A computational model of thin sheets crumpled via twisting

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Buckling as a failure

Localizes damage

Alters properties

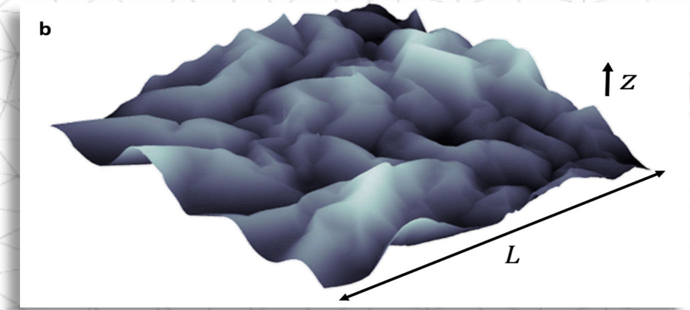


Fig. 1 of *A state variable for crumpled sheets* by
O. Gottesman, J. Andrejevic, C.H. Rycroft, and S.M. Rubenstein

Introduces frustration

Buckling as a failure or a feature

Localizes damage

Alters properties

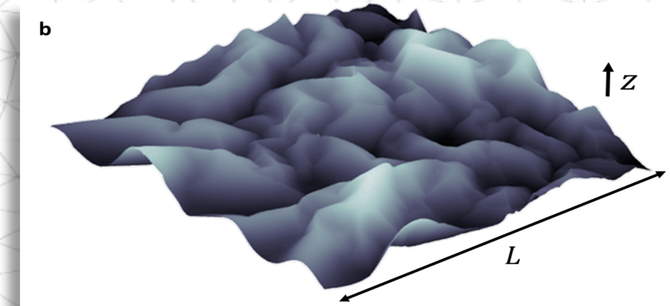


Fig. 1 of *A state variable for crumpled sheets* by O. Gottesman, J. Andrejevic, C.H. Rycroft, and S.M. Rubenstein

Introduces frustration

These consequences could be seen as failures of a material; alternatively, if we can learn to predict the transitions, these could be exploited as features in materials designed to deform controllably.

Steps to realizing design dreams

Statistical progression of crease formation

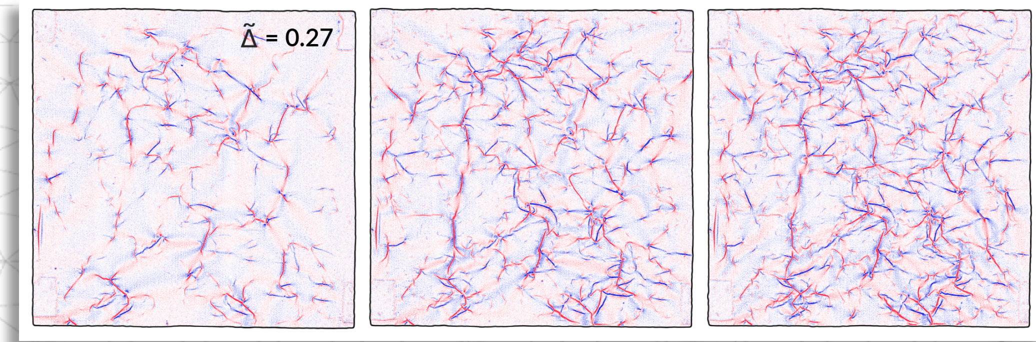
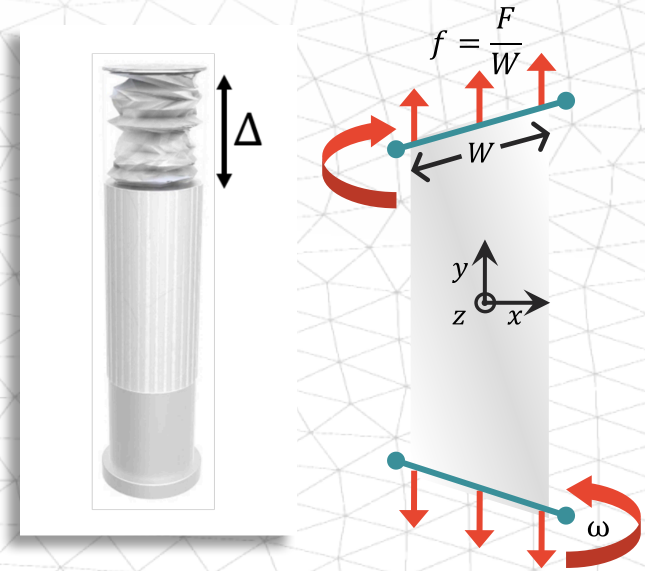


Fig. 3a of *A model for the fragmentation kinetics of crumpled thin sheets* by J. Andrejevic, L.M. Lee, S.M. Rubenstein, and C.H. Rycroft

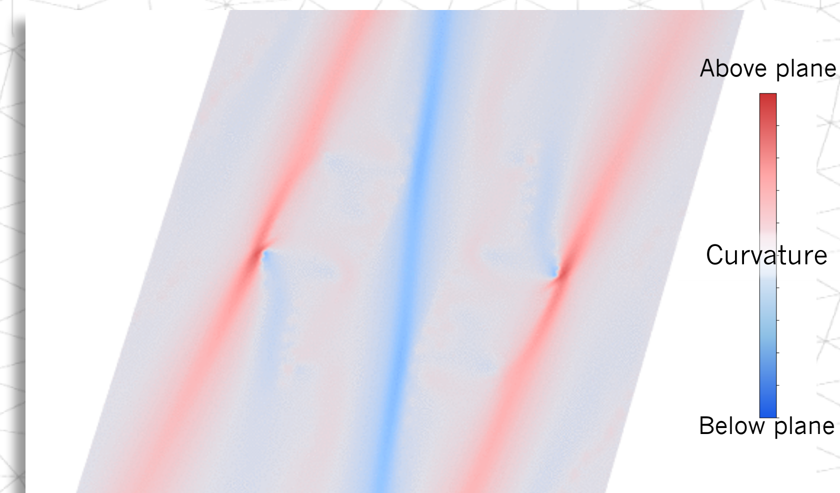


Two simulated crumpling procedures

What are the energetic trade-offs which result in damage accumulation?

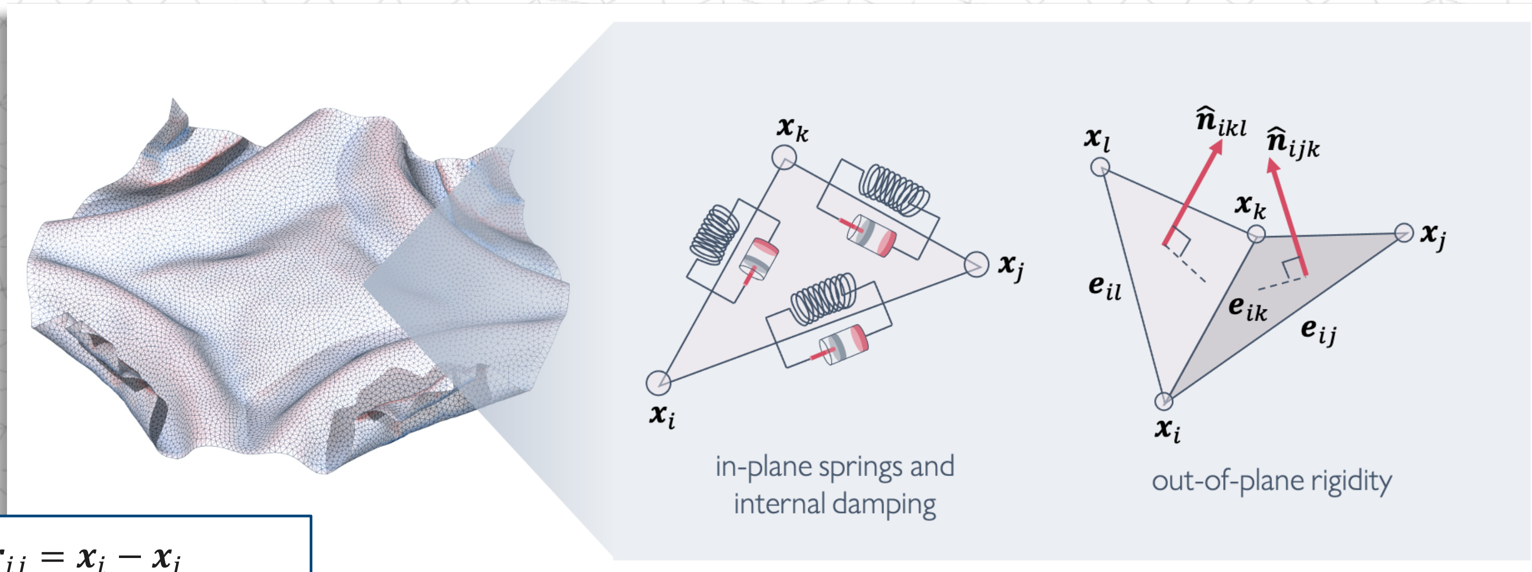
How does geometry of confinement dictate facet fragmentation?

Dynamic fragmentation kinetics



Facet evolution during crumpling via twisting

Computational model for elasticity



$$\boldsymbol{r}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$$

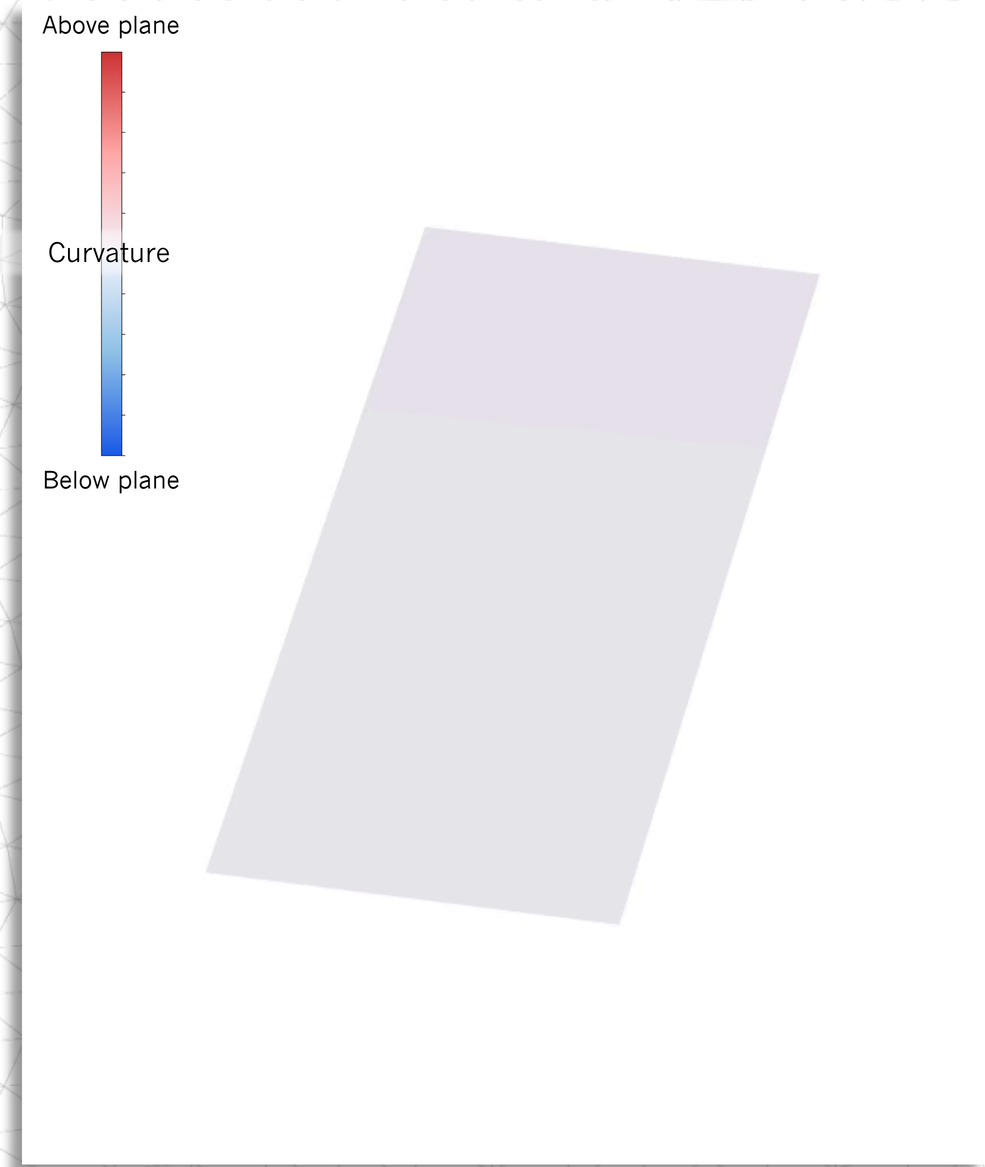
$$\kappa_{ij} \equiv angle\ of\ normals$$

In−plane springs: $E_s = \frac{1}{2} k_s^{ij} \left(\boldsymbol{r}_{ij}^0 - \boldsymbol{r}_{ij} \right)^2 \implies Y_{2D} = \frac{2}{\sqrt{3}} k_s \, , \nu = {}^1/_3$

Out−of−plane rigidity: $E_b = \frac{1}{2} k_b^{ij} \left(\kappa_{ij} \right)^2 \implies \mathrm{B} = \frac{\sqrt{3}}{2} k_b \, , h = \sqrt{\frac{8\, k_b}{k_s}}$

Triangular **and** random lattices can be mapped to a continuous elastic sheet with this model.

Computational model for elasticity



Stats:

Regular triangular lattice,

$n_{nodes} = 30551, n_{springs} = 90880,$

$W \times L \times h = 10 \times 26 \times 0.01\ cm,$

$Y_{3D} = 1.5\ GPa$, $\mathrm{f} = 86\ N/m,$

2.26×10^6 time steps,

36 hours using 16 threads

Features:

- Cross sections of deflection
- Strain tensor at each facet
- Bulk and local energy analysis
- Fine temporal resolution of buckling transitions and stress-focusing

Elastic deformation modes

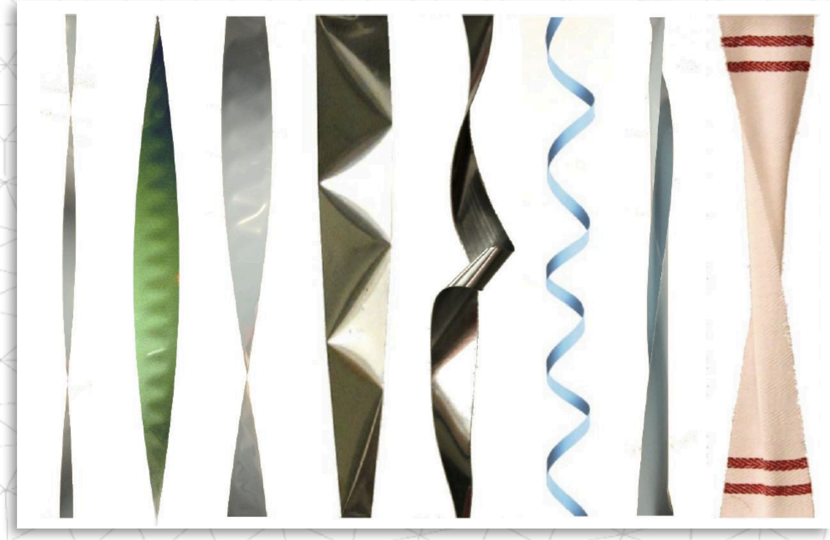
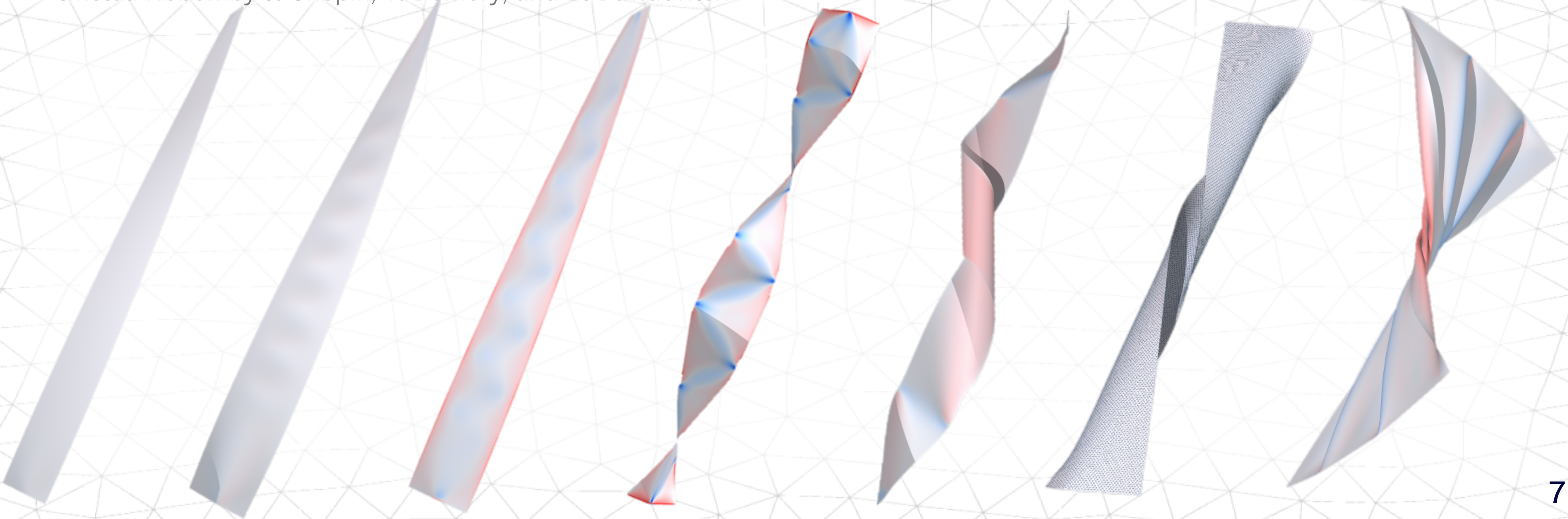


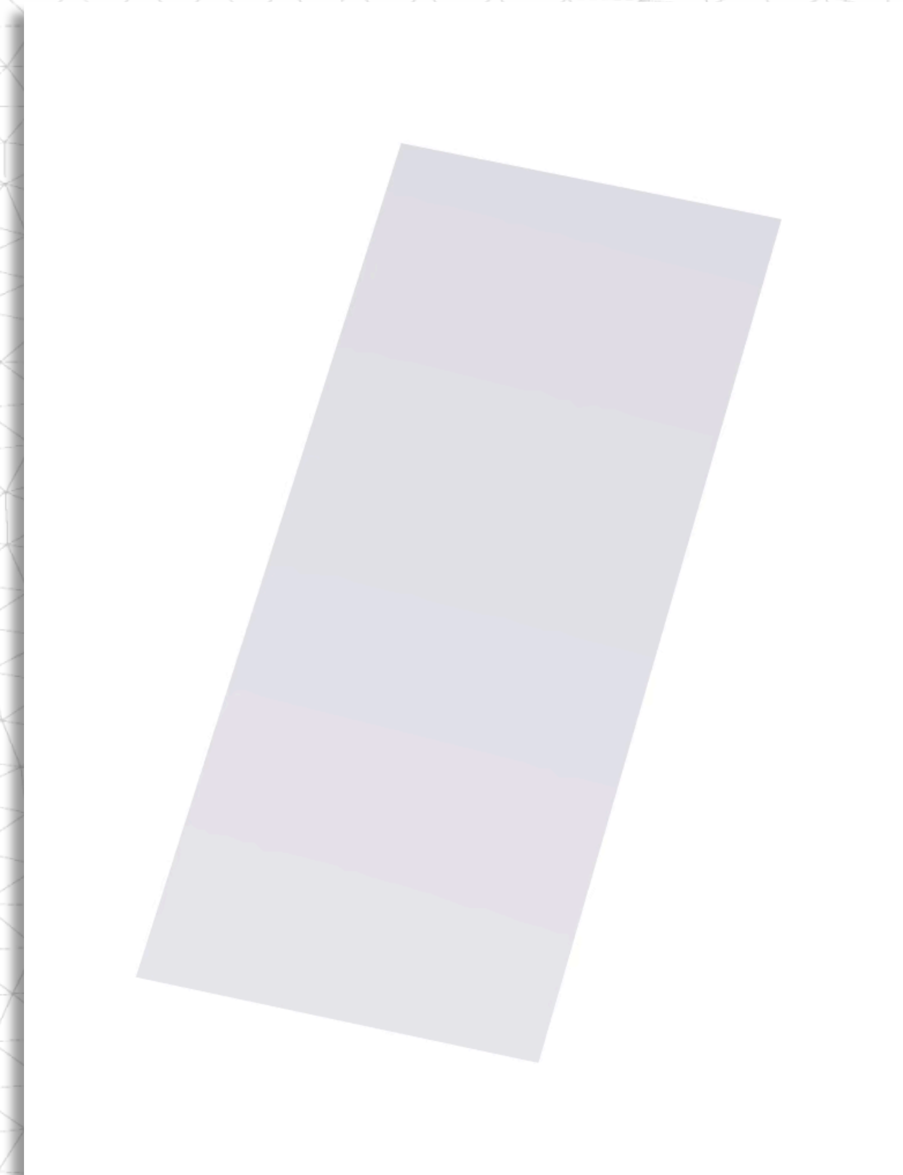
Fig. 2 of Roadmap to the morphological instabilities of a stretched twisted ribbon by J. Chopin, V. Démery, and B. Davidovitch

The sprawling zoo of elastic ribbon morphologies can be replicated with our model.

We gain insight to the energetic trade-offs which drive these transitions.



Computational model for plasticity



Stats:

Regular triangular lattice,

$n_{nodes} = 30551, n_{springs} = 90880,$

$W \times L \times h = 10 \times 26 \times 0.013\ cm,$

$Y_{3D} = 0.996\ GPa,\ \mathrm{f} = 30\ N/m,$

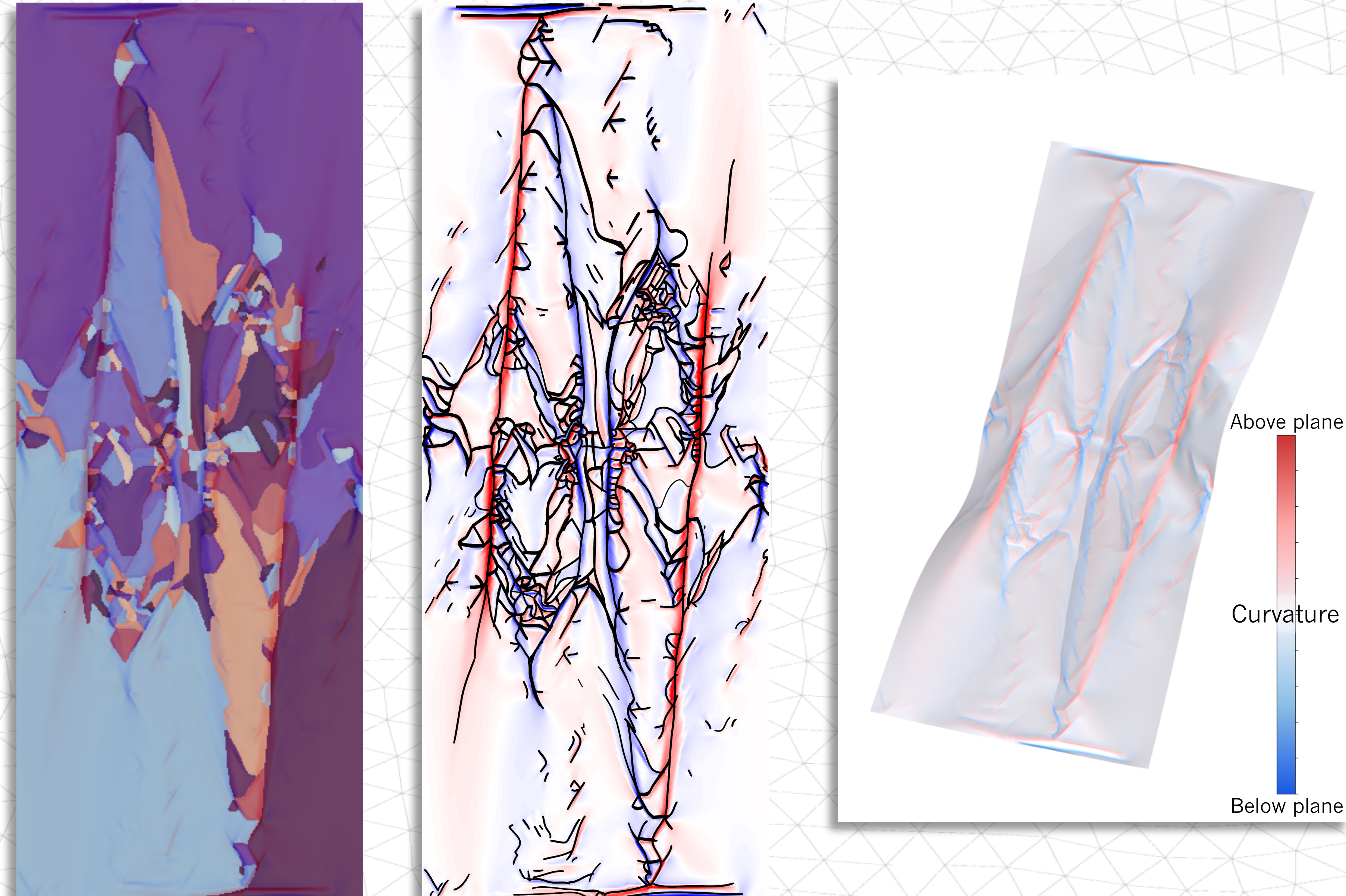
1.76×10^7 time steps,

~6 days using 16 threads

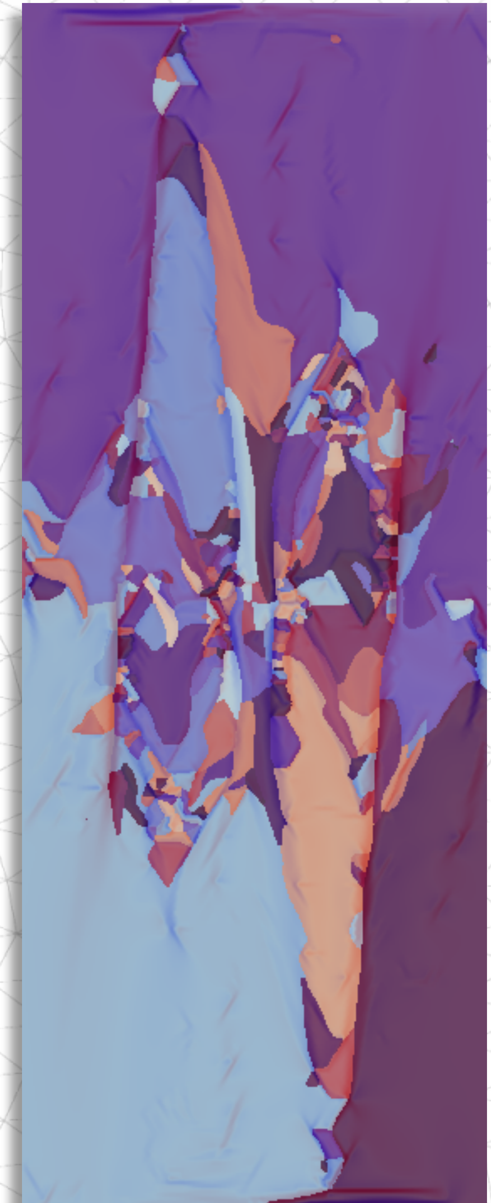
Features:

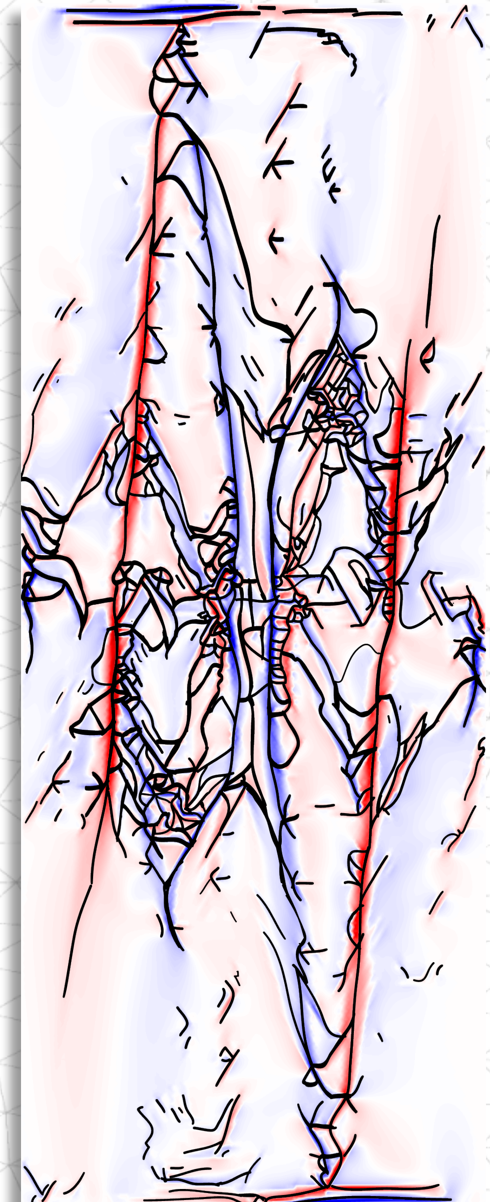
- All the elastic model features
- Precise maps of plastic damage
- ~Continuous evolution of facet fragmentation and ridge "mileage"

Simulated data to augment experiments

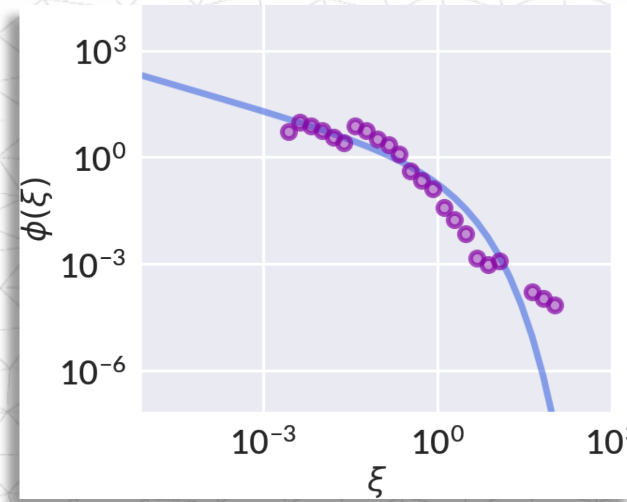


Simulated data to augment experiments

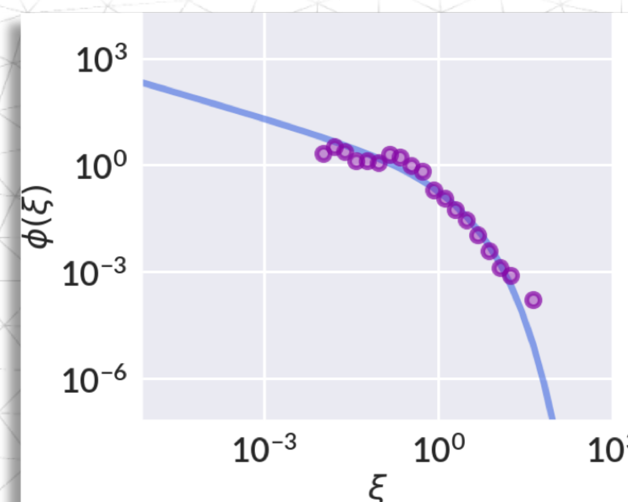




Distribution of all facets



Distribution excluding four largest facets



Summary and acknowledgements

- Simply−motivated model, with easy mapping from discrete mesh to continuous sheet
- Access to internal dynamics and ability to generate large volume of data
- Fine temporal resolution of buckling transitions and crumpling evolution

Huge thanks to Jovana Andrejevic, Prof. Chris Rycroft, Prof. Arshad Kudrolli, and Rycroft Group!

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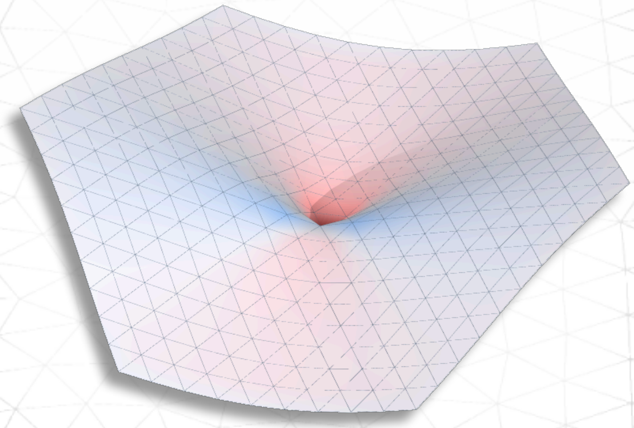
Characterizing material defects

With this mass–spring–model we precisely control our mesh topology. We can study defects, puckers, and dimples, individually or in arrays.

Characterizing how various defect models respond to loads can improve how we simulate material imperfections.



Buckled 5–7 dislocation

Puckered dilation defects

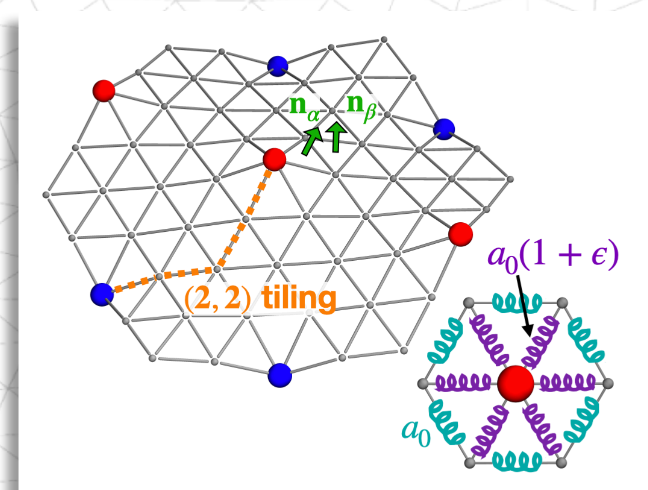
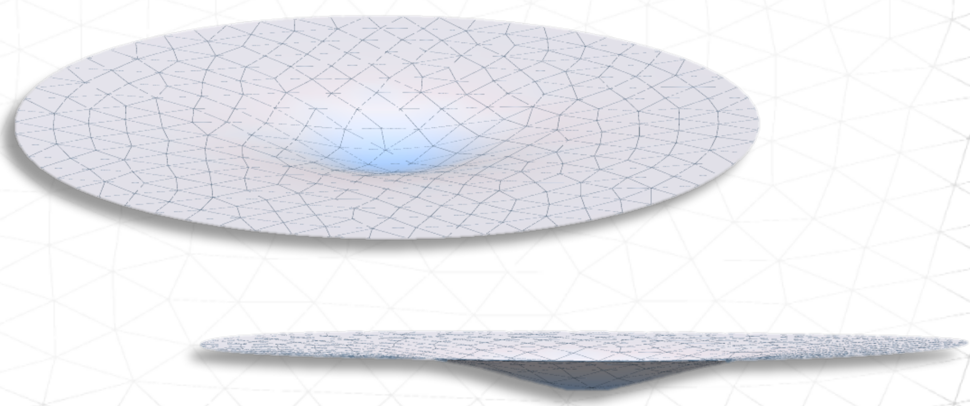


Fig. 1 of *Buckling and metastability in membranes with dilation arrays* by A. Plummer and D.R. Nelson



Plastically deformed dimples

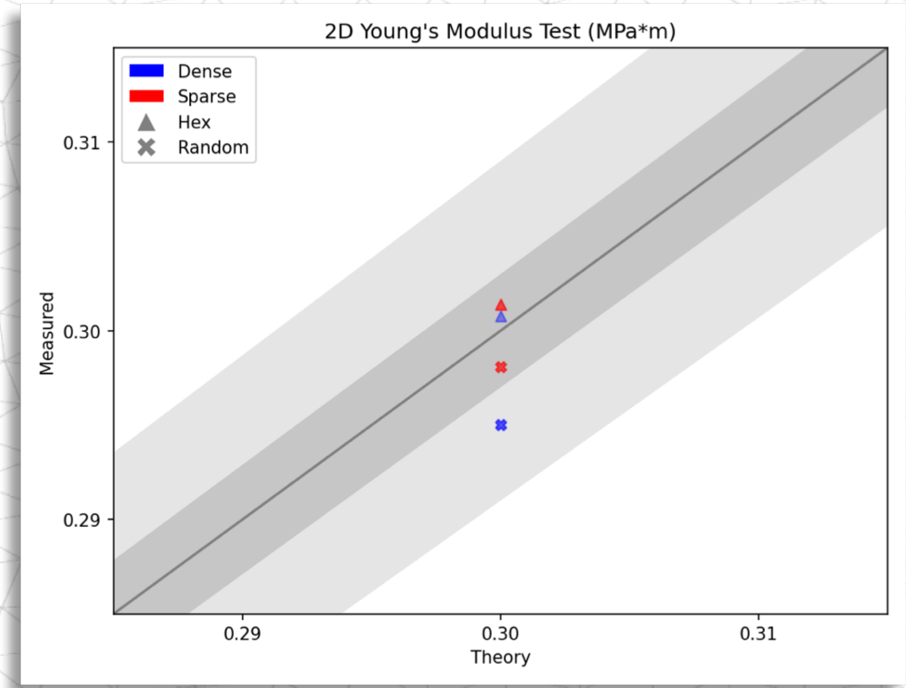
Model validation tests

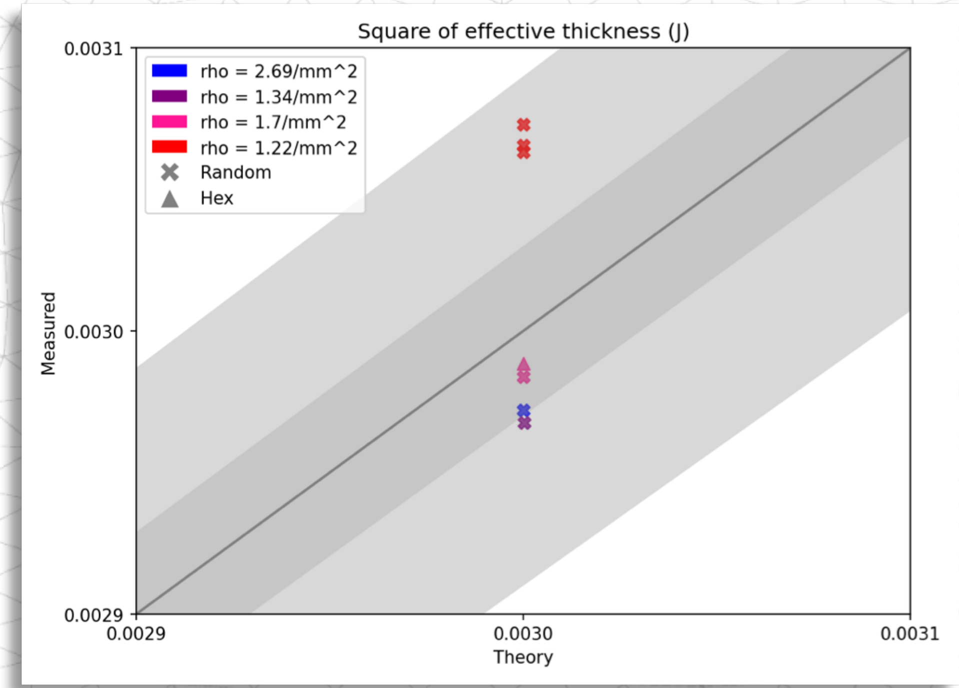
$$E_s = \frac{1}{\sqrt{3}} \frac{\bar{A}}{\|\boldsymbol{e}_{ik}\|^2} k_s (l_{ik} - \|\boldsymbol{e}_{ik}\|)^2$$

$$E_b = \frac{\sqrt{3}}{4} \frac{\|\boldsymbol{e}_{ik}\|^2}{\bar{A}} k_b \left\| \hat{n}_{ijk} - \hat{n}_{ikl} \right\|^2$$

$$Y_{2D} = \frac{2}{\sqrt{3}} k_s$$

$$\mathrm{h}_{eff}^2 = \frac{16}{\sqrt{3}} k_b$$





Dark gray band indicates 1% error, lighter gray band 3% error

Computational model for plasticity

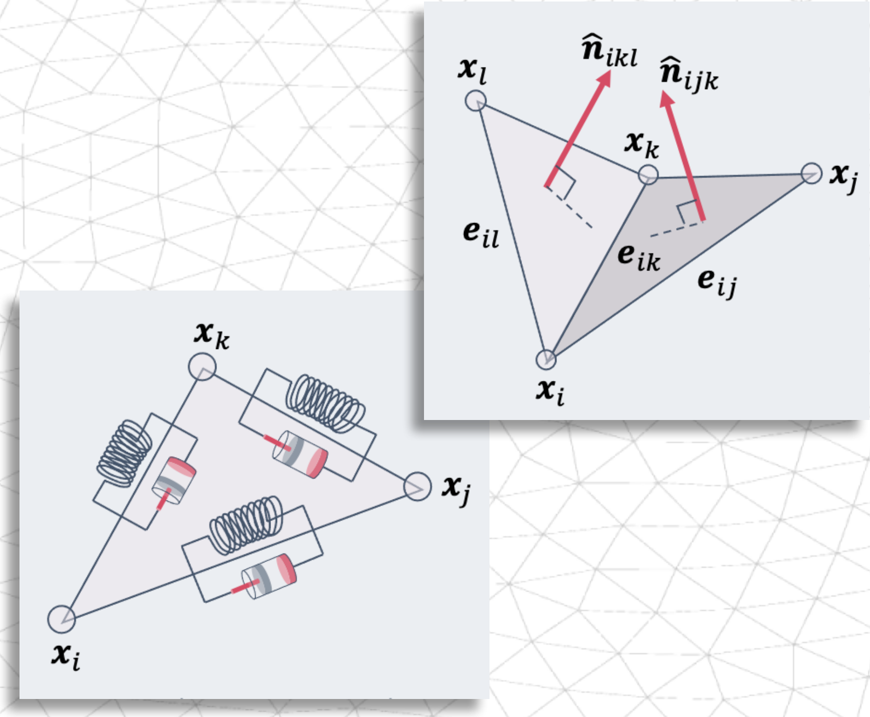
$$\boldsymbol{r}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$$
$$\kappa_{ij} \equiv angle\ of\ normals$$

In−plane springs: $E_s = \frac{1}{2} k_s^{ij} \left(\boldsymbol{r}_{ij}^0 - \boldsymbol{r}_{ij} + q_{ij} \right)^2$

Out−of−plane bending: $E_b = \frac{1}{2} \mathrm{k}_\mathrm{b}^\mathrm{ij} \left(\kappa_{ij} - p_{ij} \right)^2$

q_{ij} and p_{ij} store local stretching and bending damage respectively, effectively changing the rest length or angle at an edge.

They are signed quantities and are updated using a strain−hardening model.



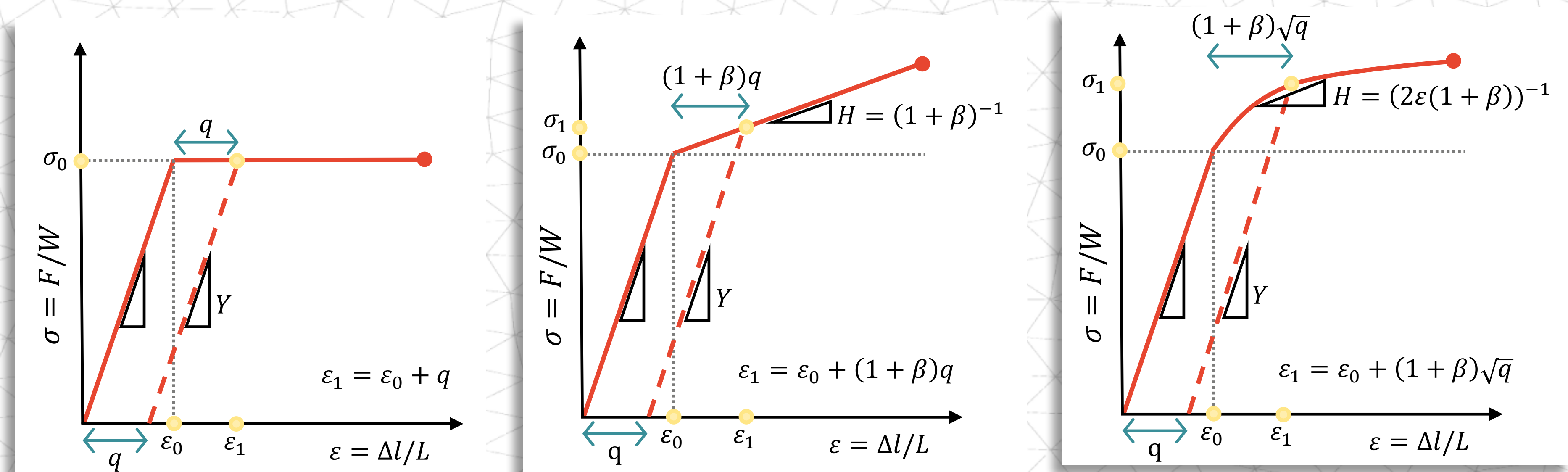
Strain models for plasticity

Current strain–hardening model

$$\frac{dq}{dt} = \begin{cases} g(l-q) & l-q>0 \\ -g(-l+q) & l-q\le 0 \end{cases}$$

$$g(\lambda) = \begin{cases} 0 & \lambda < l_0 \\ \frac{1}{1+\gamma}(\lambda - l_0) & \lambda \ge l_0 \end{cases}$$

- Isotropic hardening: yield surface is modified ($\sigma_1^t = -\sigma_1^c$)
- Kinematic hardening: yield surface is shifted ($|\sigma_1^t - \sigma_1^c| = 2Y\varepsilon_0$)



Perfectly plastic   Linear hardening   Nonlinear hardening