Thomas Piketty
Capital in the 21st Century

Program on the Study of Capitalism

March 6, 2015

Stephen Marglin

A classic in economics is a book to which everybody alludes but nobody reads.

Piketty's *Capital in the 21st Century* is right up there with Smith's *Wealth of Nations* and Keynes's *General Theory*

Hawking Index (HI)

Jordan Ellenberg

Take the page numbers of a book's five top highlights, average them, and divide by the number of pages in the whole book. The higher the number, the more of the book we're guessing most people are likely to have read.

"A Brief History of Time" by Stephen Hawking: 6.6%
"Capital in the Twenty-First Century" by Thomas Piketty : 2.4%

Mr. Piketty's book is almost 700 pages long, and the last of the top five popular highlights appears on page 26. Stephen Hawking is off the hook; from now on, this measure should be known as the Piketty Index.

http://www.wsj.com/articles/the-summers-most-unread-book-is-1404417569

Piketty’s Principal Policy Recommendation—a Progressive Wealth Tax—Is Largely Independent of the Analysis in *Capital*

Piketty’s Fundamental Contradiction of Capitalism

$$g_Y < r$$

When the rate of return on capital exceeds the rate of growth of output and income,… capitalism automatically generates arbitrary and unsustainable inequalities that radically undermine the meritocratic values on which democratic societies are based. (p 1)

When the rate of return on capital significantly exceeds the growth rate of the economy…, then it logically follows that inherited wealth grows faster than output and income. People with inherited wealth need save only a portion of their income from capital to see that capital grow more quickly than the economy as a whole. (p 26)

The Central Contradiction of Capitalism

…

The inequality r > g implies that wealth accumulated in the past grows more rapidly than output and wages. This inequality expresses a fundamental logical contradiction. The entrepreneur inevitably tends to become a rentier, more and more dominant over those who own nothing but their labor. Once constituted, capital reproduces itself faster than output increases. The past devours the future. (p 571)

Piketty’s Laws are Laws of Arithmetic

Piketty's Arithmetic Laws

1

$$\alpha = r\beta$$

$$\frac{rK}{Y} = r\frac{K}{Y}$$

2

$$\beta = \frac{s}{g_K}$$

$$\frac{K}{Y} = \frac{\frac{\Delta K}{Y}}{\frac{\Delta K}{K}}$$

2′

$$\beta' = \frac{s}{g_Y}$$

$$\frac{\Delta K}{\Delta Y} = \frac{\frac{\Delta K}{Y}}{\frac{\Delta Y}{Y}}$$

2 (restated)

$$g_K = \frac{s}{\beta} = \frac{sr}{\alpha}$$

$$\frac{\Delta K}{K} = \frac{\frac{\Delta K}{Y}}{\frac{K}{Y}} = \frac{\frac{\Delta K}{Y}r}{\frac{rK}{Y}}$$

2′ (restated)

$$g_Y = \frac{s}{\beta'} = \frac{sr}{\alpha}\frac{\beta}{\beta'}$$

$$\frac{\Delta Y}{Y} = \frac{\frac{\Delta K}{Y}}{\frac{\Delta K}{\Delta Y}} = \frac{\frac{\Delta K}{Y}r\frac{K}{Y}}{\frac{rK}{Y}\frac{\Delta K}{\Delta Y}}$$

$$\beta' = \frac{\Delta K}{\Delta Y}$$

$$\frac{\beta'}{\beta} = \frac{g_K}{g_Y}$$

If $\lim_{t\to\infty} \beta'$ exists, then $\lim_{t\to\infty} \beta = \lim_{t\to\infty} \beta'$

3

$$g_K < r \iff s < \alpha$$

3′

$$g_Y < r \iff s\frac{\beta}{\beta'} < \alpha$$

1

Capital Share = Rate of Return on Capital x Capital:Output Ratio

$$\alpha = r\beta$$

$$\frac{rK}{Y} = r\frac{K}{Y}$$

2

Capital:Output Ratio = Rate of Saving/Growth Rate of Capital Stock

$$\beta = \frac{s}{g_K}$$

$$\frac{K}{Y} = \frac{\frac{\Delta K}{Y}}{\frac{\Delta K}{K}}$$

2′

Incremental Capital:Output Ratio = Rate of Saving/Growth Rate of Output

$$\beta' = \frac{s}{g_Y}$$

$$\frac{\Delta K}{\Delta Y} = \frac{\frac{\Delta K}{Y}}{\frac{\Delta Y}{Y}}$$

$$\beta' = \frac{\Delta K}{\Delta Y} \qquad \frac{\beta'}{\beta} = \frac{g_K}{g_Y}$$

If $\lim_{t\to\infty} \beta'$ exists, then $\lim_{t\to\infty} \beta = \lim_{t\to\infty} \beta'$

3’

$$g_Y < r \iff s\frac{\beta}{\beta'} < \alpha$$

Piketty’s Fundamental Contradiction of Capitalism

$$g_Y < r$$

Piketty's originality, perseverance, and meticulousness with respect to the data contrast sharply with a very cavalier attitude towards theory.

Indeed, a plausible theory of growth and distribution is totally absent.

Why does β rise or fall over time?

According to the Harrod-Domar-Solow formula, in the long run the wealth-income ratio β is equal to the net saving rate s divided by the income growth rate g. So for a given saving rate s =10%, the long-run β is about 300% if g = 3% and about 600% if g = 1.5%. In short: capital is back because low growth is back… (p 2)

According to the one-good capital accumulation model and the Harrod-Domar-Solow formula $\beta = s/g$, the two key forces driving wealth-income ratios are the saving rate s and the income growth rate g. (p 20)

(Piketty and Gabriel Zucman, “Capital is Back: Wealth-Income Ratios in Rich Countries 1700-2010,” [working paper version])

2 (restated)

Growth Rate of Capital Stock = Saving Rate/Captial:Output Ratio

$$g_K = \frac{s}{\beta} = \frac{sr}{\alpha}$$

$$\frac{\Delta K}{K} = \frac{\frac{\Delta K}{Y}}{\frac{K}{Y}} = \frac{\frac{\Delta K}{Y}r}{\frac{rK}{Y}}$$

2′ (restated)

Growth Rate of Output = Saving Rate/Incremental Capital:Output Ratio

$$g_Y = \frac{s}{\beta'} = \frac{sr}{\alpha}\frac{\beta}{\beta'}$$

$$\frac{\Delta Y}{Y} = \frac{\frac{\Delta K}{Y}}{\frac{\Delta K}{\Delta Y}} = \frac{\frac{\Delta K}{Y}r\frac{K}{Y}}{\frac{rK}{Y}\frac{\Delta K}{\Delta Y}}$$

3

Rate of Return on Capital Exceeds Growth Rate of Capital Stock ⟺ Capital Share Exceeds Rate of Saving

$$g_K < r \iff s < \alpha$$

3′

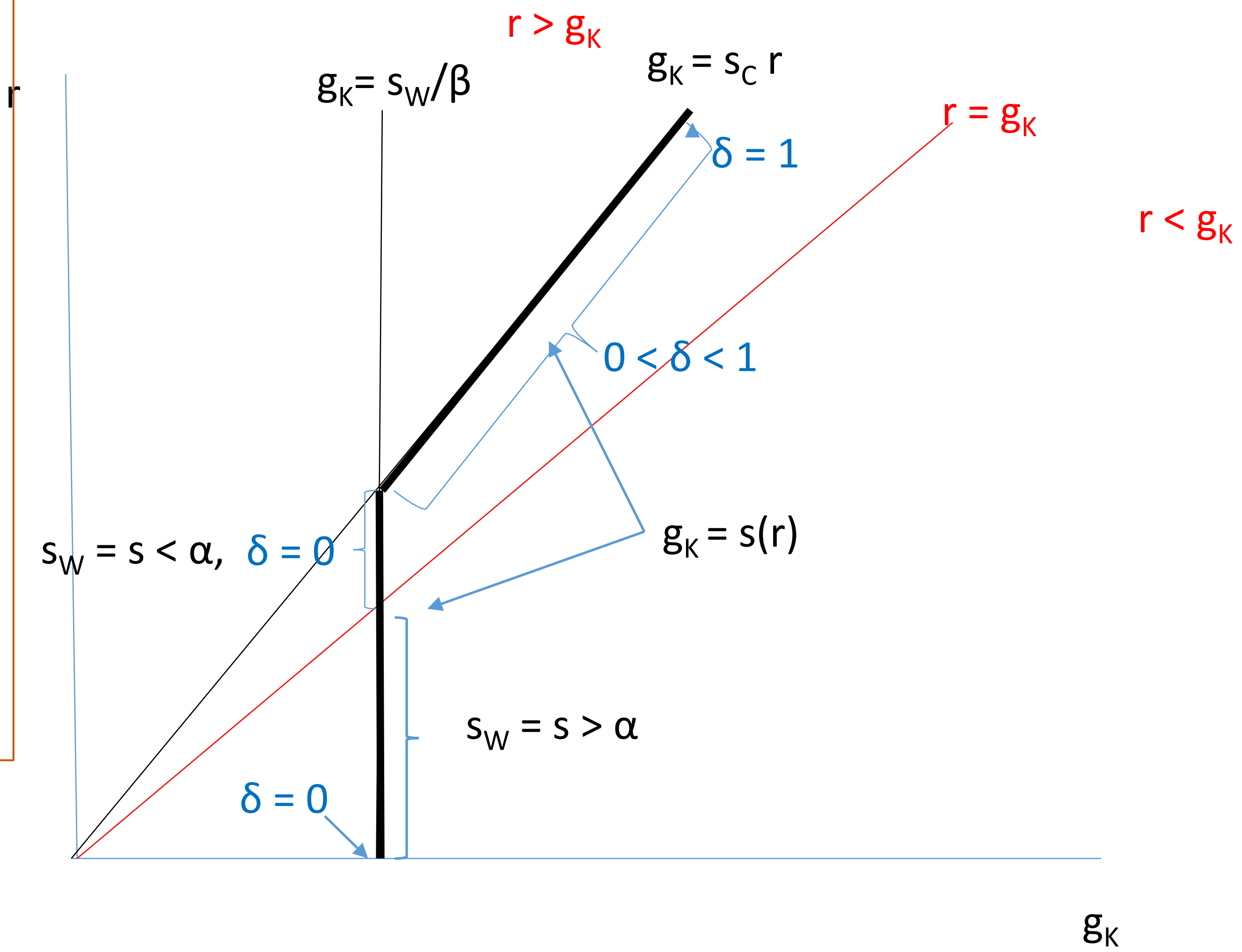
Rate of Return on Capital Exceeds Growth Rate of Output ⟺ Capital Share Exceeds Rate of Saving Multiplied by Ratio β/β′

$$g_Y < r \iff s\frac{\beta}{\beta'} < \alpha$$

A Two-Class Model of r and g_K with Rentiers Disposed to Save More than the Middle Class ($s_C > s_W$)
Fixed β (no substitution between capital and labor)

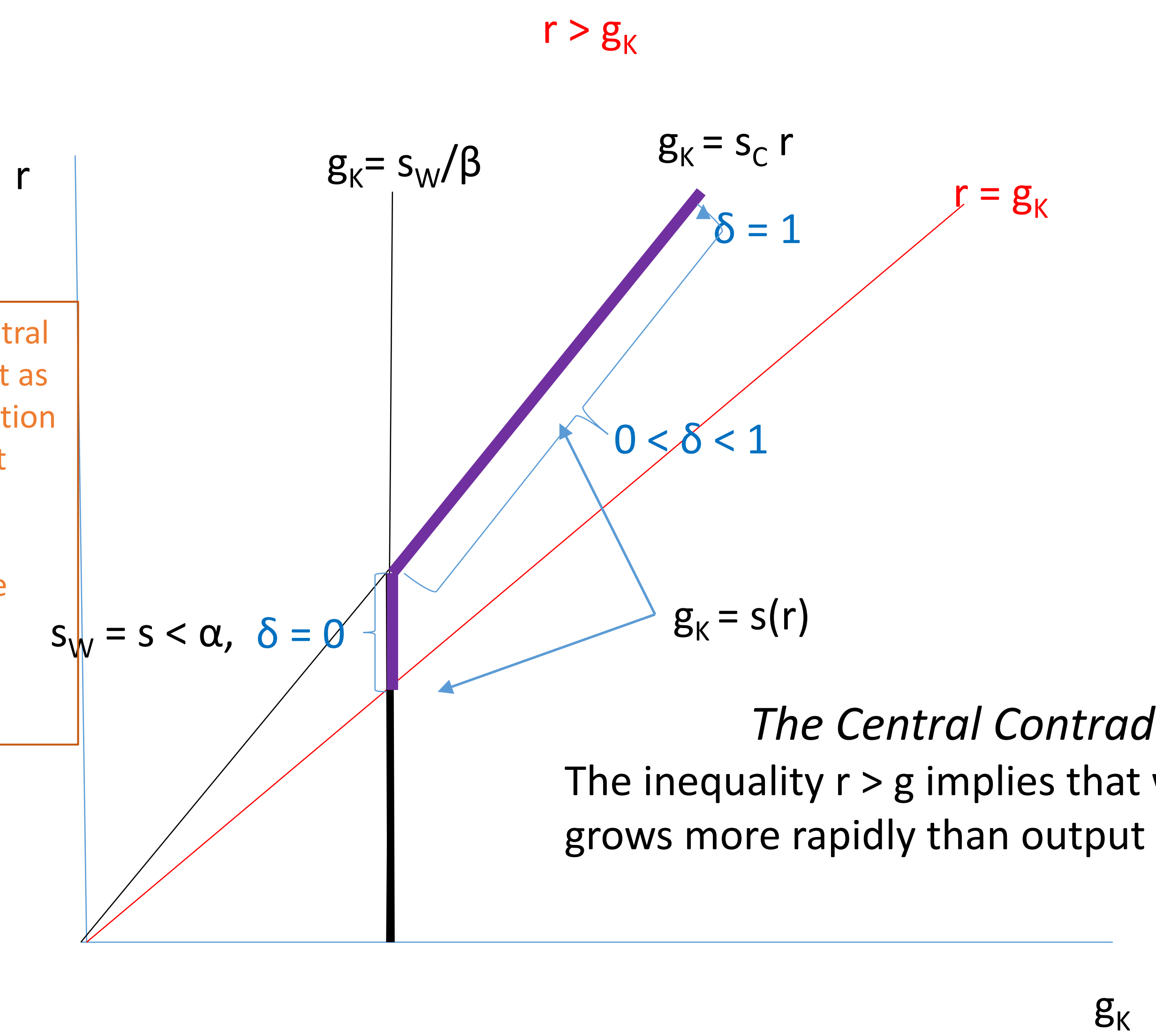
The heavy black line labeled $g_K = s(r)$ represents the relationship between the rate of return and the rate of growth for a simple two-class model with rentiers who save the fraction s_C of their income (entirely from capital) and a "middle class" which saves a lower fraction s_W of their income (salaries and capital income). Associated with each point is an overall saving rate s and a capital share α.

Each point on the schedule $g_K = s(r)$ corresponds to an equilibrium level of α; there is no endogenous mechanism to increase (or decrease) α that flows from the inequality $r > g_K$



K_W = middle class capital; K_C = rentier capital; s_W = middle-class propensity to save; s_C = rentier capital; δ = rentier share of capital

A Two-Class Model of r and g_K with Rentiers Disposed to Save More than the Middle Class ($s_C > s_W$)



The "central contradiction" is neither central nor a contradiction. [Piketty referred to it as a marketing ploy…] Along the purple portion of $g_K = s(r)$, the inequality $r > g_K$ holds, but middle class capital grows as rapidly as rentier capital, as do output and wages. Along the vertical portion of the schedule the middle class ends up owning all but a vanishing share of the capital stock.

The Central Contradiction of Capitalism

The inequality $r > g$ implies that wealth accumulated in the past grows more rapidly than output and wages.

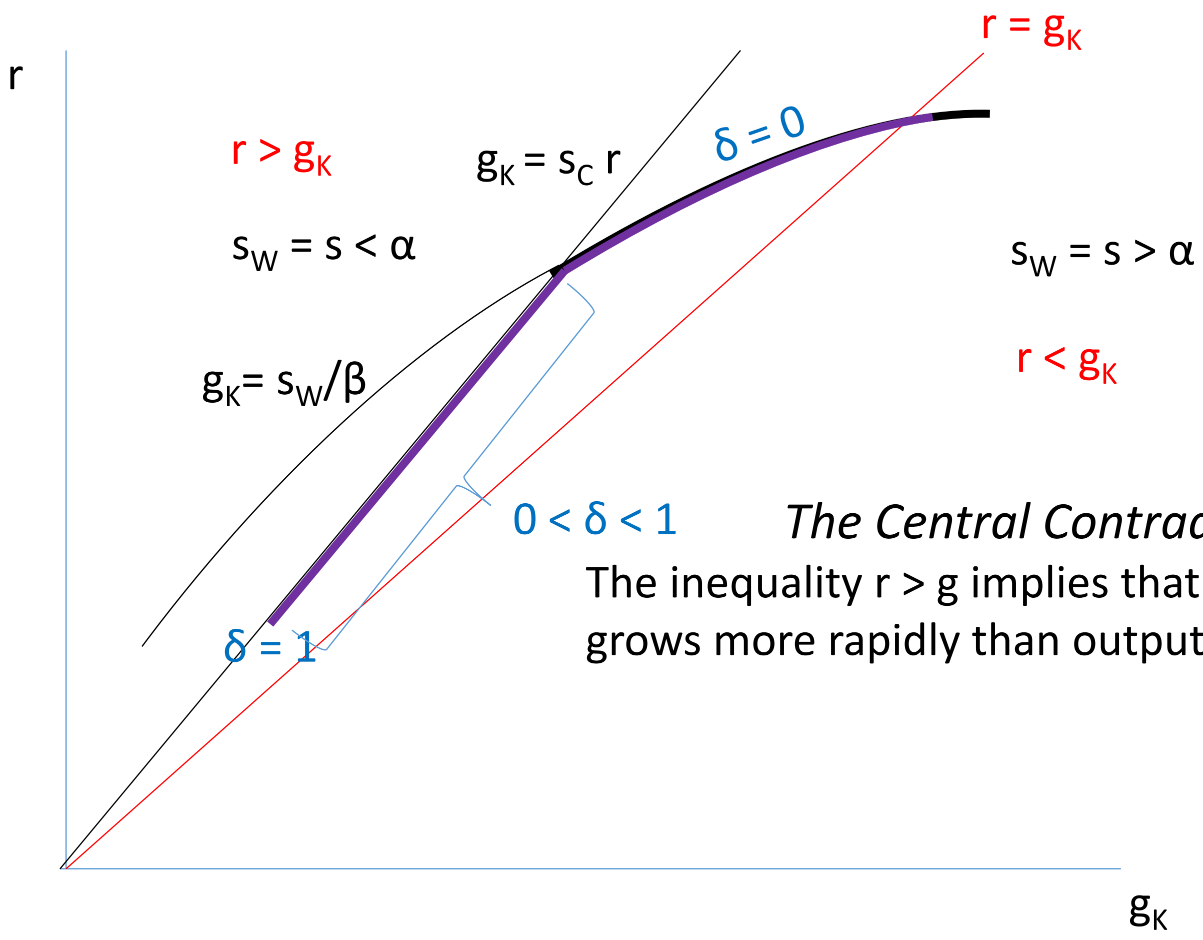
K_W = middle class capital; K_C = rentier capital; s_W = middle-class propensity to save; s_C = rentier capital; δ = rentier share of capital

A Two-Class Model of r and g_K with Rentiers Disposed to Save More than the Middle Class ($s_C > s_W$)

Elasticity of Substitution of Capital for Labor > 1

The $g_K = s(r)$ schedule changes if, as Piketty believes is the case, the elasticity of substitution of capital for labor (σ) exceeds 1. If there were a mechanism for driving α down over time--$r > g_K$ is *not* such a mechanism—then under Piketty's assumption about σ the economy would move relentlessly down the $g_K = s(r)$ schedule and rentiers would end up owning all the capital. Maybe there is such a mechanism, but Piketty has not articulated it. That's what I mean by the theory being cavalier (poor word choice). By the way, most of the empirical evidence suggests σ < 1, but I'm a consumer of this literature not a producer.

Bottom Line: the relationship between β, r, and g at best provides one piece of the necessary theory. It is analogous to having a theory of price with only a demand curve *or* a supply curve.



The Central Contradiction of Capitalism

The inequality r > g implies that wealth accumulated in the past grows more rapidly than output and wages.

K_W = middle class capital; K_C = rentier capital; s_W = middle-class propensity to save; s_C = rentier capital; δ = rentier share of capital

Why does α tend to rise more than s?

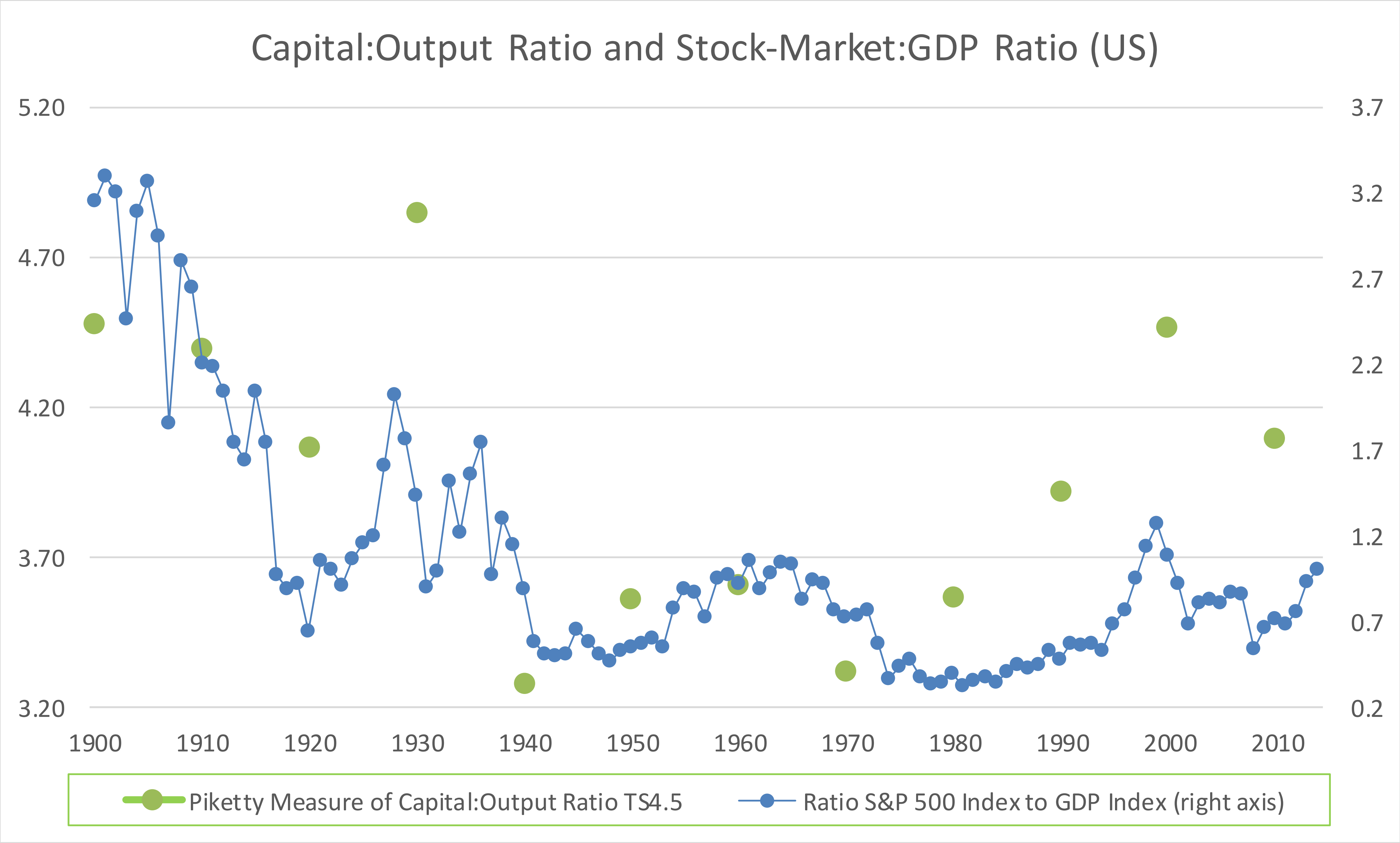
There are many uses for capital over the very long run, and this fact can be captured by noting that the long-run elasticity of substitution of capital for labor [σ] is probably greater than one. The most likely outcome is thus that the decrease in the rate of return [r] will be smaller than the increase in the capital/income ratio [β], so that capital's share [α = rβ] will increase. (*Capital in the 21st Century,* p 233)

Are we interested in physical (“real”) or value (“nominal”) ratios?

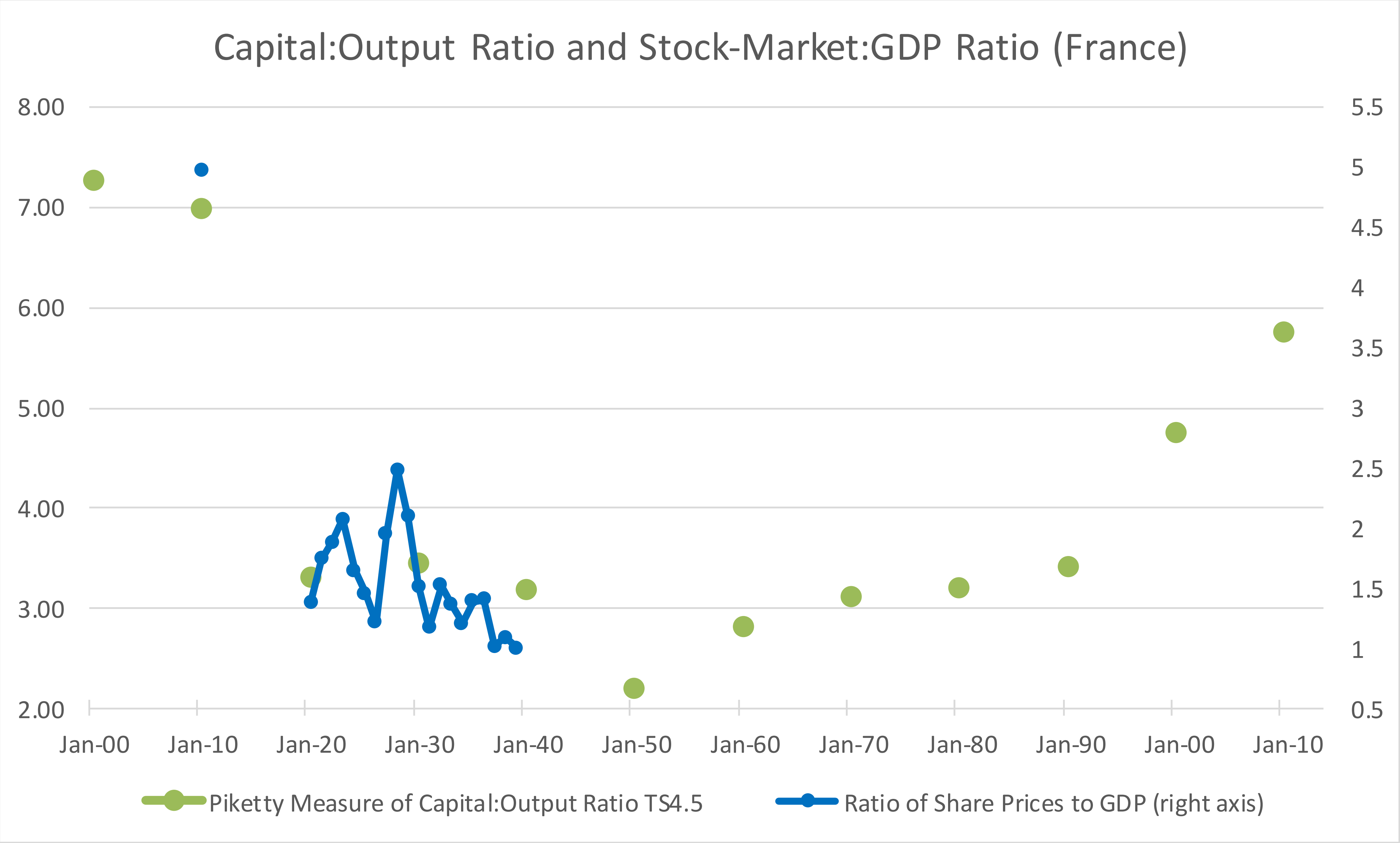
Piketty’s “laws” will hold in either case—because they are tautologies.

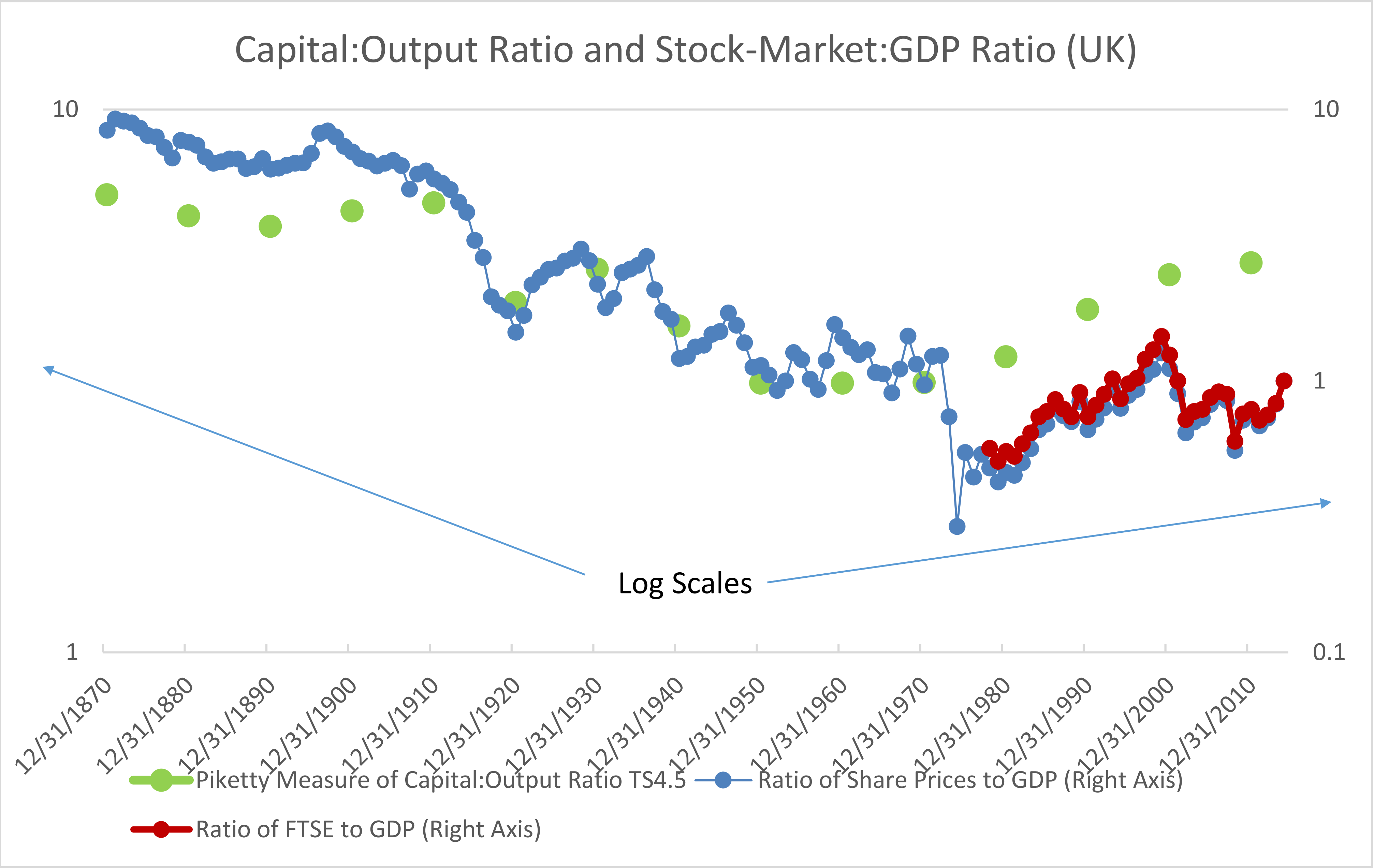
The charts in *Capital in the 21st Century* reflect nominal values but in theorizing about both the past and the future—how we got to where we are and where we might be going from here—economists normally (rightly in my view) use physical values

What happens when the relative prices of output and capital change?



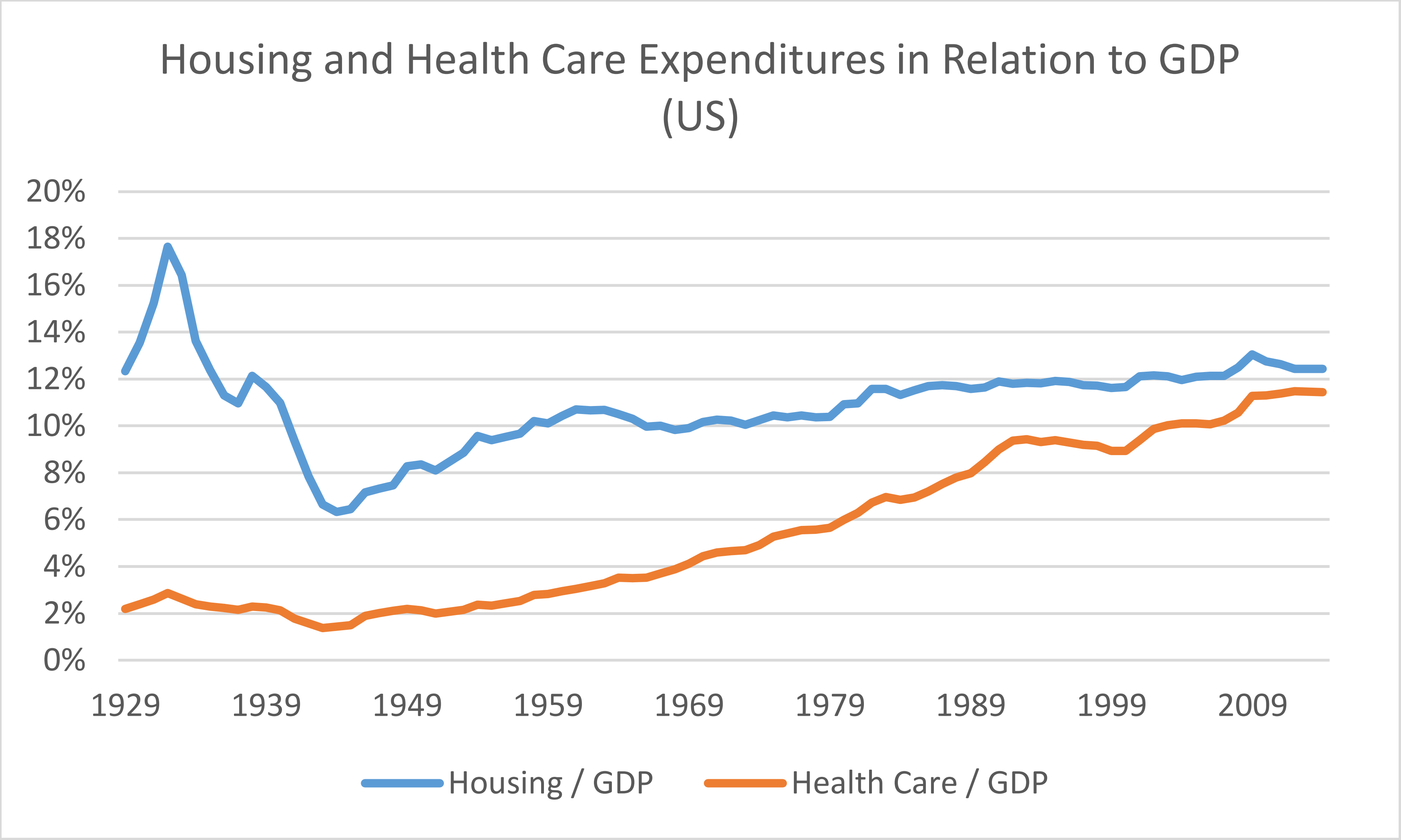
Capital:Output Ratio and Stock-Market:GDP Ratio (US)
5.20
4.70
4.20
3.70
3.20
3.7
3.2
2.7
2.2
1.7
1.2
0.7
0.2
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Piketty Measure of Capital:Output Ratio TS4.5
Ratio S&P 500 Index to GDP Index (right axis)


Capital:Output Ratio and Stock-Market:GDP Ratio (France)
8.00
7.00
6.00
5.00
4.00
3.00
2.00
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
Jan-00
Jan-10
Jan-20
Jan-30
Jan-40
Jan-50
Jan-60
Jan-70
Jan-80
Jan-90
Jan-00
Jan-10
Piketty Measure of Capital:Output Ratio TS4.5
Ratio of Share Prices to GDP (right axis)


Capital:Output Ratio and Stock-Market:GDP Ratio (UK)
10
1
10
1
0.1
Log Scales
12/31/1870
12/31/1880
12/31/1890
12/31/1900
12/31/1910
12/31/1920
12/31/1930
12/31/1940
12/31/1950
12/31/1960
12/31/1970
12/31/1980
12/31/1990
12/31/2000
12/31/2010
Piketty Measure of Capital:Output Ratio TS4.5
Ratio of Share Prices to GDP (Right Axis)
Ratio of FTSE to GDP (Right Axis)

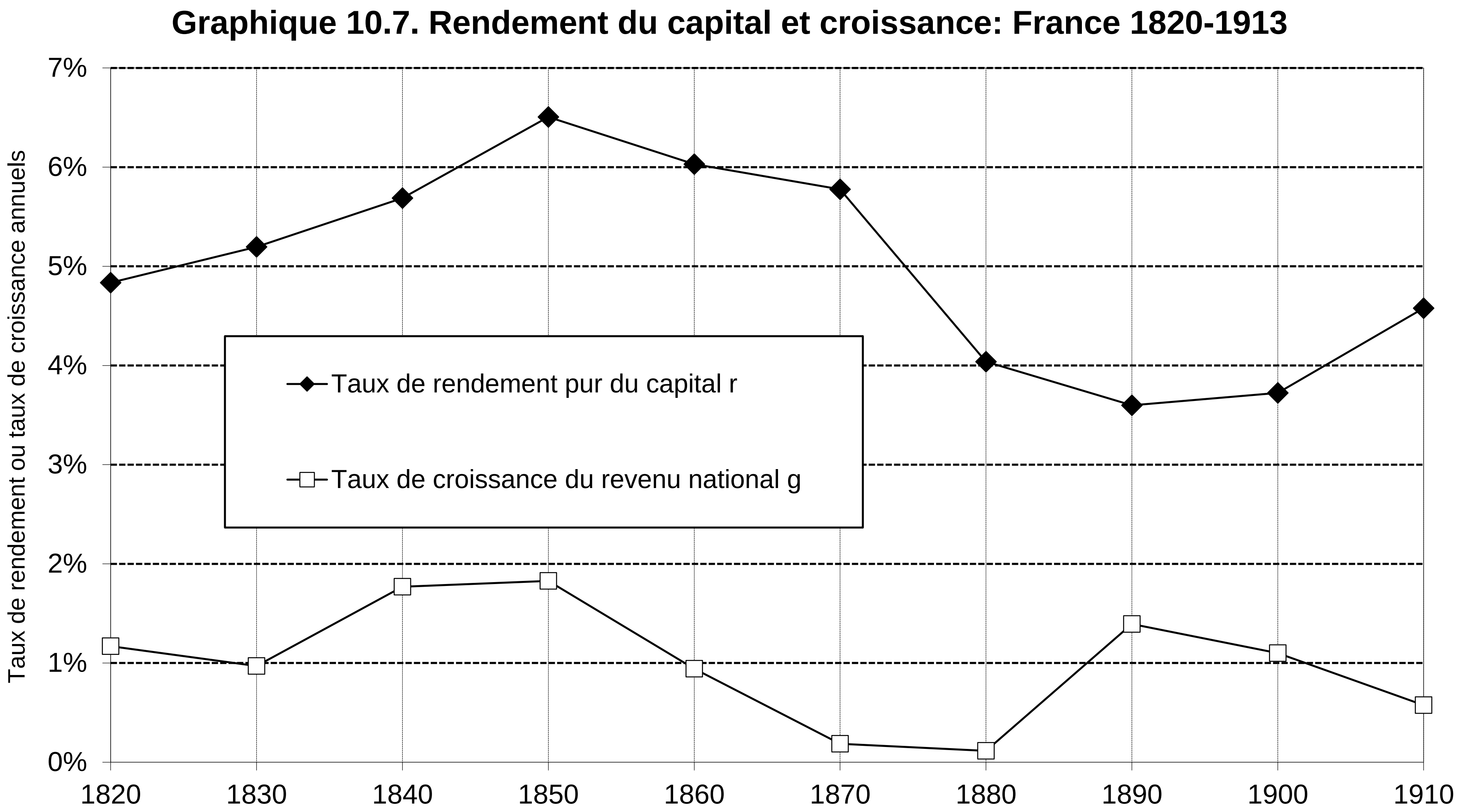
What happens when the physical composition of output changes?

Housing and health care together went from less than 10% of the economy at the end of WW II to almost 25% today. The capital:output ratio for housing services is clearly higher than for other sectors of the economy. I don't know about health care. Changes in composition of output can have large effects on β without making the rich richer.



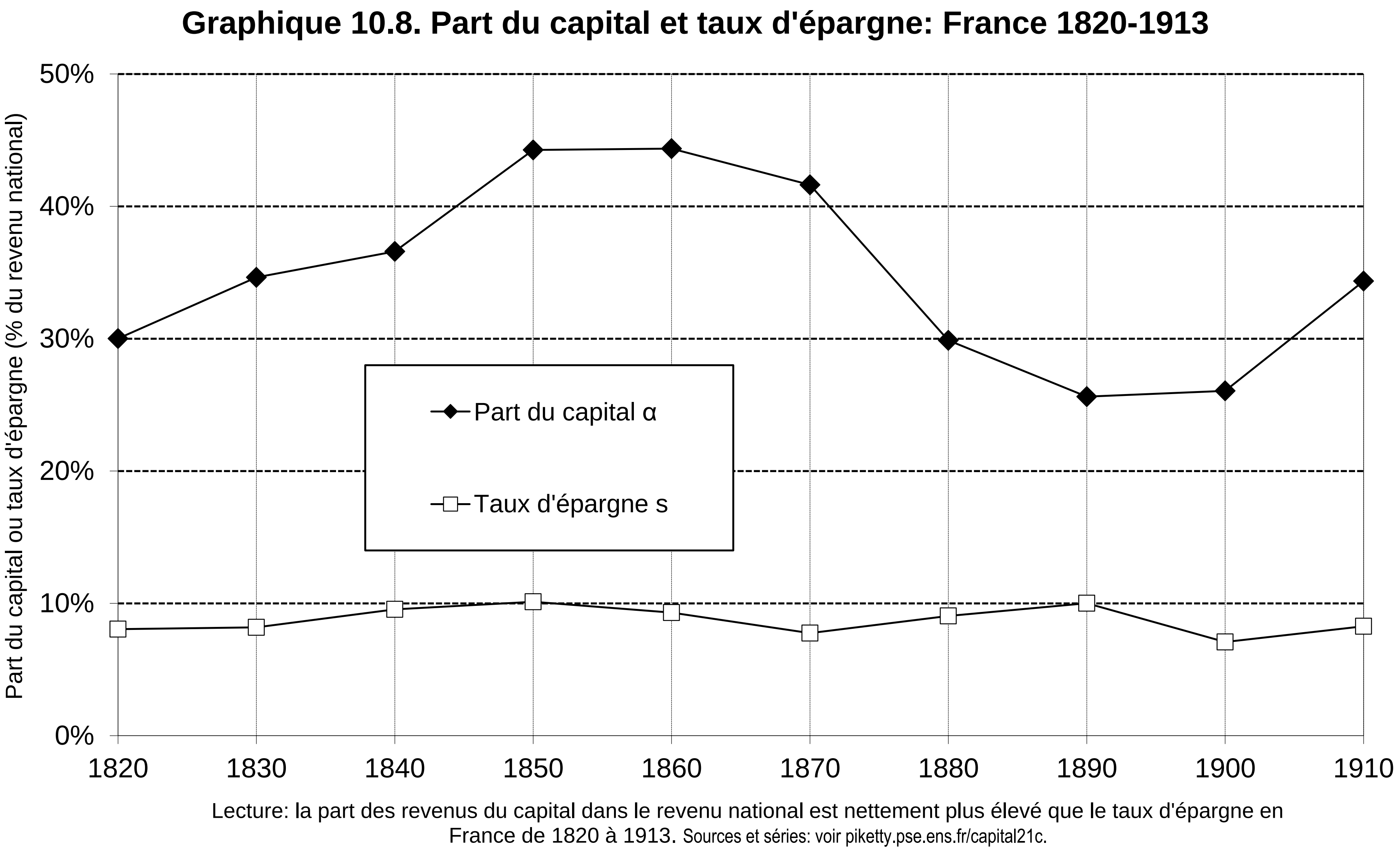
Do the numbers jibe?

Growth and Distribution in France, 1820-1910 (p 352)



Lecture: le taux de rendement du capital est nettement plus élevé que le taux de croisance en France de 1820 à 1913.
Sources et séries: voir piketty.pse.ens.fr/capital21c.

Growth and Distribution in France, 1820-1910 (p 352)



Growth and Distribution in France, 1820-1910

Averages

$s \cong .10$

$\alpha \cong .35$

$g_Y \cong .01$

$r \cong .05$

In 1820-1830

$$\left.\begin{array}{l} s \cong .10 \\ \alpha \cong .35 \\ r \cong .05 \end{array}\right\} \Longrightarrow$$

In 1820-1830:

$g_K \cong .014$

$\beta \cong 7$

$$\frac{\beta'}{\beta} = \left(\frac{sr}{\alpha} / g_Y\right) \cong 1.4$$

Source: *Capital in the 21st Century,* Figures 10.7, 10.8 (p 352)

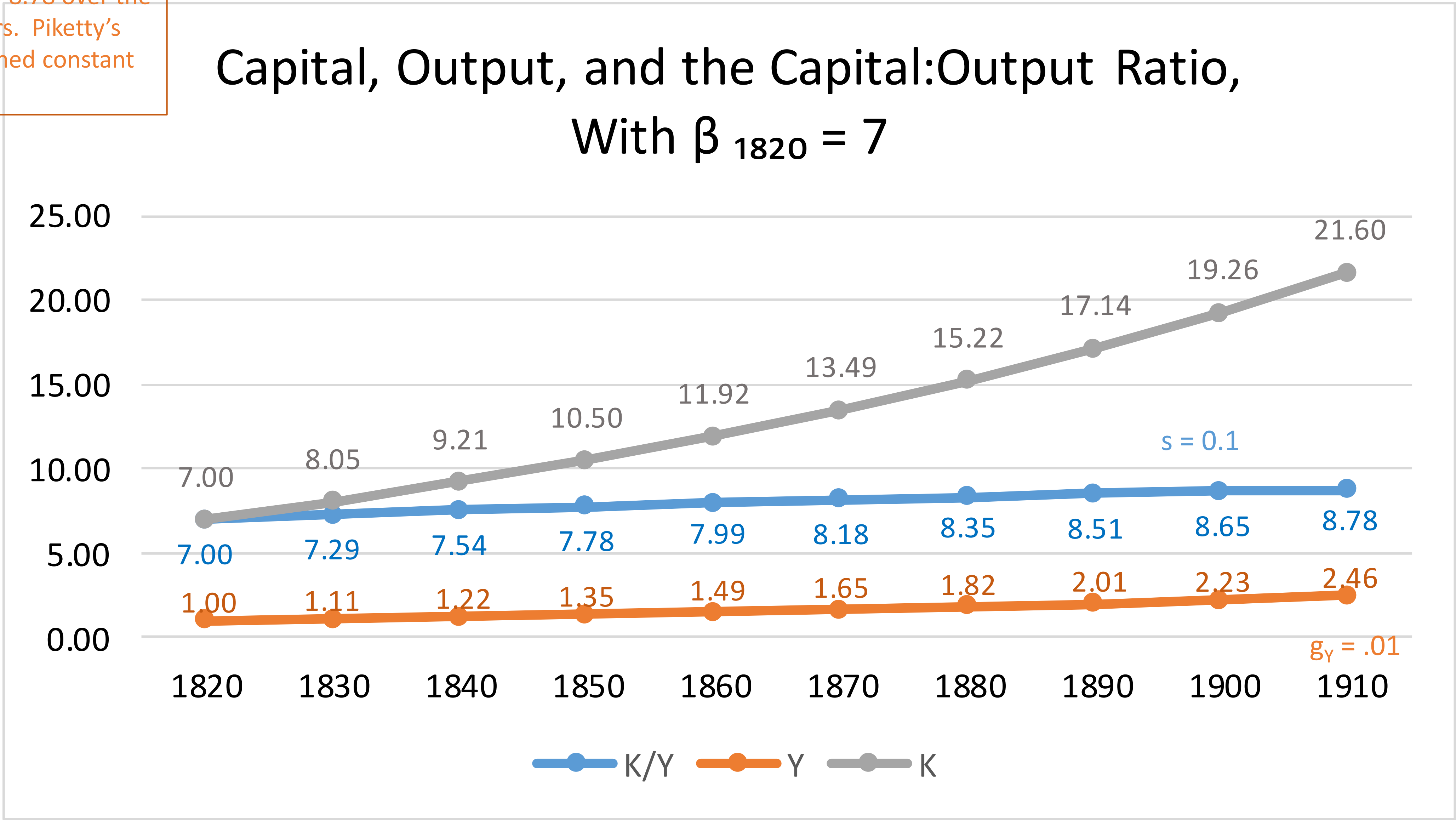
More Arithmetic Laws

$$\dot{\beta} \cong \frac{\Delta\beta}{\Delta t}$$

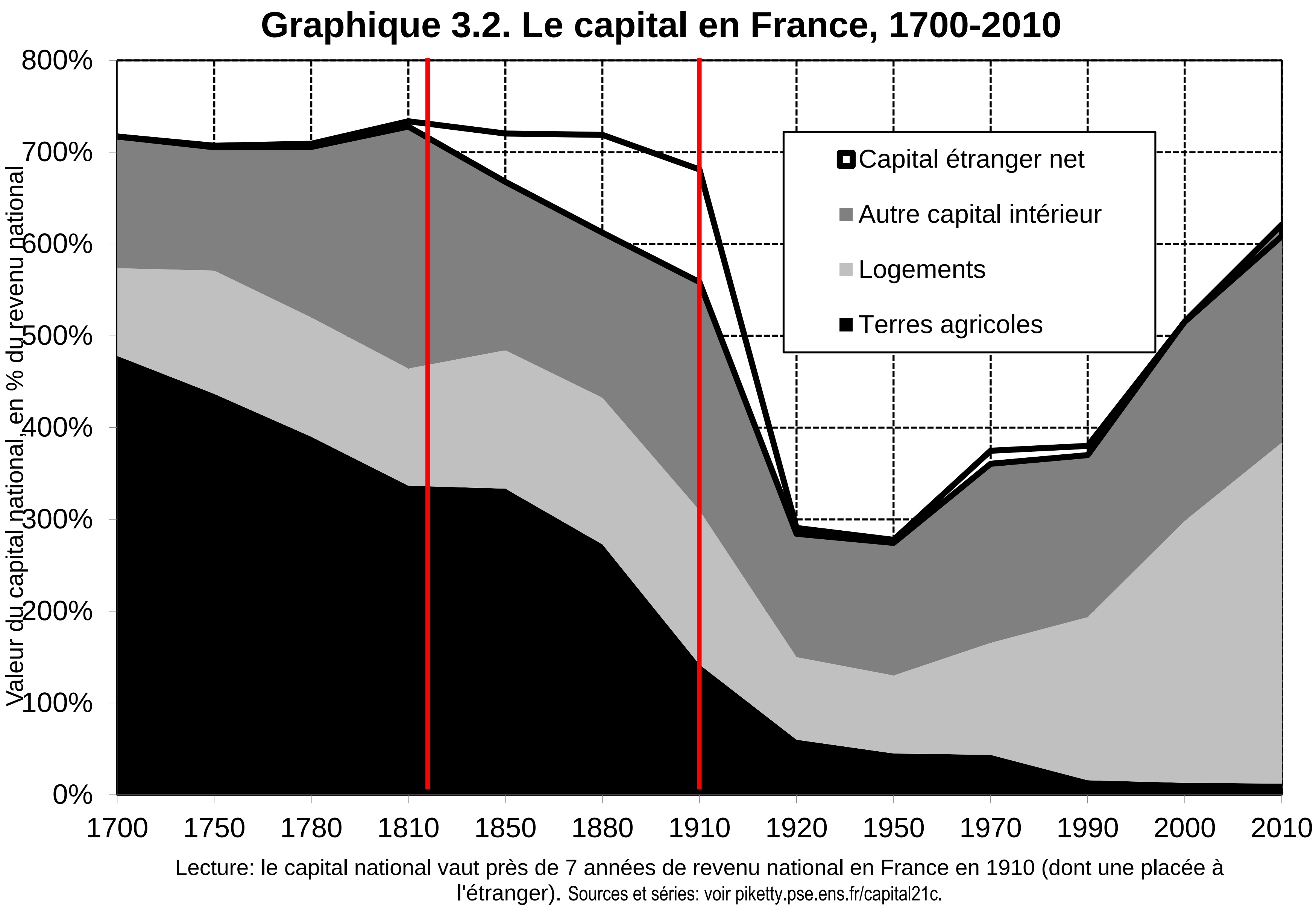
$$\dot{\beta} = s - g_Y\beta$$

$$\beta = \frac{s}{g_Y} + \left(\beta_0 - \frac{s}{g_Y}\right) e^{-g_Y t}$$

Observe the predicted rise from β = 7 to β = 8.78 over the course of 90 years. Piketty's data say β remained constant over this period.



Capital:Output Ratio in France (p 117)



Growth and Distribution in Britain, 1820-1910

p 200

Growth and Distribution in Rich Countries, 1975-2010

p 222