Discussion of PVW at Boulders 2009

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How do households decumulate their retirement savings? This is one of the most important open questions in the retirement savings literature. Poterba, Venti, and Wise (hereafter PVW) establish many interesting and important facts about the decumulation process. After resolving lots of critical technical issues that arise because of measurement errors in the HRS data, PVW show three properties. First, net worth tends to rise robustly throughout old-age in both 2-person households and 1-person households. Second, demographic transitions (e.g., widowhood) tend to slow the growth of wealth, and this wealth reduction begins long before the actual demographic transition occurs. Third, there is a very strong positive association between health and wealth. Healthy households have higher levels of wealth and higher growth rates of wealth.

These facts should lead economists to reevaluate the classical model of life-cycle consumption. Figure 1 plots the predictions of the classical model (e.g., the Lifecycle Hypothesis of Modigliani and the Permanent Income hypothesis of Friedman): a tent-shaped wealth accumulation pattern. Wealth rises smoothly during working life. Then wealth falls smoothly during retirement. However, PVW's evidence supports a more complex wealth decumulation pattern, like the pattern plotted in Figure 2. In this figure, wealth continues rising even *after* retirement, until elevated health-related expenses cause a substantial decline in wealth. At the end of this health shock, wealth resumes its rise until another health event occurs. Figure 2 illustrates a case with two (wealth-reducing) health events, but in principle many expensive health events could occur before wealth is completely spent. Moreover, these health events need not be discrete (the discrete case is illustrated in the figure).

In this discussion, I present a tractable model of such complicated decumulation dynamics. The model is in continuous-time, though the model has discrete medical events.

Let μ represent the hazard rate of arrival of one of these discrete medical events. To keep the modeling simple, assume that a medical event is *both* expensive and deadly (e.g., a retiree experiences a stroke, which leads to hospitalization, long-term care, and mortality). I summarize this by assuming that discrete medical-event utility is given by

$$M(C_M) = \frac{\zeta C_M^{1-\gamma}}{1-\gamma},$$

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where C_M is the out of pocket expenditure (endogenously chosen) during the medical event.

I assume that there is no bequest motive and no annuity market. Households have a utility function with constant relative risk aversion, γ. The flow utility for a household of size n is given by

$$n \times u\left(\frac{C}{n^\theta}\right) = n \times \frac{\left(\frac{C}{n^\theta}\right)^{1-\gamma}}{1-\gamma}.$$

To gain intuition, consider the following three benchmark cases: (i) no returns to scale, $\theta = 1$; (ii) infinite returns to scale, $\theta = 0$; and (iii) square-root returns to scale, $\theta = 1/2$. The last case is the leading empirical case.

Outside of medical events, the dynamics for wealth are smooth,

$$dW = rW - C,$$

where r is the real interest rate. During a medical event, the dynamics for W are discrete,

$$\Delta W = W - C_M.$$

With a discount rate of ρ, the continuous-time Bellman Equation for a one-person household (e.g., after the death of a spouse), is given by

$$
\begin{aligned}
\rho V_1(W) &= u(C) + EdV_1 \\
&= u(C) + \frac{\partial V_1}{\partial W}[rW - C] + \mu\left[\zeta\frac{W^{1-\gamma}}{1-\gamma} - V_1(W)\right].
\end{aligned}
$$

The continuous-time Bellman Equation for a two-person household is given by

$$
\begin{aligned}
\rho V_2(W) &= 2 \times u\left(\frac{C}{2^\theta}\right) + EdV_2 \\
&= 2 \times u\left(\frac{C}{2^\theta}\right) + \frac{\partial V_2}{\partial W}[rW - C] \\
&\quad + \mu\left[\frac{\zeta C_M^{1-\gamma}}{1-\gamma} + V_1(W - C_M) - V_2(W)\right]
\end{aligned}
$$

Using the guess-and-check method, it is easy to show that the value function for the one-person household is given by

$$V_1(W) = \frac{\varphi_1 W^{1-\gamma}}{1-\gamma},$$

where φ_1 is a constant to be solved. Applying the Envelope Theorem yields,

$$C = \varphi_1^{-\frac{1}{\gamma}} W.$$

Solving for the marginal propensity to consume (MPC), $\varphi_1^{-\frac{1}{\gamma}}$, yields,

$$\rho + \mu = \gamma\varphi_1^{-\frac{1}{\gamma}} + (1-\gamma)r + \varphi_1^{-1}\mu\zeta.$$

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We now characterize the tractable case of ln utility, which is obtained by letting $\gamma \to 1$. Now

$$\text{MPC} = \varphi_1^{-1} = \frac{\rho + \mu}{1 + \mu \zeta}.$$

Hence, the MPC rises with ρ, falls with ζ, and has an ambiguous relationship with μ. For this case, wealth will accumulate even if

$$r = \rho + \mu.$$

We can solve for medical spending during a medical event experienced by the first spouse:

$$
\begin{aligned}
\lambda^* &= \arg\max \frac{\zeta (\lambda W)^{1-\gamma}}{1-\gamma} + \frac{\varphi_1 (W - \lambda W)^{1-\gamma}}{1-\gamma} \\
&= \arg\max \frac{\zeta \lambda^{1-\gamma}}{1-\gamma} + \frac{\varphi_1 (1 - \lambda)^{1-\gamma}}{1-\gamma}
\end{aligned}
$$

The FOC implies that

$$0 = \zeta \lambda^{-\gamma} - \varphi_1 (1 - \lambda)^{-\gamma}.$$

Setting $\gamma = 1$, implies

$$
\begin{aligned}
\zeta (1 - \lambda) &= \varphi_1 \lambda \\
\lambda &= \frac{\zeta}{\varphi_1 + \zeta} \\
&= \frac{\zeta}{\frac{1 + \mu \zeta}{\rho + \mu} + \zeta}.
\end{aligned}
$$

Finally, we can also solve for two-person household by confirming that the following functional form satisfies the Bellman Equation for the two-person household.

$$V_2(W) = \frac{\varphi_2 W^{1-\gamma}}{1 - \gamma}$$

By the envelope theorem:

$$C = 2^{\frac{1 - \theta(1-\gamma)}{\gamma}} \varphi_2^{-\frac{1}{\gamma}} W.$$

Plugging this expression into the Bellman Equation, and simplifying, yields,

$$\rho = 2^{\frac{1 - \theta(1-\gamma)}{\gamma}} \varphi_2^{-\frac{1}{\gamma}} + (1 - \gamma) \left[r - 2^{\frac{1 - \theta(1-\gamma)}{\gamma}} \varphi_2^{-\frac{1}{\gamma}} \right] + \mu \left[\frac{\zeta \lambda^{1-\gamma}}{\varphi_2} + \frac{\varphi_1}{\varphi_2} (1 - \lambda)^{1-\gamma} - 1 \right]$$

Again, we'll study the special case, $\gamma = 1$. This implies

$$\text{MPC} = 2\varphi_2^{-1} = \frac{\rho + \mu}{1 + \frac{\mu}{2} \left(\zeta + \frac{1 + \mu \zeta}{\rho + \mu} \right)}.$$

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As before, the MPC rises with ρ, falls with ζ, and has an ambiguous relationship with μ. Once again, wealth will accumulate even if

$$r = \rho + \mu.$$

See Figure 3 for a calibrated/simulated path of wealth. The model has several implications. An increase in the taste for health expenditure – i.e., an increase in ζ – lowers the MPC. Wealth grows in retirement even when the discount rate equals the interest rate. Indeed, wealth grows in retirement even when the discount rate plus the mortality rate $(\rho + \mu)$ equals the interest rate. Households choose to make large proportionate reductions in wealth that coincide with medical events. These proportionate reductions in wealth are not a sign of financial distress. Rather they reflect an optimal decision to spend wealth on health services during a severe medical event. Moreover, even if retirement wealth were much greater, such expenditures would not proportionately change. As our resources rise, the model predicts that we will choose to buy better and better medical services (e.g., private hospital rooms, expensive pharmaceuticals that are not covered by insurance, home nurses, outstanding long-term care facilities, etc...).

This model provides a quantitative framework for studying wealth dynamics after retirement, and explains why households that are not experiencing medical events choose to increase their wealth throughout retirement. The empirical analysis in PVW is critical for the development of models like this that explain the surprising savings behavior of older adults.

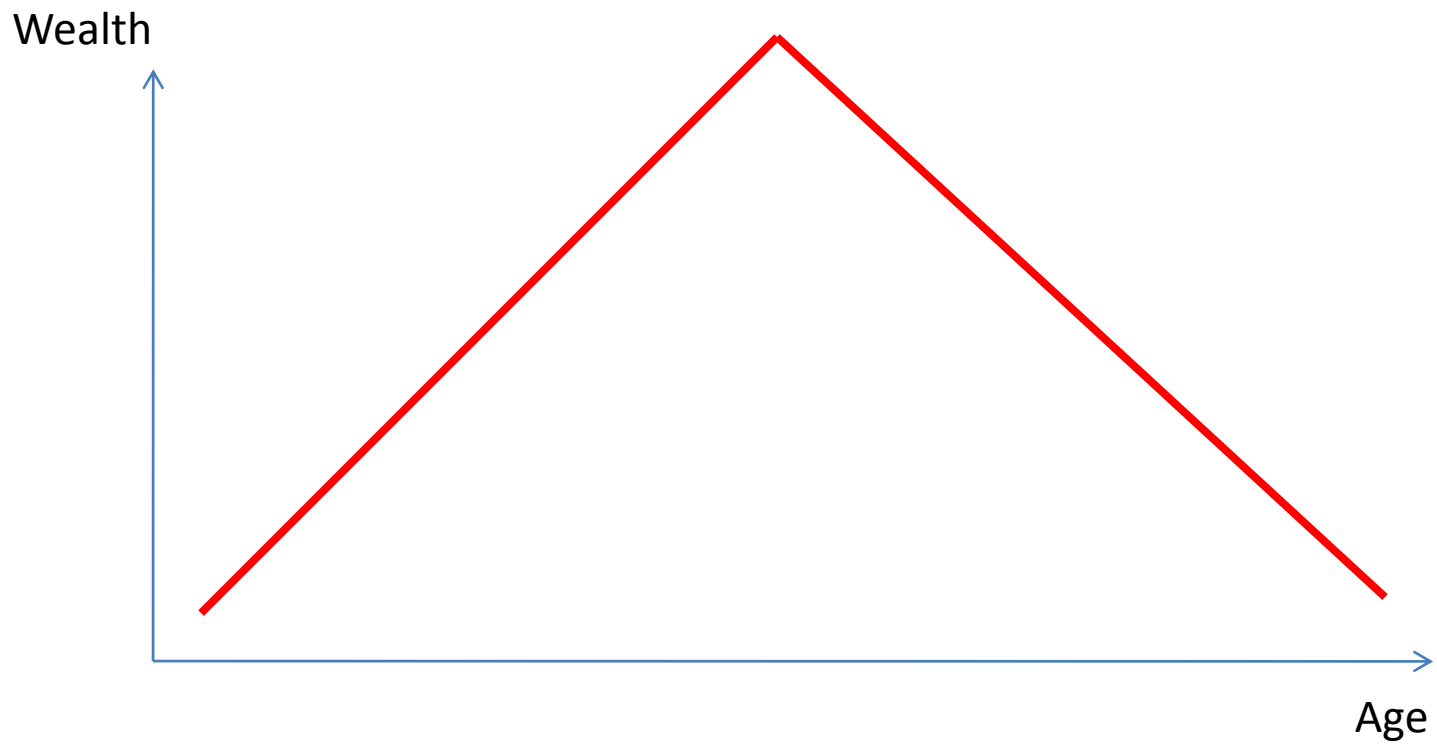Figure 1: Lifecycle wealth dynamics predicted by classical theories

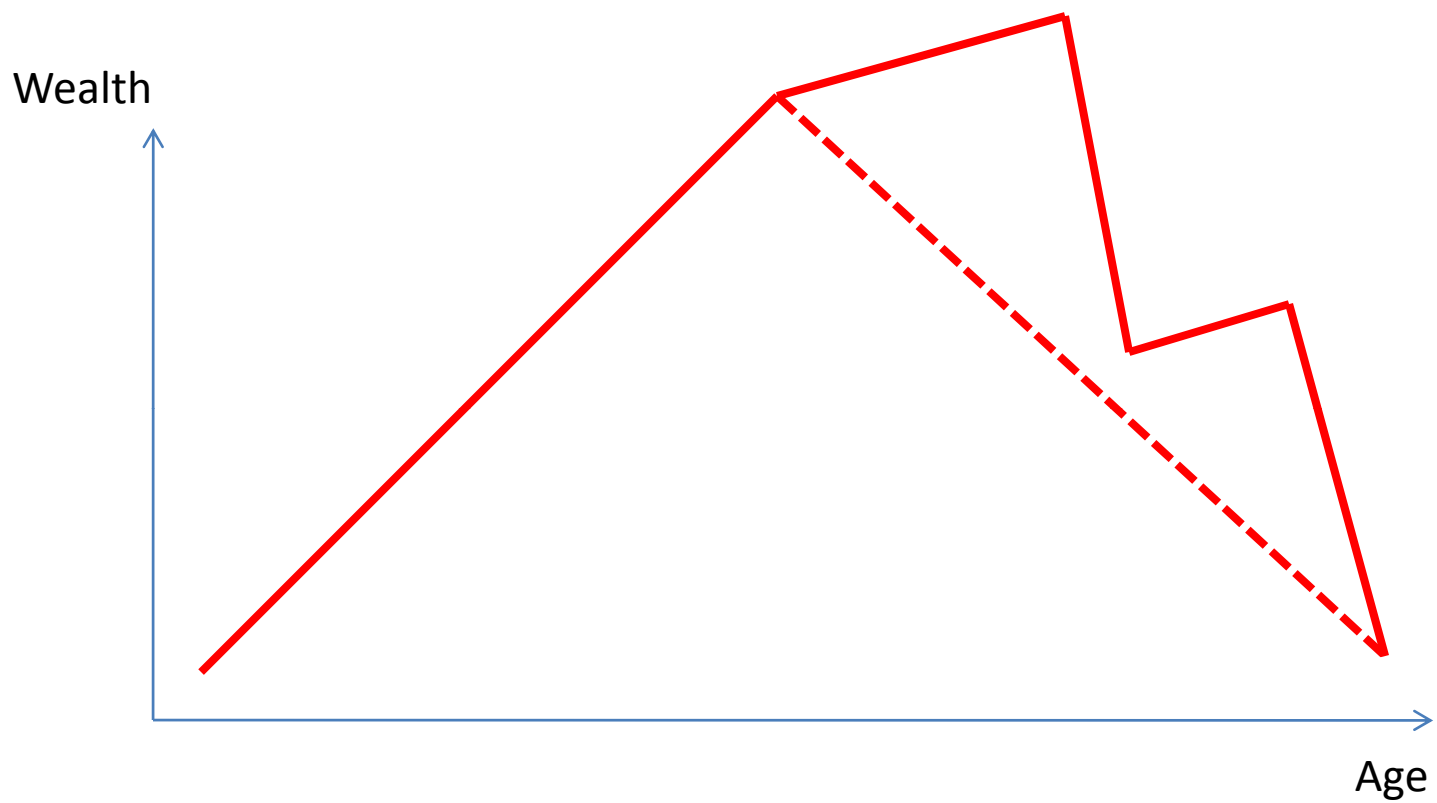# Figure 2: Stylized empirical patterns of wealth decumulation

Figure 3: Predictions of new model with medical events

Calibrated parameters: ζ = 50, ρ = 0.02, γ= 1, r = 0.03, λ = 0.5, μ = 0.02.

Wealth path with no medical events

Wealth path with two medical events

Wealth

Year into retirement