Electronic Branched Flow in Graphene: Theory and Machine Learning Prediction

Marios Mattheakis

Collaborators:
G. P. Tsironis
E. Kaxiras

School of **E**ngineering &
Applied **S**ciences (SEAS)

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Dirac Solids

Linear Energy Dispersion Relation of Dirac Solids

$$\epsilon_{\mathbf{k}} = \pm v_F \hbar |\mathbf{k}|$$

Quasi-classical dynamics: Ultra-relativistic Hamiltonian

$$\mathcal{H} = \pm v_F \sqrt{p_x^2 + p_y^2} + V(x,y)$$

Classical limit of Dirac equation

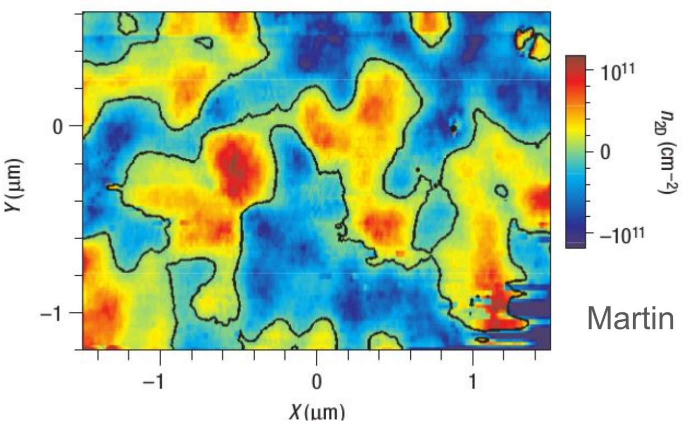
Small effective mass due to doping is neglected

$$m^* = \hbar(\pi n)^{1/2}/v_F$$

Motivation Experiments

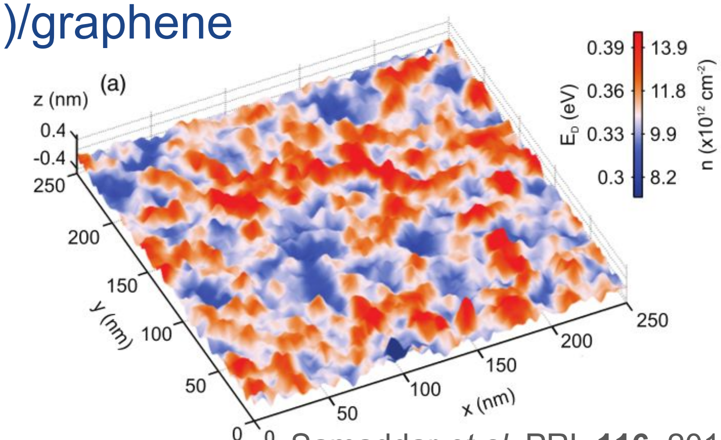
Charged impurities form disordered potential. *Charge Puddles*

hBN/graphene



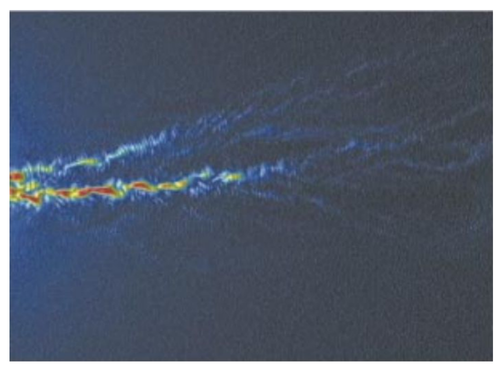
Martin *et al.*,Nature Phys **4,** 2008

Ir(111)/graphene



Samaddar *et al.* PRL **116**, 2016
Martin *et al.* PRB **91**, 2015

Branched Electronic Flow
& Caustic Formation
in parabolic 2DEGs

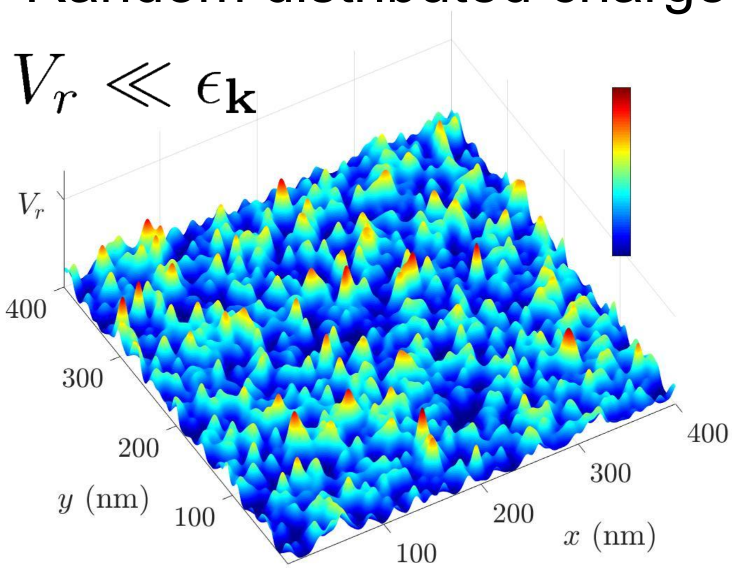


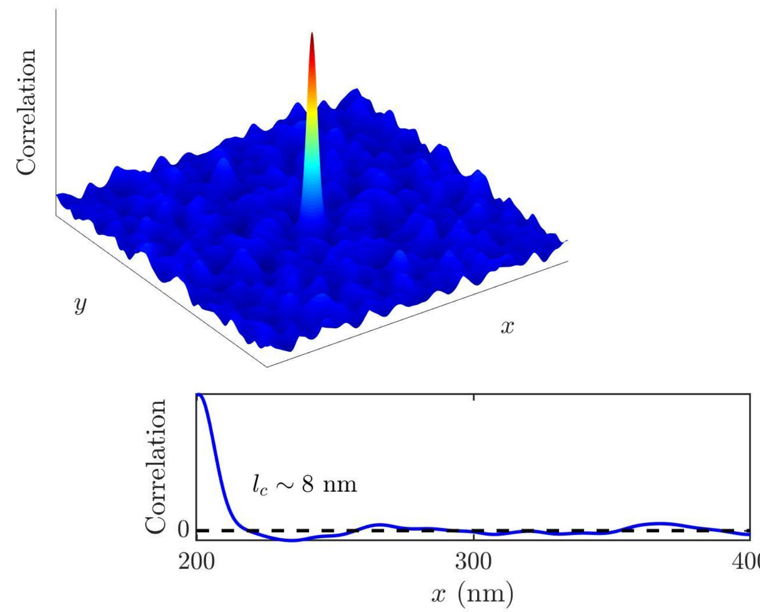
Topinka *et al.*, Nature **410,** 2001
Jura *et al.*, Nature Phys. **3,** 2007

Disordered Potential

Random distributed charge puddles of radius R = 4 nm.

$V_r \ll \epsilon_{\mathbf{k}}$



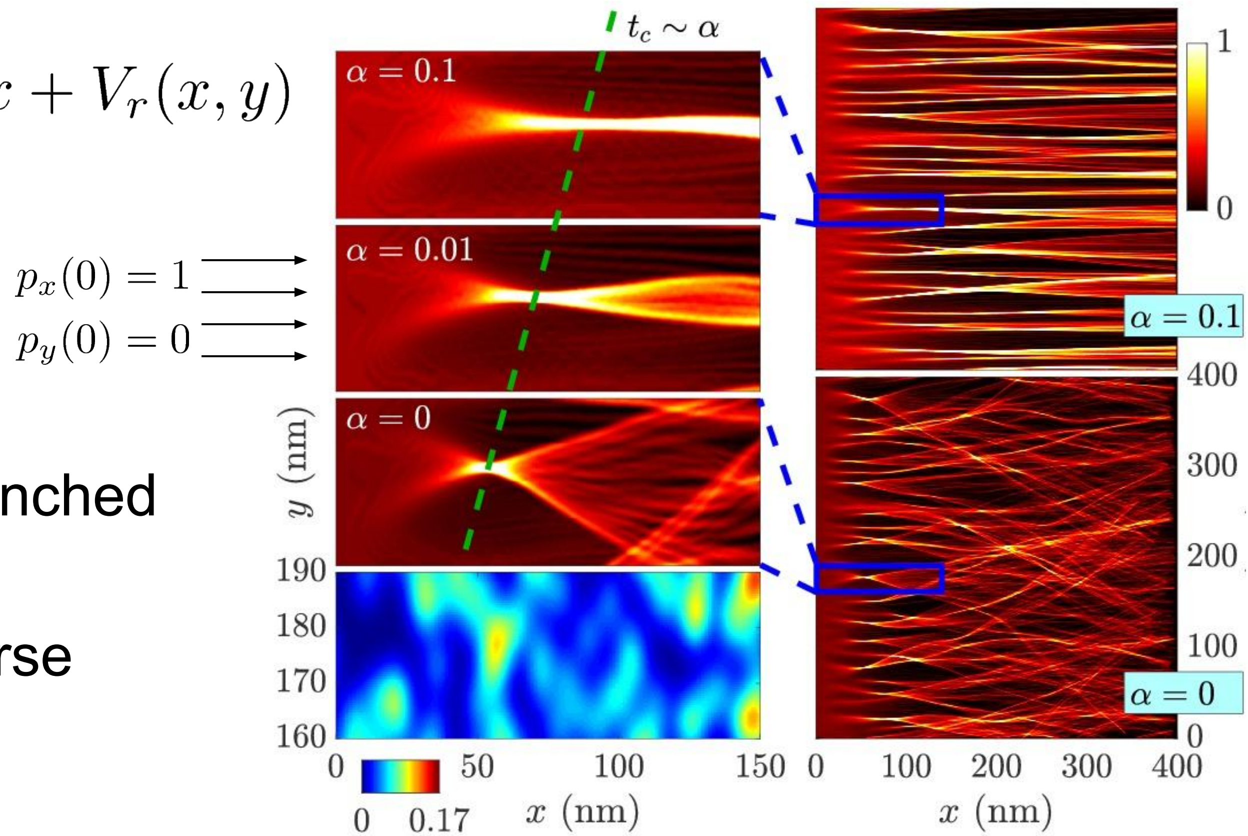


A bias potential along the direction of motion x

$$V_d = -\alpha x$$

Simulations

$$\mathcal{H} = v_F\sqrt{p_x^2 + p_y^2} - \alpha x + V_r(x,y)$$



As bias increases:

(i) the passage to branched flow delays.

(ii) the caustics disperse slower.

M.M., G.P. Tsironis, E. Kaxiras, arXiv: 1801.08217

Theoretical Model

Effective Hamiltonian for $p_x \gg p_y$

$$\mathcal{H} = p_x + \frac{p_y^2}{2p_x} - \alpha x + V_r(x, y)$$

Local curvature u equation in the quasi-2D approach $(x = t)$

$$\frac{du}{dt} + \frac{u^2}{1 + \alpha t} + \frac{\partial^2}{\partial y^2} V_r(t, y) = 0 \qquad \text{where} \qquad u(t, y) = \frac{\partial p_y}{\partial y}$$

Caustic is an area with high intensity occurs when

$$|u(t_c)| \to \infty$$

Scaling of the First Caustic

The random potential acts as white noise with variance σ^2

$$\frac{\partial^2}{\partial y^2} V_r(t,y) = \sigma^2 \xi(t) \quad \text{where} \quad \langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2\delta(t-t')$$

Langevin equation for the local curvature

$$\dot{u} = -\frac{u^2}{1+\alpha t} + \sigma^2 \xi(t)$$

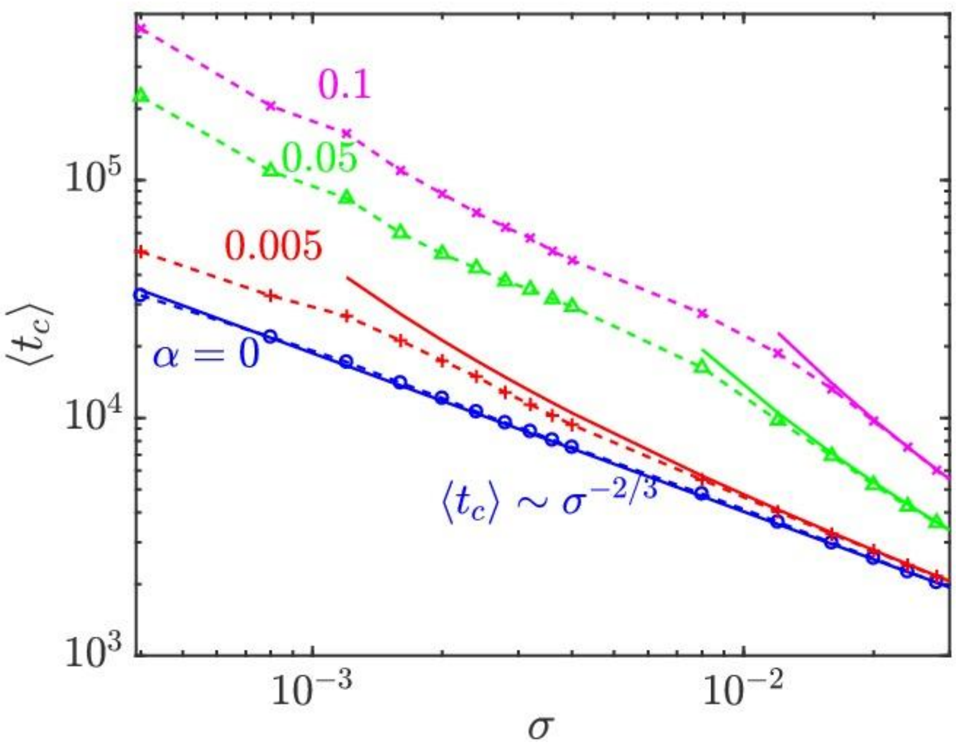
Solve approximately the first passage problem for $|u| \to \infty$

$$\langle t_c \rangle \sim \sigma^{-2/3} \left(1 + 2\tilde{\alpha} + 3\tilde{\alpha}^2 + \frac{10}{3}\tilde{\alpha}^3\right) \quad , \quad \tilde{\alpha} = 1.11\alpha\sigma^{-2/3}$$

Conventional 2D metals:

In the presence of bias the quasi-2D approach fails.

First Caustic Time

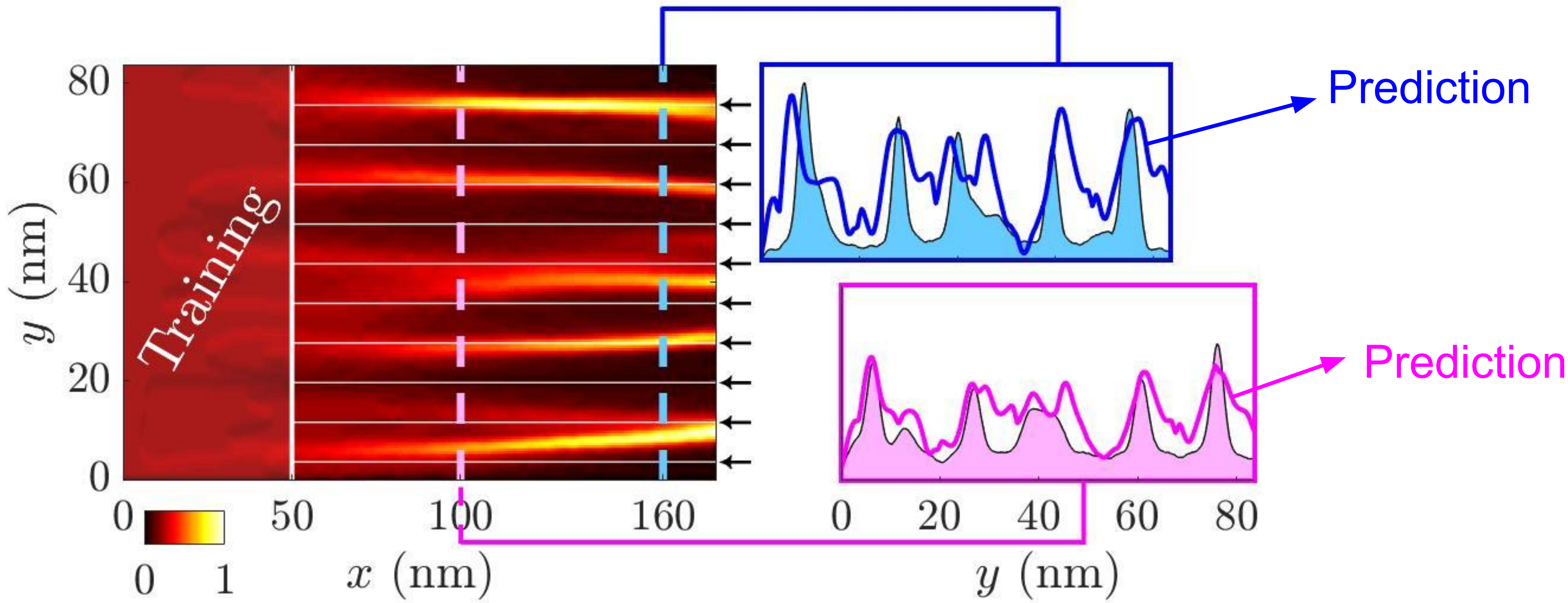


Points & Dashed lines:
Simulations

Solid lines:
Theoretical Prediction
up to $\alpha\sigma^{-2/3} < 1$

Machine Learning Predicts Caustics

The Deep Learning method *Reservoir Computing** is utilized for accurate prediction of *Singular Events* in wave dynamics.



*Lu *et al*., Chaos **27,** 2017

Conclusion

- Branched electronic flow in Dirac Solids focusing on graphene.
- A Langevin Eq. for the local curvature of an ultra-relativistic biased electronic flow is derived.
- Scaling-type relationship between the first caustic location and the statistical properties of disordered potential.
- Machine Learning prediction of singular events in wave dynamics.

arXiv: 1801.08217

mariosmat@g.harvard.edu
https://scholar.harvard.edu/marios_matthaiakis

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HARVARD FAS
RESEARCH COMPUTING

mariosmat@g.harvard.edu
https://scholar.harvard.edu/marios_matthaiakis