Collapse of Axion Stars

Madelyn Leembruggen

Department of Physics

UNIVERSITY OF Cincinnati

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Abstract

Axions are elementary particles, which have zero spin and obey Bose statistics, that have been postulated to solve the strong CP problem in quantum chromodynamics, the theory of strong interactions. At low temperatures bosons condense in the same energy state and form Bose-Einstein Condensates, due to their quantum mechanical properties. Further, these condensates, or "axion stars" are gravitationally bound. It has been proposed that axion stars could be contributing to dark matter. Previous studies [3] have found axion stars have a dilute metastable state with a critical mass of ~10^{19} kg and a radius of ~200 km. By improving previous approximations of the configuration's energy, we determine a stable dense state exists at a radius of ~10 m. Furthermore, if the mass of the axion star is supercritical (including masses much greater than 10^{19} kg), the star will begin to collapse from its dilute state to the dense state. As it contracts, the star will decay and rapidly emit relativistic axions.

Introduction

Dark matter refers to mass that cannot be visibly observed. It was first discovered in the early 20th century when scientists observed flat galactic rotation curves, where the angular velocities of galactic bodies far from the center of the galaxy do not decrease as Newtonian gravity predicts. These observations, along with evidence collected through techniques such as gravitational lensing, have led us to believe that dark matter must be over five times as abundant as luminous matter. We expect dark matter to interact gravitationally, and be thereby observable with gravitational lensing, but very weakly otherwise. Specifically, dark matter has not been observed to emit photons or interact electromagnetically, nor interact with luminous matter through nuclear forces.

Axions were proposed in the 1970s to solve a symmetry problem in a subsection of particle physics. Axions, if they exist, primarily interact gravitationally and very weakly otherwise. Individual axions are long-living, and because they are bosons, at low temperatures they will condense in the same energy state and clump together to form "stars". These characteristics make them ideal dark matter candidates. When axions condense to form stars, they become gravitationally bound, and experience self-interaction forces.

Self-Interaction Expansion [1]

Axion dynamics are described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \qquad V(\phi) = m^2 f^2\left[1-\cos\left(\frac{\phi}{f}\right)\right]$$

where ϕ is a real scalar field, $V(\phi)$ is the self-interaction potential, and m and f are the mass and decay constant of an axion. We then take the non-relativistic limit by writing ϕ in terms of a wavefunction ψ,

$$\phi = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi + e^{imt}\psi^*\right]$$

and dropping the second derivatives with respect to time and rapidly oscillating terms ($e^{\pm imt}$). Further, we introduce the gravitational potential by hand, and obtain the Hamiltonian, a conserved quantity:

$$H = \int d^3r\left[\frac{|\psi|^2}{2m} + W(\psi) + \frac{1}{2}V_{grav}|\psi|^2\right]$$

Here $W(\psi)$, the effective self-interaction potential, can be expressed in two equivalent ways:

$$W(\psi) = -m^2 f^2 \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\frac{{}^{2n}C_n}{f^{2n}}\left(\frac{\psi^*\psi}{2m}\right)^n = m^2 f^2\left[1 - \frac{\psi^*\psi}{2mf^2} - J_0\left(\sqrt{\frac{2\psi^*\psi}{mf^2}}\right)\right]$$

This gives an expression that can either be estimated with any number of terms, or kept as a complete description of the axion self-interaction.

Previous research has truncated this approximation by taking only the leading order term of the self-interaction potential. This corresponds to an attractive self-interaction. However, the self-interaction terms *alternate* between attractive and repulsive. For radii greater than ~25 meters, the leading order approximation is sufficient and there are no discernable effects on the energy landscape near the dilute minimum (Figure 1). However, in the dense range, using the improved approximation, we recover a stable configuration at a radius of ~7 meters.

From this analysis we can see the self-interaction is effectively **attractive** at large radii, but hard-core **repulsive** at very small radii. This interesting result insists higher-order terms of the self-interaction approximation cannot be ignored when considering dense axion stars.

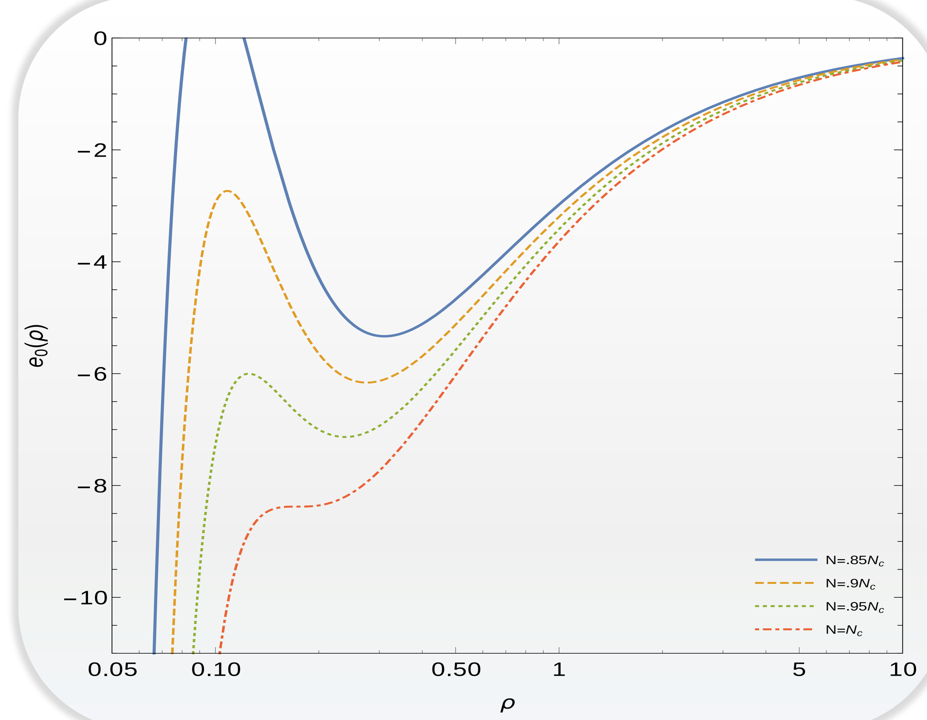

Figure 1. The energy landscape near the dilute, metastable configuration. As the mass of the dilute axion star increases, the energy well becomes shallower. Finally, when the dilute axion star reaches its critical mass, the metastable configuration can no longer exist, triggering a collapse.

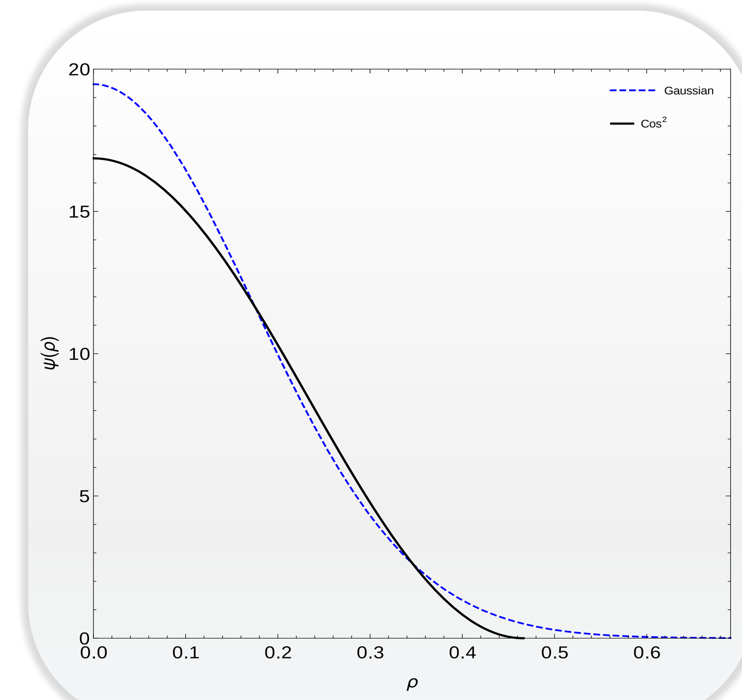

Figure 2. A comparison of the two ansätze used to analyze the stability of an axion star. The Gaussian ansatz assumes a Gaussian distribution of axion particles within the axion star. The cos^2 ansatz is similar to the Gaussian, but has a finite radius. In this plot both ansätze have been normalized to the same size, using the dimensionless variable ρ to describe the radius of the axion star.

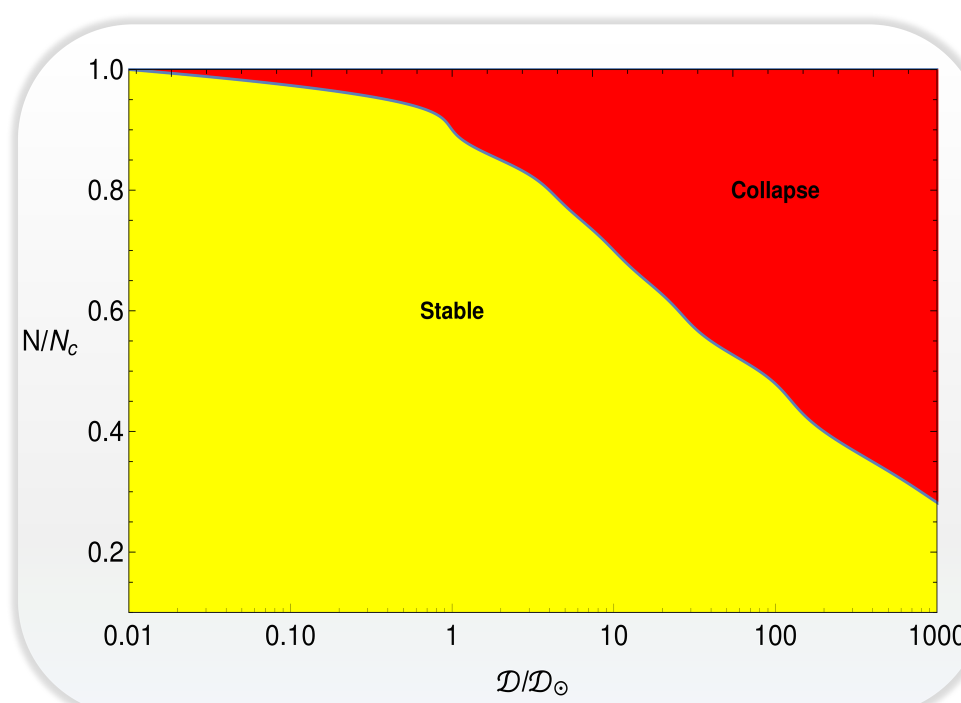

Figure 3. The two regions represented here indicate the stability of an axion star when it passes through an ordinary star. We can see that some axion stars with less than critical particle numbers could still become unstable within an ordinary star. Collapse of an axion star is dictated by the stellar density.

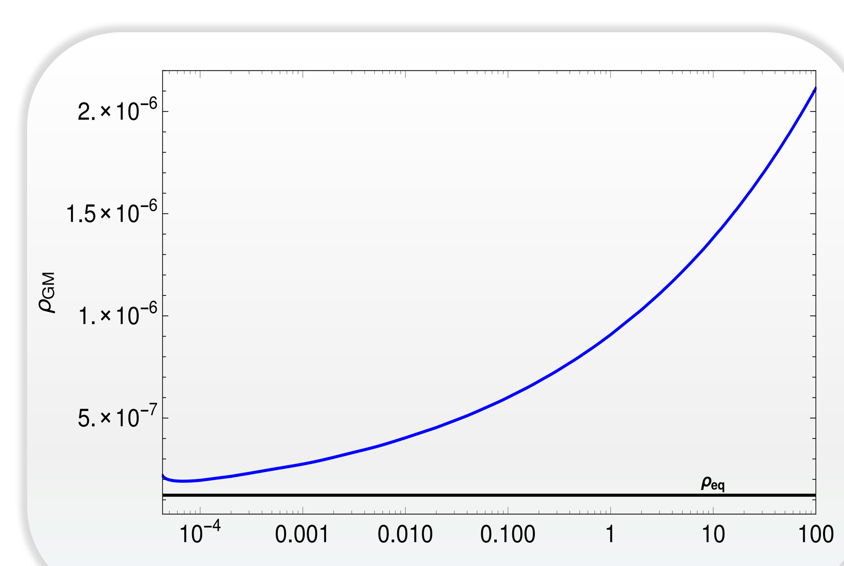

Figure 4. Here the blue line is the scaled radius of the global minimum as a function of scaled particle number. The black line is the radius at which the kinetic energy dominates. This plot demonstrates that the self-interaction energy stabilizes the dense axion star before the kinetic energy does.

Variational Method [1]

The full expression for the energy of an axion star– including the kinetic, gravitational, and self-interaction contributions– is given by:

$$E(\psi) = \int d^3r\left[\frac{1}{2m}|\nabla\psi|^2 + \frac{1}{2}V_{grav}|\psi^*\psi| + m^2f^2\left(1 - J_0\left(\sqrt{\frac{2\psi^*\psi}{mf^2}}\right)\right) - \frac{m}{2}\psi^*\psi\right]$$

To discover the axion star's stable radii, we formulate ansätze for ψ as functions of radius, and minimize the energy. Two specific ansätze were used to analyze an axion star's stability in this study.

$$\psi(r) = \frac{\sqrt{N}}{\pi^{3/4}\sigma^{3/2}}e^{-r^2/2\sigma^2} \qquad \psi(r) = \sqrt{\frac{4\pi N}{(2\pi^2-15)R^3}}\cos^2\left(\frac{\pi r}{2R}\right)$$

A comparison of these ansätze can be found in Figure 2.

Collisions with Astrophysical Sources [2]

Since an axion star near critical mass is sensitive to changes in its energy functional, the additional gravitational energy of an external astrophysical source could destabilize an axion star and trigger a collapse (Figure 3). Therefore we estimate the rate axion stars collide with various other astrophysical sources to determine the frequency of catalyzed events.

$$\Gamma_i = \frac{1}{S}\int d^3r\, n_{AS}(\vec{r})\, n_i(\vec{r})\,\langle\sigma v\rangle_i$$

Here Γ_i, the collision rate, depends on several factors: $n(\vec{r})$, the density of the population; σ, the cross section of collision; v, the relative velocity of the objects; and S, a symmetry factor which is 2 if the objects are the same, and 1 otherwise.

For a simple calculation, we can consider a constant density of axion stars and other objects throughout a galaxy. We can improve this approximation by employing specific dark matter and ordinary star distributions to the respective populations. σ is based on the cross sectional area of each object, and can be augmented by considering the gravity of a particularly massive object, such as a neutron star.

We consider three types of axion star collisions: with another axion star, an ordinary star, or a neutron star. The upper estimate of each rate is, $\mathcal{O}(10^7)$, $\mathcal{O}(3000)$, and $\mathcal{O}(200)$ collisions/year/galaxy respectively. While a collapse due to an axion star-axion star collision has P~10^{-8}, an axion star-ordinary star collision could result in total collapse if the axion star was near critical mass when the collision began. Finally, the magnetic field generated by a neutron star could be strong enough to convert an axion star to photons during an axion star-neutron star collision.

Conclusions

By choosing to approximate the self-interaction potential beyond the leading order term, we recover both *attractive* and *repulsive* self-interactions. These additional interactions, particularly the repulsive ones, reveal a **dense** stable equilibrium state with a radius of ~7 meters. Performing a numerical calculation with the full potential shows the actual radius is on the same order, at least 6 orders of magnitude larger than its Schwarzschild radius (Figure 4).

If the mass of the dilute axion star exceeds its critical mass of ~10^{19} kg, it will become unstable. In the case of destabilization, the star contracts slowly at first, then suddenly collapses to the dense state at a much smaller radius. We calculated the binding energy of the axion star as a function of its radius, and therefore as a function of time during the collapse. Based on a previous study [4] we concluded the high binding energy greatly increases the probability of decay during the final, rapid stages of the axion star's collapse. This decay, of the form $A_N \rightarrow A_{N-3} + a$ (where A_N is a condensate of N axions and a is a free axion), would cause the star to rapidly release highly relativistic axions, resulting in what has been called a Bosenova.

Since a collision with an astrophysical source could catalyze the collapse of an axion star, we examined the frequency of collisions between axion stars and other objects. We determined a collapse due to collision with another axion star would at most occur at a rate of 0.1 collapses/year/galaxy. However, a collision with an ordinary star could result in as many as 3000 collapses/year/galaxy if the ordinary star is of sufficient density. An axion star collision with neutron stars, which would result in the axions being converted to photons, could occur at a rate of ~200 collisions/year/galaxy, which is similar to the observed number of fast radio bursts.

References

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