of the portfolio managers. Note that these expectations

are close to the forward discount in line six, even though the forecasts of the fundamentalists and of the chartists are not. Since only the portfolio managers are hypothesized to take positions in the market, we can say that the magnitude of the market risk premium is small (as mean-variance optimization would predict). Finally, line five shows the actual depreciation in the simulation, which is equivalent to the rationally expected depreciation given the model above. (Of course, none of the agents has the entire model in his information set.) Notice that during the 1981–84 period, the rationally expected depreciation is not only significantly less than the forward discount, but less than zero. This pattern agrees with the results of market efficiency tests discussed earlier. But the rationally expected depreciation is increasing over time. Sometime in late 1984 or early 1985, the rationally expected rate of depreciation becomes positive and crosses the forward discount. As calculations of the Krugman-Marris type would indicate, rationally expected depreciation is now greater than the forward discount. The paradox of propositions (2) and (3) is thus resolved within the model.

All this comes at what might seem a high cost: portfolio managers behave irrationally in that they do not use the entire model in formulating their exchange rate forecasts. But another interpretation

TABLE 4

Alternative Measures of Expected Depreciation
(in per cent per annum)

Expectation from:	Line	Year					
		1981	1982	1983	1984	1985	1986
Chartists in the simulation	(1)	0	0	0	0	0	0
Fundamentalists in the simulation	(2)	7.63	9.82	11.68	11.98	10.33	7.69
Economist six-month survey	(3)	8.90	10.31	10.42	11.66	4.89	NA
Weighted average expected depreciation in the simulation	(4)	5.29	3.31	1.59	0.99	1.49	2.08
Rationally expected depreciation in the simulation	(5)	− 2.97	− 5.16	− 4.38	− 0.72	3.89	6.22
Actual forward discount	(6)	3.74	3.01	1.10	3.07	− 0.74	NA

Note: Fundamentalists in the simulation use regressively parameter of .045, implying that about 70 per cent of the contemporaneous over-valuation is expected to remain after one year. The *Economist* six-month survey includes data through December 1985. Weighted average expected depreciation in the simulation is a weighted average of chartists and fundamentalists, where the weights are those of portfolio managers. Rationally expected depreciation is the perfect foresight solution given by equations (19) and (20). The actual six-month forward discount includes data through December 1985.

of this behaviour is possible, in that portfolio managers are actually doing the best they can in a confusing world. Within this framework they cannot have been more rational; abandoning fundamentalism more quickly would not solve the problem in the sense that their expectations would not be validated by the resulting spot process in the long run. In trying to learn about the world after a regime change, our portfolio managers use convex combinations of models which are already available to them and which have worked in the past. In this context, rationality is the rather strong presumption that one of the prior models is correct. It is hard to imagine how agents, after a regime change, would know the correct model.

VI Conclusions and Extensions

This paper has posed an unorthodox explanation for the recent aerobatics of the dollar. The model we use assumes less than fully rational behaviour in the sense that none of the three classes of actors (chartists, fundamentalists and portfolio managers) condition their forecasts on the full information set of the model. In effect, the bubble is the outcome of portfolio managers' attempt to learn the model. When the bubble takes off (and when it collapses), they are learning more slowly about the model than they are changing it by revising the linear combination of chartist and fundamentalist views they incorporate in their own forecasts. But as the weight given to fundamentalists approaches zero or one, portfolio managers' estimation of the true force changing the dollar comes closer to the true one. These revisions in weights become smaller until the approximation is perfect: portfolio managers have 'caught up' by changing the model more slowly than they learn. In this sense the inability of agents with prior information to bring about immediate convergence to a rational expectations equilibrium may provide a framework in which to view 'bubbles' in a variety of asset markets.

Several extensions of the model in this paper would be worthwhile. First, it would be desirable to allow chartists to use a class of predictors richer than a simple random walk. They might form their forecasts of future depreciation by using ARIMA models, for example. Simple bandwagon or distributed lag expectations for chartists would be the most plausible since they capture a wide range of effects and are relatively simple analytically. Second, we might want to consider extensions which give the model local stability in the

neighbourhood of $\omega = 1$. Small perturbations from equilibrium would then not instantly cause portfolio managers to begin losing faith in fundamentalist counsel. Only sufficiently large or prolonged perturbations would upset portfolio managers' views enough to cause the exchange rate to break free of its fundamental equilibrium.

APPENDIX 1

In this section we consider the problem which portfolio managers face: how much weight should they give to new information concerning the 'true' level of $\omega(t)$. We obtain an explicit formulation for these optimal Bayesian weights, thus replacing equation (11) and supplying firmer foundations for the results reported in the text.

Even though in the model of the spot rate given by equation (3) the value of the currency is fully deterministic, individual portfolio managers who are unable to predict accurately *ex-ante* changes in the spot rate may view the future spot rate as random. They would then form predictions of future depreciation on the basis of observed exchange rate changes and their prior beliefs. At each point in time, portfolio managers therefore view future depreciation as the sum of their current optimal predictor and a random term.

$$\Delta s_{t+1} = \omega_t \nu(\bar{s} - s_t) + \epsilon_{t+1} \qquad (A1)$$

where ϵ_{t+1} is a serially uncorrelated normal random variable with mean 0 and variance $\nu(\bar{s} - s_{t-1})/\tau$.[13] Using Bayes' rule, the coefficient ω_t may be written as a weighted average of the previous period's estimate, ω_{t-1}, and information obtained from the contemporaneous realization of the spot rate,

$$\omega_t = [T_t/(T_t + \tau)] \omega_{t-1} + [\tau/T_t + \tau)] \\ [\Delta s_t/(\nu(\bar{s} - s_t - 1))] \qquad (A2)$$

where $T_t = T_{t-1} + \tau$ where T_0 is the precision of portfolio managers' prior information.[14]. Thus, if portfolio managers use Bayesian techniques, the weight they would give to the current period's information may be expressed as

$$\delta_t = \tau/(\tau t + T_0) \qquad (A3)$$

Equation (A3) shows that the weight which portfolio managers give to new information would fall over time

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[13] The assumption that ϵ_{t+1} exhibits such conditional heteroscedasticity results in a particularly convenient expression for δ_t (equation (A2) below). Under the assumption that ϵ_{t+1} is distributed normally $(0, \sigma^2)$, δ_t depends on all past values of the spot rate,

$$\delta_t = \tau/(\tau \nu \sum_{i=1} (\bar{s} - s_{t-i}) + T_0).$$

[14] If the prior distribution is normal, the precision is equal to the reciprocal of the variance.

as decison makers gain more confidence in their prior distribution, or as the prior distribution for the future change in the spot rate converges to the actual posterior distribution. If, however, portfolio managers suspect that the spot rate is non-stationary, past information would be discounted relative to more recent observations. Instead of combining prior information in the form of an OLS regression of actual depreciation on fundamentalist expectations (as they do above), portfolio managers might use a varying parameter technique to take into account the non-stationarity. In this case, the weight they put on new information might not decline over time to zero.

As we have shown in Appendix I of Frankel and Froot (1986b), computing δ_t using equation (A3) does not change substantially the results of the simulations presented in the text.

APPENDIX 2

In this appendix we discuss our choices of important parameters used in the simulations.

The coefficient on expected depreciation in equation (3), c, may be interpreted as the semi-elasticity of demand for domestic assets with respect to alternative (foreign) rates of return. Bilson (1985), for example, interprets c as equal to Cagan's semi-elasticity of money demand. Under the assumptions that the interest elasticity of the demand for money is .15 and that interest rates are approximately 1 per cent per month Bilson uses $c = 15$. Other possible estimates for c are much higher. An estimate of the semi-elasticity c may be obtained in a mean-variance framework; c then depends on the relative shares of assets in the market portfolio, the variance of the spot rate, and the coefficient of relative risk aversion. Estimates of c (see Frankel, 1986, Table 5a) range from 1800 to 43 800 for various currencies and estimates of portfolio shares. Our choice is somewhere in between Bilson's and Frankel's, $c = 25$. Higher values of c tend to exaggerate the rate of appreciation of the dollar and also the rate at which ω_t falls (see Table 3 in the text).

The coefficient δ measures the rate at which portfolio managers 'learn': it is the weight they give to new information about the value of ω_t. A crucial assumption of the model is that portfolio managers do put weight on their prior estimate of ω_t. If they learn too quickly, the spot rate will be stable and no bubbles will occur. In the simulations in the text, we assume that $\delta = .03$, or that portfolio managers mix the information of the current month's ω_t with data from the past three years.

The parameter v controls the speed with which the spot rate is expected to regress to $\bar{s}$. In the simulations we chose $v = .045$, which means fundamentalists expect about 60 per cent of the current deviation from $\bar{s}$ to remain after one year. Regression estimates of v from exchange rate survey data in Frankel and Froot (1985) are somewhat smaller (about .02), but in that paper the specification for expected depreciation also included a constant term (i.e. $\Delta s_t^f + 1 = \alpha + v(\bar{s} - s(t))$) which was significantly positive (about .01). After including the constant, the surveys predict a somewhat faster return to long-run

equilibrium, that only 50 per cent of the current over-valuation would remain after one year. The choice of $v = .045$ has the added advantage that the expectations of fundamentalists in the simulation appear very similar to the survey expected depreciation (see Table 4, lines two and three).

REFERENCES

Bilson, John (1978), 'The Monetary Approach to the Exchange Rate—Some Empirical Evidence', *Staff Papers,* International Monetary Fund, **25**, 48-75.

——(1985), 'Macroeconomic Stability and Flexible Exchange Rates', *American Economic Review,* Papers and Proceedings, **75**, 62-7.

Dominguez, Kathryn (1986), 'Expectations Formation in the Foreign Exchange Market: New Evidence from Survey Data', Yale University, January.

Dornbusch, Rudiger (1976), 'Expectations and Exchange Rate Dynamics', *Journal of Political Economy,* **84**, 1161-76.

——(1982), 'Equilibrium and Disequilibrium Exchange Rates', *Zeitschrift fur Wirtschafts und Sozialwissenschaften,* **102**, 573-99.

——(1983), 'Comment on Loopesko and Shafer', *Brookings Papers on Economic Activity,* **1**, 78-84.

Figlewski, Stephen (1978), 'Market "Efficiency" in a Market with Heterogeneous Information', *Journal of Political Economy,* **86**, 581-97.

——(1982), 'Information Diversity and Market Behaviour', *Journal of Finance,* **37**, 87-102.

Frankel, Jeffrey (1986), 'The Implications of Mean-Variance Optimization for Four Questions in International Macroeconomics', *Journal of International Money and Finance,* **5**, S53-S75.

Frankel, Jeffrey and Kenneth Froot (1986a), 'Using Survey Data to Test Some Standard Propositions Regarding Exchange Rate Expectations', NBER Working Paper No. 1672 (August 1985); revised, IBER, University of California, Berkeley (May 1986); forthcoming in *American Economic Review,* March 1987.

——(1986b), 'Understanding the Dollar in the Eighties: Rates of Return, Risk Premiums, Speculative Bubbles, and Chartists and Fundamentalists', Discussion Paper No. 155, Centre for Economic Policy Research, Australian National University, Canberra.

——(1986c), 'Short-Term and Long-Term Expectations of the Yen/Dollar Exchange Rate: Evidence from Survey Data', International Finance Discussion Paper No. 292, Federal Reserve Board, September.

Frenkel, Jacob (1976), 'A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence', *Scandinavian Journal of Economics,* **78**, 200-24.

Froot, Kenneth and Jeffrey Frankel (1986), 'Interpreting Tests of Unbiasedness in the Forward Discount Using Surveys of Exchange Rate Expectations', NBER Working paper No. 1963, June.

Kohlhagen, Steven (1978), 'The Behaviour of Foreign Exchange Markets—A Critical Survey of the Empirical Literature', N.Y.U. Monograph Series in Finance and Economics.

Kouri, Pentti J. K. (1976), 'The Exchange Rate and the Balance of Payments in the Short Run and the Long Run: A Monetary Approach', *Scandinavian Journal of Economics*, **78**, 280-304.

Krugman, Paul (1985), 'Is the Strong Dollar Sustainable?', in *The U.S. Dollar—Recent Developments, Outlook and Policy Options*, symposium sponsored by the Federal Reserve Bank of Kansas City, at Jackson Hole, Wyoming, 21-23 August, 103-33.

Marris, Stephen (1985), *Deficits and the Dollar: The World Economy at Risk*, Policy Analyses in International Economics, Institute for International Economics, Washington.

Mussa, Michael (1976), 'The Exchange Rate, the Balance of Payments and Monetary and Fiscal Policy under a Regime of Controlled Floating', *Scandinavian Journal of Economics*, **78**, 229-48.

Rodriguez, Carlos Alfredo (1980), 'The Role of Trade Flows in Exchange Rate Determination: A Rational Expectations Approach', *Journal of Political Economy*, **88**, 1148-58.