"Present Bias Causes and Then Dissipates Auto-Enrollment Savings Effects"

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Online Appendix

A1. Model parameter restrictions

We choose parameters so that it is optimal for an exponential household to save at least fraction m of income during working life (so it is optimal to take full advantage of the match without recycling savings during their worklife from a rollover IRA). This implies that $c \leq (1-m)y$. Given our other assumptions, this restriction can be expressed as:

$$(1+mM)y\left(1-e^{-rNT}\right)+e^{-rNT}y_R \leq (1-m)y.$$

This inequality is satisfied for the illustrative calibration that we carry through the paper, for which the savings rate is 19% and the match threshold is 6%.

A2. Exponential model calculations

Equilibrium consumption is given by

$$c=\rho\left[\int_0^{NT}(1+mM)ye^{-r\tau}\mathrm{d}\tau+\int_{NT}^{\infty}y_Re^{-r\tau}\mathrm{d}\tau\right].$$

Recalling that $\rho=r$, this equation implies

$$c=(1+mM)y\left(1-e^{-rNT}\right)+e^{-rNT}y_R.$$

Accordingly, at retirement (immediately after the final rollover), the following equation holds:

$$rw+y_R=(1+mM)y\left(1-e^{-rNT}\right)+e^{-rNT}y_R.$$

This implies

$$w=(1/r)\left[(1+mM)y-y_R\right]\left(1-e^{-rNT}\right).$$

A3. Proof that $\hat{c}(t)>y_R$

Note that

$$\hat{c}(t)=\rho\left[w(t)+z(t)+(1-e^{-r(NT-t)})(1+mM)y/r+e^{-r(NT-t)}y_R/r\right]>y_R.$$

The inequality follows because $\rho=r$, $w(t)\geq 0$, $z(t)\geq 0$, $mM\geq 0$, and $y>y_R$.

A4. Sufficient condition for full leakage before retirement.

We provide a sufficient condition for the current employer's 401(k) to be the only asset that survives at each separation. In other words, all previous retirement plan savings (now in a rollover IRA account) are consumed before the next job separation. First, we will show this property in the final employment spell. Then we will use induction to show that this property is true for all previous employment spells.

At the start of the final (Nth) employment spell, the household begins with rollover savings $(e^{rT}-1)(s+mM)y/r$ accumulated in the previous spell.

The household believes that it will immediately join the savings plan and save at the optimal rate. In other words, it believes that its permanent consumption at any time t is:

$$\hat{c} = \rho \left[w(t) + z(t) + (1 - e^{-r(NT-t)})(1+mM)y/r + e^{-r(NT-t)} y_R/r \right]$$

where $w(t) = (e^{rT} - 1)(s+mM)y/r$ at the beginning of each employment spell starting at her second employment spell ($w(0) = 0$) and $z(t) = (e^{r(t-(N-1)T)} - 1)(s+mM)y/r$ at all time points during her last employment spell. The Euler Equation leads the household to want to spend

$$c^* = \frac{1-p}{\beta} \hat{c}.$$

Its actual liquid take-home pay is $y(1-s)$. It will decumulate from its rollover IRA iff $c^* > y(1-s)$. Assuming that it is in a decumulation phase, we characterize the differential equation associated with accumulation in state variable w:

$$\dot{w} = y(1-s) - \frac{1-p}{\beta}\hat{c} + rw$$

This implies that w follows a partial differential equation. We can bound the dynamics for this PDE. Specifically,

$$\dot{w} < y(1-s) - \frac{1-p}{\beta} y_R + rw$$

because $\hat{c} > y_R$. Hence, to show that the household decumulates its IRA rollover during its next employment spell, it is sufficient to show that the bounding differential equation

$$\dot{q} = y(1-s) - \frac{1-p}{\beta} y_R + rq$$

with $q(0) = (e^{rT} - 1)(s+mM)y/r$ crosses zero before time T. The solution to the differential equation for q is

$$q(t) = \left[y(1-s) - \frac{1-p}{\beta} y_R \right] \left(e^{rt} - 1 \right) / r + e^{rt} (e^{rT} - 1)(s+mM)y/r$$

Decumulation will occur if this equation is less than or equal to zero at $t = T$. Setting this equation less than or equal to zero, we generate the sufficient condition:

$$(1-s) - \frac{1-p}{\beta} \left(\frac{y_R}{y} \right) + e^{rT}(s+mM) \leq 0$$

With a high turnover rate (so that job duration multiplied by the real interest rate, rT, is close to zero), this is approximately equal to

$$1 - \frac{1-p}{\beta} \left(\frac{y_R}{y} \right) + mM \leq 0.$$

Rearranging, we generate the approximate sufficient condition:

$$\beta \leq \left(\frac{1-p}{1+mM} \right) \left(\frac{y_R}{y} \right).$$

Without the approximation, the sufficient condition is given by:

$$\beta \leq \frac{1-p}{1 + e^{rT} mM + s(e^{rT} - 1)} \left(\frac{y_R}{y} \right)$$

Both equations imply that for the calibration in this paper, no 401(k) wealth will survive to retirement other than the savings achieved in the last employment spell.

This sufficient condition applies for the full lifecycle, because $\hat{c}$ is higher earlier in life than it is in the last employment spell. So decumulation of the rollover IRA is even faster in the first $N-1$ employment spells.

A5. Sufficient condition for partial leakage before retirement

We can also derive a sufficient condition for any leakage to occur. Consider a household near the beginning of its life, with a strictly positive balance in an IRA. This is the point where leakage is most likely to occur because the household has the most favorably biased beliefs about its own future savings behavior.

Permanent income is given by

$$\hat{c}(t) = \rho \left[\int_t^{NT} (1+mM) y e^{-r\tau} \mathrm{d}\tau + \int_{NT}^{\infty} y_R e^{-r\tau} \mathrm{d}\tau + z(t) + w(t) \right].$$

Accordingly, equilibrium consumption will exceed available liquidity if

$$(1-p)u'((1-s_D)y) > \beta u'(\hat{c}(t))$$

Hence, a sufficient condition for at least some leakage to occur is

$$\beta < \left(\frac{1-p}{1-s_D} \right) \left(\frac{\hat{c}(t)}{y} \right).$$