Graphene epsilon-near-zero plasmonic crystals

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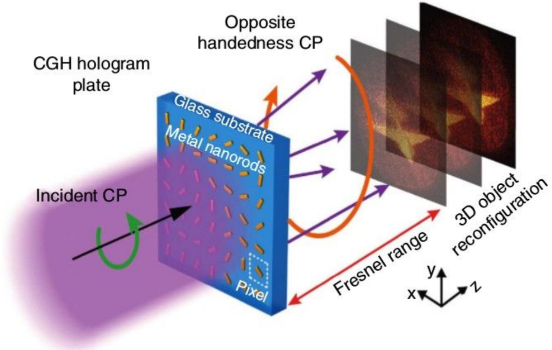
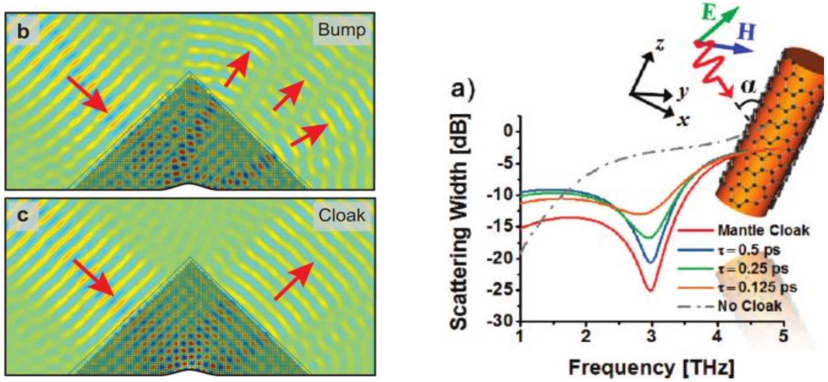
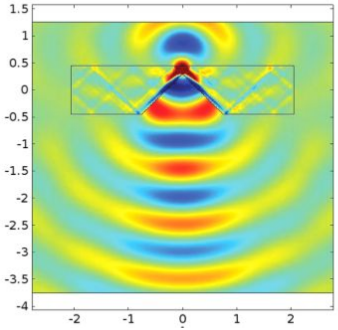
Metamaterials



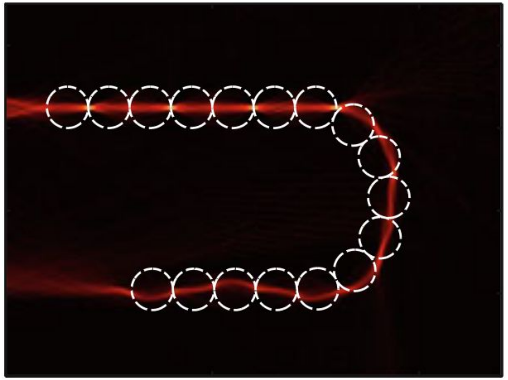
3D optical holography. Nat. Comm. **4** 2013



EM wave Cloaking. Sc. Rep. **78** 2011 & ACS Nano **7** 2011

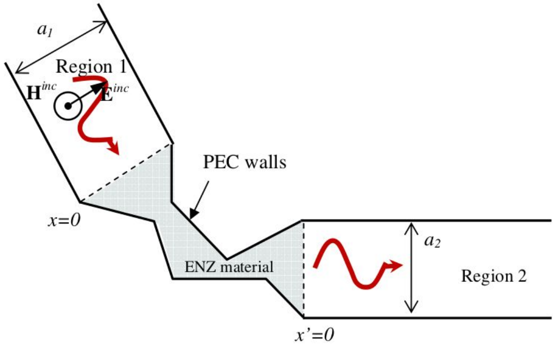


Hyperbolic antennas shape radiation pattern.
JAP **116** 2014

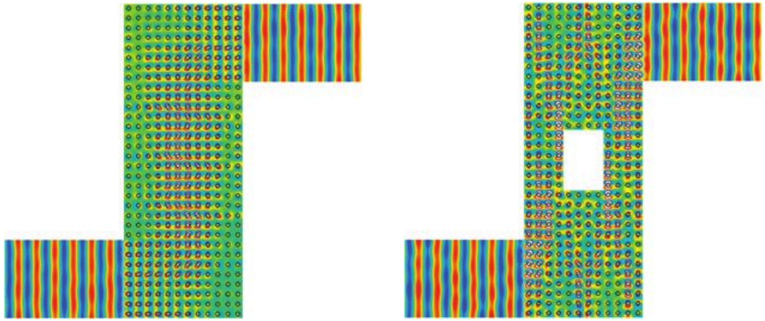


Efficient waveguides.
J. Optics **14** 2012

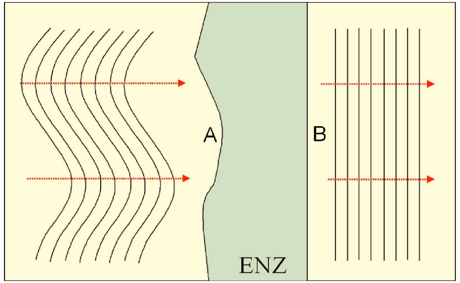
Epsilon-Near-Zero (ENZ)



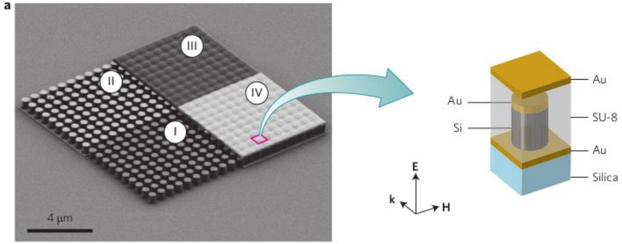
Tunneling through narrow channels. PRL **97** 2006

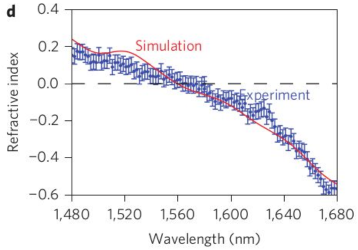


Bending and Cloaking. Nat. Matt. **10** 2011



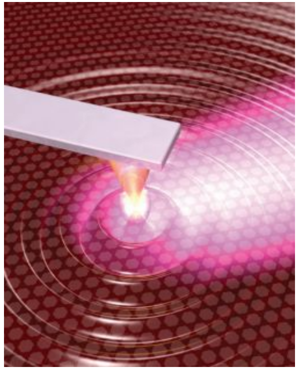
Tailoring the radiation phase pattern.
PRB **75** 2007



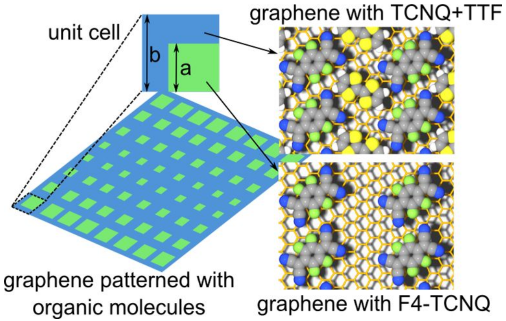


On-chip zero-index. Nat. Phot. **9** 2015

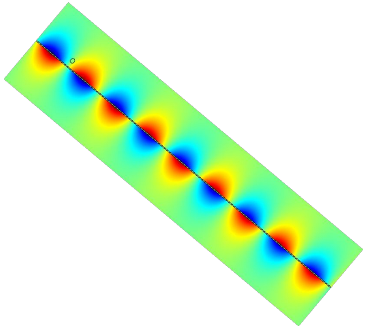
Plasmons in two-dimensional materials



Graphene, a semi-metal:
Becomes metal after doping
Large electrical conductivity
Tunable conductivity via doping
Reduced dimensions
Low optical losses

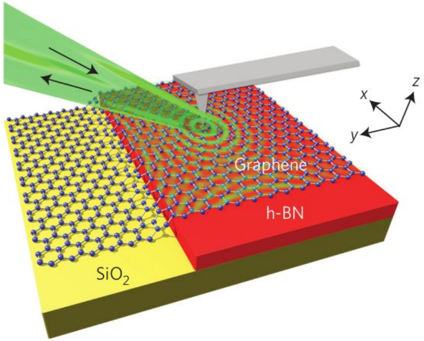


Nanoscale organic meta-lens. Nano Lett. **14** 2014



Graphene plasmonics:
Plasmons are charge oscillations
THz plasmons with tunable properties
Ultra-subwavelength plasmons

Nat. Phot. **6** 2012



Tunable hyperbolic media. Nat. NanoTech. **10** 2015

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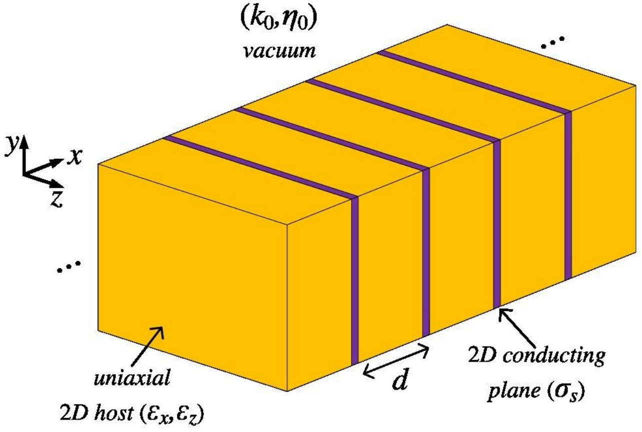
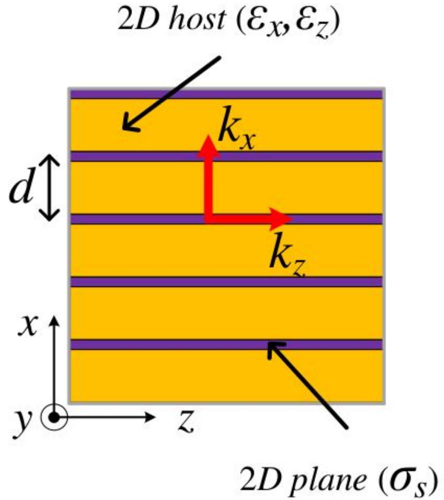
ENZ appears as a Dirac (linear) dispersion in k-space

- Anisotropic dielectric host: ENZ condition
- Space-dependent host: Universality of ENZ and Dirac dispersion
- Frequency-dependent host: Extending the ENZ frequency window
- Effective permittivity for arbitrary plasmonic crystal of 2D metals

Plasmonic Crystal

Periodic arrangement of dielectric/metal slabs





2D metals are embedded periodically in an anisotropic dielectric host

The metals carry surface current $J = \sigma E_z$

Motivation:
Can we design tunable metamaterials with desirable effective permittivity ?

Maxwell Equations

Transverse Magnetic (TM) polarization

Transversal Field

$$-\mathrm{i}\frac{\partial}{\partial z}\mathbf{\Psi} = \mathcal{M}\cdot\mathbf{\Psi} \Leftrightarrow$$

$$-\mathrm{i}\frac{\partial}{\partial z}\begin{pmatrix} E_x \\ H_y \end{pmatrix} = k_0\eta_0 \begin{pmatrix} 0 & 1+\frac{1}{k_0^2}\frac{\partial}{\partial_x}\frac{1}{\varepsilon_z}\frac{\partial}{\partial_x} \\ \frac{\varepsilon_x}{\eta_0^2} & 0 \end{pmatrix}\begin{pmatrix} E_x \\ H_y \end{pmatrix}.$$

Longitudinal Field

$$E_z = \frac{\mathrm{i}\eta_0}{k_0\varepsilon_z}\frac{\partial H_y}{\partial x}$$

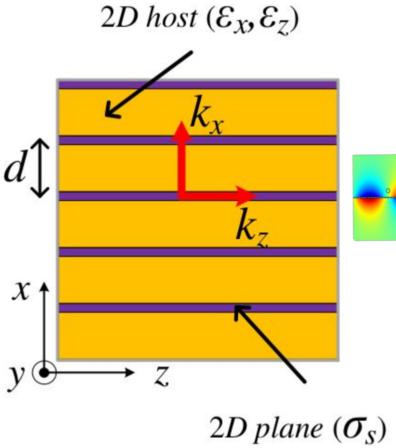
$$k_0 = \omega / c$$

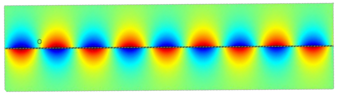
$$\eta_0 = \sqrt{\mu_0/\varepsilon_0}$$

Propagation along z direction (plasmons)

$$\mathbf{\Psi}(x,z) = \mathbf{\Psi}(x)e^{\mathrm{i}k_z z},$$

$$k_z\mathbf{\Psi} = \mathcal{M}\mathbf{\Psi}$$





Dispersion Relation

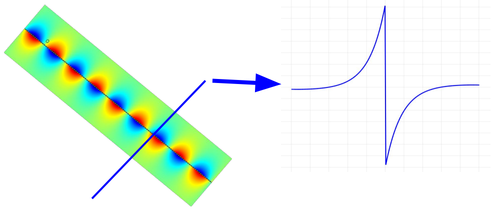
Assume periodicity along x direction (Bloch waves)

$$D(k_x, k_z) = \cos(k_x) - \left[\cosh(\kappa d) - \frac{\xi d}{2}\sinh(\kappa d)\right] = 0$$

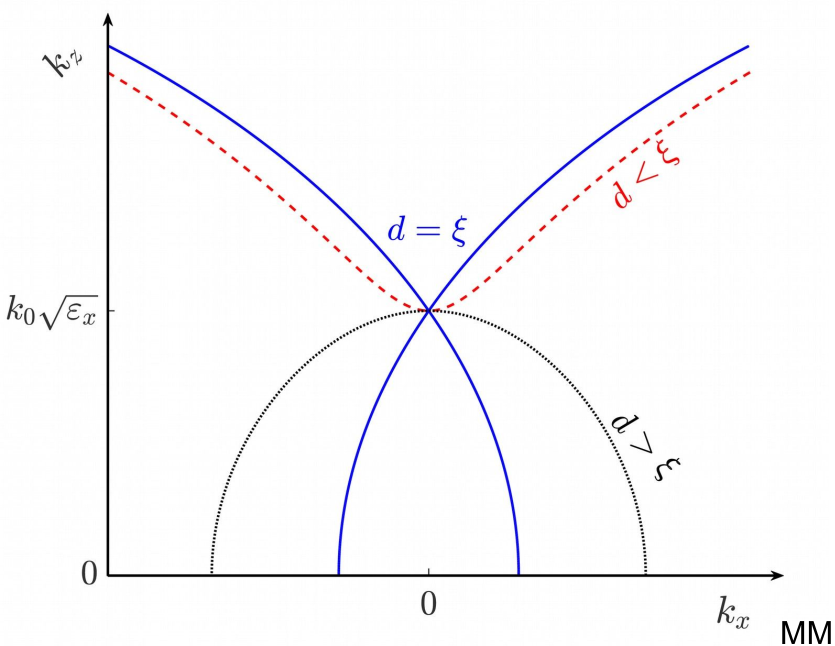
$$\kappa^2 = \frac{\varepsilon_z}{\varepsilon_x}(k_z^2 - k_0^2\varepsilon_x)$$

Plasmonic Thickness
(twice the decay length)

$$\xi = -\frac{i\sigma}{\omega\varepsilon_z}$$



Wang *et. al.* PRL **112** 2012



MM *et. al.* PRB **94** 2016

Effective dielectric function

dense grid: $d \ll \lambda$

$$\frac{k_z^2}{\varepsilon_x} + \frac{d}{(d-\xi)\varepsilon_z} k_x^2 = k_0^2$$

effective medium

$$\frac{k_z^2}{\varepsilon_x^{\text{eff}}} + \frac{k_x^2}{\varepsilon_z^{\text{eff}}} = k_0^2$$

$$\varepsilon_z^{\text{eff}} = \frac{d-\xi}{d}\varepsilon_z \qquad \varepsilon_x^{\text{eff}} = \varepsilon_x$$

ENZ condition

$$d = \xi \Rightarrow \varepsilon_z^{\text{eff}} = 0$$

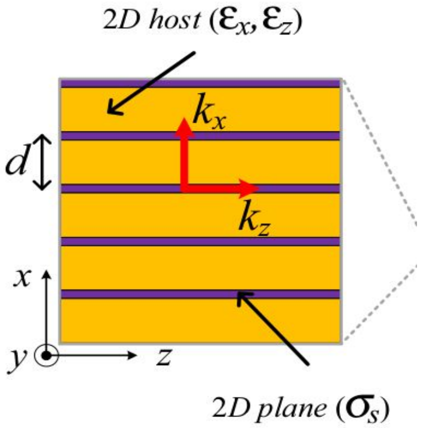
- ✓ Dirac dispersion yields ENZ behavior
- ✓ Systematic method for designing ENZ metamaterials

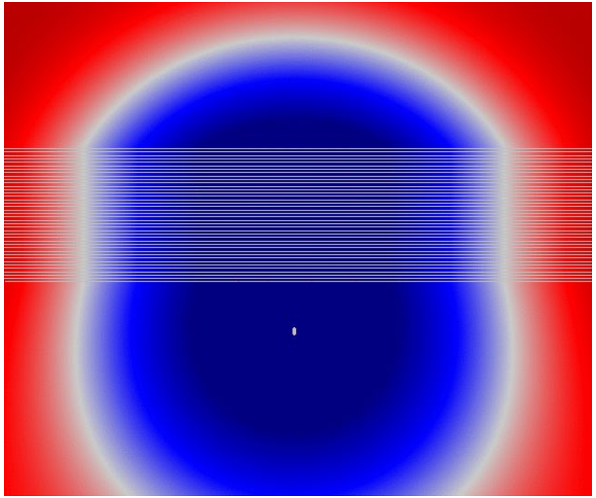
MM *et. al.* PRB **94** 2016

ENZ behavior (simulation)

- 40 graphene layers embedded in MoS_2 host (εx=3.5, εz=13, d=20.8 nm).
- Drude model for graphene conductivity.
- λ_0=12 μm (THz regime).
- 2D magnetic dipole source.

$$\sigma(\omega) = \frac{ie^2\mu_c}{\pi\hbar^2(\omega + i/\tau)}$$





Ultra fast phase transitions

No dispersion

No phase delay

MM *et. al.* PRB **94** 2016

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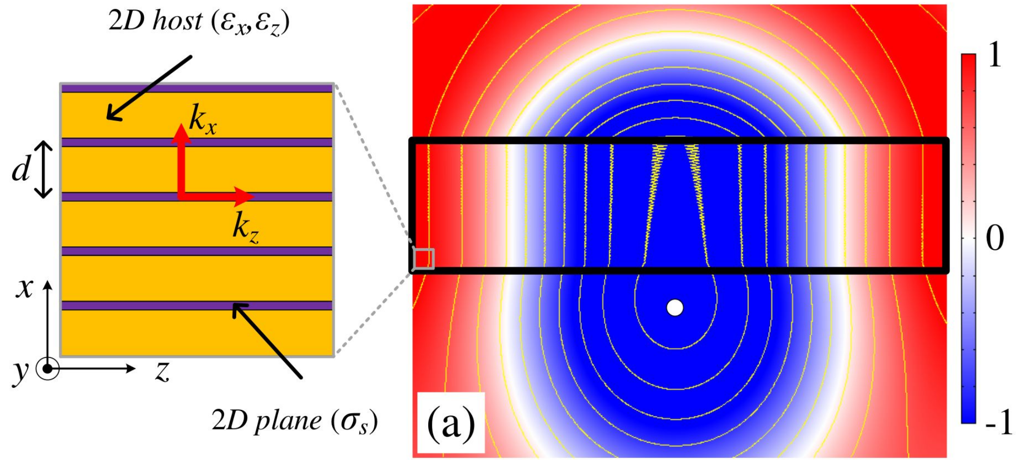

Ultra fast phase transitions

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MM *et. al.* PRB **94** 2016

Numerical EM wave simulations

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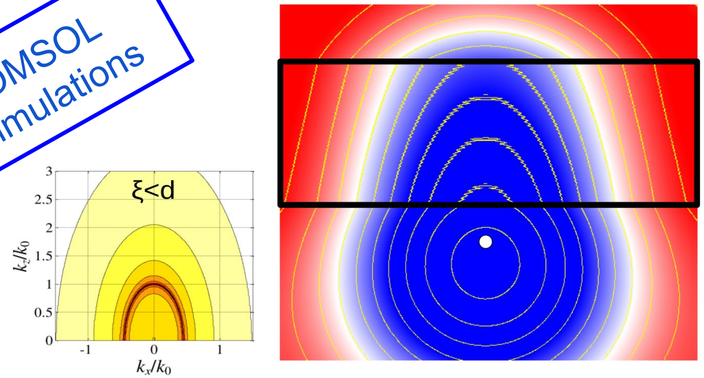
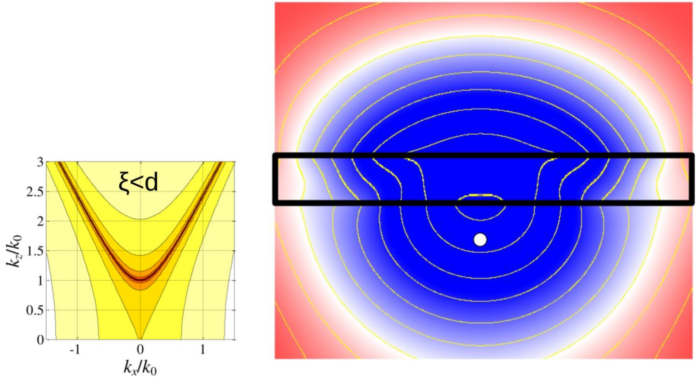
COMSOL simulations

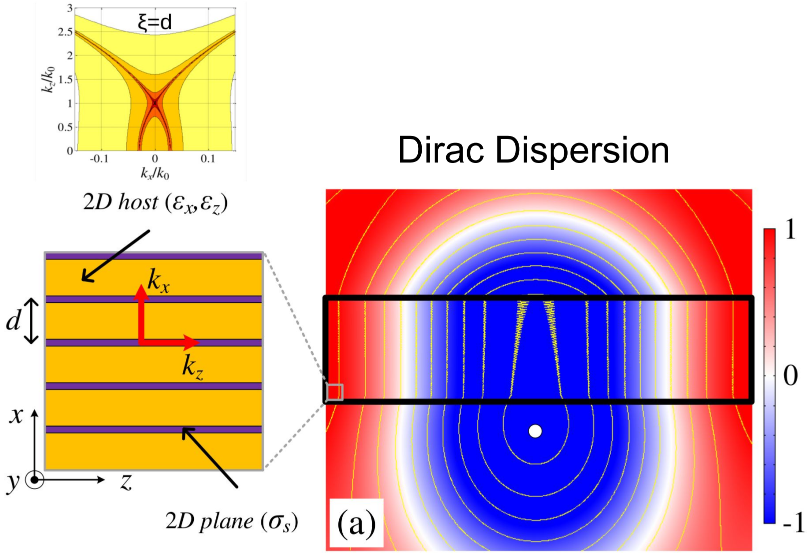
Weak plasmon coupling



Strong plasmon coupling



Dirac Dispersion



MM *et. al.* PRB **94** 2016

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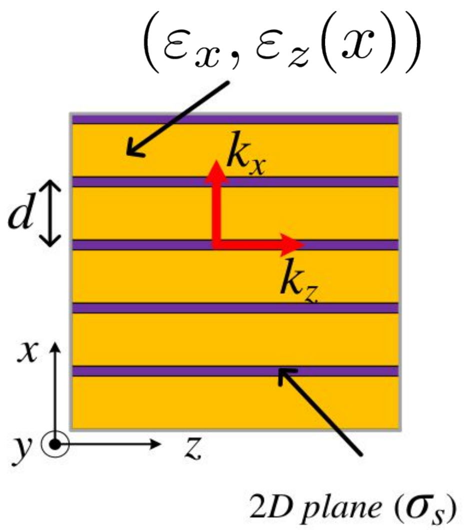
Universal behavior of a Dirac dispersion

Dirac dispersion and ENZ is a **universal property** of plasmonic crystals consisting of 2d metals in host with space-dependent permittivity



Introduce an extra degree of freedom for tuning the dielectric properties

Universal condition for ENZ

$$d_0 = \xi_0 \left[\int_0^1 f(x) dx \right]^{-1}, \quad \frac{\varepsilon_z^{\mathrm{eff}}}{\varepsilon_{z,0}} = \xi_0 \left(\frac{1}{d_0} - \frac{1}{d} \right)$$

$$\varepsilon_z(x) = \varepsilon_{z,0} f(x/d), \quad f(x) > 0$$

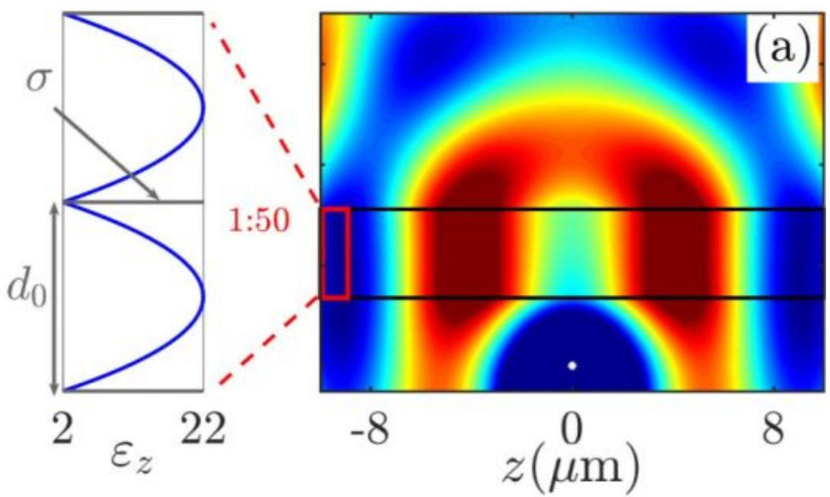
$$\xi_0 = -\frac{i\sigma}{\omega \varepsilon_{z,0}}$$

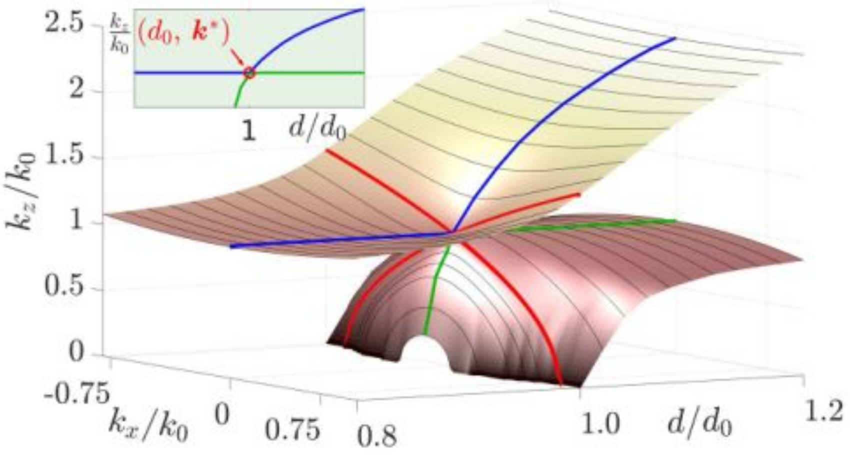
Maier, MM *et al.*, PRB **97**, 2018

Universal behavior of a dispersive Dirac cone

Parabolic profile

$$\varepsilon_z(x) = \varepsilon_{z,0}\left[1 + 6\alpha\frac{x}{d}\left(1 - \frac{x}{d}\right)\right],$$



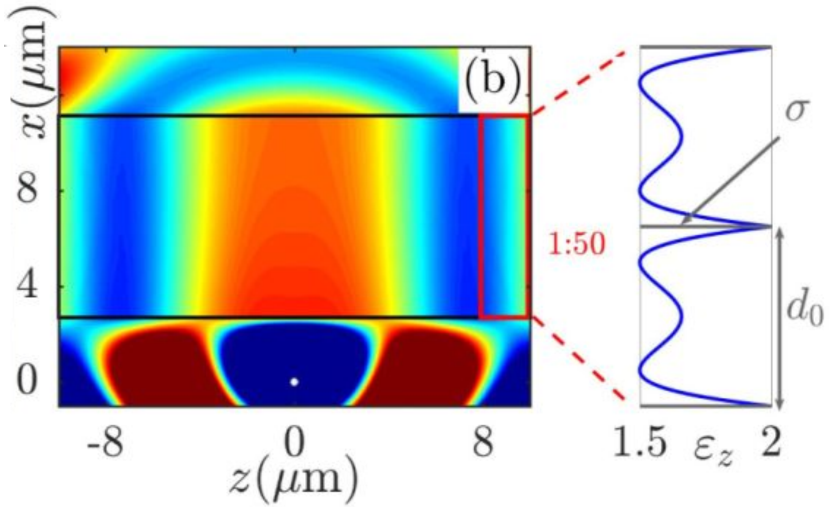


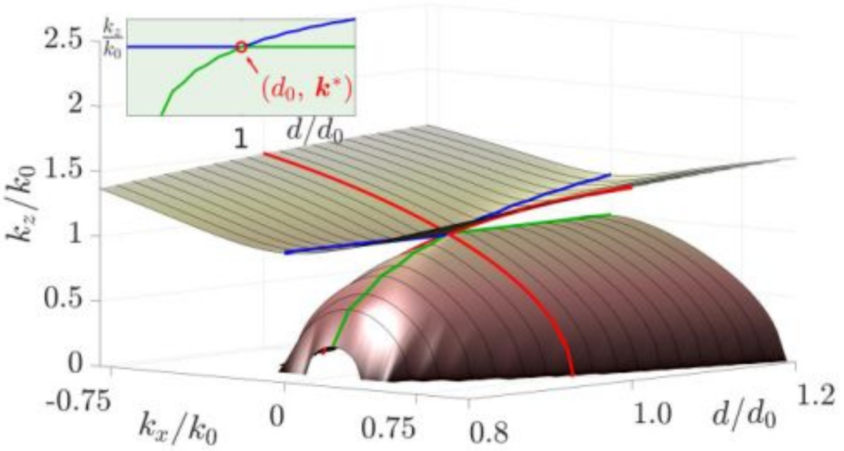
Maier, MM *et al.*, PRB **97**, 2018

Universal behavior of a dispersive Dirac cone

Double well profile

$$f_{\mathrm{dw}}(x) = 1 - 3.2x + 13.2x^2 - 20x^3 + 10x^4$$





Maier, MM *et al.*, PRB **97**, 2018

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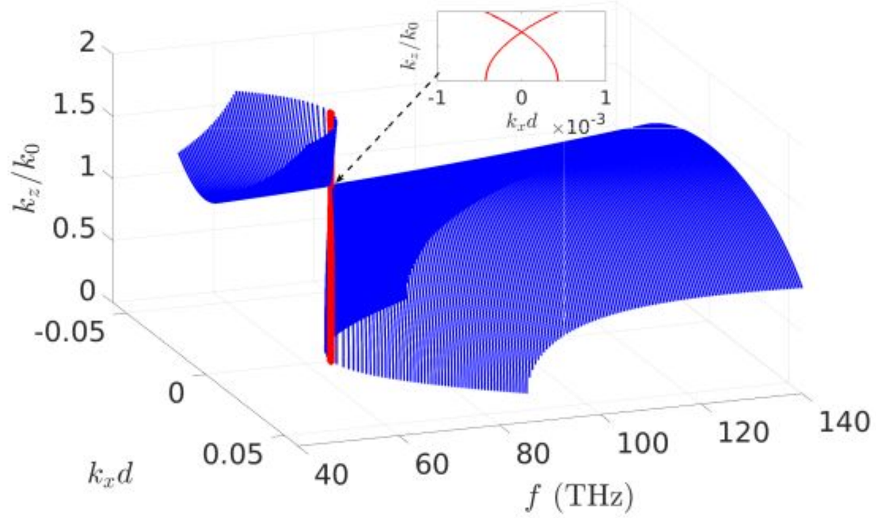
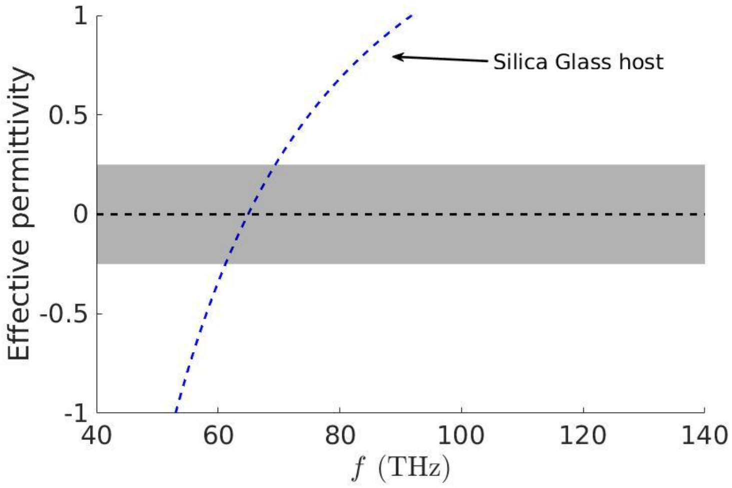
Expanding the ENZ frequency window

Very narrow frequency window with ENZ properties



$$\varepsilon = 2$$



MM *et. al.* ACM NanoCom 2019

Lorentz dispersive host

Add a new Dirac point expanding ENZ frequency range

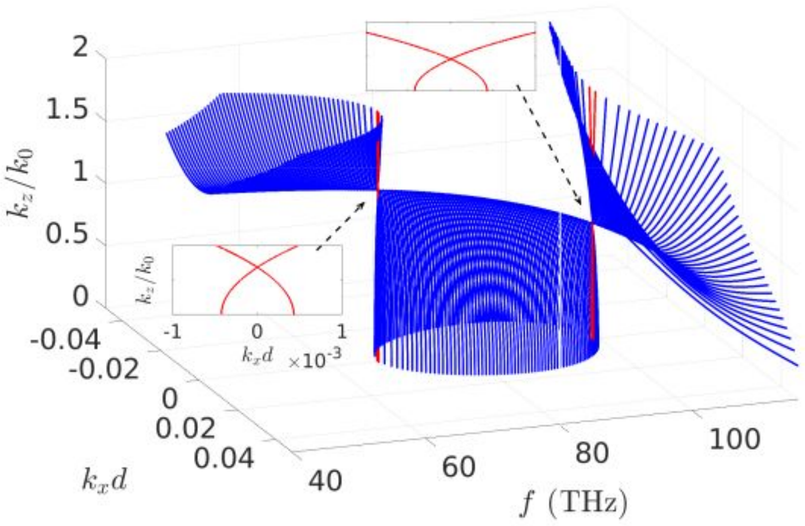
$$\varepsilon(\omega) = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)\omega_0^2}{\omega_0^2 - \omega^2 + i\omega\Gamma}$$

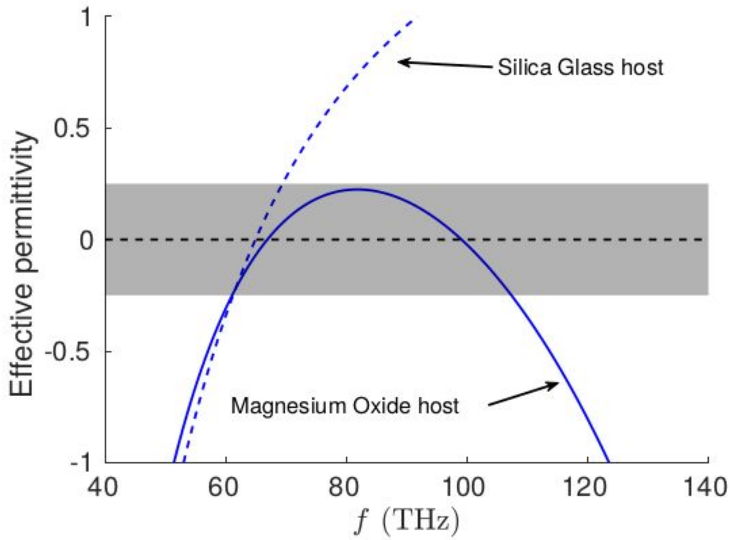
$\varepsilon_\infty = 11.2$

$\varepsilon_s = 2.6$

$\Gamma = 0$

$\omega_0 = 1\text{eV}\ (f = 242.8\text{THz})$





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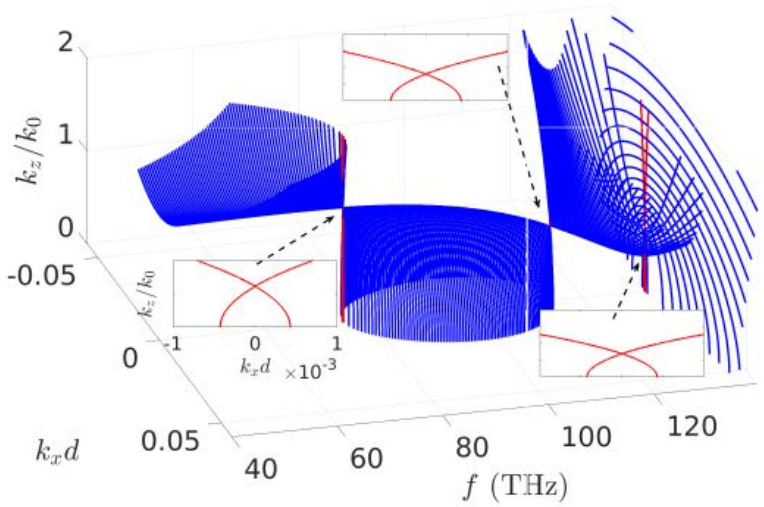
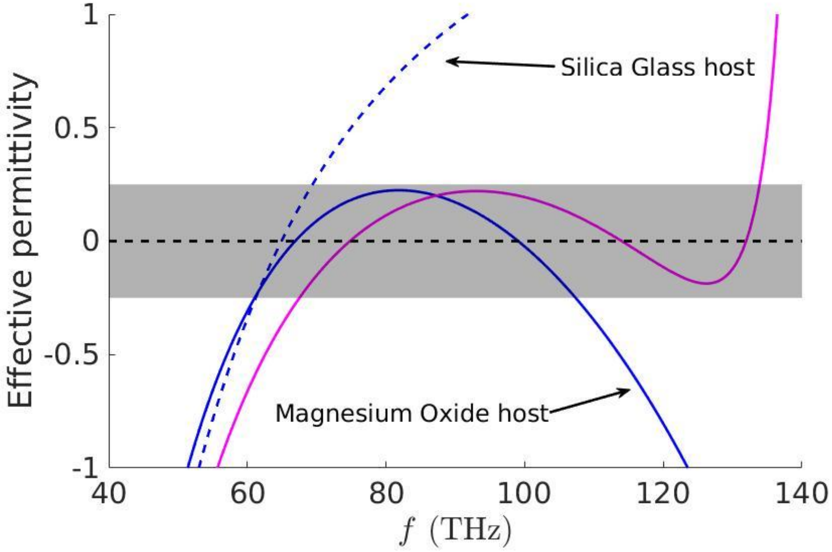
Multi-Oscillator Lorentz dispersive host

Add more Dirac points and expanding more the ENZ window

$$\varepsilon(\omega) = \varepsilon_\infty + \sum_i^N \frac{g_i \; \omega_{0i}^2}{\omega_{0i}^2 - \omega^2 + i\omega\Gamma_i},$$





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General formula for the effective medium

A rigorous formulation for the effective dielectric function

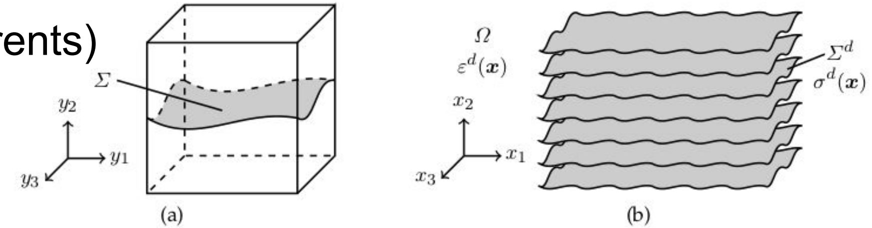
Arbitrary shape of a 2D material as the building element

Periodic structure in one, two, or three dimensions

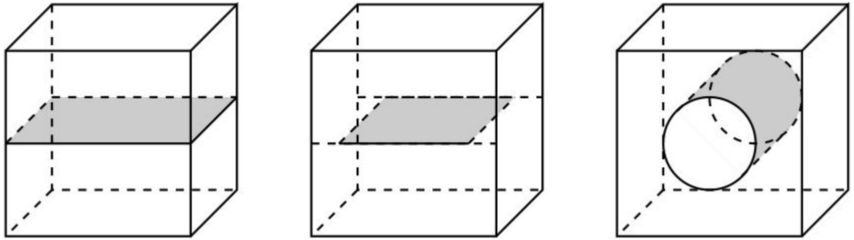
Finite and edge state effects (surface + line currents)

Finite number of structural periods

Spatial & frequency-dependent host permittivity



Choose your 2D element:
planar, ribbon, tube



Maier, MM, *et al.* RSPA 2019

Conclusion

- A systematic way to design crystals with ENZ behavior
  - Tuning dynamically the optical properties
  - Dirac dispersion indicates ENZ behavior
- Universality of Dirac dispersion and ENZ for 2D plasmonic crystals with spatial-dependent host
- Expand the ENZ frequency window with Lorentz dispersive host
- A rigorous formulation for a general case of plasmonic crystal structure:
  A general ENZ condition

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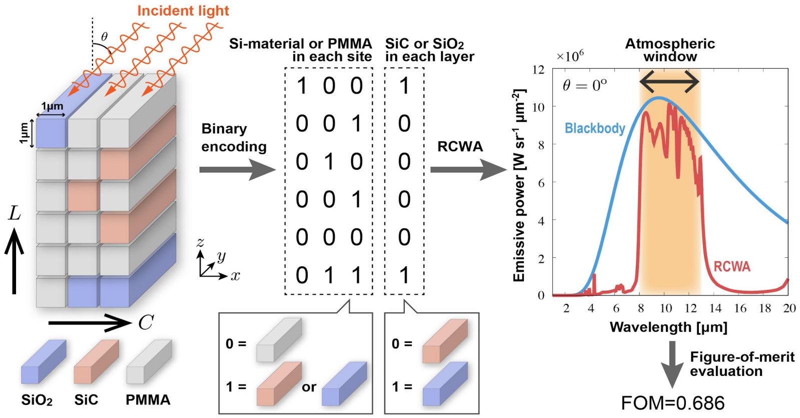
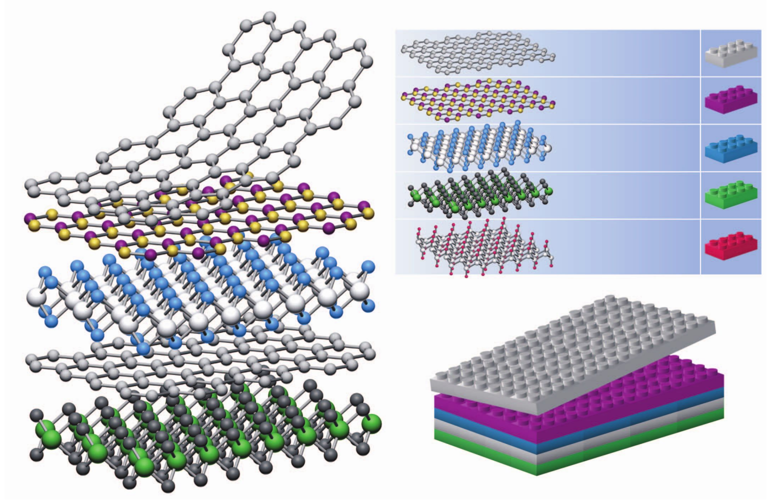
A promising direction in engineering

Machine Learning for designing new metamaterials



Kitai et al. ArxiV 1902.06573

Flatland multilayer heterostructures



Geim *et al*. Nature **499** 2019

Scheme:

Set target properties

Define a periodic structure

Choose the building elements

Let ML to design the optimal configuration

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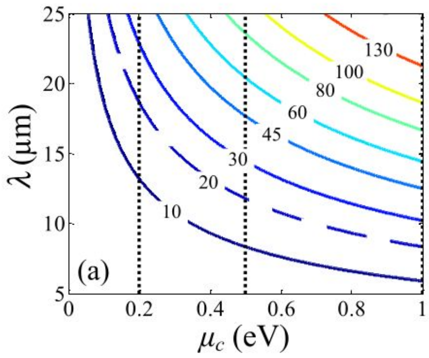
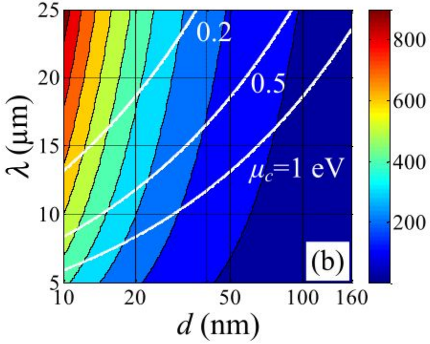
Supplementary Material

General Investigation (maps)

- ➢ Combinations of μc and λ leading to PDP & ENZ (ξ is plotted in nm).
- ✓ A structure with arbitrary **d** can be fabricated and then with suitable choice of $\boldsymbol{\mu_c}$ and **λ** we achieve ENZ behavior.



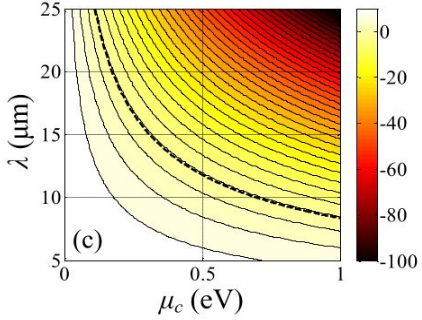


$$\frac{L}{d} = \sqrt{\frac{2}{\varepsilon_z}}\sqrt{\frac{\mathrm{Im}[\sigma_s]}{\mathrm{Re}[\sigma_s]}}\frac{1}{k_0 d}$$

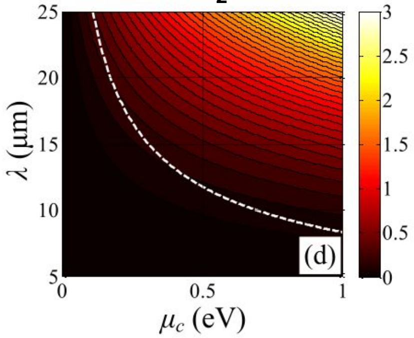
For Drude:
$L \sim \lambda^{1/2}$

The propagation distance L/d of a plasmonic mode for all **λ**, **d** and $\boldsymbol{\mu_c}$ combinations leading to ENZ.

Re[ε_z^{eff}]



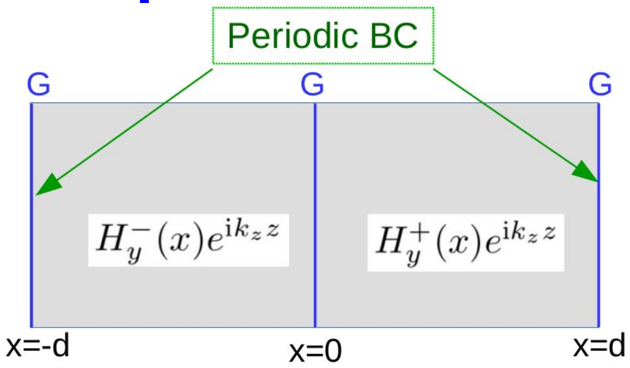
Im[ε_z^{eff}]



Effective permittivity for **λ** & $\boldsymbol{\mu_c}$ combinations and fixed period **d**=20nm.

- ● Dashed lines indicate the ENZ regime.
- ✓ Low losses in the ENZ region.
- ✓ Very negative ε achieved but accompanied with high losses.

Dispersion Relation



- Assuming that Graphene carries surface current $J=\sigma E_z$.
- Periodicity: The eigenmodes are Bloch waves and arranged in bands.

$$H_y^+(x) = H_y^-(x-d)e^{ik_x d}$$

$$F(k_x, k_z) = \cos(k_x d) - \left[\cosh(\kappa d) - \frac{\xi\kappa}{2}\sinh(\kappa d)\right] = 0$$

$$\kappa = \sqrt{\frac{\varepsilon_z}{\varepsilon_x}(k_z^2 - k_0^2\varepsilon_x)}.$$

$$\xi = -\frac{i\sigma_g \eta_0}{k_0 \varepsilon_z}$$

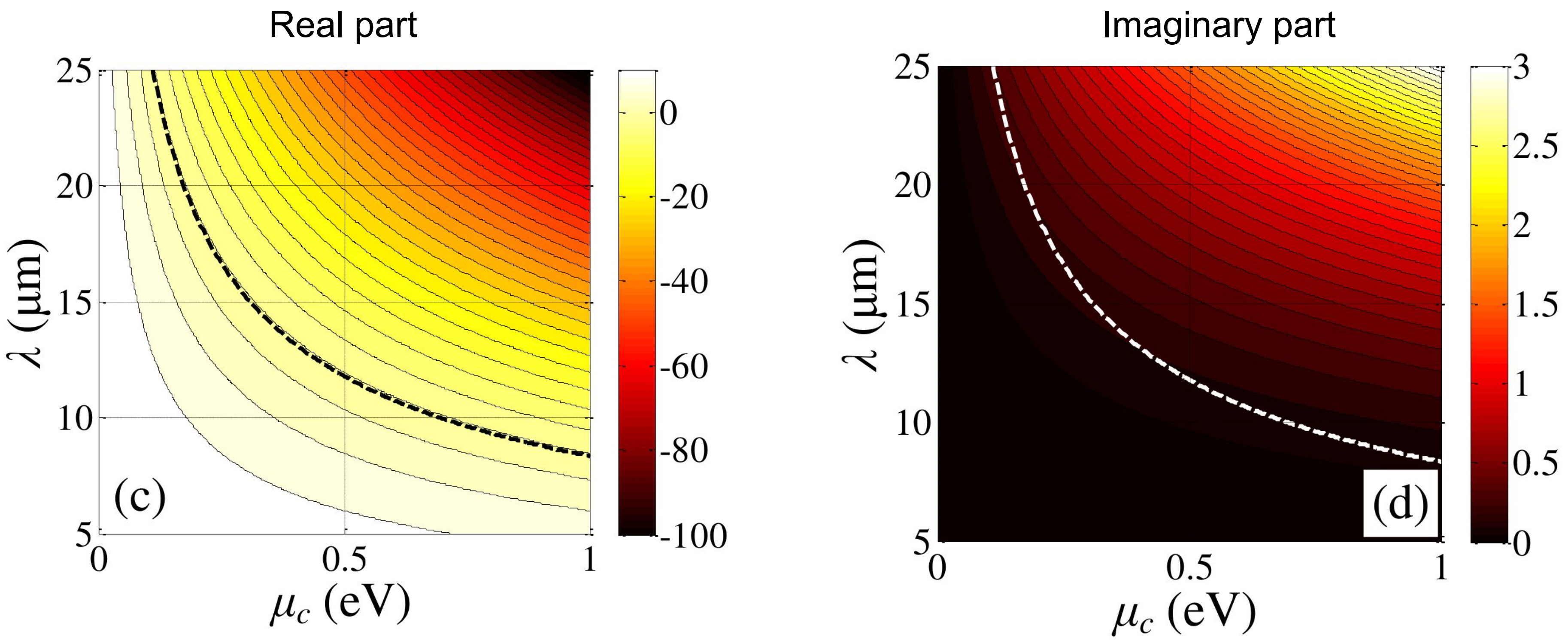
- Assume very dense grid: $\kappa d << 1$
- Around Brilloun center: $k_x \sim 0$.

$$\frac{k_z^2}{\varepsilon_x} + \frac{d}{(d-\xi)\varepsilon_z}k_x^2 = k_0^2.$$

- d>ξ: Weak Plasmon Coupling → Elliptic Band
- d=ξ: Critical Plasmon Coupling → Two Linear Bands
- d<ξ: Strong Plasmon Coupling → Hyperbolic Band

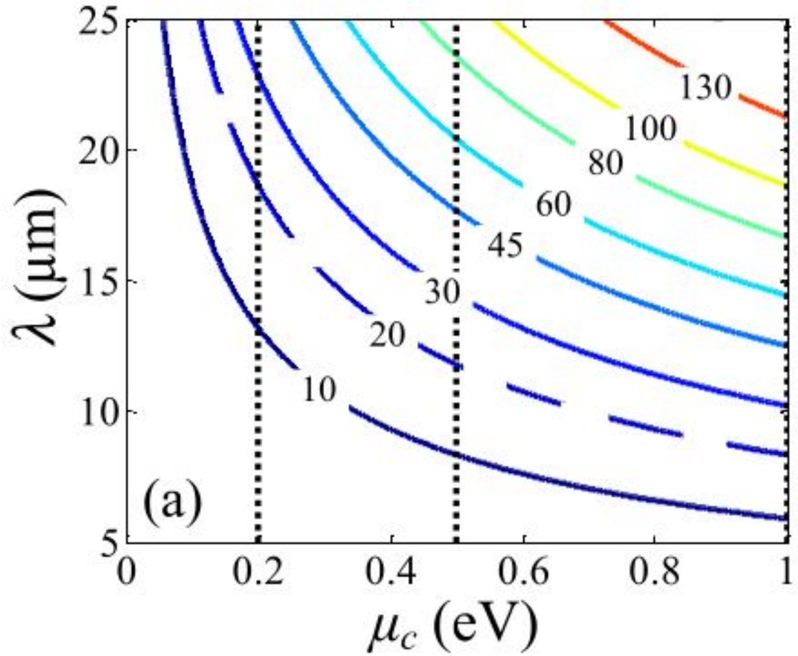
$$k_z^2 \simeq \frac{\varepsilon_x d}{\varepsilon_z(\xi - d)}k_x^2$$

Effective dielectric function

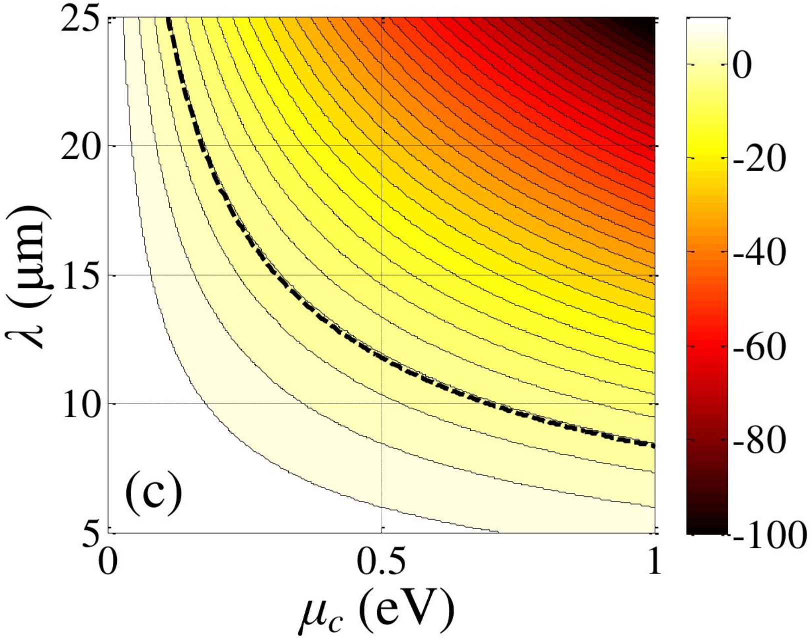


Tunable EM properties

ENZ condition



Effective permittivity



Universal behavior of a dispersive Dirac cone

Dirac dispersion and ENZ is a universal property of plasmonic crystals consisting of 2d metals in host with space-dependent permittivity

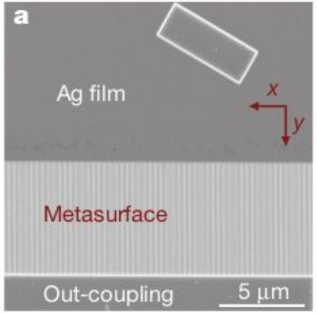
General dispersion relation:

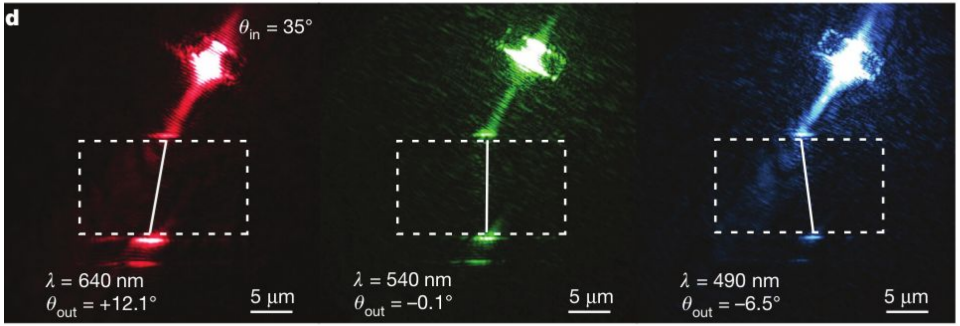
$$D[\boldsymbol{k}] = \det\left(\begin{bmatrix} \mathcal{E}_{(1)}(d) & \mathcal{E}_{(2)}(d) \\ \mathcal{E}'_{(1)}(d) & \mathcal{E}'_{(2)}(d) \end{bmatrix} - e^{ik_x d} \begin{bmatrix} 1 & 0 \\ -i(\sigma/\omega)\kappa(k_z) & 1 \end{bmatrix} \right) = 0.$$

Universal condition for ENZ

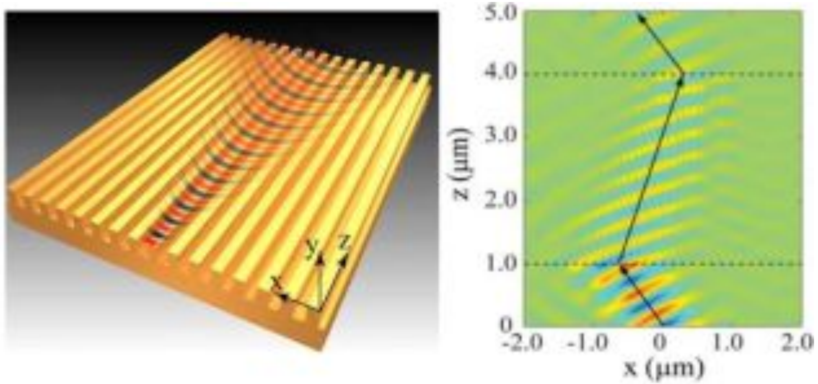
$$d_0 = \xi_0 \left[\int_0^1 f(x)dx \right]^{-1}, \quad \frac{\varepsilon_z^{\text{eff}}}{\varepsilon_{z,0}} = \xi_0 \left(\frac{1}{d_0} - \frac{1}{d} \right).$$

Multilayer Metamaterial





Hyperbolic metasurface.
Nat. Lett. **522** 2015

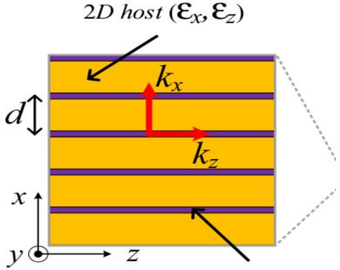


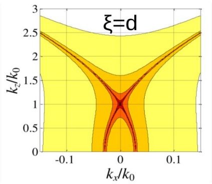
Negative refraction. APL 2013

Numerical EM wave simulations (vids)

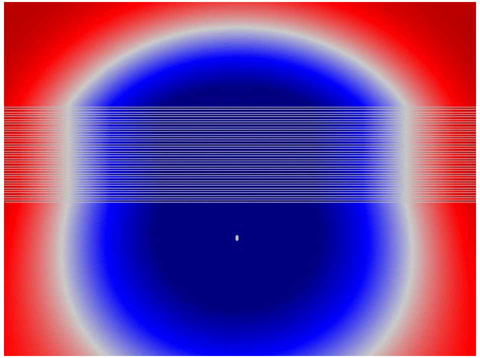
- 40 graphene layers embedded in MoS_2 host (εx=3.5, εz=13, d=20.8 nm).
- Drude model for graphene conductivity.
- λ_0=12 μm (THz regime).
- 2D magnetic dipole source.

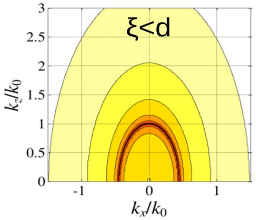
COMSOL simulations



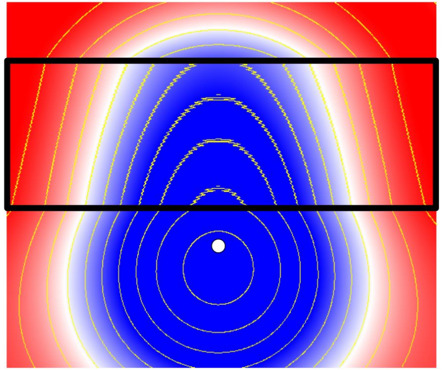


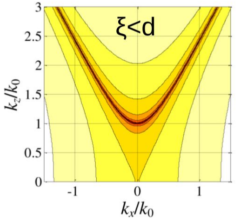
Plasmonic Dirac Point





Weak plasmon coupling





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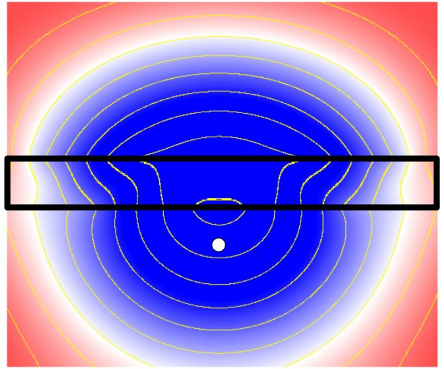

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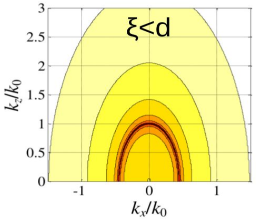

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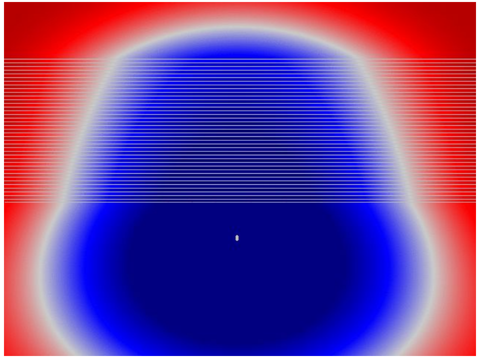

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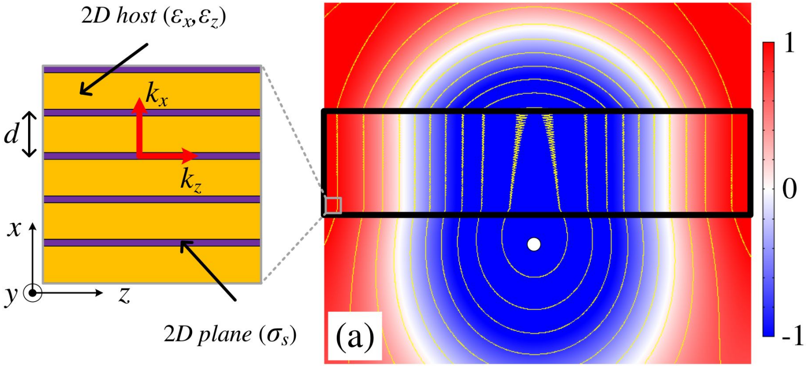

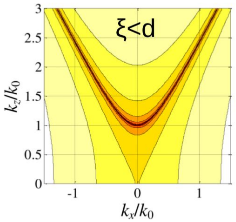

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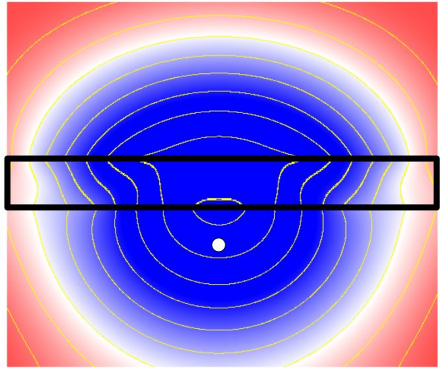

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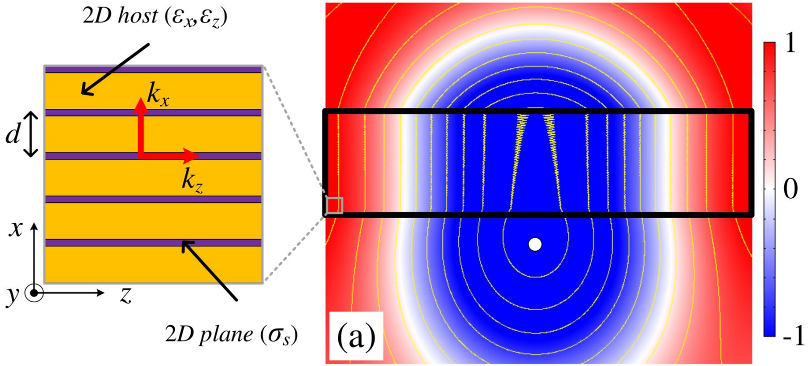

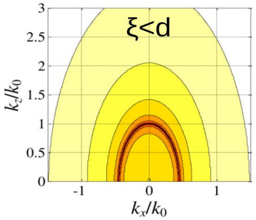

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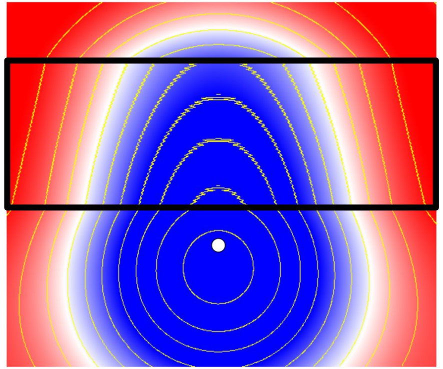

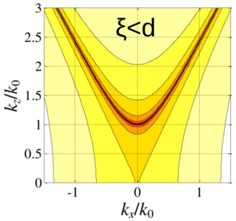

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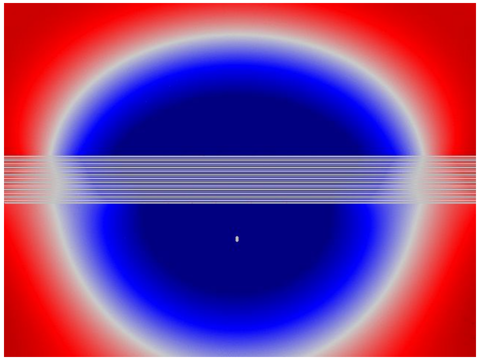

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ENZ properties

Wave propagation with no dispersion and with no phase delay

Propagation through narrow channels

Bending over arbitrary angles

Ultra fast phase transitions

Hiding objects (cloaking)