Capital Mobility in Neoclassical Models of Growth

By Robert J. Barro, N. Gregory Mankiw, and Xavier Sala-i-Martin*

The neoclassical growth model accords with empirical evidence on convergence if capital is viewed broadly to include human investments, so that diminishing returns to capital set in slowly, and if differences in government policies or other variables create substantial differences in steady-state positions. However, open-economy versions of the theory predict higher rates of convergence than those observed empirically. We show that the open-economy model conforms with the evidence if an economy can borrow to finance only a portion of its capital, for example, if human capital must be financed by domestic savings. (*JEL* O40, E10)

Several recent studies have looked for evidence on convergence, defined here as the tendency for poor economies to grow faster than rich economies. The clearest empirical support for convergence comes from economies that, except for initial conditions, appear similar. Steve Dowrick and Duc-Tho Nguyen (1989) reported convergence for OECD countries. Barro and Sala-i-Martin (1991, 1992b) found that convergence occurred for the U.S. states and the regions of Europe and Japan at a rate of about 2 percent per year. Moreover, for the U.S. states, state product per capita and state personal income per capita have converged at roughly the same rate.

The evidence from larger samples of countries is more controversial. Paul M. Romer (1987) and Sergio Rebelo (1991) emphasized the lack of correlation between initial per capita GDP and the subsequent per capita growth rate for a broad sample of about 100 countries. They interpreted this finding as evidence against the convergence implications of the neoclassical growth model that was developed by Frank Ramsey (1928), Robert M. Solow (1956), Trevor Swan (1956), David Cass (1965), and Tjalling C. Koopmans (1965). Yet an alternative interpretation is that these economies, unlike the OECD countries and U.S. states, have substantially different steady states. These differences can reflect the effects of disparities in preferences and government policies on the saving rate, fertility, and the available production technology. Barro (1991), Ross Levine and David Renelt (1992), and Mankiw et al. (1992) reported that, after controlling for differences in investment rates, initial stocks of human capital, and some other variables, countries converge *conditionally* at a rate of about 2 percent per year. That is, the various data sets confirm that conditional rates of convergence are slow but significant.

Standard theories of economic growth, however, cannot easily explain all of the empirical findings on convergence. Barro and Sala-i-Martin (1992a) and Mankiw et al. (1992) noted that the neoclassical growth model can explain the observed rate of conditional convergence if economies are closed and the capital share is about 0.8. A capital share this large is reasonable if capital is viewed broadly to include human and physical components. (A more conventional capital share of around 0.3 implies much faster convergence than that observed in the data.)

*Barro: Department of Economics, Harvard University, Cambridge, MA 02138; Mankiw: Department of Economics, Harvard University; Sala-i-Martin: Department of Economics, Yale University. This research has been supported by grants from the National Science Foundation. Sala-i-Martin was supported by the European Fund for Regional Development (FEDER).

Yet the assumption of a closed economy is more difficult to justify, especially when applied to economies like the U.S. states. Interest rates are about the same in each state, and substantial borrowing and lending seems to flow across state borders. However, if one assumes that economies are open, then the neoclassical growth model predicts that capital will move quickly to equalize marginal products and, hence, that convergence of output per worker will be rapid.

Standard endogenous-growth models for closed economies, as described by Rebelo (1991), also cannot account fully for the evidence. The one-sector, AK, model without diminishing returns can explain differences in output per person without differences in real interest rates. But this type of model is inconsistent with convergence if each economy has a fixed technology parameter, A. Two-sector endogenous-growth models can explain convergence based on imbalances between physical and human capital (see Casey B. Mulligan and Sala-i-Martin, 1993). But real interest rates would differ across closed economies, and the imbalances would vanish instantaneously across open economies.

The main purpose of this paper is to construct a model of economic growth that is consistent with the growing body of evidence on convergence. In particular, we want to explain gradual convergence in output and income per person while allowing for an international credit market that equates the real interest rates across economies. The key to our model is that capital is only partially mobile: borrowing is possible to finance accumulation of physical capital, but not accumulation of human capital. We show that this assumption of partial capital mobility, imbedded in an open-economy version of the neoclassical growth model, can explain the evidence on convergence.

I. The Model

Output is produced with three inputs: physical capital, human capital, and a nonreproducible factor, which we view as raw labor. We assume that the production function is Cobb-Douglas:

$$Y = AK^{\alpha}H^{\eta}(Le^{gt})^{1-\alpha-\eta}$$

where $\alpha > 0$, $\eta > 0$, $\alpha + \eta < 1$, Y is output, K is the stock of physical capital, H is the stock of human capital, L is the quantity of raw labor, and A is a fixed technology parameter.[1] Raw labor grows at the constant, exogenous rate n, and g is the constant, exogenous rate of labor-augmenting technological progress. We assume a one-sector production technology in which physical capital, human capital, and consumables are perfectly substitutable uses of output. If we work as is customary in units of effective labor ($y \equiv Y/Le^{gt}$, $k \equiv K/Le^{gt}$, $h \equiv H/Le^{gt}$), then the production function is given in intensive form by

$$(1) \qquad y = Ak^{\alpha}h^{\eta}.$$

The households own the three inputs and rent them to firms at competitive rental prices. Firms pay a proportional tax at rate τ on output. We interpret this tax to include various elements that affect the incentives to accumulate capital; for example, τ includes the risk of expropriation by the government, strong labor unions, or foreign invaders. The after-tax cash flow for the representative firm is given in units of effective labor by

$$(2) \qquad \pi = (1-\tau)Ak^{\alpha}h^{\eta} - w - R_{k}K - R_{h}h$$

where w is the wage rate, R_k is the rental price of physical capital, and R_h is the rental price of human capital. In the absence of adjustment costs, the maximization of the present value of future cash flows is equivalent to the maximization of profit in each period. The firms therefore equate the

[1]Most of the analysis goes through with a more general production function that satisfies the usual neoclassical properties, including constant returns to scale.

marginal products to the rental prices:

$$\text{(3a)} \quad R_k = (1-\tau)\alpha A k^{\alpha-1} h^{\eta} = (1-\tau)\alpha y/k$$

$$\text{(3b)} \quad R_h = (1-\tau)\eta A k^{\alpha} h^{\eta-1} = (1-\tau)\eta y/h$$

$$\text{(3c)} \quad w = (1-\tau)A k^{\alpha} h^{\eta} - R_k k - R_h h.$$

These conditions imply $\pi = 0$ in equation (2).

Households are represented by the standard, infinitely-lived, Ramsey consumer with preferences

$$\text{(4)} \quad U = \int_0^{\infty} \left(\frac{C^{1-\theta} - 1}{1-\theta} \right) e^{-(\rho-n)t} dt$$

where C is per capita consumption, ρ is the subjective rate of time preference, and θ is the inverse of the intertemporal elasticity of substitution. Population, e^{nt}, corresponds to the labor force.

The households own the physical and human capital and also have the net stock of debt, d, per unit of effective labor. They receive income from wages and rentals and spend this income on accumulation of physical capital, accumulation of human capital, and consumption. Hence, the budget constraint is

$$\text{(5)} \quad \dot{h} + \dot{k} - \dot{d} = w + (R_k - n - g - \delta)k - (r - n - g)d + (R_h - n - g - \delta)h - c$$

where r is the real interest rate, $c \equiv Ce^{-gt}$, and a dot over a variable represents its time derivative. (Recall that firms' profits, π, equal zero and therefore do not appear in the equation.) Equation (5) assumes that the relative prices of consumables, physical capital, and human capital are always fixed at unity and that physical and human capital depreciate at the same rate, δ. We also assume that none of the taxes collected are remitted to households, although our results would not change if these revenues showed up as lump-sum transfers or as government services that did not affect productivity or interact with the choices of consumption.

Households can borrow and lend at the real interest rate r on the domestic bond market. In a closed economy, the debt (d) is zero for the representative household, and r is determined by the equilibrium of saving and investment at the national level. In an open economy, r is determined at the world level, and d (the foreign debt per effective worker) can be positive or negative.

To simplify the exposition, we integrate the households and firms by substituting the first-order conditions from equation (3) into the budget constraint in equation (5) to get

$$\text{(6)} \quad \dot{h} + \dot{k} - \dot{d} = (1-\tau)A k^{\alpha} h^{\eta} - (\delta + n + g)(k + h) - (r - n - g)d - c.$$

Households maximize utility from equation (4) subject to equation (6), given $k(0) > 0$, $h(0) > 0$, and $d(0)$. In the following sections, we consider this maximization problem under environments that involve different degrees of capital mobility. These differences entail various restrictions on the path of debt, d.

II. The Closed Economy

A. *Laws of Motion*

The first environment is the closed economy, in which households can borrow and lend on the domestic capital market at the rate r but cannot borrow or lend on international markets. Hence, $d = 0$, and the only difference from the standard neoclassical growth model is that the technology involves two kinds of capital. The rate of return, r, must equal the net returns on the two kinds of capital, $R_k - \delta$ and $R_h - \delta$, where the rental prices are given in equations (3a) and (3b). These equations imply $k/h = \alpha/\eta$ at all points in time. If the initial value of k/h differs from α/η, then households "jump" to the desired ratio. Since we assume no adjustment costs or irreversibility constraints, this jump is feasible.[2] We can then rewrite the budget constraint from equation (6) in terms of a broad capital stock, $z \equiv k + h$:

$$(7) \quad \dot{z} = (1-\tau)\tilde{A}z^{\alpha+\eta} - (\delta + n + g)z - c$$

where $\tilde{A} \equiv A\alpha^{\alpha}\eta^{\eta}(\alpha+\eta)^{-(\alpha+\eta)}$.

The household's problem now corresponds to the standard formulation of the neoclassical growth model, except that the production function is less concave: the capital share is $\alpha + \eta$, which corresponds to physical and human capital, rather than α, which corresponds only to physical capital. Diminishing returns therefore set in more slowly.

The Euler equation that characterizes the solution is familiar:

$$(8) \quad \dot{c}/c = (1/\theta)[(1-\tau)\tilde{A}(\alpha+\eta)z^{\alpha+\eta-1} - (\delta + \rho + \theta g)]$$

where $(1-\tau)\tilde{A}(\alpha+\eta)z^{\alpha+\eta-1}$ is the after-tax marginal product of capital, which equals $r + \delta$. Equations (7) and (8) and the transversality condition fully describe the transition of the economy toward the steady state.

B. *The Steady State and the Feldstein-Horioka Puzzle*

The steady-state growth rate of the variables in units of effective labor is zero. The per capita variables grow accordingly at the rate of productivity growth, g, and the level variables grow at the rate of growth of population plus productivity, $n + g$. The steady-state stock of broad capital in units of effective labor is given by

$$(9) \quad z^* = \left[\frac{(1-\tau)\tilde{A}(\alpha+\eta)}{\delta+\rho+\theta g}\right]^{1/(1-\alpha-\eta)}$$

[2] If physical or human capital is irreversible (as is realistic in most situations), then the model involves transitional dynamics, as described by Mulligan and Sala-i-Martin (1993) and Barro and Sala-i-Martin (1994 Chapters 4, 5).

Hence, z^* is a decreasing function of ρ, θ, δ, and τ, and an increasing function of the level of technology, $\tilde{A}$. The breakdown of broad capital into its two components is given in the steady state by

$$(10) \quad k^* = z^*\alpha/(\alpha+\eta)$$
$$h^* = z^*\eta/(\alpha+\eta).$$

This analysis for closed economies provides an interpretation of the puzzle raised by Martin S. Feldstein and Charles Horioka (1980) regarding the strong correlation between saving and investment rates across countries. Suppose that all countries have the same technology and preferences and differ only in the level of the tax rate, τ, which should be interpreted broadly to include various disincentives to invest. The steady-state ratio of gross investment (on physical and human capital), i, to GDP equals the full depreciation rate, $\delta + n + g$, times the ratio of total capital to output, $(z/y)^*$. Equation (8) implies that $(z/y)^*$ equals $(1-\tau)(\alpha+\eta)/(\delta+\rho+\theta g)$. Since the gross saving rate, s, equals the investment ratio, the steady-state values are therefore

$$(11) \quad (i/y)^* = s^* = \frac{(1-\tau)(\delta+n+g)(\alpha+\eta)}{\delta+\rho+\theta g}.$$

Hence, closed economies with high tax rates—that is, high disincentives for investment—have low steady-state investment and savings rates.

Now consider the incipient capital movements. The steady-state real interest rate is $\rho + \theta g$ and is therefore independent of the tax rate. If economies differ only in their tax rates, then the steady-state real interest rates are the same for all countries. Economies with low steady-state capital intensities have high marginal products of capital, but they also have high tax rates. The exact offset of these two effects implies that the after-tax marginal products of capital and, therefore, the real interest rates are independent of the capital intensities. If we opened up international capital markets, then capital would not flow across countries because the after-tax returns are already equalized. Thus, domestic investment and national saving rates would be perfectly correlated in a cross section of countries even with full capital mobility.

C. *Convergence During the Transition*

To quantify the convergence implications of the model, we follow Barro and Sala-i-Martin (1992a) and log-linearize the system, equations (7) and (8), around steady state. The growth rate of y (output per effective worker) between times 0 and T can be written as

$$(12)\quad (1/T)\{\log[y(T)/y(0)]\} \approx \left(\frac{1-e^{-\beta T}}{T}\right)\{\log[y^*/y(0)]\}.$$

That is, the growth rate is a negative function of initial per capita income, $y(0)$, after controlling for the steady-state level, y^*. The convergence rate, β, is a function of the underlying parameters:[3]

$$(13)\quad 2\beta = \left\{\varphi^2 + 4\left(\frac{1-\alpha-\eta}{\theta}\right)(\delta+\rho+\theta g) \times\left[\frac{\delta+\rho+\theta g}{\alpha+\eta} - (\delta+n+g)\right]\right\}^{1/2} - \varphi$$

where $\varphi = \rho - n - (1-\theta)g > 0$.

Equation (13) implies that the convergence rate, β, depends inversely on the share of broad capital, $\alpha+\eta$. If $\alpha+\eta=1$ (i.e., if all inputs can be accumulated), then $\beta=0$. This outcome corresponds to the one-sector linear endogenous-growth model of Rebelo (1991). Another important property is that β is independent of the level of technology, A, and the tax rate, τ.

We can assess the convergence speed quantitatively if we specify parameter values that seem reasonable for the U.S. economy. We assume $n = 0.01$ per year to correspond to average population growth in recent decades and $g = 0.02$ per year to match the long-term growth rate of real per-capita GDP. National-accounts measures of depreciation of stocks of physical capital suggest $\delta \approx 0.05$ per year. The values $\alpha = 0.3$ and $\eta = 0.5$ are consistent with shares of physical and human capital estimated by Dale W. Jorgenson et al. (1987).

If we assume $\rho = 0.02$ per year and $\theta = 2$, then the steady-state interest rate (which equals $\rho + \theta g$) is 0.06 per year, which fits with long-term averages of real rates of return on the stock market. If $\tau = 0.3$, then the implied steady-state gross saving rate, s^*, is determined from equation (11) as 0.41, which is satisfactory for a broad concept of saving that includes gross expenditures on human capital. Values of θ that are much above 2 imply counterfactually low saving rates; for example, $\theta = 10$ implies $s^* = 0.12$.

Our baseline specification of parameter values is therefore

$$(14)\quad \begin{array}{lll} n = 0.01 & g = 0.02 & \delta = 0.05 \\ \theta = 2 & \rho = 0.02 & \alpha = 0.3 \\ & \eta = 0.5. & \end{array}$$

These values imply a rate of convergence, β, from equation (13) of 0.014 per year. This value is close to the low end of the range of estimates of convergence coefficients (roughly 0.015–0.030 per year) from the empirical literature mentioned in the Introduction.

The coefficient β is not very sensitive to variations in ρ, n, g, and δ. For example, if we maintain the values of the other parameters at their baseline values, then β rises from 0.014 to 0.015 if ρ increases to 0.03 or n increases to 0.02, and it rises to 0.017 if g increases to 0.03 or δ increases to 0.07. The coefficient β also rises if θ declines; for example, β rises to 0.020 if θ falls to 1.

[3]This formula corresponds to the one given in Barro and Sala-i-Martin (1992a), except that the capital share is now $\alpha+\eta$ rather than α.

The convergence speed is more sensitive to variations in the broad capital share, $\alpha+\eta$. For example, β rises to 0.024 if $\alpha+\eta$ falls to 0.7 and to 0.035 if $\alpha+\eta$ falls to 0.6. This sensitivity underlies the conclusion that the neoclassical growth model is consistent with empirical estimates of convergence speeds only if the capital share, $\alpha+\eta$, is high.

The ratio of capital stocks, k/h, remains constant during the transition, but the ratio of each stock to GDP, k/y and h/y, rises as the economy approaches the steady state. The transitional increase in k/y conflicts with the view associated with Nicholas Kaldor (1961) that the capital–output ratio changes little during the process of development. If $\alpha+\eta=0.8$, however, then diminishing returns set in slowly, and the model predicts only moderate changes in k/y: if k and h double, then k/y rises by 15 percent. Since k/y is not precisely constant empirically, the prediction of a slowly rising k/y is not a shortcoming of the theory.

The theory also implies that the marginal products of physical and human capital, and hence the real interest rate r, would decline over time. If $\alpha+\eta=0.8$ and r begins at 8 percent for example, then a doubling of k and h implies that r would fall to 6.9 percent. This slow decline of the real interest rate—a reflection of the slow onset of diminishing returns—does not conflict with empirical evidence. Barro (1993 table 11.1) shows, for example, that the real interest rate for the United States in the 19th century was higher than that in the 20th century.

The expression for β in equation (13) simplifies if, following Solow (1956) and Swan (1956), we assume a constant gross saving rate. This assumption amounts to a restriction on the parameters of the model (see Mordecai Kurz, 1968; Barro and Sala-i-Martin, 1994 Ch. 2). The gross saving rate is constant if

$$(15)\quad \theta=\frac{\delta+\rho}{(\alpha+\eta)(\delta+n)-g(1-\alpha-\eta)}$$

and the corresponding gross saving rate is then

$$(16)\quad s=(1-\tau)/\theta.$$

In this case, the accumulation constraint in equation (7) can be written as

$$(17)\quad \dot{z}/z=s(1-\tau)\tilde{A}z^{\alpha+\eta-1}-(\delta+n+g)$$

and the steady-state capital intensity is

$$z^*=\left[\frac{s\tilde{A}(1-\tau)}{\delta+n+g}\right]^{1/(1-\alpha-\eta)}$$

Equation (12) still provides a log-linear approximation to the growth rate, but the convergence coefficient simplifies to

$$(18)\quad \beta=(1-\alpha-\eta)(\delta+n+g)$$

which is the expression given in Mankiw et al. (1992). As before, β is a decreasing function of the capital share, $\alpha+\eta$, and equals zero when $\alpha+\eta=1$.[4] Also, β is again independent of the level of technology, $\tilde{A}$, and the tax rate, τ.

If the parameters (α, η, δ, ρ, n, and g) take on the values assumed in equation (14) and if $\tau=0.3$, then equations (15) and (16) imply that the gross saving rate is constant at the value $s=0.44$ if $\theta=1.59$. In this case, the value of β from equation (18) is 0.016.

Note that the high gross saving rate includes saving in human capital. The saving rate that corresponds to physical capital is given by $s\alpha/(\alpha+\eta)$, which equals 0.17 for the specified parameter values. We also have to consider that only a fraction of the expenditure on human capital appears in measured GDP: a significant portion corresponds to forgone wages. If we accept John

[4] The condition for a constant saving rate, which is necessary for equation (18) to hold, cannot arise in the Ramsey model if $\alpha+\eta<g/(\delta+n+g)$. This conclusion follows from equation (15) and the inequality $\theta>0$. Within the Ramsey model, the convergence coefficient approaches infinity as $\alpha+\eta\to 0$, contrary to the finite limit suggested by equation (18). Also, equation (18) shows how β depends on α, η, δ, n, and g, only when ρ or θ move in a compensating way in equation (15) to maintain a constant saving rate.

W. Kendrick's estimate that about half of spending on human capital appears in GDP (Kendrick, 1976 tables A-1 and B-2), then the predicted saving rate that corresponds to physical capital becomes 0.23. This figure corresponds well to observed ratios of physical investment to GDP. For the United States, for example, the ratio of real gross domestic investment to real GDP averaged 0.21 from 1960 to 1990.[5]

The major deficiency of the neoclassical growth model is the assumption of a closed economy, an assumption that is especially unappealing for the U.S. states and the regions of European countries. Therefore, we now extend the analysis to allow for capital mobility across economies.

III. The Open Economy with Perfect Capital Mobility

Suppose now that households can borrow and lend at the going interest rate on world capital markets. We assume that the country is small relative to the rest of the world and faces a constant world real interest rate, r^{ω}, which pegs the domestic rate, r.[6] The rate r^{ω} would be constant if the world were in the kind of steady state that we described above for a closed economy. Goods are tradable internationally, but labor cannot migrate.

The interest rate, r, again equals the net returns on the two kinds of capital, $R_k - \delta$ and $R_h - \delta$, where the rental prices are given in equations (3a) and (3b). But since $r = r^{\omega}$, a constant, the implied values of k and h—and hence, y—are also constant. In other words, the model predicts that a small open economy will jump instantaneously to its steady-state levels of output, physical capital, and human capital per effective worker and will remain there forever. The predicted rates of convergence for output and capital are infinite, a result that conflicts sharply with the empirical evidence discussed earlier. We could eliminate the infinite speeds of convergence by introducing adjustment costs and irreversibility conditions for physical and human capital. Plausible modifications along these lines do not, however, eliminate the counterfactual prediction that convergence rates would be rapid in an open economy with perfect capital mobility.[7] We therefore now turn to a model that allows for imperfect capital mobility.

IV. The Open Economy with Partial Capital Mobility

A. *Laws of Motion*

We now assume that the amount of debt, d, cannot exceed the quantity of physical capital, k. This assumption introduces an asymmetry between the two capital stocks: k can be used as collateral for international borrowing, whereas h cannot.[8] We are as-

[5]The ratio was computed from data in *Citibase*, defining gross investment to include public investment for nonmilitary and military purposes. The average ratio for private investment alone is 0.17. We may also want to add consumer-durables purchases, which averaged 8 percent of GDP from 1960 to 1990. A further small adjustment would modify GDP to include the service flows from government capital (to the extent that these flows were not already reflected in private output) and consumer durables.

[6]We assume $r^{\omega} > g + n$, a condition which ensures that the present value of future wages is bounded. The small-country assumption also requires $r^{\omega} \leq \rho + \theta g$; otherwise, the domestic country's assets and consumption will grow faster than the world's, and the country will eventually not be small.

[7]See Barro and Sala-i-Martin (1994 Ch. 3) for a discussion. The model has other unappealing implications. As first conjectured by Ramsey (1928), if countries differ in their effective discount rates, $\rho + \theta g$, then the most patient country asymptotically owns all the assets in the world, including all claims on human capital and raw labor. For the rest of the counties, c approaches zero, and the debt eventually mortgages all domestic capital and raw labor. The model also predicts, counterfactually, that GDP would typically behave very differently from GNP.

[8]Daniel Cohen and Jeffrey D. Sachs (1986) developed a model with one capital good in which the borrowing constraint amounts to $d \leq \nu k$, where $0 \leq \nu \leq 1$. In other words, only the fraction ν of capital serves as collateral. Our model differs in that k and h are imperfect substitutes as inputs to production, and the choices between k and h determine the fraction of broad capital, $k + h$, that constitutes collateral.

suming implicitly that domestic residents own the physical capital buy may obtain part or all of the financing for this stock by issuing bonds to foreigners. The results would be the same if we allowed for direct foreign investment, in which case the foreigners would own part of the physical capital rather than bonds. The important assumption is that domestic residents cannot borrow with human capital or raw labor as collateral and that foreigners cannot own domestic human capital or raw labor. We are, in particular, ruling out any international migration of labor.[9]

There are various ways to motivate the borrowing constraint. Physical capital is more easily repossessed and more readily monitored than human capital and is therefore more easily financed with debt. Physical capital is also more amenable to direct foreign investment: a person can own a factory but not someone else's stream of labor income. Finally, one can abandon the terms "physical capital" and "human capital" and recognize that not all investments can be financed through perfect capital markets. The key distinction between k and h is not the physical nature of the capital, but whether the cumulated goods serve as collateral for borrowing on world markets. In this sense, the k in the model is likely to be much narrower than literal physical capital.

We still assume that the world interest rate, r^{ω}, is constant at its steady-state value. We assume also that $r^{\omega} = \rho + \theta g$, the steady-state interest rate that would apply if the domestic economy were closed. That is, the home economy is neither more nor less impatient than the world as a whole. The initial quantity of assets per effective worker is $k(0) + h(0) - d(0)$, and the key consideration is whether this quantity is greater or less than the steady-state amount of human capital, h^*. If $k(0) + h(0) - d(0) \geq h^*$, then the borrowing constraint is not binding, and the economy jumps to the steady-state values of k, h, and y. In contrast, if $k(0) + h(0) - d(0) < h^*$, then the constraint is binding (i.e., $d = k$ applies), and we obtain some new results. We therefore focus on this situation.[10]

Since physical capital serves as collateral, the net return on this capital, $R_k - \delta$, still equals the world interest rate, r^{ω}, at all points in time. The formula for R_k from equation (3a) therefore implies

$$k = (1 - \tau)\alpha y / (r^{\omega} + \delta). \tag{19}$$

Equation (19) ensures that the ratio of physical capital to GDP, k/y, remains constant throughout the transition to the steady state. In contrast, k/y rose steadily during the transition for the closed economy that we considered earlier. (The ratio of human capital to GDP, h/y, rose over time for the closed economy and will turn out still to rise for the open economy.) We mentioned before that the rough constancy over time of k/y is one of Kaldor's (1961) stylized facts about economic development. (Angus Maddison [1980] provides some confirmation of this regularity.) The consistency of the credit-constrained open-economy model with this "fact" is therefore notable.[11]

The result for k from equation (19) can be combined with the production function, $y = Ak^{\alpha}h^{\eta}$, to express y as a function of h:

$$y = Bh^{\varepsilon} \tag{20}$$

[9] Juan Braun (1993) and Barro and Sala-i-Martin (1994 Ch. 9) assess the role of migration in an extended version of the neoclassical growth model. If people with the average quantity of human capital in their economy tend to move, then migration speeds up convergence. If the cost of moving is highly sensitive to the volume of migration, however, then the effect on the convergence speed is small. If people with above-average amounts of human capital tend to move, then the effect on the convergence speed is further reduced and could be eliminiated.

[10] If $r^{\omega} < \rho + \theta g$, then the domestic economy must eventually become constrained on the world credit market. Hence, our analysis of a debt-constrained economy applies at some time in the future even if not at the initial date. If $r^{\omega} > \rho + \theta g$, then the assumption of a small economy is violated eventually, and r^{ω} would have to change (see footnote 6).

[11] The precise constancy of k/y in the model depends on the fixity of the world interest rate, r^{ω}, and on the assumption that the production function is Cobb-Douglas.

where

$$B \equiv A^{1/(1-\alpha)}[(1-\tau)\alpha/(r^{\omega}+\delta)]^{\alpha/(1-\alpha)}$$

and $\varepsilon \equiv \eta/(1-\alpha)$ are constants. The condition $0<\alpha+\eta<1$ implies $0<\varepsilon<\alpha+\eta<1$. Thus, the reduced-form production function in equation (20) expresses y as a function of h with positive and diminishing marginal product. The convergence implications of this model are therefore similar to those of the closed economy: both models involve the accumulation of a capital stock under conditions of diminishing returns.

The budget constraint from equation (6) can be combined with the reduced-form production function from equation (20), the borrowing constraint $d=k$, and the condition $(r^{\omega}+\delta)k=(1-\tau)ay$ from equation (19) to get the revised budget constraint:

$$(21)\quad \dot{h}=(1-\alpha)(1-\tau)Bh^{\varepsilon} -(\delta+n+g)h-c.$$

Note that the term $\alpha(1-\tau)Bh^{\varepsilon}=\alpha(1-\tau)y$ corresponds to the flow of rental payments on physical capital, $(r^{\omega}+\delta)k$ [see equation (19)]. Since $d=k$, this term corresponds to the factor payments to foreigners and therefore to the difference between GNP and GDP. GDP exceeds GNP because the country is constrained on the international credit market and therefore has the positive foreign debt per effective worker, $d=k$.

Households now maximize utility, given by equation (4), subject to the budget constraint in equation (21) and a given initial stock of human capital, $h(0)>0$. [The initial value $h(0)$ equals the given amount of initial assets, which was assumed to be less than h^*.] The Euler equation is

$$(22)\quad \dot{c}/c =(1/\theta)[(1-\tau)(1-\alpha)B\varepsilon h^{\varepsilon-1}-(\delta+\rho+\theta g)]$$

where

$$(1-\tau)(1-\alpha)B\varepsilon h^{\varepsilon-1}=(1-\tau)B\eta h^{\varepsilon-1}$$

is the after-tax marginal product of human capital. Equations (21) and (22) and the transversality condition fully describe the transitional dynamics of this model.

Since we assumed $r^{\omega}=\rho+\theta g$, the steady state is the same as that for the closed economy. In particular, $h^*=z^*\eta/(\alpha+\eta)$, as in equation (10), where z^* is the steady-state quantity of broad capital for the closed economy, given in equation (9). Hence, the opportunity to borrow on the world credit market does not influence the steady state, but it will turn out to affect the speed of convergence.[12]

B. *Convergence Along the Transition*

The system described by equations (21) and (22) and the transversality condition has the usual saddle-path characteristics. Compare the debt-constrained open economy with the closed economy: equation (21) corresponds to equation (7), and equation (22) to equation (8). The only differences are that equation (21) contains $(1-\alpha)B$ as a proportional constant in the production function, whereas equation (7) has A; the capital-stock variable is h rather that $z \equiv h+k$; and the exponent on the capital stock is $\varepsilon \equiv \eta/(1-\alpha)$ rather than $\alpha+\eta$. Since ε and $\alpha+\eta$ are positive and less than 1 (i.e., both models feature diminishing returns), the dynamics of the models are essentially the same.

Equation (13) determines the convergence coefficient, β, for the closed economy. The only difference in the credit-constrained open economy is that $\alpha+\eta$ has to be replaced by $\varepsilon \equiv \eta/(1-\alpha)$. (Recall that the level of the production technology does not influence the rate of convergence.) Hence, the convergence coefficient for the log-linearized version of the credit-con-

[12] If we had assumed $r^{\omega}<\rho+\theta g$—so that the home economy is more impatient than the rest of the world (see footnote 10)—then the availability of foreign borrowing would also affect the steady-state position. The open economy would have higher steady-state capital intensities, h^* and k^*, than the closed economy.

strained open economy is given by

$$(23)\quad 2\beta_{\text{open}} = \left\{\varphi^2 + 4\left(\frac{1-\varepsilon}{\theta}\right)(\delta+\rho+\theta g) \times\left[\frac{\delta+\rho+\theta g}{\varepsilon} - (\delta+n+g)\right]\right\}^{1/2} - \varphi$$

where φ is again equal to $\rho - n - (1-\theta)g > 0$.[13] The coefficient determined from equation (23) is the same value that would arise in a closed economy that had the broad capital share ε, rather that $\alpha+\eta$. Since $\varepsilon \equiv \eta/(1-\alpha)$, it follows from $\alpha+\eta<1$ that $\varepsilon < \alpha+\eta$. *The credit-constrained open economy therefore works like a closed economy with a broad capital share that is less than* $\alpha+\eta$. Recall that the rate of convergence depends inversely on the capital share (because a smaller capital share means that diminishing returns set in more rapidly). The credit-constrained open economy therefore has a higher rate of convergence than the closed economy. Note, however, that $(\alpha+\eta)\to 1$ implies $\varepsilon \to 1$, and therefore, $\beta_{\text{open}} \to 0$ in equation (23). Thus, if diminishing returns to broad capital do not apply ($\alpha+\eta=1$), then the model still does not exhibit the convergence property.[14]

We can understand why the partially open economy converges faster than the closed economy by thinking about the tendency for diminishing returns to set in as human capital, h, is accumulated. For given exponents of the production function, α and η, the key issue is the transitional behavior of the ratio k/h. In the closed economy, k/h stays constant, whereas in the open economy, k/h falls during the transition. That is, k is relatively high at the outset in an open economy because the availability of foreign finance makes it easy to acquire k quickly. The fall in k/h over time causes diminishing returns to h to set in faster than otherwise; hence, the speed of convergence is greater in the open economy than in the closed economy.

Although the credit-constrained open economy converges faster than the closed economy, the speed of convergence is now finite for the open economy, and the difference from the closed economy is not large for plausible parameter values. If we use the baseline parameter values shown in equation (14), then the convergence coefficient implied by equation (23) is 0.022, compared with 0.014 for the closed economy. The value 0.022 accords well with empirical estimates of convergence coefficients for open economies, such as the U.S. states and the regions of some Western European countries and Japan.

The sensitivity of the convergence speed in the credit-constrained open economy to variations in the underlying parameters in similar to that discussed before for a closed economy. The coefficient β_{open} in equation (23) is not very sensitive to changes in ρ, n, g, and δ. For example, if the other parameters remain at their baseline values, then β_{open} rises from 0.022 to 0.024 if ρ increases to 0.03 or n increases to 0.02. The coefficient rises to 0.026 if g increases to 0.03 and to 0.027 if δ increases to 0.07. It rises to 0.030 if θ declines to 1.

The convergence speed is again more sensitive to the broad capital share, $\alpha+\eta$. If we hold fixed the fraction of capital that serves as collateral, $\alpha/(\alpha+\eta)$, at $\frac{3}{8}$, then β_{open} rises to 0.036 if $\alpha+\eta$ falls to 0.7 and rises to 0.051 if $\alpha+\eta$ falls to 0.6.

A new effect involves changes in $\alpha/(\alpha+\eta)$ for a given value of $\alpha+\eta$, that is, changes in the fraction of total capital that serves as collateral on foreign loans. We have as-

[13] We have assumed throughout a Cobb-Douglas production function, so that the elasticity of substitution between human and physical capital equals 1. In a previous version of this paper (Barro et al., 1992) we assumed that y depended on h and k in a constant-elasticity-of-substitution form, and we analyzed how the elasticity of substitution affects the rate of convergence. For the plausible case in which the steady-state stock of human capital exceeds that of physical capital, variations of the elasticity of substitution cause only minor changes in the rate of convergence.

[14] If $\alpha=0$, so that no capital constitutes collateral, then $\varepsilon=\eta$ and β_{open} in equation (23) corresponds to the value from equation (13) for a closed economy. If $\eta=0$, so that all capital serves as collateral, then $\varepsilon=0$ and β_{open} in equation (23) becomes infinite, as in the open economy with perfect capital mobility.

sumed thus far that $\alpha = 0.3$, a reasonable value if α corresponds to the share of physical capital in output. Since the collateral for foreign borrowing is likely to be less than the quantity of physical capital, it may be better to assume a smaller value of α. For example, if $\alpha = 0.2$ and $\eta = 0.6$, so that $\alpha/(\alpha+\eta) = \frac{1}{4}$, then $\beta_{\text{open}} = 0.019$. If $\alpha = 0.1$ and $\eta = 0.7$, so that $\alpha/(\alpha+\eta) = \frac{1}{8}$, then $\beta_{\text{open}} = 0.016$. Thus, if a credit-constrained open economy can amass a foreign debt that is only about one-eighth of its total capital, then the convergence speed is very close to the value (0.014) that would apply if the economy were closed.

The transition to the steady state features a monotonic increase in human capital per effective worker, h, from its initial value, $h(0)$, to its steady-state value, h^*. Equation (20) implies that the growth rate of y is ε times the growth rate of h, where ε is between 0 and 1. The ratio h/y therefore rises steadily during the transition. Recall, however, that equation (19) implies that the ratio k/y is constant. Therefore, k grows at the same rate as y, and the ratio h/k increases during the transition. Note that, although physical capital serves fully as collateral, k nevertheless rises gradually toward its steady-state value, k^*. The reason is the constraint of domestic saving on the accumulation of human capital and the complementarity between h and k in the production function. When h is low, the schedule for the marginal product of physical capital is low; hence, $k < k^*$ follows even though domestic producers can finance all acquisitions of physical capital with foreign borrowing. The gradual increase of human capital impacts positively on the marginal product of physical capital and leads thereby to an expansion of k.

Foreign borrowing occurs only on loans secured by physical capital, and the interest rate on these loans is pegged at the world rate, r^{ω}. We can also introduce a domestic credit market, although the setting with a representative domestic agent always ends up with a zero volume of borrowing on this market. For loans that are secured by physical capital, the shadow interest rate on the domestic market must also be r^{ω}. If we assume that human capital and raw labor do not serve domestically as collateral, then the shadow interest rate on the domestic market with these forms of security is infinity (or at least high enough to drive desired borrowing to zero), just as it is on the world market.

We might assume instead that human capital and raw labor serve as collateral for domestic borrowing but not for foreign borrowing. This situation would apply if the legal system enforces loan contracts based on labor income when the creditor is domestic, but not when the creditor is foreign. In this case, the shadow interest rate on domestic lending, collateralized by labor income, equals the net marginal product of human capital. This net marginal product begins at a relatively high value [corresponding to the low starting stock, $h(0)$] and then falls gradually toward the steady-state value, r^{ω}. Thus, the transition features a decrease in the spread between this kind of domestic interest rate and the world rate, r^{ω}. A possible example is the curb market for informal lending in Korea (see Susan M. Collins and Won-Am Park, 1989 p. 353). The spread between curb-market interest rates and world interest rates was 30–40 percentage points in the 1960's and 1970's, but fell by the mid-1980's to about 15 percentage points.

We can again simplify the formula for the rate of convergence if we assume a constant gross saving rate. The required value of θ for a constant gross saving rate is now

$$\theta = (\delta+\rho)/[\varepsilon(\delta+n) - g(1-\varepsilon)] \tag{24}$$

and the corresponding gross saving rate (expressed relative to GNP) is $s = (1-\tau)/\theta$. The rate of convergence is then

$$\beta_{\text{open}} = (1-\varepsilon)(\delta+n+g). \tag{25}$$

For the parameter values used before, the critical value of θ is 1.9, and the corresponding values of s and β_{open} are 0.37 and 0.023, respectively.

Another interesting implication of this model is that, despite the existence of international borrowing and lending, the convergence properties of gross national product and gross domestic product are the same. As noted before, the factor income from abroad is $-(r^{\omega}+\delta)k = -(1-\tau)\alpha y$ [see equation (19)]. Therefore,

$$\text{(26)}\quad \text{GNP (per unit of effective labor)}$$
$$= y-(1-\tau)\alpha y$$
$$= y[1-\alpha(1-\tau)].$$

Since GNP is proportional to GDP, which corresponds to y, the convergence rates for GNP and GDP are the same. This result suggests that data sets that involve GDP are likely to generate similar rates of convergence as those that involve GNP or measures of national income. Some confirmation of this prediction comes from the study of the U.S. states by Barro and Sala-i-Martin (1991): the rates of convergence are similar for gross state product per capita and state personal income per capita.

The gap between GDP and GNP for a credit-constrained open economy equals $\alpha(1-\tau)y$ and would therefore be about 20 percent of GDP for the parameter values assumed before ($\alpha = 0.3$, $\tau = 0.3$). The current-account deficit, which equals the change in physical capital, is correspondingly large. In the steady state, the ratio of the current-account deficit to GDP equals $(\delta + n + g)k/y$. Since $k/y = \alpha(1-\tau)/(r^{\omega}+\delta)$ [from equation (19)], the current-account ratio can be expressed as

$$\alpha(1-\tau)(\delta+n+g)/(r^{\omega}+\delta).$$

For the parameter values that we have been using (including $r^{\omega} = 0.06$), this ratio is 15 percent.

It is unusual to find developing countries that have values this high for the GDP–GNP gap and the current-account deficit.[15] We can reconcile the theory with this observation by noting first that many developing countries are insufficiently productive to be credit-constrained and, second, that the collateral for international debt is likely to be substantially smaller than the quantity of physical capital. If the coefficient α were less than 0.3, then the predicted ratios for the GDP–GNP gap and the current-account deficit would be correspondingly smaller.

[15]One counterexample is Singapore: its current-account deficit was between 10 percent and 20 percent of GDP throughout the 1970's (see International Monetary Fund, 1991).

V. Conclusion

Economists have long known that capital mobility tends to raise the rate at which poor and rich economies converge. The main message of this paper is that the quantitative impact of this effect is likely to be small. If there are some types of capital, such as human capital, that cannot be financed by borrowing on world markets, then open economies will converge only slightly faster than closed economies. This prediction accords with the empirical literature, which finds that samples of open economies, such as the U.S. states, converge only slightly faster than samples of more closed economies, such as the OECD countries.

REFERENCES

Barro, Robert J. "Economic Growth in a Cross Section of Countries." *Quarterly Journal of Economics*, May 1991, *106*(2), pp. 407–43.

______. *Macroeconomics*, 4th Ed. New York: Wiley, 1993.

Barro, Robert J. and Sala-i-Martin, Xavier. "Convergence Across States and Regions." *Brookings Papers on Economic Activity*, 1991, (1), pp. 107–58.

______. "Convergence." *Journal of Political Economy*, April 1992a, *100*(2), pp. 223–51.

______. "Regional Growth and Migration: A Japan–U.S. Comparison." *Journal of the Japanese and International Economies*, December 1992b, *6*(4), pp. 312–46.

______. *Economic growth.* New York: McGraw-Hill, 1994.

Barro, Robert J.; Mankiw, N. Gregory and Sala-i-Martin, Xavier. "Capital Mobility in Neoclassical Models of Growth." National

Bureau of Economic Research (Cambridge, MA) Working Paper No. 4206, November 1992.

Braun, Juan. "Essays on Economic Growth and Migration." Ph.D. dissertion, Harvard University, May 1993.

Cass, David. "Optimum Growth in an Aggregative Model of Capital Accumulation." *Review of Economic Studies*, July 1965, *32*(91), pp. 233–40.

Cohen, Daniel and Sachs, Jeffrey D. "Growth and External Debt Under Risk of Debt Repudiation." *European Economic Review*, June 1986, *30*(3), pp. 1138–54.

Collins, Susan M. and Park, Won-Am. "External Debt and Macroeconomic Performance in South Korea," in Jeffrey D. Sachs, ed., *Developing country debt and economic performance*. Chicago: University of Chicago Press, 1989, pp. 121–40.

Dowrick, Steve and Nguyen, Duc-Tho. "OECD Comparative Economic Growth 1950–85: Catch-Up and Convergence." *American Economic Review*, December 1989, *79*(5), pp. 1010–30.

Feldstein, Martin S. and Horioka, Charles. "Domestic Savings and International Capital Flows." *Economic Journal*, June 1980, *90*(358), pp. 314–29.

International Monetary Fund. *International financial statistics yearbook*. Washington, DC: International Monetary Fund, 1991.

Jorgenson, Dale W.; Gollop, Frank M. and Fraumeni, Barbara M. *Productivity and U.S. economic growth*. Cambridge, MA: Harvard University Press, 1987.

Kaldor, Nicholas. "Capital Accumulation and Economic Growth," in F. A. Lutz and D. C. Hague, eds., *The theory of capital*. New York: St. Martin's, 1961, pp. 177–222.

Kendrick, John W. *The formation and stocks of total capital*. New York: Columbia University Press, 1976.

Koopmans, Tjalling C. "On the Concept of Optimal Economic Growth," in Pontificia Academia Scientiarum, *The econometric approach to development planning*. Amsterdam: North-Holland, 1965, pp. 225–87.

Kurz, Mordecai. "The General Instability of a Class of Competitive Growth Processes." *Review of Economic Studies*, April 1968, *35*(102), pp. 155–74.

Levine, Ross and Renelt, David. "A Sensitivity Analysis of Cross-Country Growth Regressions." *American Economic Review*, September 1992, *82*(4), pp. 942–63.

Maddison, Angus. *Phases of capitalist development*. Oxford: Oxford University Press, 1982.

Mankiw, N. Gregory; Romer, David and Weil, David N. "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics*, May 1992, *107*(2), pp. 407–37.

Mulligan, Casey B. and Sala-i-Martin, Xavier. "Transitional Dynamics in Two-Sector Models of Endogenous Growth." *Quarterly Journal of Economics*, August 1993, *108*(3), pp. 739–73.

Ramsey, Frank P. "A Mathematical Theory of Saving." *Economic Journal*, December 1928, *38*(152), pp. 543–59.

Rebelo, Sergio. "Long-Run Policy Analysis and Long-Run Growth." *Journal of Political Economy*, June 1991, *99*(3), pp. 500–21.

Romer, Paul M. "Crazy Explanations for the Productivity Slowdown," in Stanley Fischer, ed., *NBER macroeconomics annual: 1987*. Cambridge, MA: MIT Press, 1987, pp. 163–202.

Solow, Robert M. "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics*, February 1956, *70*(1), pp. 65–94.

Swan, Trevor W. "Economic Growth and Capital Accumulation." *Economic Record*, November 1956, *32*(63), pp. 334–61.