

Optimal Defaults and Active Decisions*

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December 3, 2004

JEL classification: D0, E21, G23

Keywords: behavioral economics, active decisions, 401(k), defaults, defined contribution pension, retirement savings, consumption.

*We thank Hewitt Associates for their help in providing the data. We are particularly grateful to Lori Lucas, Jim McGhee, Scott Peterson, Deborah Bonus and Yan Xu, some of our many contacts at Hewitt. We also appreciate many helpful conversations with Harlan Fleece and the exceptional research assistance of Nelson Uhan, Jared Gross, John Friedman, Hongyi Li, Laura Serban, Keith Ericson, Ben Grossman, and Fuad Faridi. Richard Thaler, Mark Iwry, Shlomo Benartzi, Annamaria Lusardi and seminar participants at the University of Chicago, Harvard, MIT, Berkeley, Cornell, USC, LSE, Wharton, Dartmouth, Stanford, the New York Federal Reserve, the Russell Sage Foundation and the NBER have provided much useful feedback. Choi acknowledges financial support from a National Science Foundation Graduate Research Fellowship and the Mustard Seed Foundation. Choi, Laibson, and Madrian acknowledge individual and collective financial support from the National Institute on Aging (grants R01-AG-16605, R29-AG-013020, and T32-AG00186). Laibson also acknowledges financial support from the Sloan Foundation.

ABSTRACT: Defaults can have a dramatic influence on consumer decisions. We identify an overlooked but practical alternative to defaults: requiring individuals to make an explicit choice for themselves. We study such “active decisions” in the context of 401(k) saving. We find that compelling new hires to make active decisions about 401(k) enrollment raises the initial fraction that enroll by 28 percentage points relative to a standard opt-in enrollment procedure, producing a savings distribution three months after hire that would take three years to achieve under standard enrollment. We also present a model of 401(k) enrollment and derive conditions under which the optimal enrollment regime is automatic enrollment (i.e., default enrollment), standard enrollment (i.e., default non-enrollment), or active decisions (i.e., no default and compulsory choice). Active decisions are optimal when consumers have a strong propensity to procrastinate and savings preferences are highly heterogeneous. Naive beliefs about future time-inconsistency strengthen the normative appeal of the active decision enrollment regime.

Economists have studied two kinds of 401(k) enrollment. Under “standard enrollment,” employees are by default not enrolled and can choose to opt into the plan. Under “automatic enrollment,” employees are by default enrolled and can choose to opt out. In this paper, we analyze an overlooked third alternative: requiring employees to make an explicit choice for themselves. In this “active decision” regime there is *no* default to fall back on.

Ex ante, it might seem that a default should not matter if agents believe it is arbitrarily chosen and if opting out of the default is easy. In practice, defaults—even bad defaults—tend to be sticky; employees rarely opt out.¹ This perverse property of defaults has been documented in a wide range of settings: organ donation decisions (Johnson and Goldstein 2003, Abadie and Gay 2004), car insurance plan choices (Johnson et al 1993), car option purchases (Park, Yun, and MacInnis 2000), and consent to receive e-mail marketing (Johnson, Bellman, and Lohse 2003).

In light of this inertia, defaults are socially desirable when agents have a shared optimum and the default leads them to it (e.g., a low-fee index fund). But even a well-chosen default may be undesirable if agents have heterogeneous needs.² For example, in a firm whose workforce includes young, cash-strapped single parents and older employees who need to quickly build a retirement nest egg, one 401(k) savings rate isn’t right for everyone.

Active decisions are an intriguing, though imperfect, alternative to defaults. On the positive side, an active decision mechanism avoids the biases introduced by defaults because it does not corral agents into a single default choice. The active decision mechanism encourages agents to think about an important decision and thereby avoid procrastinating. On the negative side, an active decision compels agents to struggle with a potentially time-consuming decision—which they may not be qualified to make—and then explicitly express their choice. Some people would welcome a benign third party who is willing to make and automatically implement that decision for them. In addition, social engineers might prefer a default that aggressively encourages some social goal, like organ donation or retirement saving.³

¹For example, about three-quarters of 401(k) participants in firms with automatic enrollment retain both the default contribution rate and the default asset allocation (Madrian and Shea 2001, Choi et al 2002, 2004). These “choices” are puzzling because most companies with automatic enrollment have very conservative defaults; a typical firm might have a default contribution rate of 2% of income, even though contributions up to 6% of income garner matching contributions from the employer.

²One could design defaults that are tailored to each employee, but these would be construed as advice in the current regulatory environment. Firms are not allowed to give financial advice to their employees, although they are allowed to give employees access to an independent financial advisor.

³See Hurst (2004) and Warshawsky and Ameriks (2000) for evidence that many U.S. households are

The current paper lays the groundwork for a debate about active decisions by describing how an active decision 401(k) enrollment regime worked at one large firm. In addition, we present a model that provides a formal framework for evaluating the relative efficacy of different choice mechanisms, including defaults and active decisions.

Our empirical study exploits a natural experiment in 401(k) enrollment. One large firm (unintentionally) used active decisions and then switched to a standard enrollment regime. This change in 401(k) enrollment procedures occurred as a by-product of the transition from a paper-and-pencil administrative system to a phone-based administrative system. The firm did not anticipate that the transition to a phone-based system with a default of non-enrollment would transform the psychology of 401(k) participation. Rather, the change in administrative systems was motivated solely by the convenience and efficiency of phone-based enrollment. The loss of active decision effects was a collateral consequence of that transition.

We find that active decisions raise the initial fraction of employees enrolled by 28 percentage points relative to what is obtained with a standard default of non-enrollment. Active decisions raise average savings rates and accumulated balances by accelerating decision-making. We show that conditional on demographics, each employee under active decisions will on average *immediately* choose a savings rate similar to what she would otherwise take up to three *years* to attain under standard enrollment. There is also no evidence that employees make more haphazard savings choices under active decisions. Because the typical worker will change jobs several times before retirement, accelerating the 401(k) savings decision by three years at the beginning of each job transition can have a large impact on accumulated wealth at retirement.

In our model of the enrollment process, defaults matter for three key reasons. First, employees face a cost of opting out of the employer's chosen default. Second, this cost varies over time, creating option value to waiting for a low-cost period to take action. Third, employees with present-biased preferences may procrastinate in their decision to opt out of the default. We derive conditions under which the optimal enrollment regime is automatic enrollment, standard enrollment, or active decisions. Active decisions are socially optimal

undersaving. However, this is an open question with research on both sides of the debate. See Aguiar and Hurst (2004) and Engen, Gale, and Uccello (1999) for evidence that the lifecycle model with liquidity constraints matches U.S. data.

when consumers have highly heterogeneous savings preferences and a strong propensity to procrastinate.

The rest of this paper proceeds as follows. Section 1 describes the details of the two 401(k) enrollment regimes at the company we study. Section 2 describes our data. Section 3 compares the 401(k) savings decisions of employees hired under the active decision regime to those hired under the standard enrollment regime. Section 4 then models the enrollment decision of a time-inconsistent employee who has rational expectations, derives the socially optimal enrollment mechanism for such agents, and also considers the case of employees with naive expectations about their future time inconsistency. Section 5 discusses the key implications of the model. Section 6 concludes and briefly discusses practical considerations in the implementation of active decision mechanisms.

1 The Natural Experiment

We use employee-level data from a large, publicly-traded Fortune 500 company in the financial services industry. In December 1999, this firm had offices in all 50 states, as well as the District of Columbia and Puerto Rico. This paper will consider the 401(k) savings decisions of employees at the firm from January 1997 through December 2001.

Until November 1997, all newly hired employees at the firm were *required* to submit a form within 30 days of their hire date stating their 401(k) participation preferences, *regardless* of whether they wished to enroll or not. Although there was no tangible penalty for not returning the 401(k) form, human resource officers report that only 5% of employees failed to do so.⁴ We believe that this high compliance rate arose because the form was part of a packet that included other forms whose submission was legally required (e.g., employment eligibility verification forms, tax withholding forms, other benefits enrollment forms, etc.). Moreover, employees who did not return the form were reminded to do so by the human resources department.

Employees who declined to participate in the 401(k) plan during this initial enrollment period could not subsequently enroll in the plan until the beginning (January 1) of succeeding calendar years. Later in the paper, we will show that this delay played no role in the active decision effect.

⁴A failure to return the form was treated as a negative 401(k) election.

At the beginning of November 1997, the company switched from a paper-based 401(k) enrollment system to a telephone-based system. Employees hired after this change no longer received a 401(k) enrollment form when hired. Instead, they were given a toll-free phone number to call if and when they wished to enroll in the 401(k) plan. We call this new system the “standard enrollment” regime because its non-enrollment default is used by most companies. The telephone-based system also allowed employees to enroll on a daily basis, rather than only at the beginning of each calendar year as had previously been the case. This change applied not only to employees hired after November 1997, but to all employees working at the company.

A number of other 401(k) plan features also changed at the same time. We believe that these additional changes made 401(k) participation more attractive, so our estimates of the active decision effect are a lower bound on the true effect. These other changes include a switch from monthly to daily account valuation, the introduction of 401(k) loans, the addition of two new funds as well as employer stock to the 401(k) investment portfolio,⁵ and a switch from annual to quarterly 401(k) statements. Table 1 summarizes the 401(k) plan rules before and after the November 1997 changes.

2 The Data

We have two types of employee data. The first dataset is a series of cross-sections at year-ends 1998, 1999, 2000, and 2001. Each cross-section contains demographic information for everybody employed by the company at the time, including birth date, hire date, gender, marital status, state of residence, and salary. For 401(k) plan participants, each cross-section also contains the date of enrollment and year-end information on balances, asset allocation, and the terms of any outstanding 401(k) loans. The second dataset is a longitudinal history of *every* individual transaction in the plan from September 1997 through April 2002: savings rate elections, asset allocation elections for contributions, trades among funds, loan-based withdrawals and repayments, financial hardship withdrawals, retirement withdrawals, and rollovers.

To analyze the impact of active decisions, we compare the behavior of two employee

⁵Prior to November 1997, employer stock was available as an investment option only for after-tax contributions.

groups: employees hired between January 1, 1997 and July 31, 1997 under the active decision regime,⁶ and employees hired between January 1, 1998 and July 31, 1998 under the standard enrollment regime.⁷ We refer to the first group as the “active decision cohort” and the second group as the “standard enrollment cohort.”

The active decision cohort is first observed in our cross-sectional data in December 1998, 18 to 24 months after hire, and in the longitudinal data starting in September 1997, 3 to 9 months after hire. The longitudinal data only contain participants. The standard enrollment cohort is also observed in our cross-sectional data starting in December 1998, but this is only 6 to 12 months after their hire date. In the longitudinal data, 401(k) participants from this cohort are observed as soon as they enroll.

Since 401(k) participants are less likely to subsequently leave their employer,⁸ our data structure introduces selection effects that are stronger for the active decision cohort than the standard enrollment cohort. To equalize the sample selectivity of the active decision and standard enrollment cohorts, we restrict the standard enrollment cohort sample to those employees who were still employed by the company in December 1999. We have no reason to believe that the turnover rates of employees from these two cohorts were different over these time horizons. The economic environment faced by these two groups of employees was similar until the start of the 2001 recession. In addition, company officials reported no material changes in hiring or employment practices during this period.

Table 2 presents demographic statistics on the active decision and standard enrollment cohorts at the end of December in the year *after* they were hired. The cohorts are similar in age, gender composition, income, and geographical distribution. The dimension along which they differ most is marital status, and even here the differences are not large: 57.2% of the active decision cohort is married, while this is true for only 52.2% of the standard enrollment cohort. The third column of Table 2 shows that the new-hire cohorts are different from

⁶We exclude employees hired prior to January 1, 1997 because the company made two substantive plan changes that took effect January 1, 1997. First, the company eliminated a one-year service requirement for 401(k) eligibility. Second, the company changed the structure of its 401(k) match. Although active decisions were used until the end of October 1997, we do not include employees hired from August through October to avoid any confounds produced by the transition to standard enrollment. For example, an enrollment blackout was implemented for several weeks during the transition.

⁷For both groups, we restrict our sample to employees under age 65 who are eligible to participate in the plan.

⁸See Even and MacPherson (1999).

employees at the company overall. As expected, the new-hire cohorts are younger, less likely to be married, and paid less on average. The last column reports statistics from the Current Population Survey, providing a comparison between the company’s employees and the total U.S. workforce. The company has a relatively high fraction of female employees, probably because it is in the service sector. Employees at the company also have relatively high salaries. This is partially due to the fact that the company does not employ a representative fraction of very young employees, who are more likely to work part-time and at lower wages.

3 Empirical Results

3.1 401(k) Enrollment

We first examine the impact of the active decisions on enrollment in the 401(k). Figure 1 plots the fraction enrolled in the 401(k) after three months of tenure for employees who were hired in the first seven months of 1997 (the active decision cohort) and the first seven months of 1998 (the standard enrollment cohort). We use the third month of tenure because it could take up to three months for enrollments to be processed in the active decision regime.⁹ The average three-month enrollment rate is 69% for the active decision cohort, versus 41% for the standard enrollment cohort, and this difference is statistically significant at the 99% level for every hire month.

Figure 2 plots the fraction of employees who have enrolled in the 401(k) plan against tenure. The active decision cohort’s enrollment fraction grows more slowly than the standard enrollment cohort’s, so the enrollment gap decreases with tenure. Nonetheless, the active decision cohort’s enrollment fraction exceeds the standard enrollment cohort’s by 17 percentage points at 24 months of tenure, and by 5 percentage points at 42 months. These differences are statistically significant at the 99% level for every tenure level.

Figures 1 and 2 could be misleading if enrollees under the active decision regime are subsequently more likely to stop contributing to the 401(k) plan. However, attrition rates from the 401(k) plan are indistinguishable under the active decision regime and the standard

⁹Enrollments were only processed on the first of each month under the active decision regime. Since employees had 30 days to turn in their form, an employee who was hired late in a month and turned in her form just before the deadline could be enrolled three months after her hire.

enrollment regime. Indeed, 401(k) participation is a nearly-absorbing state under either enrollment regime.¹⁰

We ascribe the active decision effect to the fact that active decision employees had to express their 401(k) participation decision during their first month of employment, rather than being able to delay action indefinitely. However, there is another distinction between the active decision and standard enrollment regimes as implemented at the company. Under the standard enrollment regime, employees could enroll in the 401(k) plan at any time. Under the active decision regime, if employees did not enroll in the plan in their first 30 days at the company, their next enrollment opportunity did not come until January 1 of the following calendar year.¹¹ Therefore, in addition to the required affirmative or negative enrollment decision, the active decision cohort faced a narrower enrollment window than the standard enrollment cohort. In theory, this limited enrollment window could cause higher initial 401(k) enrollment rates by accelerating the enrollment of employees who would have otherwise enrolled between the third month of their tenure and the following January.

However, the enrollment differences between the cohorts are too large to be explained by a window effect. If only a window effect were operative, enrollment fractions for the two groups should be equal after twelve months of tenure. In fact, the enrollment fraction of the active decision cohort at three months of tenure is not matched by the standard enrollment cohort until it approaches three years of tenure.

There is also no evidence that the window effect can even *partially* explain the active decision effect. If the window effect were important, then enrollment during the initial 30-day eligibility period under active decisions should be higher for employees hired earlier in the year than employees hired later in the same year, since employees hired earlier have a longer wait until the next January 1 enrollment opportunity. We find no such relationship; instead, the correlation between the three-month enrollment fraction and the time from hire month to the following January 1 is negative (-0.20). This result is corroborated in data from pre-1997 cohorts hired under the active decision regime.

Although active decisions induce earlier 401(k) enrollment, this may come at the cost

¹⁰These calculations are available from the authors.

¹¹In fact, the active decision cohort we analyze (January to June 1997 hires) was able to enroll in the 401(k) plan any time after November 1997, when the company switched to the phone-based daily enrollment system. At hire, however, the active decision employees were not aware of this impending change and would have believed January 1, 1998 to be their next enrollment opportunity.

of more careful and deliberate thinking about how much to save for retirement. We now turn our focus to other 401(k) savings outcomes to see what impact active decisions have on them.

3.2 401(k) Contribution Rate

Figure 3 plots the relationship between tenure and the average 401(k) contribution rate for both the active decision and standard enrollment cohorts. The averages include both participants (who have a non-zero contribution rate) and non-participants (who have a zero contribution rate). Because our longitudinal data do not start until September 1997, the contribution rate profile cannot be computed for the entire active decision cohort until 9 months of tenure.

The active decision cohort contributes 4.8% of income on average at month 9, and this slowly increases to 5.5% of income by the fourth year of employment. In contrast, the standard enrollment cohort contributes only 3.6% of income on average at month 9, and it takes three years for it to match the active decision cohort’s nine-month savings rate. At each tenure level in the graph, the difference between the groups’ average contribution rates is statistically significant at the 99% level.

Figure 4 plots the average contribution rate of employees who have a non-zero contribution rate (i.e., 401(k) participants). In contrast to Figure 3, active decision participants have a lower average contribution rate than standard enrollment participants until the fourth year of tenure.¹² To gain insight into this pattern, we plot the 25th, 50th, 75th, and 90th percentile contribution rates for the standard enrollment and active decision cohorts in Figure 5. Non-participants are assigned a zero contribution rate and are included in these distributions. We see that at each of these points in the distribution, the active decision cohort’s contribution rate matches or exceeds the standard enrollment cohort’s contribution rate at virtually every tenure level. There is nearly no gap between the two cohorts’ contribution rates at the 90th percentile, where enrollment in the 401(k) occurs immediately for both groups. As we move down the savings distribution, the difference between the two cohorts tends to increase, and most of this difference is due to active decision cohort employees

¹²These differences are statistically different at the 99% level through the 29th month of tenure, and at the 95% level through the 30th month of tenure.

signing up for the 401(k) plan earlier in their tenure. Overall, it seems that employees save at roughly the same rate under both regimes once they have enrolled. Therefore, the lower average contribution rate among active decision participants is not due to active decisions lowering the savings rates of those who would have otherwise contributed more under standard enrollment. Rather, active decisions bring employees with weaker savings motives into the participant pool earlier in their tenure.

Table 3 presents the results of a regression of the two regimes' contribution rates on demographic variables. Both active decision and standard enrollment employees are included in the regression, regardless of participation status. If the employee was hired under the standard enrollment regime, the dependent variable is equal to the contribution rate at 36 months after hire. If the employee was hired under the active decision regime, the dependent variable is equal to an estimate of the contribution rate at 3 months after hire. This estimate is constructed by taking the earliest available contribution rate (which may be as late as 9 months after hire) for the active decision employee and setting that contribution rate to zero if the employee had not enrolled in the plan within 3 months of hire. The explanatory variables are a constant, marital status, log of salary, and age dummies. The effect of these variables is allowed to vary between the active decision and standard enrollment cohorts.¹³

The regression coefficients suggest that in expectation, there is little difference between the savings rate an employee chooses immediately after hire under active decisions and the rate she would have in effective three years after hire under standard enrollment. The only variable we can statistically reject having the same effect under both regimes is marital status; active decisions seem to eliminate the savings gap between married and single individuals. Furthermore, there is no evidence that savings choices under active decisions are more haphazard than savings rates under standard enrollment; a Goldfeld-Quandt test cannot reject the null hypothesis that the regression residuals for the two cohorts were drawn from distributions with the same variances.¹⁴ If the active decision deadline causes people to make more mistakes when choosing their savings rate, then these mistakes are almost perfectly predicted by their demographics.

In sum, active decisions cause employees to immediately choose a savings rate that they

¹³The number of data points in the regression is less than the total number of employees in the two cohorts because some employees are missing demographic data.

¹⁴The ratio of the mean squared error for the standard enrollment cohort to the mean squared error for the active decision cohort is 1.018.

would otherwise take up to three years to attain under standard enrollment.

3.3 401(k) Asset Allocation

The effect of active decisions on asset allocation cannot be cleanly inferred because the menu of investment fund options changed in November 1997, the same time that the company switched from active decisions to the standard enrollment regime. Prior to the change, employer stock was only available as an investment option for after-tax contributions, which are rare because they are generally less tax-efficient than pre-tax contributions.¹⁵ During the transition to standard enrollment, employer stock was added as an investment option for pre-tax 401(k) contributions. Subsequently, the average allocation to employer stock more than doubled and the average allocation to all other asset classes correspondingly decreased. It is impossible to determine how much of this increase was caused by the standard enrollment regime, and how much was caused by the ten-fold increase in the fraction of employees for whom employer stock was a viable investment option.

The impact of active decisions on asset allocation is an important open question, since employees have low levels of financial knowledge about different asset classes (John Hancock 2002) and tend to make poor asset allocation choices (Benartzi and Thaler 2001, Cronqvist and Thaler 2004). In section 4, we explain why active decisions are likely to be better suited for contribution rate choices than for asset allocation choices. We believe asset allocation decisions are best treated with a clear default option.

3.4 401(k) Asset Accumulation

We next consider the impact of active decisions on asset accumulation. Asset accumulation analysis is confounded by time effects, since asset returns are highly volatile. Moreover, the investment fund menu changed over time, as explained above, further confounding this analysis. Nonetheless, it is the level of asset accumulation that will ultimately drive retirement timing and consumption levels. Studying asset accumulation also gives us insight

¹⁵Pre-tax contributions are more tax-efficient unless the contributor has a short investment horizon and expects tax rates to rise sharply in the future. At the company, 12% of 401(k) participants made after-tax contributions during 1998. Participants who make after-tax contributions tend to be at their pre-tax contribution limit.

into whether increased 401(k) loan activity offsets increased contribution rates under active decisions.¹⁶

To measure asset accumulation, we divide 401(k) balances by annual base pay. Our measure of 401(k) balances excludes outstanding principal from 401(k) loans and any balances an employee rolled over from a previous employer.

Figure 6 reports balance-to-pay ratios at the 25th, 50th, 75th, and 90th percentiles of the balance-to-pay distribution for the active decision and standard enrollment cohorts. The impact of the market downturn in 2001 appears around the 48th month of tenure for the active decision cohort and the 36th month of tenure for the standard enrollment cohort.

It is apparent that the balance-to-pay ratio paths are nearly identical for the two cohorts at both the 75th and 90th percentiles. By contrast, the 25th percentile active decision employee has a much higher balance-to-pay ratio because participation begins two years earlier in her tenure than it does for the 25th percentile standard enrollment employee. The 50th percentile active decision employee has a slightly higher balance-to-pay ratio, but the effects of the 2001 stock market downturn muddy the picture. Overall, it appears that active decision enrollment only affects asset accumulation in the bottom half of the accumulation distribution. This is consistent with the results of the contribution rate analysis.

4 A Model of 401(k) Enrollment

The empirical analysis in Section 3 shows that active decisions accelerate 401(k) enrollment. But these results do not enable us to evaluate the welfare consequences of the active decision regime. We now present a structural model that provides a framework for setting socially optimal defaults by modeling an employee's response to an enrollment regime as well as the planner's initial choice of the enrollment regime. In the model, an active decision regime is equivalent to a default that is so far from all employees' optimal savings rate that everybody feels compelled to opt out immediately. Although we focus on 401(k) enrollment, the model generalizes to other situations where defaults matter.

The model has four key features: (1) a time-varying transaction cost for opting out of a

¹⁶The active decision cohort did not have 401(k) loans available to them at the time they made their initial contribution rate decision. However, after November 1997, they were able to borrow against their 401(k) balances.

default, which creates an option value for delaying action and generates inertia at a default; (2) present-bias, which generates self-defeating procrastination and exacerbates this inertia; (3) heterogeneity in optimal savings rates across workers, which implies that a single default will not be optimal for all workers; and (4) asymmetric information of the form where each worker knows more than the planner about the worker’s optimal savings rate.

The third and fourth assumptions are critical for our welfare results. If the planner knows the worker’s true optimum, then the planner should simply set that optimum as the default, saving the worker the time and effort of making the choice for himself. There is a growing body of evidence that planners make better *asset allocation* choices than workers (Benartzi and Thaler 2001, Cronqvist and Thaler 2004). However, survey evidence suggests that workers have idiosyncratic *savings* needs, and that workers understand this individual variation (Choi et al 2004). Hence, the model that follows is best thought of as a model of saving rates choices and not of asset allocation choices. For asset allocation choices, well-chosen defaults are likely to dominate active decisions.

Our model has two stages. First, a benign planner creates an enrollment mechanism (a default or active decision). Then, individual workers make a sequence of savings decisions. Our analysis solves the model through backwards induction. We begin by solving the worker’s problem, holding fixed the enrollment mechanism. Then we solve the planner’s problem by identifying the enrollment mechanism that maximizes social surplus.

4.1 The Sophisticated Worker’s Problem

Workers have quasi-hyperbolic preferences, so they have the discount function $1, \beta\delta, \beta\delta^2, \dots$ where $0 < \beta \leq 1$ (Phelps and Pollak 1968, Laibson 1997). Workers suffer from a dynamic inconsistency problem; they have impatient preferences for the present but patient preferences for the future. For simplicity, we set $\delta = 1$, eliminating long-run discounting. We begin by assuming that workers are sophisticated and understand their dynamically inconsistent preferences. Unlike naive agents, whose actions we characterize later, sophisticated actors make current decisions based on rational expectations of future actions.

Each worker has an exogenously determined optimal savings rate s that is known to the worker but not observed by the planner.¹⁷ The planner only knows the probability density

¹⁷For simplicity, we assume that each worker’s optimal savings rate is constant over time.

function for s , denoted $f(s)$. In period 0, the planner sets the default contribution rate d (which could equal 0) at which all new employees are automatically enrolled in the company savings plan. We will show below that an active decision regime is equivalent to an extreme default choice far outside the support of $f(s)$.

In every subsequent period, each worker has the opportunity to change her savings rate from the default to her optimal level. Should she act, she incurs a transaction cost c drawn independently each period from a uniform distribution on the interval $[\underline{c}, \bar{c}]$, where $\underline{c} < \bar{c}$. This cost is determined at the beginning of the period and is known to the worker at the time of her decision. If the worker is not at her optimal savings rate at the end of the period, she suffers a flow loss $L(s, d) > 0$ at the beginning of the next period. Once the worker is at her optimal savings rate, she experiences no further flow losses.

Given d and c , the worker minimizes the current discounted loss function W . Let c, c' and c'' denote the opt-out costs drawn in the current and next two periods, respectively. The variables s and d are fixed from the perspective of a single worker, so we simplify notation by temporarily defining $L \equiv L(s, d)$. Thus, the current discounted loss function can be represented as

$$W(L, c) = \begin{cases} c & \text{if the worker opts out of the default} \\ \beta [L + E V(L, c')] & \text{if the worker delays} \end{cases}. \quad (1)$$

The function $V(\cdot)$ represents the worker's (rational) expected continuation losses if she delays opting out of the default. Since workers have no long run discounting, this continuation loss function can be written,

$$V(L, c') = \begin{cases} c' & \text{if the worker opts out tomorrow} \\ L + E V(L, c'') & \text{if the worker delays tomorrow} \end{cases}. \quad (2)$$

The discount factor β only appears in the equation for W because the worker views tradeoffs between the current period and the immediate future differently than tradeoffs between any two future periods.

The equilibrium for this game takes the form of a cutoff rule; workers opt out of the default if and only if the stochastic transaction cost falls below some point c^* . Workers at

this cutoff must be indifferent (in the current period) between opting out and delaying:

$$c^* = \beta [L + E[V(L, c')]] . \quad (3)$$

Combining (3) with the fixed point condition for the future loss function, we obtain the following proposition.

Proposition 1 *A sophisticated worker will opt out of the default if and only if she draws a transaction cost $c < c^*$, where*

$$c^* = \frac{\underline{c} + \sqrt{\underline{c}^2 [1 - (2 - \beta)\beta] + 2\beta(2 - \beta)(\bar{c} - \underline{c})L}}{2 - \beta} . \quad (4)$$

This function is increasing in L , the per-period cost of deviation from a worker's optimal savings rate; the higher these flow losses, the more willing the worker is to incur the transaction cost of opting out of the default. While c^* is always above \underline{c} , it need not lie below \bar{c} . Agents may face such large flow losses that they will act in the first period for all costs in the support of c .

The cutoff threshold c^* increases with β . As β rises, the worker becomes less present-biased and hence more willing to incur c now to reduce future flow losses. The expression simplifies when $\beta = 1$, which generates the benchmark cutoff threshold $\underline{c} + \sqrt{2(\bar{c} - \underline{c})L}$. This is the cutoff that the agent would prefer to use in all *future* periods. When $\beta < 1$ (the leading case in our analysis), c^* will be *below* this benchmark threshold. Because the worker's short-run preference places extra weight on the present, the worker is prone to delay incurring transaction costs and therefore stay at the default "too long." We will refer to this "excessive" delay as "procrastination." Of course, it is only excessive to the extent that one places normative weight on the worker's long-run preferences.

4.2 The Planner's Optimization Problem

The benign planner acting in the interest of workers sets the default contribution rate d in order to minimize the average loss function V for the workforce. Hence, the planner takes the long-run perspective and minimizes the exponentially discounted stream of losses (using

the exponential discount factor $\delta = 1$).¹⁸

The planner's decision problem takes the form

$$d^* = \arg \min_d \int l(s, d) f(s) ds,$$

where $l(s, d)$ is the expected value of a worker's loss function (integrating over the distribution of current opt-out costs):

$$l(s, d) \equiv E_c V(L(s, d), c).$$

Alternatively, we can write $l(s, d)$ using the Bellman Equation:

$$l(s, d) = \begin{cases} E(c|c \leq c^*) & \text{with probability } F(c^*) \\ L(s, d) + l(s, d) & \text{with probability } 1 - F(c^*) \end{cases}.$$

For analytical tractability, we assume $L(s, d) = \kappa(s - d)^2$.¹⁹ This functional form allows us to abuse notation and write $l(\Delta) = l(s, d)$, where $\Delta \equiv s - d$. Solving for $l(\Delta)$ yields the following proposition.

Proposition 2 *The worker's expected loss function from the planner's perspective is*

$$l(\Delta) = \begin{cases} 0 & \text{if } \Delta = 0 \\ \frac{\sqrt{2(\bar{c}-\underline{c})\beta(2-\beta)\kappa\Delta^2+(1-\beta)^2\underline{c}^2}}{\beta(2-\beta)} - \kappa\Delta^2 + \frac{\underline{c}}{\beta(2-\beta)} & \text{if } 0 < |\Delta| < \sqrt{\frac{\bar{c}(2-\beta)-\underline{c}\beta}{2\beta\kappa}} \\ \frac{\bar{c}+\underline{c}}{2} & \text{otherwise} \end{cases}. \quad (5)$$

Figure 7 graphs this function for three different parameter sets.²⁰ We now provide intuitive explanations for the shapes of the graphs. (We will discuss the shaded areas in the figure later.)

¹⁸To motivate this long-run perspective, it is enough to assume that regulations established by the planner in period t don't take effect until period $t+1$. Then every worker at every point in time will want the planner to minimize the loss function V .

¹⁹In firms that match employee 401(k) contributions up to a threshold, a savings rate below one's optimum may be more costly than a savings rate above it. In such cases, the analytically useful symmetry of our theoretical loss function is unrealistic.

²⁰Figure 7 also contains a discontinuity at 0. When $\Delta = 0$, the worker suffers no loss ($l(\Delta) = 0$), since the default savings rate is her optimal savings rate ($d = s$). Any worker for whom $\Delta \neq 0$ will eventually opt out of the default to avoid perpetually incurring flow costs. When the worker does opt out, she will incur some stochastic cost c , creating the discontinuity.

When $|\Delta|$ is in a neighborhood of zero, increasing $|\Delta|$ increases expected losses because the worker is moved further from her optimal savings rate. These losses plateau at the left and right boundaries of each panel in Figure 7 because above a certain value of $|\Delta|$, the flow losses are so large that the worker always opts out of the default in the first period. The worker then incurs an expected loss equal to the unconditional transaction cost expectation, $(\underline{c} + \bar{c})/2$. However, she does not suffer any flow losses, and hence her expected loss is insensitive to Δ . This corresponds to the case discussed above where $c^* \geq \bar{c}$.

If $\beta = 1$, there is no present-bias, and the expected loss function is weakly increasing in $|\Delta|$ everywhere. That is, workers are always weakly better off if the default is closer to their optimum. (See the left panel of Figure 7.) But if $\beta < 1$, there is an intermediate region of $|\Delta|$ where losses are above the plateaus. (See the center and right panels of Figure 7.) A worker in one of these “humps” is made better off if the default is moved further away from his optimum. This is because time-inconsistent workers have a propensity to excessively delay opting out. A bad default is like a “kick in the pants,” motivating a procrastinating worker to opt out of the default more quickly. For workers in the humps, the reduced procrastination losses from a worse default exceed the direct effect of higher flow losses. In particular, these workers are better off if the default is moved so far away from their optimum that they are in one of the plateau regions and are compelled to opt out immediately.

The lower β is, the worse is the tendency to procrastinate, and the larger are the humps. In the right panel of Figure 7, β is so small that *all* workers (except those for whom $s = d$) are weakly better off if they were compelled to opt out immediately rather than given the option to delay.

To derive the planner’s choice of d , we make a parametric assumption for the optimal savings rate distribution $f(s)$. For analytic tractability, we assume that s is uniformly distributed on the interval $[\underline{s}, \bar{s}]$, where $\underline{s} < \bar{s}$. Thus, the planner solves

$$d^* = \arg \min_d \int_{\underline{s}-d}^{\bar{s}-d} l(\Delta) d\Delta. \quad (6)$$

The planner’s optimization problem reduces to finding the limits of integration that minimize an area of fixed width under the expected loss function, corresponding to the shaded areas in Figure 7. The width of the region is the span of the optimal savings distribution, $\bar{s} - \underline{s}$. The location of the region is determined by the choice of the savings default d .

4.3 Characterizing Optimal Default Policies

The two key elements in the planner's problem are the shape of the expected loss function, determined largely by β , and the width of the integral bounds, $\bar{s} - \underline{s}$. We will present the optimal solutions graphically and give intuitions for the results before providing a precise mathematical characterization of the solutions.

There are only four classes of optimal solutions. Figure 8 shows the regions where each solution class obtains in $[\bar{s} - \underline{s}] \times \beta$ space. The planner is indifferent between default regimes on the boundaries between the regions. We will discuss each solution class in turn.

First, consider the southeast region of Figure 8. In this corner, agents have weak dynamic inconsistency problems (β is close to 1) and relatively homogeneous optimal savings rates ($\bar{s} - \underline{s}$ is close to zero). The optimal solution here is a “center” default where d is in the middle of the optimal savings rate support ($d^* = (\bar{s} + \underline{s})/2$), minimizing the average distance from workers' optimal savings rates. The left panel of Figure 7 illustrates the center default. The shaded region represents the social losses over the support of Δ values associated with the optimal default d^* . (Recall that $\Delta = s - d$ and $s \sim f(s)$.)

As β falls, the humps in the loss function grow, and eventually, so many workers fall in the humps under the center default that it is no longer optimal. Recall that workers in the humps are made better off if the default is moved further away from their optimum. The larger $\bar{s} - \underline{s}$ is, the faster losses in the humps accrue, so the western boundary of the center default region in Figure 8 is increasing in β . For low β values, the optimal default is an “active decision” default, where d is set far above or below the range of optimal savings rates. The flow loss from not opting out is large for all workers, so everybody is “forced” to act in the first period. In other words, this extreme choice of d is equivalent to requiring agents to make an active savings decision in the first period. Active decisions are optimal when β is low because potential procrastination losses are large relative to the option value of waiting for a low opt-out cost. By forcing everybody to act in the first period, the planner eliminates those procrastination losses at the cost of foregoing option value. Note that for sufficiently low values of β , the active decision regime is optimal for *all* values of $\bar{s} - \underline{s}$. The right panel in Figure 7 presents this case, where all workers (for whom $\Delta \neq 0$) are better off if forced to act immediately.

We call the third solution class “offset” defaults. In this solution class, the planner sets the default within the support of optimal worker saving rates but to one extreme of this

range so that workers fall into only one of the two humps. Offset defaults are optimal when both $\bar{s} - \underline{s}$ and β are large (the northeast region of Figure 8). The center panel of Figure 7 illustrates such a scenario. The offset default is a compromise between the active decision and center solutions. Because agents show little dynamic inconsistency, it is not efficient to force everyone to opt out in the first period when many could gain from being allowed to wait for a low-cost period to act. On the other hand, the range of optimal savings rates is large enough that a center solution would place too many workers in the humps. By using an offset default, the planner beneficially moves population mass from one of the humps to a plateau, while still letting those with optimal rates near the new default exploit the option value of waiting.

We can also characterize the boundaries of the offset default region. Figure 8 shows that the indifference curve between the offset and active decision regions is vertical. This is because if the heterogeneity of agents' optimal savings rates increased slightly, the marginal saver at the boundary of the Δ support would be compelled to act immediately under both regimes. Thus, changing $\bar{s} - \underline{s}$ cannot make one regime more attractive than the other; only β can change this relation.

Now consider the boundary of this region when $\beta = 1$. In this case, the agent is time-consistent, so the expected loss function is weakly increasing in $|\Delta|$, as in the left panel of Figure 7. The center default is *always* a solution for this scenario. But if $\bar{s} - \underline{s}$ is large enough that some workers always opt out immediately under the center default, the planner can move the default slightly away from the center without affecting the average loss function, since workers at both boundaries of Δ 's support are still acting immediately. Thus, the solution set when $\beta = 1$ and $\bar{s} - \underline{s}$ is large includes a neighborhood around the center default.

Another interesting property of these optimal policies is the existence of a “global indifference point” at which the center default, offset default, and active decision regimes produce equal total expected losses for workers. We can also prove that the active decision region grows when the loss from a given deviation from one's optimal savings rate—controlled by the parameter κ —increases. Increasing the volatility of opt-out costs (i.e., raising $\bar{c} - \underline{c}$) increases the option value to waiting for a low-cost period and hence decreases the attractiveness of extreme defaults that compel immediate action.

In the following subsection, we formalize the heuristic analysis presented above. Readers not interested in the mathematical details of the optimal defaults should skip this subsection.

4.3.1 Optimal Defaults: A Formal Characterization

The following theorem characterizes the optimal default policies for different values of $\bar{s} - \underline{s}$ and β .

Theorem 3 *Suppose workers are sophisticated and optimal savings rates are uniformly distributed between \underline{s} and \bar{s} . Then there are four sets of candidate solutions for the optimal default d^* : the center default $D_c = \{\frac{\bar{s} + \underline{s}}{2}\}$, the offset defaults $D_o = \{\underline{s} + \sqrt{\frac{\bar{c}\beta - \underline{c}(2-\beta)}{2(2-\beta)\kappa}}, \bar{s} - \sqrt{\frac{\bar{c}\beta - \underline{c}(2-\beta)}{2(2-\beta)\kappa}}\}$, the active decision defaults $D_a = \mathbb{R} \setminus (\underline{s} - \sqrt{\frac{\bar{c}(2-\beta) - \underline{c}\beta}{2\beta\kappa}}, \bar{s} + \sqrt{\frac{\bar{c}(2-\beta) - \underline{c}\beta}{2\beta\kappa}})$, and the center neighborhood defaults $D_{cn} = [\underline{s} + \sqrt{(\bar{c} - \underline{c})/2\kappa}, \bar{s} - \sqrt{(\bar{c} - \underline{c})/2\kappa}]$. Let $\beta^* = 1 - \frac{\bar{c} - \underline{c}}{\bar{c} + \underline{c}}$, and define $\hat{\beta}$ implicitly with the equation*

$$\int_{-\sqrt{\frac{\bar{c}\beta - \underline{c}(2-\beta)}{2(2-\beta)\kappa}}}^{\sqrt{\frac{\bar{c}(2-\beta) - \underline{c}\beta}{2\beta\kappa}}} \left(\frac{\bar{c} + \underline{c}}{2} - l(\Delta) \right) d\Delta = 0.$$

Also, implicitly define the function $[\bar{s} - \underline{s}]^{a/c}(\beta)$ over the range $\beta \in (\beta^*, \hat{\beta}]$ by the equality

$$\int_{-\frac{\bar{s} - \underline{s}}{2}}^{\frac{\bar{s} - \underline{s}}{2}} \left(\frac{\bar{c} + \underline{c}}{2} - l(\Delta) \right) d\Delta = 0$$

and the function $[\bar{s} - \underline{s}]^{o/c}(\beta)$ over the range $\beta \in [\hat{\beta}, 1)$ by

$$\int_{\frac{\bar{s} - \underline{s}}{2}}^{\sqrt{\frac{\bar{c}(2-\beta) - \underline{c}\beta}{2\beta\kappa}}} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta - \int_{-\frac{\bar{s} - \underline{s}}{2}}^{-\sqrt{\frac{\bar{c}\beta - \underline{c}(2-\beta)}{2(2-\beta)\kappa}}} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta = 0.$$

Then

$$d^* \in \begin{cases} D_a & \text{if } \beta \in (0, \beta^*] \text{ or } \beta \in (\beta^*, \hat{\beta}] \text{ and } \bar{s} - \underline{s} \geq (\bar{s} - \underline{s})^{a/c}(\beta) \\ D_o & \text{if } \beta \in [\hat{\beta}, 1) \text{ and } \bar{s} - \underline{s} \geq (\bar{s} - \underline{s})^{o/c}(\beta) \\ D_c & \text{if } \beta \in (\beta^*, \hat{\beta}] \text{ and } \bar{s} - \underline{s} \leq (\bar{s} - \underline{s})^{a/c}(\beta) \text{ or } \beta \in (\hat{\beta}, 1) \text{ and } \bar{s} - \underline{s} \leq (\bar{s} - \underline{s})^{o/c}(\beta) \\ D_{cn} & \text{if } \beta = 1 \text{ and } \bar{s} - \underline{s} \geq (\bar{s} - \underline{s})^{o/c}(1) \end{cases}$$

$[\bar{s} - \underline{s}]^{a/c}(\beta)$ is monotonically increasing in β . At the global indifference point $(\hat{\beta}, [\bar{s} - \underline{s}]^{o/c}(\hat{\beta}))$, the planner is indifferent among the center default, an offset default, and active decisions.

The theoretical appendix contains the proof of this theorem, but we provide a heuristic proof here.

The proof begins by establishing basic properties for $l(\Delta)$, including symmetry, continuity, and differentiability at all points except the knots. Next, we identify those regions of $l(\Delta)$ which lie above $\frac{\bar{c}+\underline{c}}{2}$. We also calculate the derivatives of l and prove that $l(\Delta)$ has global maxima at $\Delta = \pm\sqrt{\frac{(\bar{c}-\underline{c})^2-\underline{c}^2(1-\beta)^2}{2(\bar{c}-\underline{c})\beta(2-\beta)\kappa}}$ and a global minimum at $\Delta = 0$. The next step shows that if a and b are on different sides of the same hump and $l(a) = l(b)$, then $|l'(a)| < |l'(b)|$; in other words, the outer portion of the humps is steeper than the portion closer to $\Delta = 0$. This fact is critical because it implies that, starting from the maximum of l at the top of each hump, the expected loss for workers falls faster when they move away from the default than when they move towards it. After showing that $\frac{\partial l}{\partial \beta} \leq 0$, we complete the preliminary analysis by solving for the first and second order conditions of the planner's maximization problem and identifying the three candidate solution sets.

We then characterize the optimal solutions. When $\beta \in (0, \beta^*]$, we prove that active decisions are the only possible solution. Because $l(\Delta) > \frac{\bar{c}+\underline{c}}{2}$ for all $\Delta \neq 0$, the proof is trivial. Similarly, when $\beta = 1$, the center default—as well as those defaults in the immediate neighborhood for large enough $\bar{s} - \underline{s}$ —must always be optimal because of the weak monotonicity of $l(\Delta)$ in $|\Delta|$.

The final step of the proof is to choose among the multiple candidate solutions in the large intermediate range where $\beta \in (\beta^*, 1)$ by determining the boundaries between the regions. In addition, we formalize the intuition given above for the positive monotonicity of the boundary between the active decision and center default regions. We also prove the existence of the global indifference point by transitivity of the planner's preferences.

4.4 The Case of Naive Workers

Our analysis to this point has assumed that workers understand their own time inconsistency. Our qualitative conclusions do not change if we assume instead that workers are naive about their self-control problems and believe they will be time-consistent in the future (O'Donoghue and Rabin 1999a,b). While the mathematics behind the optimal default policy for naive workers is different from that for sophisticated agents, the intuition is similar. Workers who exhibit quasi-hyperbolic discounting may benefit from being forced to act in

the first period regardless of the stochastic opt-out cost.

Theorem 4 *For given parameter values, if active decisions are optimal for sophisticated workers, then active decisions are also optimal for naive workers.*

This theorem follows directly from naifs’ mistaken belief that their future selves will not suffer from dynamic inconsistency. Because of this belief, naifs’ expectation of future losses from inaction today is lower than sophisticates’ correct expectation. Therefore, the naif is less willing than the sophisticate to incur the stochastic opt-out cost today. Now suppose the parameter values are such that an active decision regime is optimal for sophisticates. This implies that sophisticates’ self-control problem is bad enough that compelling them to immediate action is welfare-improving. Naifs, however, have a worse self-control problem than sophisticates, so an active decision regime must be optimal for naifs as well.

5 Model Discussion

Having characterized the optimal 401(k) enrollment regime, we now connect our theoretical results to real-world 401(k) institutions. We classify 401(k) enrollment regimes into three types: standard enrollment, automatic enrollment, and active decisions.²¹ Under standard enrollment, employees have a default savings rate of zero and are given the option to raise this savings rate. Under automatic enrollment, employees have a default savings rate that is strictly positive and are given the option to change that savings rate (including opting out of the plan). Under active decisions, employees face no default and instead must affirmatively pick their own savings rate.

In our theoretical framework, the standard enrollment regime is an example of an offset default, since a 0% savings rate lies at one end of the optimal savings rate distribution.²² The automatic enrollment regime, as typically implemented, is an example of an offset default with a low contribution rate, although in some firms with higher default contribution rates

²¹The highly successful SMT Plan (Bernartzi and Thaler 2001) is another important type of 401(k) enrollment regime, but its complexity makes it hard to apply to our model. The SMT Plan has many key features, including an automatic accelerator that increases the contribution rate at each pay raise.

²²The standard enrollment default savings rate is on the boundary of the action space, but this location is consistent with the concept of an offset default if savings preferences cross the boundary because some households would like to dissave.

it is more like a center default.²³ Finally, a real-world active decision enrollment regime is equivalent to picking a default contribution rate that is so costly to all workers that everyone chooses to take action immediately and move to his or her individually optimal savings rate.

When β is close to one, a center default is optimal if employee savings preferences are relatively homogeneous. In practice, employees will have relatively homogeneous savings preferences when the workforce is demographically homogeneous (e.g., has a narrow range of ages) or if a generous employer match causes most employees to want to save at the match threshold. As savings preferences become more heterogeneous, offset defaults such as standard enrollment and automatic enrollment with conservative defaults become more attractive. Standard enrollment and automatic enrollment with conservative defaults are also more attractive when a substantial fraction of employees have a low optimal savings rate in the 401(k). This may be the case if the company offers a generous defined benefit pension, if the company employs many low-wage workers who will have a high Social Security replacement rate, or if the company employs primarily young workers who would like to dissave at the present because they expect high future income growth.

If employees have a strong tendency to procrastinate (β is far below one), then active decisions are optimal regardless of how heterogeneous savings preferences are.²⁴ The long-run stickiness of the default savings rate and asset allocation under automatic enrollment (Madrian and Shea 2001; Choi et al 2004) supports the concern that many employees have an excessive tendency to delay opting out of defaults; it typically takes more than two years for the median employee to opt out of a 2 or 3% savings rate default. Active decisions eliminate the procrastination problem at the expense of losing the option value of waiting for a low-cost period to act.

6 Conclusion

This paper identifies and analyzes the active decision alternative to default-based 401(k) enrollment processes. The active decision approach forces employees to explicitly choose

²³Choi et al (2004) report that three-quarters of companies with automatic enrollment set their default contribution rate at 2% or 3% of pay, which is much lower than the 7% average 401(k) savings rate selected by employees when they make an affirmative choice.

²⁴However, in the limit case of zero heterogeneity, the planner should adopt automatic enrollment with a default at the universal optimal savings rate.

between the options of enrollment and non-enrollment in the 401(k) plan without advantaging either of these outcomes. We find that the fraction of employees who enroll in the 401(k) three months after hire is 28 percentage points higher under an active decision regime than under a standard opt-in enrollment regime. The active decision regime also raises average saving rates and accumulated 401(k) balances. The distribution of new employees' savings rates under active decisions matches the distribution it takes three years to achieve under standard enrollment.

We also present a model of the employee's 401(k) enrollment choice. Under this framework, three socially optimal 401(k) enrollment procedures emerge. We describe conditions under which the optimal enrollment regime is either automatic enrollment (i.e., default enrollment), standard opt-in enrollment (i.e., default non-enrollment), or active decisions (i.e., no default and compulsory choice). The active decision regime is socially optimal when consumers have heterogeneous savings preferences and a strong tendency to procrastinate.

Although active decision 401(k) enrollment regimes are not widely utilized today, we anticipate that the evidence reported in this paper will lead to broader adoption of such schemes. In the current technological environment, an active decision enrollment regime need not necessarily take the form of the paper-and-pencil system that we studied. Instead, active decision systems could be designed to take advantage of the efficiencies available with electronic enrollment. For example, a firm could require new employees to visit a Web site where they would actively elect to enroll in or opt out of the 401(k) plan, perhaps in conjunction with electing other benefits or providing other information relevant to the company.²⁵ Firms could also compel non-participating employees to make an active decision during each annual open enrollment period. This would ensure that non-participating employees rethink their non-participation in the 401(k) at least once a year.

The active decision approach to increasing 401(k) participation has some attractive features relative to other savings promotion schemes. Compared to financial education, requiring an active decision is a more cost-effective way to boost 401(k) participation. Active decisions are also more cost-effective than increasing the employer match rate, since increases in the match rate apply even to employees who would have enrolled with little or no match. Finally, requiring individuals to make an active decision represents a weaker alternative to the standard paternalism implicit in specifying a default that will advantage one particular

²⁵Workers without access to computers could submit paper forms.

course of action over another. Active decision interventions are designed principally to force a decision-maker to think about a problem. This is still a type of paternalism, but it does not presuppose an answer to the problem.²⁶

We should note that we are not opposed to financial education, employer matching, or automatic enrollment. Rather, we view all of these, along with active decisions, as potentially complementary approaches to fostering increased and higher-quality 401(k) participation. These tools, when implemented well, can greatly enhance the retirement income security of a company's current and future employees.

Active decision interventions will be useful in many situations where consumer heterogeneity implies that one choice isn't ideal for everyone (e.g., the selection of a health plan or automobile insurance²⁷) and firms or governments feel uncomfortable implementing employee-specific defaults (e.g., if such employee-specific defaults are viewed as "advice" with fiduciary consequences).²⁸ In contrast, defaults will have a natural role to play in cases where a large degree of homogeneity is appropriate and household decision-makers have limited expertise (e.g., portfolio allocation).²⁹ Future research should explore active decision experiments in other decision domains and compare the relative efficacy of active decision and default-based systems, as well as hybrid systems which may turn out to be the most useful approach of all.

²⁶We view active decisions as an example of libertarian paternalism (Sunstein and Thaler 2003).

²⁷The active decision approach to purchasing automobile insurance is widely used. Drivers cannot, in general, register their cars without obtaining insurance. But the government does not specify a default insurance contract for drivers; rather, it requires drivers to obtain their own insurance—to make an active decision. The model in the paper suggests that there is a good justification for this approach: there is likely to be substantial heterogeneity in individual preferences over insurance policy types and companies.

²⁸An example of an intriguing employee-specific default is a default savings rate that increases with the employee's age.

²⁹See Benartzi and Thaler (2003) and Cronqvist and Thaler (2004) for evidence on poor asset allocation choices.

7 Appendix

7.1 The Sophisticated Worker's Optimization Problem

Proposition 1 *A sophisticated worker will opt out of the default if and only if she draws a transaction cost $c < c^*$, where*

$$c^* = \frac{\underline{c} + \sqrt{\underline{c}^2 [1 - (2 - \beta)\beta] + 2\beta(2 - \beta)(\bar{c} - \underline{c})L}}{2 - \beta}. \quad (7)$$

Proof. Fix d . Note that if $s \neq d$, then $c^* > \underline{c}$; otherwise the agent would never act and would lose $W = L + \beta \sum_{t=1}^{\infty} L = \infty$. Also, the agent must be indifferent between acting and not acting in the present, so

$$c^* = \beta [L + EV(L, c', c^*)] \quad (8)$$

where

$$V(L, c', c^*) = \begin{cases} c' & \text{if } c' < c^* \\ L + E_{+1}V(L, c'', c^*) & \text{if } c' \geq c^* \end{cases}, \quad (9)$$

c' is the cost draw for the current period, and c'' is the cost draw for the following period. Letting $\phi = EV(L, c', c^*)$, at c^* the function $V(\cdot)$ must also be a fixed point for the future value function. Generalizing over periods and then integrating over draws of c implies that

$$\phi = \frac{c^{*2} - 2Lc^* + 2L\bar{c} - \underline{c}^2}{2(c^* - \underline{c})}. \quad (10)$$

Substituting (10) into (8) gives us a quadratic equation in c^* . Using the quadratic formula and choosing the upper root so that $c^* > \underline{c}$, we obtain equation (7). ■

Proposition 2 *The worker's expected loss function from the planner's perspective is*

$$l(\Delta) = \begin{cases} 0 & \text{if } \Delta = 0 \\ \frac{\sqrt{2(\bar{c} - \underline{c})\beta(2 - \beta)\kappa\Delta^2 + (1 - \beta)^2\underline{c}^2}}{\beta(2 - \beta)} - \kappa\Delta^2 + \frac{\underline{c}}{\beta(2 - \beta)} & \text{if } 0 < |\Delta| < \sqrt{\frac{\bar{c}(2 - \beta) - \underline{c}\beta}{2\beta\kappa}} \\ \frac{\bar{c} + \underline{c}}{2} & \text{if } \sqrt{\frac{\bar{c}(2 - \beta) - \underline{c}\beta}{2\beta\kappa}} < |\Delta| \end{cases}. \quad (11)$$

Proof. The expected loss function for workers from the planner's perspective is exactly

$EV(L, c', c^*)$ if $\underline{c} < c^* < \bar{c}$. Substituting (7) into (10) yields

$$l(\Delta) = \frac{\sqrt{2(\bar{c} - \underline{c})\beta(2 - \beta)\kappa\Delta^2 + (1 - \beta)^2\underline{c}^2}}{\beta(2 - \beta)} - \kappa\Delta^2 + \frac{\underline{c}}{\beta(2 - \beta)}.$$

If $c^* \geq \bar{c}$, the agent will act immediately regardless of the opt-out cost. In this case, $l(\Delta) = \frac{\bar{c} + \underline{c}}{2}$, the average opt-out cost drawn in the first period. The condition $c^* \geq \bar{c}$ holds when

$$|\Delta| < \sqrt{\frac{\bar{c}(2 - \beta) - \underline{c}\beta}{2\beta\kappa}}.$$

Finally, the agent never opts out or incurs any flow losses when $\Delta = 0$, so $l(\Delta) = 0$. ■

7.2 The Planner's Optimization Problem

Before proving Theorem 3, we establish several useful lemmas. For notational ease, we denote the knot point of the function l by $K \equiv \sqrt{\frac{\bar{c}(2 - \beta) - \underline{c}\beta}{2\beta\kappa}}$.

Lemma 5 *The function $l(\Delta)$ is symmetric around $\Delta = 0$, continuous when $\Delta \neq 0$, and twice-differentiable with respect to all its parameters when $\Delta \neq 0$ and $\Delta \neq \pm K$.*

Going forward, we assume $\Delta \geq 0$; the case $\Delta < 0$ is equivalent by the symmetry of the function $l(\Delta)$. We now prove some lemmas regarding the shape of l . It is useful to define $\beta^* = 1 - \frac{\bar{c} - \underline{c}}{\bar{c} + \underline{c}}$. We first characterize l for $\beta \in (0, \beta^*]$ and then move on to the case where $\beta \in (\beta^*, 1]$.

Lemma 6 *If $\beta \in (0, \beta^*]$, then $l(\Delta) > \frac{\bar{c} + \underline{c}}{2}$ when $0 < \Delta < K$.*

Proof. The only roots of $l(\Delta) = \frac{\bar{c} + \underline{c}}{2}$ over the domain $\Delta \in [0, K]$ satisfy the condition

$$\kappa\Delta^2 = \frac{\bar{c}\beta - \underline{c}(2 - \beta)}{2(2 - \beta)} \text{ or } \frac{\bar{c}(2 - \beta) - \underline{c}\beta}{2\beta} \quad (12)$$

Note that the one of the roots is K and the other does not exist in the proper region of Δ when $\beta \in (0, \beta^*]$. Because

$$l'(\Delta)|_{\Delta \rightarrow K_-} = -2\kappa K \left(\frac{(1 - \beta)\bar{c}}{(2 - \beta)\bar{c} - \underline{c}} \right) < 0,$$

the lemma follows immediately. ■

At this point, we define $E \equiv \sqrt{\frac{\bar{c}\beta - \underline{c}(2-\beta)}{2(2-\beta)\kappa}}$ and $M \equiv \sqrt{\frac{(\bar{c}-\underline{c})^2 - \underline{c}^2(1-\beta)^2}{2(\bar{c}-\underline{c})\beta(2-\beta)\kappa}}$ for notational convenience when $\beta \in (\beta^*, 1]$. Note that if $\beta \in (\beta^*, 1)$, then $0 < E < M < K$. When $\beta = 1$, $E = M = K$.

Lemma 7 *If $\beta \in (\beta^*, 1)$, then*

- $l'(\Delta) > 0$ when $0 < \Delta < M$
- $l'(M) = 0$
- $l'(\Delta) < 0$ when $M < \Delta < K$
- $l'(\Delta) = 0$ when $K < \Delta$

The point $\Delta = M$ is a global maximum. When $\beta = 1$,

- $l'(\Delta) > 0$ when $0 < \Delta < K$
- $l'(\Delta) = 0$ when $K \leq \Delta$

Proof. The signing of the derivatives is straightforward. Note that M only exists as a critical point if

$$(\bar{c} - \underline{c})^2 - \underline{c}^2(1 - \beta)^2 \geq 0.$$

But $\beta \in (\beta^*, 1]$ implies that

$$(\bar{c} - \underline{c})^2 - \underline{c}^2(1 - \beta)^2 > (\bar{c} - \underline{c})^2 - \underline{c}^2(1 - \beta^*)^2 = 1 - \frac{\underline{c}^2}{(\bar{c} + \underline{c})^2} > 0,$$

so the critical point always exists in this range of parameters. It is apparent from the signs of $l'(\Delta)$ over $\Delta \in (0, \infty)$ that M is a global maximum. When $\beta = 1$, we get $M = K = \sqrt{\frac{\bar{c}-\underline{c}}{2\kappa}}$. The part of the function with a downward slope is “squeezed out.”

■

Lemma 8 *If $\beta \in (\beta^*, 1]$, then*

- $l(\Delta) < \frac{\bar{c} + \underline{c}}{2}$ when $0 \leq \Delta < E$

- $l(E) = \frac{\bar{c}+c}{2}$
- $l(\Delta) > \frac{\bar{c}+c}{2}$ when $E < \Delta < K$

Proof. We know from (12) that $l(\Delta) = \frac{\bar{c}+c}{2}$ when $\Delta = K$ or $\Delta = E$. Since $\beta > \beta^*$, the latter root exists. Lemma 7 and the continuity of l implies that $l(\Delta) > \frac{\bar{c}+c}{2}$ when $E < \Delta < K$ and $l(\Delta) < \frac{\bar{c}+c}{2}$ when $0 < \Delta < E$. That $l(0) < \frac{\bar{c}+c}{2}$ is obvious. ■

Lemma 9 *Let $\Delta \in (M, K)$ and $\beta \in (\beta^*, 1)$. Define the function $y : (M, K) \rightarrow (E, M)$ implicitly by $l(\Delta) = l(y(\Delta))$. Then $l'(y(\Delta)) < -l'(\Delta)$.*

Proof. Since $l(\cdot)$ is strictly increasing on (E, M) and strictly decreasing on (M, K) , we may define inverses $l_{s-}^{-1}(\cdot)$ onto (E, M) and $l_{s+}^{-1}(\cdot)$ onto (M, K) . Notice that $l_{s-}^{-1}(\zeta) = y(l_{s+}^{-1}(\zeta))$. Since $l(\Delta)$ is of the form $b\sqrt{a^2 + \Delta^2} - c\Delta^2 + d$, it is clear that $\forall \Delta \in (E, K)$, $l'''(\Delta) < 0$. Therefore,

$$\forall y \in (E, M) \text{ and } \Delta \in (M, K), l''(y) > l''(\Delta). \quad (13)$$

Now, we claim that

$$\exists \varepsilon > 0 \text{ such that } \forall \Delta \in (M, M + \varepsilon), l'(y(\Delta)) < -l'(\Delta). \quad (14)$$

To prove this, consider the series expansion of $l(\Delta)$ about M . Since our desired result is invariant under translation, assume without loss of generality that $M = 0$ and $l(M) = l(0) = 0$. Then noting that $l'(0) = 0, l''(0) < 0, l'''(0) < 0$, we get

$$l(\Delta) = \alpha\Delta^2 + \gamma\Delta^3 + O(\Delta^4); \alpha, \gamma < 0.$$

Note that for small but nonzero values of $|\Delta|$, $l(\Delta) < 0$. It is then straightforward to verify that

$$\begin{aligned} l_+^{-1}(\zeta) &= \sqrt{\frac{\zeta}{\alpha}} - \frac{\gamma\zeta}{2\alpha^2} + O(\zeta^{3/2}) \\ l'(\Delta) &= 2\alpha\Delta + 3\gamma\Delta^2 + O(\Delta^3) \\ l'(l_+^{-1}(\zeta)) &= -2\sqrt{\alpha\zeta} + \frac{2\gamma}{\alpha}\zeta + O(\zeta^{3/2}). \end{aligned}$$

Similarly,

$$l'(l_-^{-1}(\zeta)) = 2\sqrt{\alpha\zeta} + \frac{2\gamma}{\alpha}\zeta + O(\zeta^{3/2})$$

Setting $\Delta = l_+^{-1}(\zeta)$, for small but nonzero values of Δ (which implies small negative values of ζ),

$$l'(y(\Delta)) = l'(l_+^{-1}(\zeta)) < -l'(l_+^{-1}(\zeta)) = -l'(\Delta),$$

which proves (14). Now assume a contradiction to the lemma: $\exists \Delta \in (M, K)$ such that $l'(y(\Delta)) \geq -l'(\Delta)$. Let $X_{\geq} \equiv \{\Delta : \Delta \in (M, K), l'(y(\Delta)) \geq -l'(\Delta)\}$ and $\Delta_{\geq} \equiv \inf(X_{\geq})$. From (14), $\Delta_{\geq} > M$. It follows from continuity that $l'(y(\Delta_{\geq})) = -l'(\Delta_{\geq})$. Combining this with (13) and noting that $\frac{l''(\Delta)}{l'(\Delta)} = \frac{dl'(\Delta)}{d\Delta} \frac{d\Delta}{dl(\Delta)} = \frac{dl'(\Delta)}{dl(\Delta)}$,

$$\frac{l''(y(\Delta_{\geq}))}{l'(y(\Delta_{\geq}))} = \frac{dl'(y(\Delta_{\geq}))}{dl(y(\Delta_{\geq}))} > -\frac{dl'(\Delta_{\geq})}{dl(\Delta_{\geq})} = -\frac{l''(\Delta_{\geq})}{l'(\Delta_{\geq})}. \quad (15)$$

Consider a perturbation of Δ that increases the value of $l(\Delta)$. From (15) and $l'(\Delta_{\geq}) < 0$, we see that $\exists \varepsilon > 0$ such that $l'(y(\Delta_{\geq} - \varepsilon)) > -l'(\Delta_{\geq} - \varepsilon)$. But this contradicts the definition of Δ_{\geq} . ■

The final preliminary step characterizes the relationship between l and β .

Lemma 10 $\frac{\partial l}{\partial \beta} \Big|_{\Delta} < 0$ when $\Delta \in (0, K)$. $\frac{\partial l}{\partial \beta} \Big|_{\Delta} = 0$ when $\Delta \notin (0, K)$.

Proof. When $\Delta = 0$, $l = 0$ always, so $\frac{\partial l}{\partial \beta} \Big|_{\Delta=0} = 0$. For $0 < \Delta < K$, algebra yields

$$\frac{\partial l}{\partial \beta} = -\frac{2\underline{c}(1-\beta)}{\beta^2(2-\beta)^2} - \frac{(1-\beta)[2(\bar{c}-\underline{c})\beta(2-\beta)\kappa\Delta^2 + \underline{c}^2(2(1-\beta)^2 + \beta(2-\beta))]}{\beta^2(2-\beta)^2\sqrt{2(\bar{c}-\underline{c})\beta(2-\beta)\kappa\Delta^2 + \underline{c}^2(1-\beta)^2}},$$

which is negative since both fractions are positive. For $\Delta \geq K$, $\frac{\partial l}{\partial \beta} = \frac{\partial(\frac{\bar{c}+\underline{c}}{2})}{\partial \beta} = 0$. ■

We now solve the planner's problem and characterize the optimal solution d^* . The result is presented in a number of lemmas which together imply Theorem 3.

Lemma 11 *A solution d^* to the planner's problem must satisfy the conditions*

$$l(\bar{s} - d^*) = l(\underline{s} - d^*) \text{ and } l'(\bar{s} - d^*) \geq l'(\underline{s} - d^*). \quad (16)$$

Proof. Define $Z(d) \equiv \int_{\underline{s}-d}^{\bar{s}-d} l(\Delta) d\Delta$. The planner will solve the program $d^* = \arg \min_d Z(d)$. The two conditions are the first order condition ensuring a stationary point and the second order condition ensuring a local minimum. ■

Lemma 12 *If $\beta \in (0, \beta^*]$, then $d^* \in D_a$, where $D_a = (-\infty, \underline{s} - K] \cup [\bar{s} + K, \infty)$.*

Proof. By Lemma 6, $l(\Delta) > \frac{\bar{c} + c}{2}$ for all $0 < |\Delta| < K$. Thus, if $d \notin D_a$, then $\int_{\underline{s}-d}^{\bar{s}-d} l(\Delta) d\Delta > (\bar{s} - \underline{s}) \frac{\bar{c} + c}{2}$. But if $d \in D_a$, then $\int_{\underline{s}-d}^{\bar{s}-d} l(\Delta) d\Delta = (\bar{s} - \underline{s}) \frac{\bar{c} + c}{2}$. All $d \in D_a$ trivially satisfy the first and second order conditions in (16). ■

Lemma 13 *If $\beta \in (\beta^*, 1)$, then all the defaults satisfying the conditions of Lemma 11 are members of the set $D_c \cup D_o \cup D_a$, where $D_c = \{(\bar{s} + \underline{s})/2\}$, $D_o = \{\underline{s} + E, \bar{s} - E\}$, and $D_a = \mathbb{R} \setminus (\underline{s} - K, \bar{s} + K)$.*

Proof. Points in $D_c \cup D_a$ satisfy the first order condition in (16). Points in D_o satisfy the first order condition if $\bar{s} - \underline{s} \geq K + E$. Lemma 9 implies points in these three sets are the only ones that can satisfy the second order condition. ■

Lemma 14 *Suppose that $\beta = 1$. If $\bar{s} - \underline{s} \leq 2K$ then $d^* = \frac{\bar{s} + \underline{s}}{2}$. If $\bar{s} - \underline{s} > 2K$ then $d^* \in [\underline{s} + K, \bar{s} - K]$.*

Proof. It is easy to verify that the conditions of Lemma 11 are satisfied for the proposed d^* in both cases. Lemma 7 states that $l(\Delta)$ is weakly increasing in $|\Delta|$, so the center default $\frac{\bar{s} + \underline{s}}{2}$ is always an optimum. If $\bar{s} - \underline{s} > 2K$, the endpoints of the integral $Z(\frac{\bar{s} + \underline{s}}{2})$ are outside $[-K, K]$. Because $l'(\Delta) = 0 \forall \Delta \notin (-K, K)$, the planner is indifferent among defaults in the interval $[\underline{s} + K, \bar{s} - K]$. ■

Lemma 15 *Let $\bar{s} - \underline{s} \geq K + E$, so that points in D_o satisfy the conditions of Lemma 11. Then there exists $\hat{\beta} \in (\beta^*, 1)$ such that if $\beta > \hat{\beta}$, the planner always prefers a default in D_o to any default in D_a . If $\beta < \hat{\beta}$, the planner always prefers a default in D_a to any default in D_o . $\hat{\beta}$ is implicitly defined by the equality*

$$\int_{-E(\hat{\beta})}^{K(\hat{\beta})} l(\Delta) d\Delta = \frac{\bar{c} + c}{2}(K + E). \quad (17)$$

Proof. The difference between total expected losses for workers when $d^* = d_a \in D_a$ and $d^* = d_o \in D_o$ is

$$Z(d_a) - Z(d_o) = \int_{-E}^K \left(\frac{\bar{c} + c}{2} - l(\Delta) \right) d\Delta.$$

Then

$$\frac{\partial (Z(d_a) - Z(d_o))}{\partial \beta} = - \int_{-E}^K \frac{\partial}{\partial \beta} l(\Delta) d\Delta > 0,$$

where the last inequality is implied by Lemma 10. We know from Lemma 14 that when $\beta = 1$, d_o is preferred to d_a , and from Lemma 12 that the opposite is true when $\beta \leq \beta^*$. Since $Z(d_a) - Z(d_o)$ is continuous and monotonic with respect to β , there is exactly one $\hat{\beta} \in (\beta^*, 1)$ such that $Z(d_o) = Z(d_a)$. When $0 < \beta < \hat{\beta}$, $Z(d_a) < Z(d_o)$, and when $\hat{\beta} < \beta \leq 1$, the opposite is true. ■

Lemma 16 Define the function $[\bar{s} - \underline{s}]^{o/c}(\beta)$ implicitly with the equality

$$\int_{\frac{\bar{s}-\underline{s}}{2}}^K \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta - \int_{-\frac{\bar{s}-\underline{s}}{2}}^{-E} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta = 0 \quad (18)$$

for $\beta \in [\hat{\beta}, 1)$. Along the implicit function the planner is indifferent between defaults $d_o \in D_o$ and $d_c = \frac{\bar{s} + \underline{s}}{2}$. For a given β in this range, the planner strictly prefers d_o over d_c if and only if $\bar{s} - \underline{s} > [\bar{s} - \underline{s}]^{o/c}(\beta)$.

Proof. When $\bar{s} - \underline{s} < K + E$, d_o does not satisfy the conditions in Lemma 11, and $Z'(d)$ implies that d_c will be preferred over d_o . When $\bar{s} - \underline{s} \geq K + E$, the indifference curve between the center default and the offset default is defined by

$$\begin{aligned} Z(d_o) - Z(d_c) &= \int_{-E}^{\bar{s}-\underline{s}-E} l(\Delta) d\Delta - \int_{-\frac{\bar{s}-\underline{s}}{2}}^{\frac{\bar{s}-\underline{s}}{2}} l(\Delta) d\Delta \\ &= \int_{-E}^K \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta - \int_{-\frac{\bar{s}-\underline{s}}{2}}^{\frac{\bar{s}-\underline{s}}{2}} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta \\ &= \int_{\frac{\bar{s}-\underline{s}}{2}}^K \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta - \int_{-\frac{\bar{s}-\underline{s}}{2}}^{-E} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta = 0. \end{aligned}$$

Let $[\bar{s} - \underline{s}]^{o/c}(\beta)$ denote the implicit function of β defined by the indifference curve above. It is clearly continuous in β . Note that for a fixed β ,

$$\frac{\partial [Z(d_o) - Z(d_c)]}{\partial (\bar{s} - \underline{s})} \Big|_{\bar{s}-\underline{s}=[\bar{s}-\underline{s}]^{o/c}} = \left[\frac{\bar{c} + \underline{c}}{2} - l\left(\frac{\bar{s} - \underline{s}}{2}\right) \right]_{\bar{s}-\underline{s}=[\bar{s}-\underline{s}]^{o/c}} \leq 0, \quad (19)$$

with strict inequality when $\frac{\bar{s}-\underline{s}}{2} < K$. Also, when $\beta \in (\beta^*, 1)$,

$$\begin{aligned} Z(d_o) - Z(d_c)|_{\bar{s}-\underline{s} \geq 2K} &= - \int_{-K}^{-E} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta < 0 \\ Z(d_o) - Z(d_c)|_{\bar{s}-\underline{s} = K+E} &= \left(\int_{\frac{K+E}{2}}^K - \int_{-\frac{K+E}{2}}^{-E} \right) l(\Delta) d\Delta > 0, \end{aligned} \quad (20)$$

with the latter inequality following from Lemma 9. (19) and (20) imply that $[\bar{s} - \underline{s}]^{o/c}(\beta)$ exists and is well-defined. So when $\bar{s} - \underline{s}$ is above the indifference curve, the offset default is preferred, and the opposite holds for the region below the curve. ■

Lemma 17 Define the function $[\bar{s} - \underline{s}]^{a/c}(\beta)$ implicitly with the equality

$$\int_{-\frac{\bar{s}-\underline{s}}{2}}^{\frac{\bar{s}-\underline{s}}{2}} \left(\frac{\bar{c} + \underline{c}}{2} - l(\Delta) \right) d\Delta = 0. \quad (21)$$

for $\beta \in [\beta^*, \hat{\beta}]$. Along the implicit function the planner will be indifferent between defaults $d_a \in D_a$ and $d_c = \frac{\bar{s} + \underline{s}}{2}$. For a given β in this range, the planner strictly prefers d_a over d_c if and only if $\bar{s} - \underline{s} > [\bar{s} - \underline{s}]^{a/c}(\beta)$. $[\bar{s} - \underline{s}]^{a/c}(\beta^*) = 0$, and $[\bar{s} - \underline{s}]^{a/c}$ is monotonically increasing with respect to β .

Proof. The indifference curve between the center default and active decisions is defined by

$$Z(d_a) - Z(d_c) = (\bar{s} - \underline{s}) \frac{\bar{c} + \underline{c}}{2} - \int_{-\frac{\bar{s}-\underline{s}}{2}}^{\frac{\bar{s}-\underline{s}}{2}} l(\Delta) d\Delta = \int_{-\frac{\bar{s}-\underline{s}}{2}}^{\frac{\bar{s}-\underline{s}}{2}} \left(\frac{\bar{c} + \underline{c}}{2} - l(\Delta) \right) d\Delta = 0.$$

Let $[\bar{s} - \underline{s}]^{a/c}$ denote the implicit function of β defined by the indifference curve above. Now

$$\frac{\partial [Z(d_a) - Z(d_c)]}{\partial (\bar{s} - \underline{s})} \Big|_{\bar{s}-\underline{s}=[\bar{s}-\underline{s}]^{a/c}} = \left[\frac{\bar{c} + \underline{c}}{2} - l\left(\frac{\bar{s} - \underline{s}}{2}\right) \right]_{\bar{s}-\underline{s}=[\bar{s}-\underline{s}]^{a/c}} < 0, \quad (22)$$

since $l(\frac{\bar{s}-\underline{s}}{2})$ must be greater than $\frac{\bar{c} + \underline{c}}{2}$ due to the planner's indifference between the center default and active decisions. Also, (20) holds for $\beta \in (\beta^*, \hat{\beta})$, which gives us

$$Z(d_a) - Z(d_c)|_{\bar{s}-\underline{s} \geq 2K} < Z(d_o) - Z(d_c) = - \int_{-K}^{-E} \left(l(\Delta) - \frac{\bar{c} + \underline{c}}{2} \right) d\Delta < 0, \quad (23)$$

where the first inequality comes from Lemma 15. In addition,

$$Z(d_a) - Z(d_c)|_{\bar{s}-\underline{s} \ll E} \approx (\bar{s} - \underline{s}) \left(\frac{\bar{c} + \underline{c}}{2} - l(0) \right) > 0. \quad (24)$$

(22), (23), and (24) establish existence and uniqueness of $[\bar{s} - \underline{s}]^{a/c}(\beta)$. For a fixed β , d_c is optimal when $\bar{s} - \underline{s}$ is below $[\bar{s} - \underline{s}]^{a/c}(\beta)$ and d_a is optimal when $\bar{s} - \underline{s}$ is above $[\bar{s} - \underline{s}]^{a/c}(\beta)$. When $\beta = \beta^*$, $\lim_{\Delta \rightarrow 0} l(\Delta) = \frac{\bar{c} + \underline{c}}{2}$, so by Lemma 6, $Z(d_a) < Z(d_c)$ if $\bar{s} - \underline{s} > 0$. Therefore, we may define $[\bar{s} - \underline{s}]^{a/c}(\beta^*) = 0$. Using the Implicit Function Theorem, (22), and Lemma 10, we have

$$\frac{d[\bar{s} - \underline{s}]^{a/c}}{d\beta} = -\frac{\partial(Z(d_a) - Z(d_c))/\partial\beta}{\partial(Z(d_a) - Z(d_c))/\partial(\bar{s} - \underline{s})} > 0.$$

Thus the indifference boundary is well-defined, and monotonically increasing in β . ■

Lemma 18 *If $\beta \in (\beta^*, 1)$, then $[\bar{s} - \underline{s}]^{a/c}(\beta) = [\bar{s} - \underline{s}]^{o/c}(\beta)$ if and only if $\beta = \hat{\beta}$. That is, the three indifference boundaries intersect at a unique global indifference point in the $(\beta, \bar{s} - \underline{s})$ space.*

Proof. We know from the proof of Lemma 16 that the planner is indifferent between the center default and the offset default along $[\bar{s} - \underline{s}]^{o/c}$ and that $[\bar{s} - \underline{s}]^{o/c}(\hat{\beta}) \geq K + E$. We know from Lemma 15 that when $\bar{s} - \underline{s} \geq K + E$ and $\beta = \hat{\beta}$, the planner is indifferent between the offset default and active decisions. Therefore, by transitivity, the planner must be indifferent between the center default and active decisions at $(\hat{\beta}, [\bar{s} - \underline{s}]^{o/c}(\hat{\beta}))$. This global indifference point must be unique because the planner is indifferent between active decisions and the offset default only when $\beta = \hat{\beta}$, and the indifference curves take only one value each at $\hat{\beta}$. ■

Theorem 3 *See the body of the paper for the full statement of this theorem.*

Proof. This theorem, as it appears in the body of the paper, combines the Lemmas in this Appendix into one statement. Lemma 13 establishes the three classes of optimal defaults when $\beta < 1$. The indifference curves for the region $\beta \in (\beta^*, 1)$ are defined in Lemmas 15, 16, and 17. The regions where each default class is optimal are also described in these Lemmas. The optimal default policies for the region $\beta \in (0, \beta^*)$ and $\beta = 1$ are derived in Lemmas 12 and 14. Lemma 17 proves the monotonicity of $[\bar{s} - \underline{s}]^{a/c}(\beta)$. Lemma 18 establishes the existence of a unique global indifference point. ■

The following theorem states a comparative static on optimal defaults as κ increases.

Theorem 19 *The optimal default class for a set of parameter values including loss constant κ and optimal savings range $\bar{s} - \underline{s}$ is also the optimal default class for a setting where the loss constant is $\lambda\kappa$, the optimal savings rate range is $(\bar{s} - \underline{s})/\sqrt{\lambda}$, and all other parameters are unchanged. The set of pairs $\{\beta, \bar{s} - \underline{s}\}$ where active decisions are optimal strictly increases as one increases κ . Formally, $\frac{\partial \kappa^{1/2}[\bar{s} - \underline{s}]^{a/c}}{\partial \kappa} = 0$ and $\frac{\partial \beta^*}{\partial \kappa} = \frac{\partial \hat{\beta}}{\partial \kappa} = 0$.*

Proof. Since κ only appears in l as $\kappa\Delta^2$, if we define $\kappa' \equiv \lambda\kappa$ and $\Delta' \equiv \Delta/\sqrt{\lambda}$, then $l(\Delta)|_{\kappa} = l(\Delta')|_{\kappa'} \forall \Delta$, provided $\lambda > 0$. It follows that $\forall d$,

$$\int_{\underline{s}-d}^{\bar{s}-d} l(\Delta)|_{\kappa} d\Delta = \int_{(\underline{s}-d)/\sqrt{\lambda}}^{(\bar{s}-d)/\sqrt{\lambda}} l(\Delta')|_{\kappa'} \sqrt{\lambda} d\Delta' \quad (25)$$

This implies that if d is optimal when the loss constant is κ , the maximum optimal savings rate is \bar{s} , and the minimum optimal savings rate is \underline{s} , then $d/\sqrt{\lambda}$ will be optimal when the loss constant is $\lambda\kappa$, the maximum optimal savings rate is $\bar{s}/\sqrt{\lambda}$, and the minimum optimal savings rate is $\underline{s}/\sqrt{\lambda}$. It is easy to verify that d and $d/\sqrt{\lambda}$ belong to the same default class within their respective parameter settings. It is also trivial to show that holding all other parameters fixed, the optimal default class depends only on the distance between the maximum and minimum optimal savings rates, not their values individually. Applying (25) to the definition of $[\bar{s} - \underline{s}]^{a/c}$ in (21) yields the result that $[\bar{s} - \underline{s}]^{a/c}$ is inversely proportional to $\sqrt{\kappa}$, so $\frac{\partial \kappa^{1/2}[\bar{s} - \underline{s}]^{a/c}}{\partial \kappa} = 0$. It is obvious from the definition of β^* that $\frac{\partial \beta^*}{\partial \kappa} = 0$. To establish that $\frac{\partial \hat{\beta}}{\partial \kappa} = 0$, we show that if a given value of β satisfies (17) under parameter value κ , then it also satisfies the relation under parameter value κ' . Note that the values of E and K are functions of κ , $E(\kappa) = \sqrt{\lambda}E(\kappa')$, and $K(\kappa) = \sqrt{\lambda}K(\kappa')$. Then for a given value of $\hat{\beta}$,

$$\begin{aligned} \int_{-E(\kappa')}^{K(\kappa')} l(\Delta')|_{\kappa'} d\Delta' &= \frac{1}{\sqrt{\lambda}} \int_{-E(\kappa)}^{K(\kappa)} l(\Delta)|_{\kappa} d\Delta \\ &= \frac{\bar{c} + c}{2} \left(\frac{K(\kappa) + E(\kappa)}{\sqrt{\lambda}} \right) \\ &= \frac{\bar{c} + c}{2} (K(\kappa') + E(\kappa')), \end{aligned}$$

where the second equality results from (17) being true under parameter values κ and $\hat{\beta}$. ■

7.3 The Case of Naive Workers

Lemma 20 $\frac{\partial c^*}{\partial \beta} > 0$.

Proof.

$$\frac{\partial c^*}{\partial \beta} = \frac{\underline{c} \left(\sqrt{\underline{c}^2 (1 - (2 - \beta)\beta) + 2\beta(1 - \beta)(\bar{c} - \underline{c})L} - \underline{c}(1 - \beta) \right) + 2(2 - \beta)(\bar{c} - \underline{c})L}{(2 - \beta)^2 \sqrt{\underline{c}^2 (1 - (2 - \beta)\beta) + 2\beta(1 - \beta)(\bar{c} - \underline{c})L}} \quad (26)$$

is positive, since both parts of the numerator are positive. ■

Theorem 4 *For given parameter values, if active decisions are optimal for sophisticated workers, then active decisions are also optimal for naive workers.*

Proof. Consider the naive workers' future optimization problem from their current perspective. Naive workers believe they will act as if they were time-consistent exponential discounters ($\beta = 1$) in the future. Therefore, they anticipate using a cutoff cost c_e^* in the future when deciding whether to opt out, where

$$c_e^* = \arg \min_{c'} E(V(L, c')) = \arg \min_{c'} \begin{cases} c' & \text{if } c' \leq \hat{c}^* \\ L + E(V(L, c'')) & \text{if } c' > \hat{c}^* \end{cases} \quad (27)$$

Sophisticates understand that they will use the same cutoff cost c_s^* both today and tomorrow. Thus, $E(\widehat{V(L)}|naive) \leq E(\widehat{V(L)}|soph)$, where $E(\widehat{V(L)})$ represents the worker's perceived future loss expectation for herself. The inequality is strict if $\beta < 1$ by Lemma 20.

The naifs' current cutoff cost will be chosen such that

$$c_n^* = \beta(L + E(\widehat{V(L)}|naive)).$$

Since $E(\widehat{V(L)})$ is lower for naive workers than for sophisticated workers, $c_n^* \leq c_s^*$, with the inequality strict if $\beta < 1$. Furthermore, we know that $c_s^* < c_e^*$ when $\beta < 1$ by Lemma 20.

Note that agents using a cutoff rule c^* will act with probability $p = \frac{c^* - \underline{c}}{\bar{c} - \underline{c}}$ and pay an average transaction cost $\tilde{c} = \frac{c^* + \underline{c}}{2}$ when they act. Thus,

$$\begin{aligned} l(c^*, L) &= p\tilde{c} + (1 - p)p(L + \tilde{c}) + (1 - p)^2 p(2L + \tilde{c}) + \dots \\ &= \frac{c^* + \underline{c}}{2} + \frac{\bar{c} - c^*}{c^* - \underline{c}} L. \end{aligned}$$

Note that c_e^* is the point which minimizes this function. Let $l_e(\Delta)$, $l_s(\Delta)$, and $l_n(\Delta)$ be the expected loss functions for agents using cutoff values c_e^* , c_s^* , and c_n^* , respectively. Then the convexity of l and the ordering $c_n^* \leq c_s^* \leq c_e^*$ implies that $l_e(\Delta) \leq l_s(\Delta) \leq l_n(\Delta)$ when $0 < |\Delta| < K$, with strict inequalities when $\beta < 1$. We have assumed in the statement of the theorem that active decisions are optimal for sophisticates (which only holds when $\beta < 1$). Therefore, $\forall d \in (\underline{s} - K, \bar{s} + K)$, $\int_{\underline{s}-d}^{\bar{s}-d} l_n(\Delta) d\Delta > \int_{\underline{s}-d}^{\bar{s}-d} l_s(\Delta) d\Delta > (\bar{s} - \underline{s}) \frac{\bar{c}+c}{2}$, where the last term is the total loss under active decisions. Hence, when active decisions are best for sophisticates, they are best for naifs. ■

8 References

- Abadie, Alberto, and Sebastien Gay (2004). “The Impact of Presumed Consent Legislation on Cadaveric Organ Donation: A Cross Country Study,” NBER Working Paper No. 10604.
- Aguiar, Mark, and Erik Hurst (2004). “Consumption vs. Expenditure,” University of Chicago Working Paper.
- Benartzi, Shlomo, and Richard H. Thaler (2001). “Naive Diversification Strategies in Defined Contribution Saving Plans,” *American Economic Review* 91(1): 79-98.
- Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick (2004). “For Better or For Worse: Default Effects and 401(k) Savings Behavior,” in David A. Wise, ed., *Perspectives in the Economics of Aging* (Chicago, IL: University of Chicago Press): 81-121.
- Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick (2002). “Defined Contribution Pensions: Plan Rules, Participant Decisions, and the Path of Least Resistance,” in James M. Poterba, ed., *Tax Policy and the Economy* 16, (Cambridge, MA: MIT Press): 67-113.
- Cronqvist, Henrik, and Richard H. Thaler (2004). “Design Choices in Privatized Social-Security Systems: Learning from the Swedish Experience,” *American Economic Review* 94(2): 424-428.

- Engen, Eric M., William G. Gale, and Cori E. Uccello (1999). "The Adequacy of Household Saving," *Brookings Papers on Economic Activity* 0(2): 65-165.
- Even, William E., and David MacPherson (1999). "Employee Participation in 401(k) Plans," Florida State University Working Paper.
- Hurst, Erik (2004). "Grasshoppers, Ants and Pre-Retirement Wealth: A Test of Permanent Income Consumers," University of Chicago Working Paper.
- John Hancock (2002). "Eighth Defined Contribution Plan Survey: Insight Into Participant Knowledge & Behavior." (Boston, MA: John Hancock Financial Services).
- Johnson, Eric J., and Daniel Goldstein (2003). "Do Defaults Save Lives?," *Science* 302(November 21): 1338-1339.
- Johnson, Eric J., John Hershey, Jacqueline Meszaros, and Howard Kunreuther (1993). "Framing, Probability Distortions, and Insurance Decisions," *Journal of Risk and Uncertainty* 7(1): 35-53.
- Johnson, Eric J., Steven Bellman, and Gerald L. Lohse (2002). "Defaults, Framing and Privacy: Why Opting In-Opting Out," *Marketing Letters* 13(1): 5-15.
- Laibson, David (1997). "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics* 112(2): 443-477.
- Madrian, Brigitte C., and Dennis Shea (2001). "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior," *Quarterly Journal of Economics* 116(4): 1149-1187.
- O'Donoghue, Ted, and Matthew Rabin (1999a). "Procrastination in Preparing for Retirement," in Henry J. Aaron, ed., *Behavioral Dimensions of Retirement Economics* (Washington, D.C.: Brookings Institution Press; New York, NY: Russell Sage Foundation): 125-156.
- O'Donoghue, Ted, and Matthew Rabin (1999b). "Doing It Now or Later," *American Economic Review* 89(1): 103-124.

- Park, C. Whan, Sung Y. Jun, and Deborah J. MacInnis (2000). "Choosing What I Want Versus Rejecting What I Do Not Want: An Application of Decision Framing to Product Option Choice Decisions," *Journal of Marketing Research* 37(2): 187-202.
- Phelps, Edmund S., and Robert A. Pollak (1968). "On Second-Best National Savings and Game-Equilibrium Growth," *Review of Economic Studies* 35(2): 185-199.
- Sunstein, Cass R., and Richard H. Thaler (2003). "Libertarian Paternalism," *American Economic Review* 93(2): 175-179.
- Thaler, Richard H., and Shlomo Benartzi (2004). "Save More Tomorrow™: Using Behavioral Economics to Increase Employee Saving," *Journal of Political Economy* 112(1): S164-187.
- Warshawsky, Mark J., and John Ameriks (2000). "How Prepared Are Americans for Retirement?," in Olivia S. Mitchell, P. Brett Hammond, and Anna M. Rappaport, eds., *Forecasting Retirement Needs and Retirement Wealth* (Philadelphia, PA: University of Pennsylvania Press): 33-67.

Table 1. 401(k) plan features by effective date

	Effective January 1, 1997	Effective November 23, 1997
<i>Eligibility</i>		
Eligible employees	U.S. employees, age 18+	U.S. employees, age 18+
First eligible	Immediately upon hire	Immediately upon hire
Employer match eligible	Immediately upon hire	Immediately upon hire
<i>Enrollment</i>	First 30 days of employment or January 1 of succeeding calendar years	Daily
<i>Contributions</i>		
Employee contributions	Up to 17% of compensation	Up to 17% of compensation
Non-discretionary employer match	50% of employee contribution up to 5% of compensation	50% of employee contribution up to 5% of compensation
Discretionary employer match	Up to 100% of employee contribution depending on company profitability (50% for bonus-eligible employees); 100% in 1997.	Up to 100% of employee contribution depending on company profitability (50% for bonus-eligible employees); varied from 0% to 100% for 1997-2000. ^a
<i>Vesting</i>	Immediate	Immediate
<i>Other</i>		
Loans	Not available	Available; 2 maximum
Hardship withdrawals	Available	Available
Investment choices	6 options. Employer stock also available, but only for after-tax contributions.	8 options + employer stock (available for before- and after-tax contributions)

^a Actual discretionary match rates for 1995-2001 were 20% (1995), 20% (1996), 100% (1997), 100% (1998), 27% (1999), 33% (2000), 0% (2001).

Table 2. Comparison of worker characteristics

	Study company			U.S. workforce (3/98 CPS)
	Active decision	Standard	All	
	cohort on 12/31/98	enroll. cohort on 12/31/99	workers on 12/31/99	
<i>Average age (years)</i>	34.1	34.0	40.5	38.8
<i>Gender</i>				
Male	45.4%	43.4%	45.0%	53.1%
Female	54.6%	56.6%	55%	46.9
<i>Marital Status</i>				
Single	42.8%	47.8%	32.4%	39.0%
Married	57.2%	52.2%	67.6%	61.0%
<i>Compensation</i>				
Avg. monthly base pay	\$2,994	\$2,911	\$4,550	--
Median monthly base pay	\$2,648	\$2,552	\$3,750	--
Avg. annual income ^a	\$34,656	\$34,001	\$52,936	\$32,414
Median annual income ^a	\$30,530	\$29,950	\$42,100	\$24,108
<i>Geography</i>				
East	10.0%	8.4%	12.1%	18.9%
Midwest	37.9%	39.8%	35.3%	24.1%
South	37.1%	39.0%	37.8%	34.7%
West	15.0%	12.6%	14.7%	22.4%
<i>Number of Employees</i>	2,205	2,344	46,822	--

The samples in the first three columns are taken from individuals employed at the study company as of the dates indicated in the column title. The sample in the last column is all individuals in the March 1998 Current Population Survey who worked in the previous year (weighted).

^aThe annual income measure that is reported to us for the study company is the employee's annual taxable (W2) income. Annual income for the U.S. workforce calculated from the CPS is total annual labor earnings in the previous calendar year, some of which may be non-taxable.

Table 3. Regression of contribution rates under two 401(k) enrollment regimes

Intercept	-8.125** (1.998)
<i>Married</i>	1.239** (0.251)
$\text{Log}(\text{Base pay})$	1.985** (0.223)
$0 \leq \text{Age} < 30$	-4.003** (1.061)
$30 \leq \text{Age} < 40$	-3.311** (1.060)
$40 \leq \text{Age} < 50$	-2.688** (1.076)
$50 \leq \text{Age} < 60$	-1.755 (1.105)
<i>Active decision cohort</i>	3.258 (3.410)
<i>Active decision cohort</i> \times <i>Married</i>	-0.963** (0.396)
<i>Active decision cohort</i> \times $\text{Log}(\text{Base pay})$	-0.064 (0.385)
<i>Active decision cohort</i> \times $(0 \leq \text{Age} < 30)$	-2.381 (1.571)
<i>Active decision cohort</i> \times $(30 \leq \text{Age} < 40)$	-2.047 (1.573)
<i>Active decision cohort</i> \times $(40 \leq \text{Age} < 50)$	-1.988 (1.598)
<i>Active decision cohort</i> \times $(50 \leq \text{Age} < 60)$	-0.517 (1.667)
R^2	0.12
N	2,422

If the employee is in the active decision cohort, the dependent variable is the 401(k) contribution rate (in percentage points) three months after hire; if the employee is in the standard enrollment cohort, the dependent variable is the contribution rate three years after hire. Dependent variables are calculated as of the contribution rate date. *Married* is a dummy that equals one if the employee is married. The variables $n_1 \leq \text{Age} < n_2$ are dummies that equal one if the employee's age falls within the specified range. *Active decision cohort* is a dummy that equals one if the employee is in the active decision cohort. Standard errors are reported in parentheses under the point estimates.

*Significant the 5% level

**Significant at the 1% level

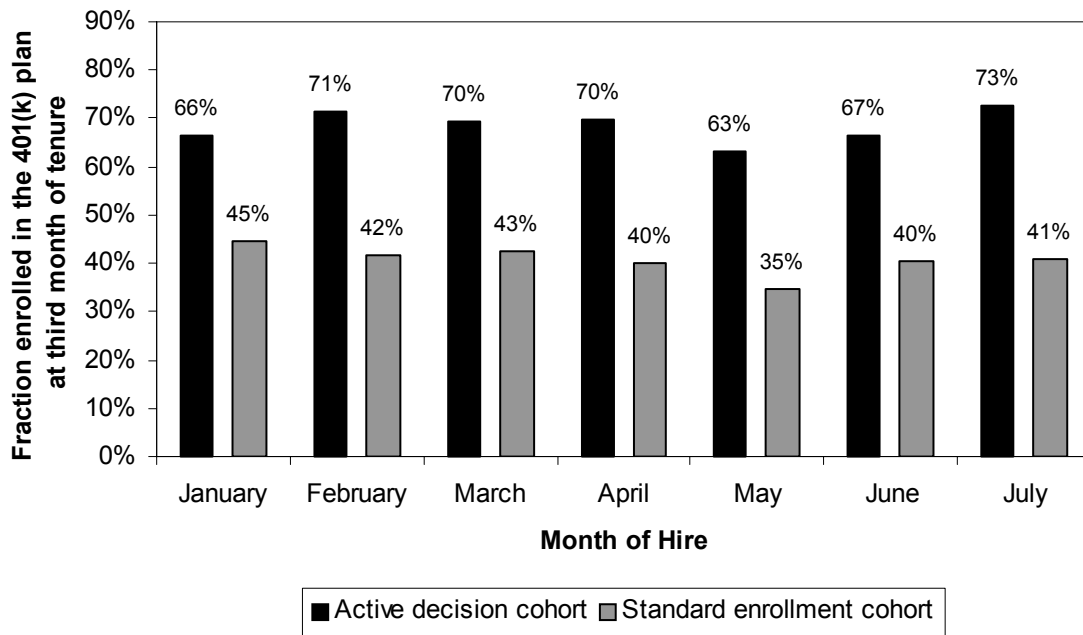


Figure 1. Fraction of employees enrolled in the 401(k), by hire month. The fraction displayed is as of the end of the third month of tenure at the company. The active decision cohort was hired between January and July 1997. The standard enrollment cohort was hired between January and July 1998.

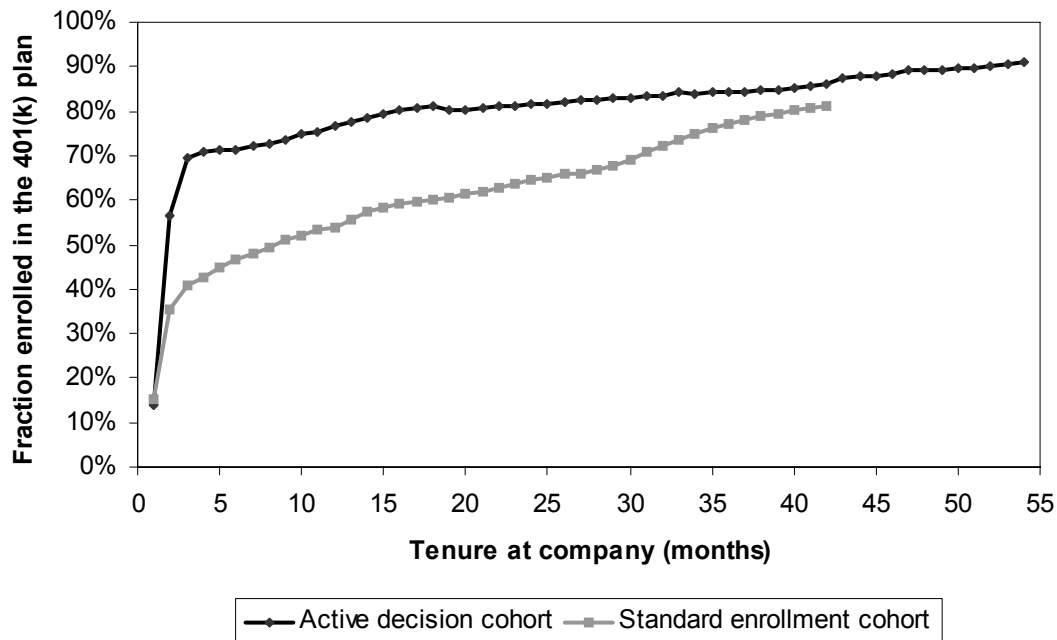


Figure 2. Fraction of employees enrolled in the 401(k) plan, by tenure at company. An employee is counted enrolled in the 401(k) plan even if he or she is not currently contributing to the plan. The series are not monotonically rising because they are constructed from multiple cross-sections, so the samples are not fixed over time.

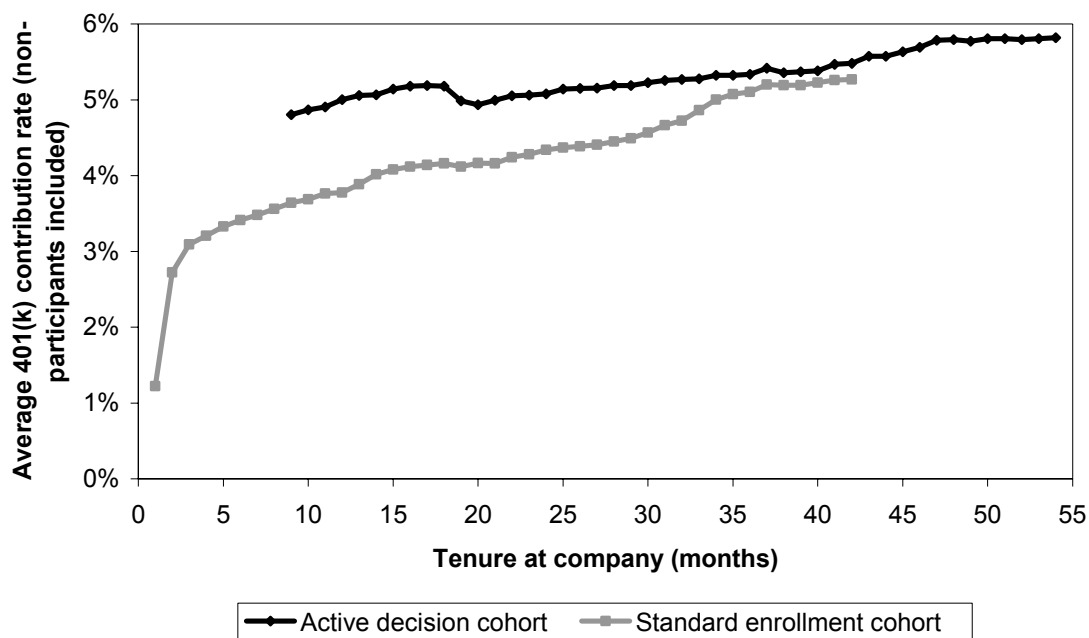


Figure 3. Average 401(k) contribution rate by tenure at company. At each point, the averages include employees not currently contributing to the 401(k) plan; their contribution rate is zero.

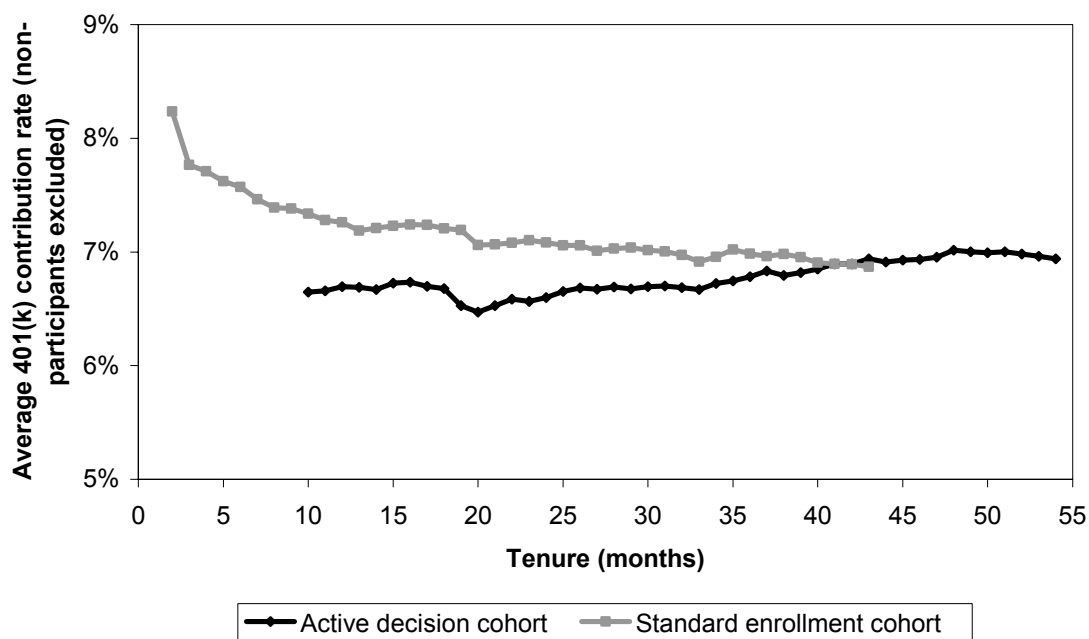


Figure 4. Average 401(k) contribution rate among 401(k) participants by tenure at company. At each point, the averages exclude employees not currently contributing a positive amount to the 401(k) plan.

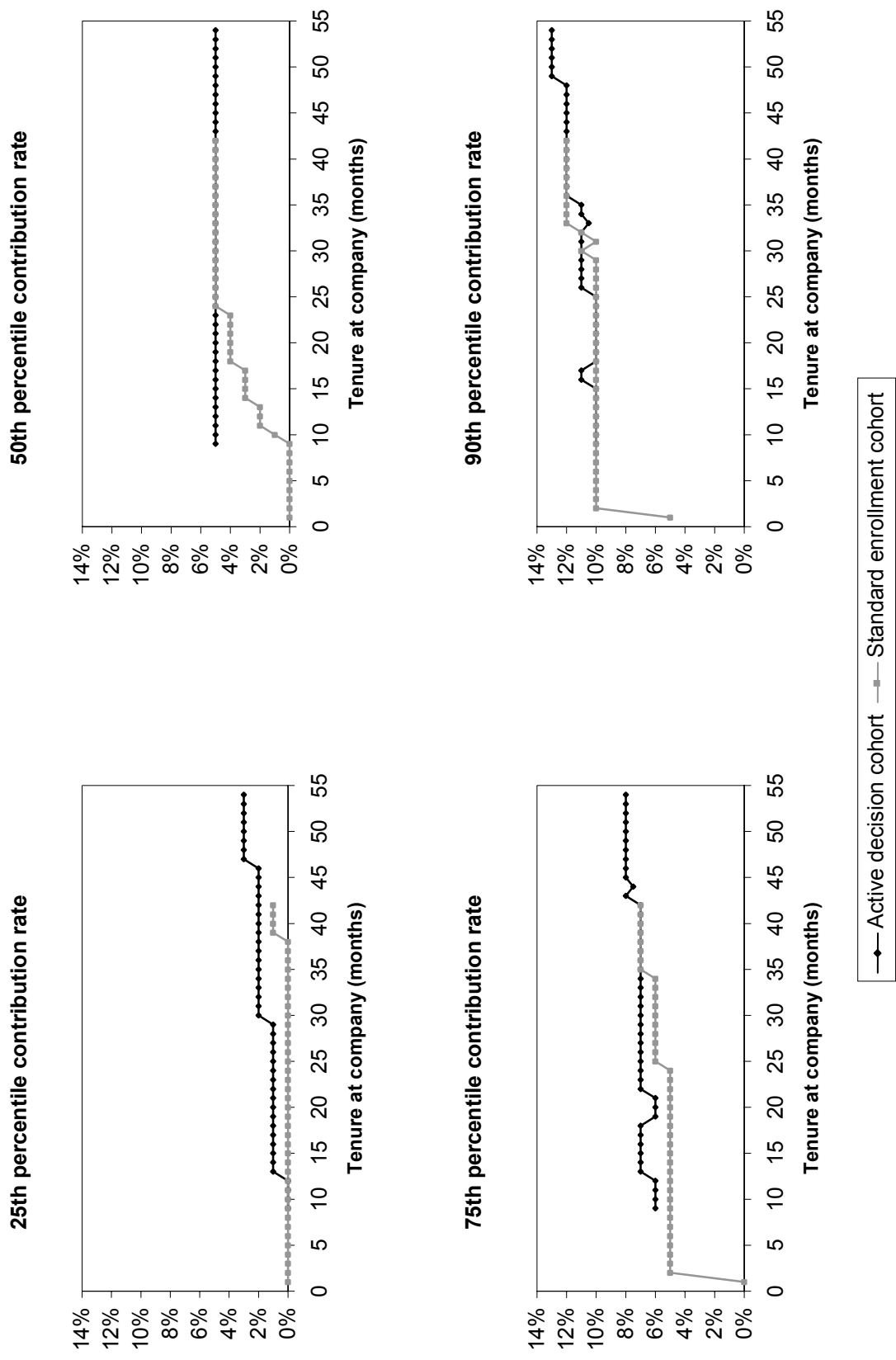


Figure 5. 401(k) contribution rates at different contribution rate percentiles. The percentile breakpoints are calculated separately for each cohort at each point in time.

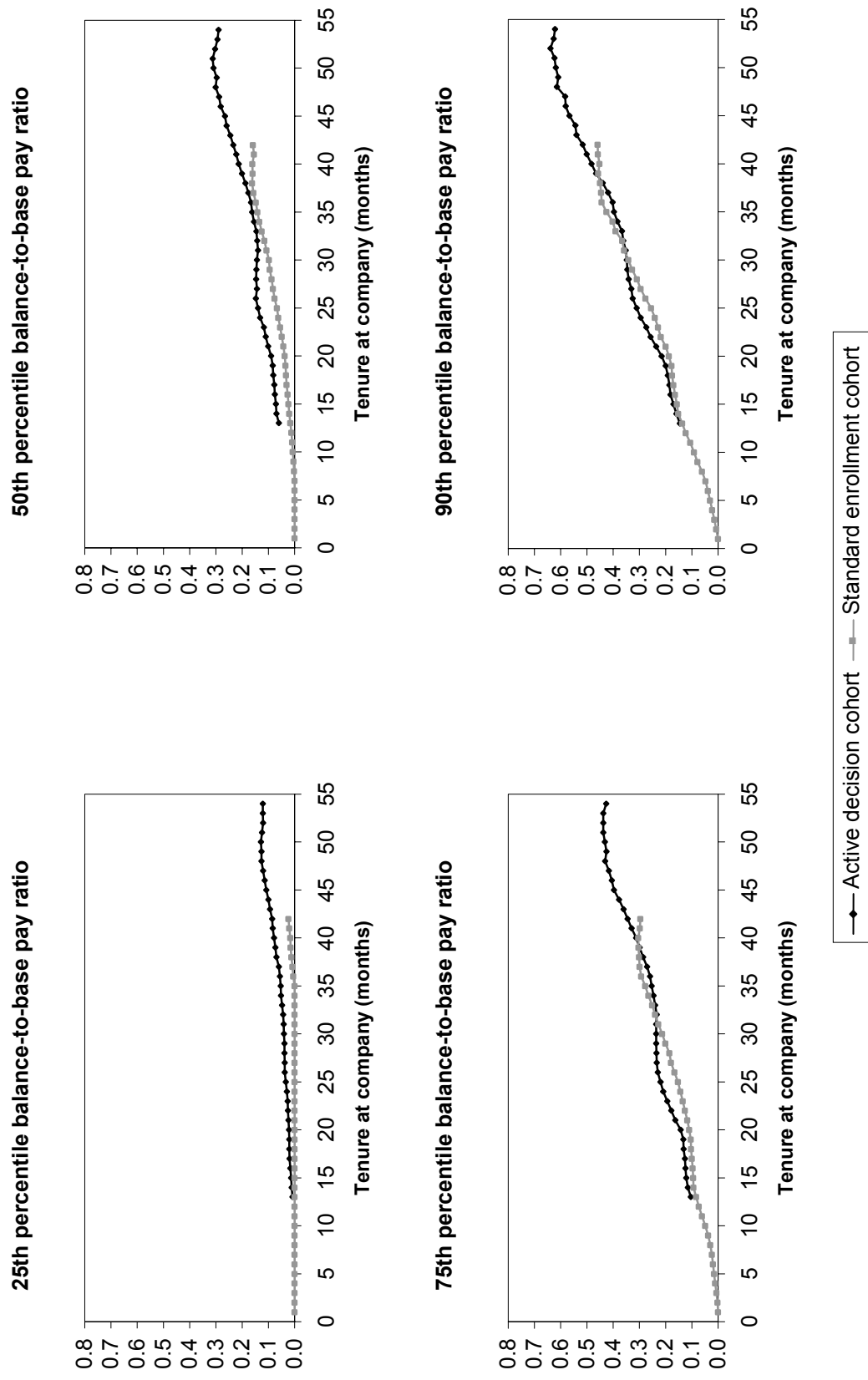


Figure 6. 401(k) balance-to-base pay ratios at different balance-to-base pay percentiles. The balances exclude any money rolled into the account from a former employer. The percentile breakpoints are calculated separately for each cohort at each point in time.

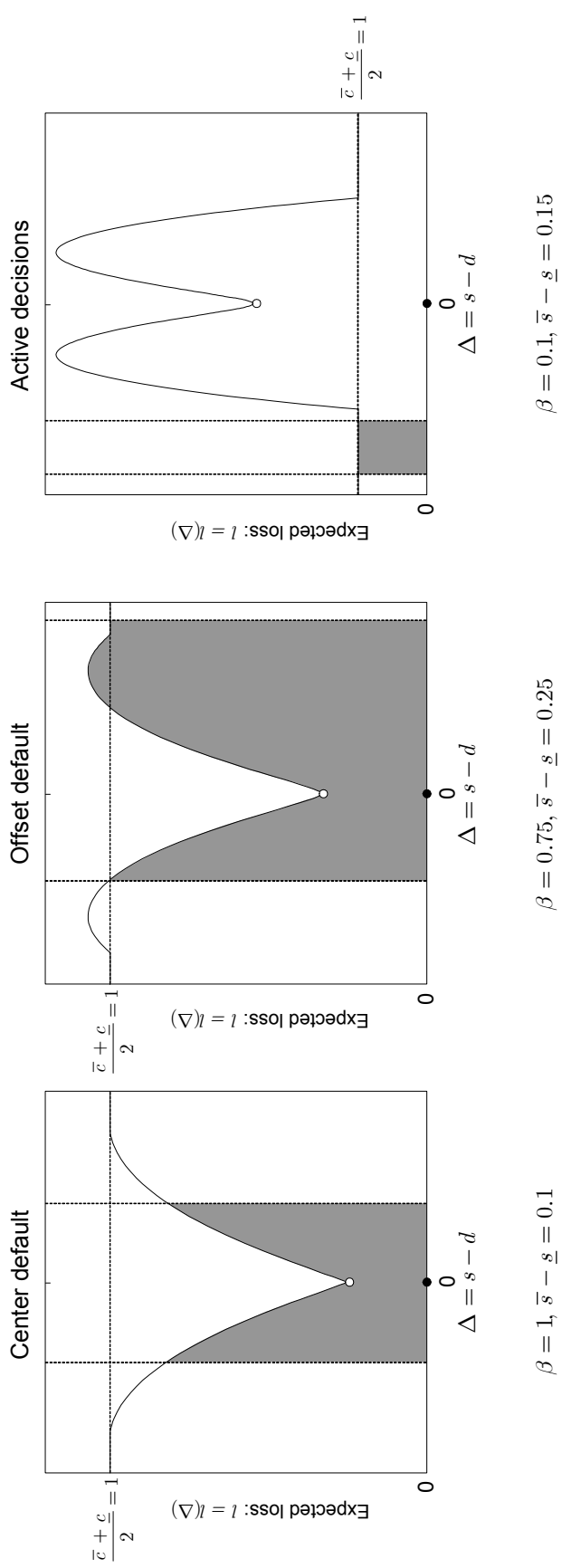


Figure 7. Possible optimal default regimes. The panels illustrate parameter settings that support the three classes of optimal defaults: the center default, the offset default, and active decisions. The shaded area in each panel represents the social welfare losses generated by the corresponding default regime. The parameter values common to the three panels are the average opt-out cost $(\bar{c} + \underline{c})/2 = 1$, the range of opt-out costs $\bar{c} - \underline{c} = 1.5$, and the loss function scaling factor $\kappa = 100$. The parameters specific to each panel, the quasi-hyperbolic discount factor β and the range of optimal savings rates $\bar{s} - \underline{s}$, appear below each figure.

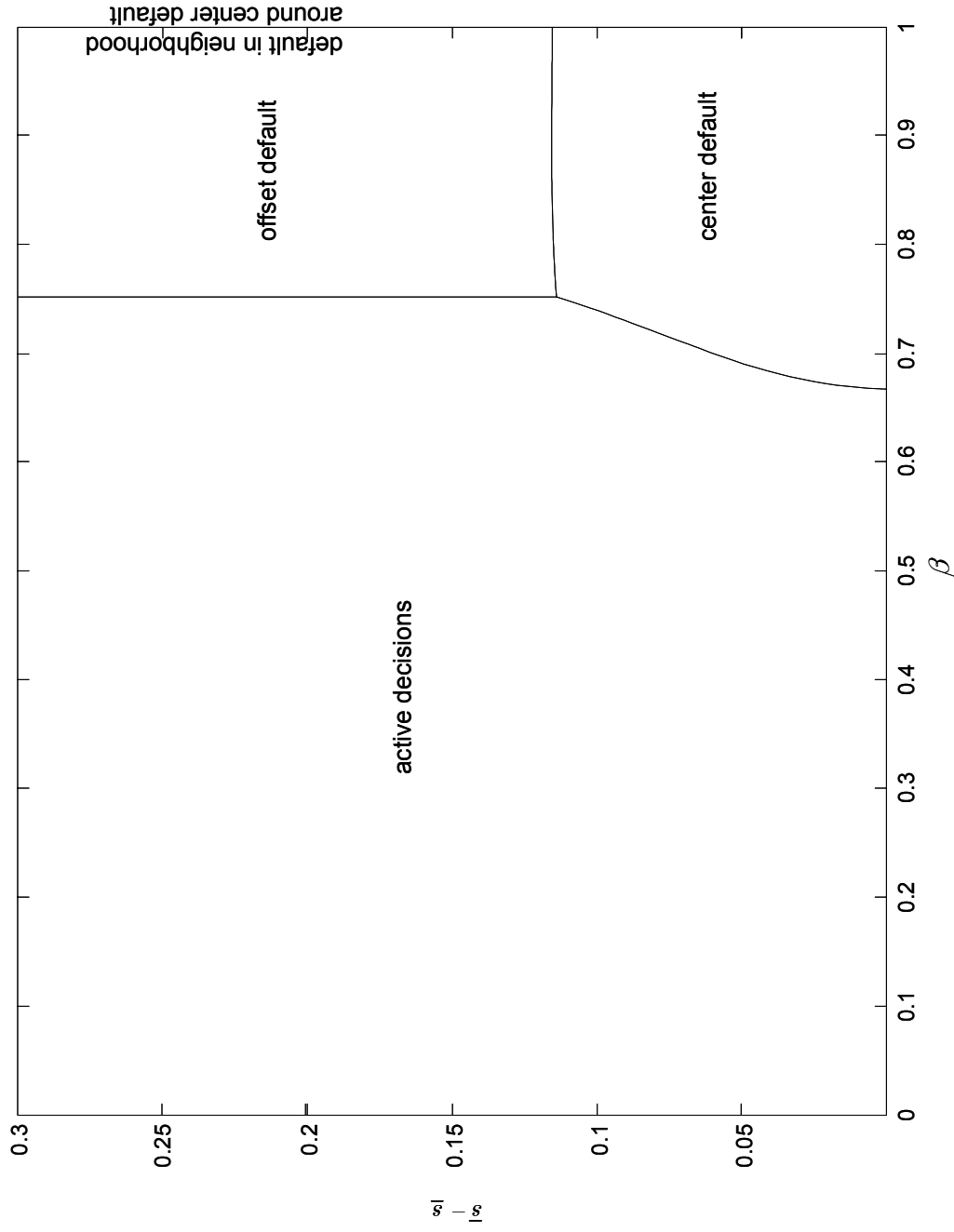


Figure 8. Characterization of optimal default regimes. This figure shows the boundaries of the optimal default regimes as a function of the quasi-hyperbolic discount factor β and the range of optimal savings rates $\bar{s} - s$ for the following sets of parameters: average opt-out cost $(\bar{c} + \underline{c})/2 = 1$, range of opt-out costs $\bar{c} - \underline{c} = 0.666$, loss function scaling factor $\kappa = 100$.