Electromagnetic wave propagation in gradient index metamaterials, plasmonic systems and optical fiber networks

Marios Mattheakis

Department of Physics, University of Crete, Heraklion

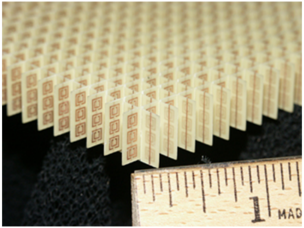
31 October 2014

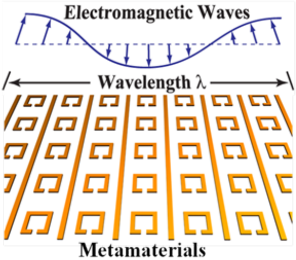
Outline

Outline

Metamaterials (MMs)

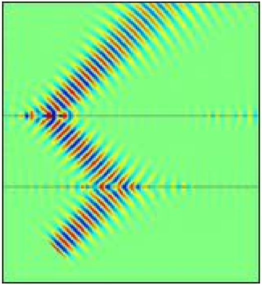
- Artificial subwavelength structures
- Macroscopic properties obtained by the microscopic structure and properties of the compositional materials
- Provide properties that have not been found in nature, such as
  - Negative refractive index
  - Cloaking
  - Flat slab perfect imaging
  - Gradient refractive index (GRIN)



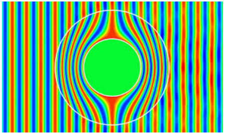


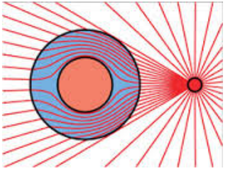
Metamaterials (MMs)

Negative Index

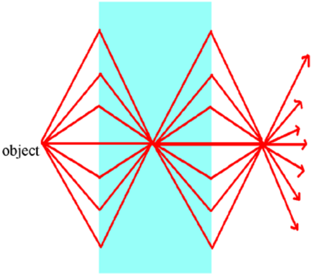


Cloaking





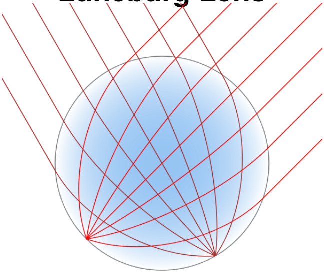
Perfect Imaging



Gradient Refractive Index (GRIN) metamaterials

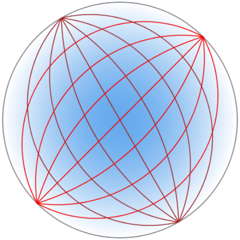
- Formed via spatial variation of the refractive index
- Lead to enhanced light manipulation in a variety of circumstances

Luneburg Lens



$$n(r) = \sqrt{2 - \left(\frac{r}{R}\right)^2}$$

Maxwell fisheye



$$n(r) = \frac{n_0}{1+\left(\frac{r}{R}\right)^2}$$

Outline

Quasi 2D ray tracing

Polar coordinates

- Fermat Principle of least time for a refractive index $n(r)$

$$\mathcal{S} = \int_A^B n(r)ds = \int_A^B n(r)\sqrt{1 + r^2\dot{\phi}^2}dr \quad , \qquad \delta\mathcal{S} = 0$$

- Optical Lagrangian $\mathcal{L}$ and Hamiltonian $\mathcal{H}$

$$\mathcal{L}(\phi, \dot{\phi}, r) = n(r)\sqrt{1 + r^2\dot{\phi}^2} \ , \ \mathcal{H} = -\frac{\sqrt{n^2 r^2 - p_\phi^2}}{r}$$

- First integral of motion

$$\int d\phi = \int \frac{p_\phi}{r\sqrt{n^2 r^2 - p_\phi^2}}dr \quad \text{where} \quad p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \text{constant}$$

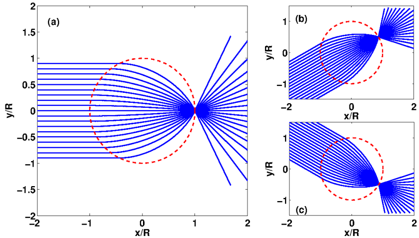
Ray tracing solution for a Luneburg Lens (LL)

- Complete ray solution for an LL for rays with initial angle θ.

$$y(x) = \frac{(2x_0 y_0 + R^2 \sin(2\theta))\, x}{2x_0^2 + (1+\cos(2\theta))\, R^2} + \frac{\sqrt{2} R y_0 \cos(\theta) \sqrt{(1+\cos(2\theta))\, R^2 + 2x_0^2 - 2x^2}}{2x_0^2 + (1+\cos(2\theta))\, R^2} - \frac{x_0 \sin(\theta) \sqrt{(1+\cos(2\theta))\, R^2 + 2x_0^2 - 2x^2}}{2x_0^2 + (1+\cos(2\theta))\, R^2}$$

- For rays parallel to the propagation axis x ($\theta = 0$)

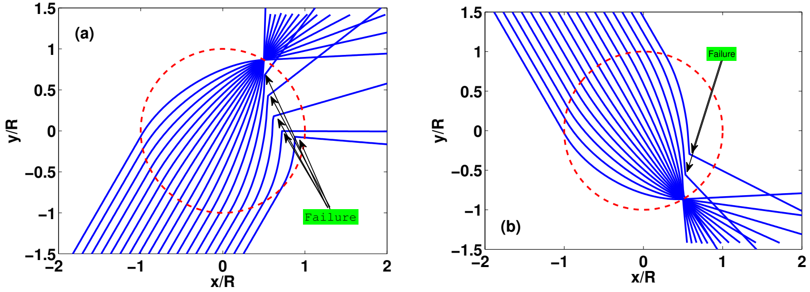
$$y(x) = \frac{y_0}{x_0^2 + R^2} \left(x_0 x + R\sqrt{R^2 + x_0^2 - x^2} \right)$$



M. Mattheakis *et. al.*, “Luneburg lens waveguide networks” *J. Opt.* **14** *(2012) 114005*

Break down of quasi 2D approach

- Quasi 2D method failures to describe backscattered rays
- This failure is due to the assumption that the radial coordinate plays the role of time



Parametric 2D ray tracing

In Cartesian coordinates

- Fermat Principle of least time for a refractive index $n(x, y)$, for $x(\tau)$ and $y(\tau)$ where τ the generalized time parameter

$$\mathcal{S} = \int_A^B n(x, y)\sqrt{\dot{x}^2 + \dot{y}^2} d\tau \ , \qquad \delta\mathcal{S} = 0$$

- Optical Lagrangian $\mathcal{L}$ and Hamiltonian $\mathcal{H}$

$$\mathcal{L} = n\sqrt{\dot{x}^2 + \dot{y}^2} \ , \ \mathcal{H} = \frac{k_x^2 + k_y^2}{2} - \frac{n^2}{2} \ , \ \left(k_q = \frac{\partial\mathcal{L}}{\partial\dot{q}}\right)$$

- Ray tracing equation

$$\boxed{\ddot{\vec{r}} = \frac{1}{2}\nabla n(\vec{r})^2} \quad (\text{where } \ \vec{r} = (x, y))$$

Helmholtz wave equation approach

In Cartesian coordinates

- Helmholtz wave equation and standard assumption

$$\left[\vec{\nabla}^2 + (nk_0)^2\right] u(x,y) = 0, \quad u(x,y) = A(x,y)e^{i\phi(x,y)}$$

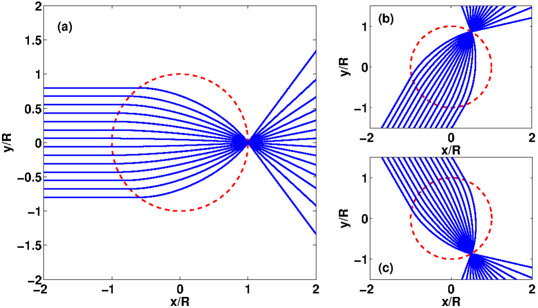
- Resulting equations and derivation of Hamiltonian $\mathcal{H}$

$$\left.\begin{array}{c} (\nabla\phi)^2 - (nk_0)^2 = \frac{\nabla^2 A}{A} \to 0 \\ \vec{k} \equiv (\nabla\phi) \end{array}\right\} \Rightarrow \mathcal{H} = \frac{\vec{k}^2}{2k_0} - \frac{k_0}{2} n(\vec{r})^2$$

The term $\frac{\nabla^2 A}{A}$ is called Helmholtz potential and preserves the wave behavior in the ray tracing equation. In geometric optic approach can be neglected

Application of 2D ray solution to Luneburg index

$$\left.\begin{aligned}\ddot{\vec{r}} &= \frac{1}{2}\nabla n(\vec{r})^2 \\ n(\vec{r}) &= \sqrt{2-\left(\frac{\vec{r}}{R}\right)^2}\end{aligned}\right\}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}\cos\left(\frac{t}{R}\right) + \begin{pmatrix} k_{x0} \\ k_{y0} \end{pmatrix} R \sin\left(\frac{t}{R}\right)$$



M. Mattheakis *et. al.*, "Luneburg lens waveguide networks" *J. Opt.* **14** *(2012) 114005*

Finite Difference in Time Domain (FDTD)

- FDTD solves numerically the time dependent Maxwell's equations
- Discretization both in space and time with grid unit cells $(\Delta x, \Delta y)$ and Δt respectively
- Stability criterion

$$\Delta x << \lambda_{min} \text{ and } \Delta y << \lambda_{min}$$

- Courant limit

$$\Delta t < \frac{\sqrt{(\Delta x)^2 + (\Delta y)^2}}{c_{max}}$$

Application of FDTD to a Luneburg lens

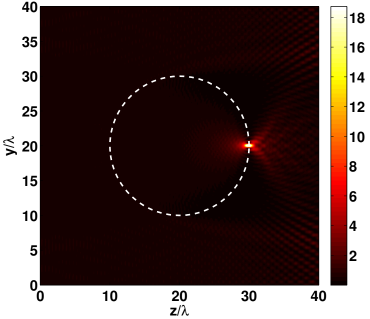
- FDTD is applied for a monochromatic EM plane wave source with wavelength λ
- Assumed the vacuum as bulk material ($\varepsilon = 1$)
- An LL is used with $\varepsilon = 2 - (r/R)^2$ with $R = 10\lambda$
- The steady state of the electric intensity is plotted

Transverse Magnetic waves (TM polarization)

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\frac{\partial E_z}{\partial y}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

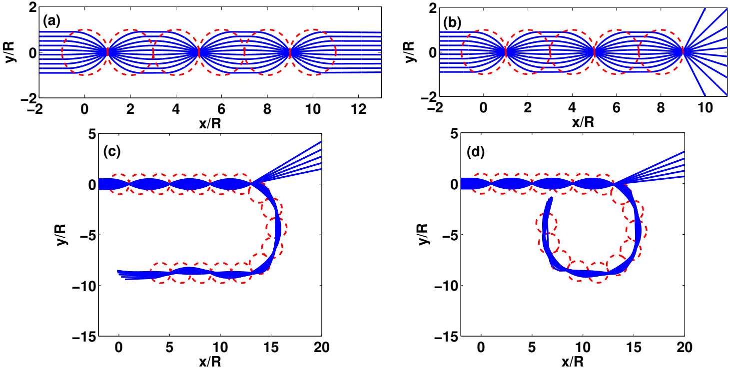


Outline

Luneburg lens waveguides

Ray tracing solutions

Electromagnetic waveguides can be formed by LLs.

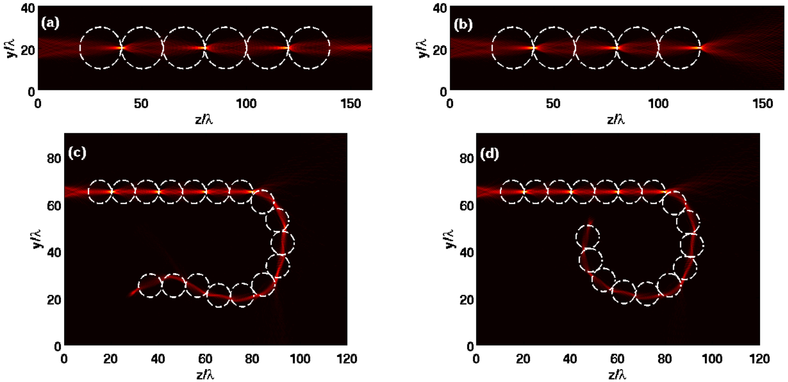


M. Mattheakis *et. al.*, “Luneburg lens waveguide networks” *J. Opt.* **14** *(2012) 114005*

Luneburg lens waveguides

FDTD wave simulations

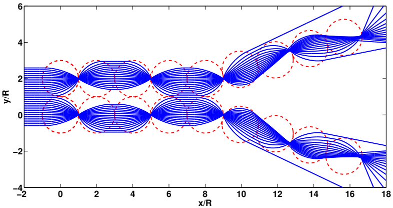
Electromagnetic waveguides can be formed by LLs

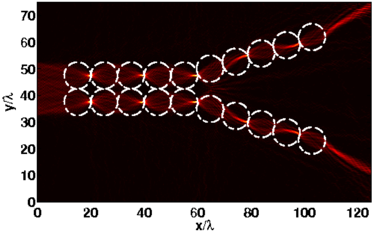


M. Mattheakis *et. al.*, “Luneburg lens waveguide networks” *J. Opt.* **14** *(2012) 114005*

Beam splitter formed by Luneburg lens

A beam splitter can be formed by LLs





The losses are $\sim$ 10%. 90% of the incoming rays are split and guided through LLs configuration.

Outline

Caustics

- Caustics are areas with high intensity
- Caustics and branched flow can arise in light propagation through random fluctuated refractive index

Statistics of caustics

- Derivation of a Hamilton-Jakobi equation by an ordinary Hamiltonian with unit mass ($m = 1$)

$$\left.\begin{aligned} \mathcal{H} = \frac{p^2}{2} + V(t,y) \\ \vec{p} = \frac{\partial S}{\partial y} \end{aligned}\right\} \Rightarrow \frac{\partial}{\partial t} S(t,y) + \frac{1}{2}\left(\frac{\partial S}{\partial y}\right)^2 + V(t,y) = 0$$

- Definition of curvature of classical action S

$$u \equiv \frac{\partial p}{\partial y} = \frac{\partial^2 S}{\partial y^2}$$

- The singularities of curvature ($u \to \infty$) denote caustics

Statistics of caustics

- Orinary Differential Equations for curvature u derived by HJE

$$\frac{d}{dt}u + u^2 + \frac{\partial^2}{\partial y^2}V(t,y) = 0$$

- Potential acts as delta correlated noise $\Gamma(t)$

$$\frac{\partial^2}{\partial y^2}V(t,y) = \Gamma(t) \quad , \quad \langle \Gamma(t)\Gamma(t') \rangle = 2\sigma\delta(t-t')$$

- Relation between standard deviation σ and Diffusion coefficient D

$$D = 2\sigma^2$$

- Ordinary Stochastic Differential Equation for curvature u

$$\frac{du}{dt} = -u^2 - \sigma\Gamma(t)$$

Statistics of caustics

Scaling law

- Fokker-Plank equation (FPE) for probability density P

$$\frac{\partial}{\partial t}P(u,t) = \left[\frac{\partial}{\partial u}u^2 + \frac{\partial^2}{\partial u^2}\frac{D}{2}\right]P(u,t)$$

- Backward Fokker-Plank equation (BFPE) for probability density p_f

$$\frac{\partial}{\partial t}p_f(u,t) = \left[-u_0^2\frac{\partial}{\partial u_0} + \frac{D}{2}\frac{\partial^2}{\partial u_0^2}\right]p_f(u,t)$$

- Looking for the average first time that $u \to \infty$ for arbitrary initial curvature u_0, obtaining a scaling law for the mean time of the first caustic onset

$$\langle t_c \rangle \sim \sigma^{-2/3}$$

Simulations setup

- Monochromatic electromangetic waves propagate through disordered Luneburg networks
- 150 randomly located generalized LLs each radius with $R = 10\lambda$ in lattice with dimensions $460\lambda \times 360\lambda$
- Generalized LL refraction index

$$n(r) = \sqrt{\alpha \left(n_L^2 - 1\right) + 1}$$

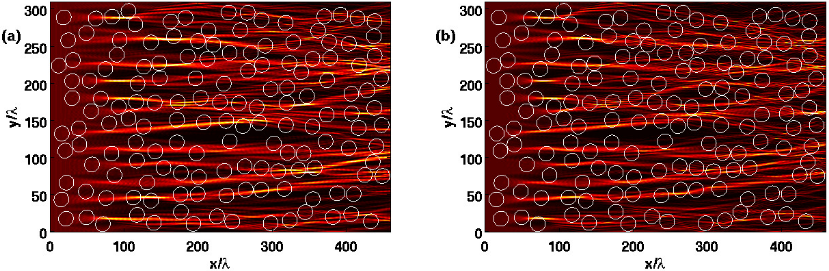
- α is called "strength" parameter and it is proportional to random potential standard deviation σ

$$\sigma \simeq 0.1\alpha$$

FDTD simulations

Simulation for different values of strength parameter α

- Left figure: $\alpha = 0.07$
- Right figure: $\alpha = 0.1$



Mattheakis *et. al.*, “Branched flow through optical complex systems” *(working paper)*

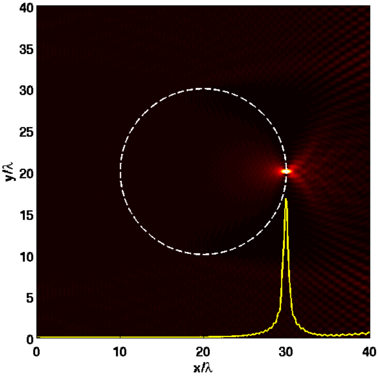
Scintillation index

- The scintillation index σ_I shows the deviation of the Intensity I of the mean value of intensity $\langle I \rangle$

$$\sigma_I^2 = \frac{\langle I(x)^2 \rangle}{\langle I(x) \rangle^2} - 1$$

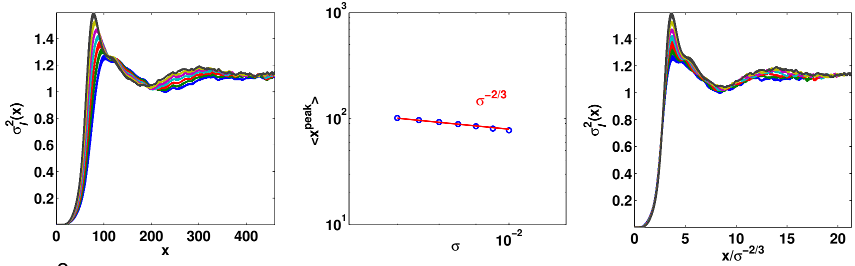
- Since caustics are high intensity areas, a maximum of σ_I shows caustic formation

Yellow curve denotes the σ_I



Numerical results

- Simulation for several values of strength parameter α are taken place, resulting to the correspond scintillation indexes plots.



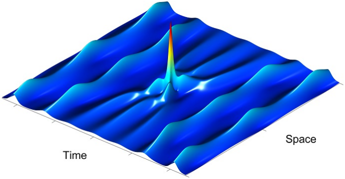
(a) $\sigma_I^2(x)$ for different values of σ
(b) maximum position of the $\sigma_I^2(x)$
(c) $\sigma_I^2(x)$ are plotted in rescaled x axis

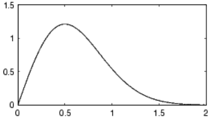
M. Mattheakis *et. al.*, "Branched flow through optical complex systems" *(working paper)*

Outline

Definition of Rogue Waves (RWs)

- RWs are relatively large and spontaneous surface waves
- Waves with height at least two times greater than Significant Wave Height (SWH)
- SWH is the mean wave height of the highest (statistical) third of the waves
- Long tailed height distribution instead of Rayleigh distribution
- RWs have been found in
  - Ocean water surface waves
  - Microwaves propagation
  - Financial systems



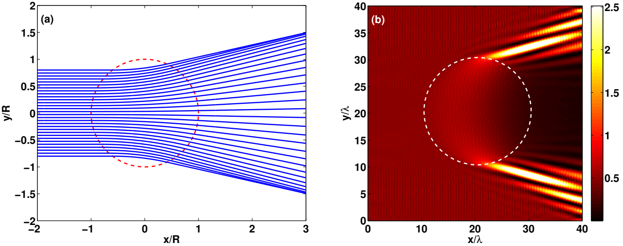


Luneburg Hole (LH) lens consists random photonic networks

- Luneburg Hole (LH) is a new GRIN lens with index

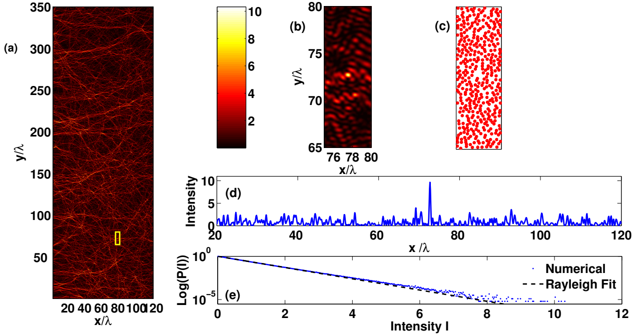
$$n(r) = \sqrt{1 + \left(\frac{r}{R}\right)^2}$$

- LH has purely defocussing properties



FDTD Results

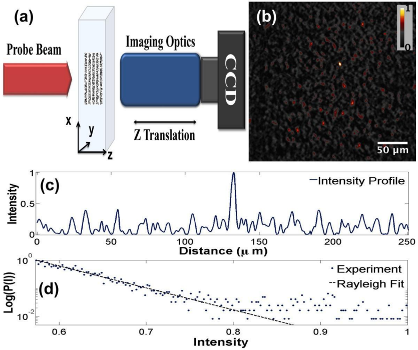
- 400 randomly located LH lenses, each radius with $R = 3.5\lambda$, consist a photonic disordered lattice
- Filling factor of the arrangement $f = 0.17$



M. Mattheakis *et. al.*, “Linear and nonlinear photonic rogue waves in complex transparent media” (*working paper)*

Experimental Results

- Photonic disordered lattices ($(250 \times 250)\mu m^2$) consist of five superposed layers of 400 LHs for each layer



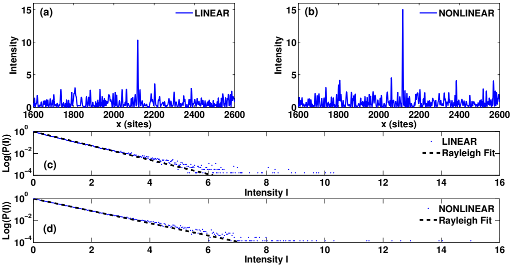
M. Mattheakis *et. al.*, “Linear and nonlinear photonic rogue waves in complex transparent media” (*working paper)*

Nonlinear permittivity function

- Introduce focusing nonlinearity (Kerr effect) in the permittivity function

$$\varepsilon = n^2 = \varepsilon_L + \chi|E|^2 \quad (\chi \text{ varying from } 10^{-7} \text{ to } 10^{-6})$$

- Linear RW position and statistics are not affected by the presence of relatively small nonlinearity

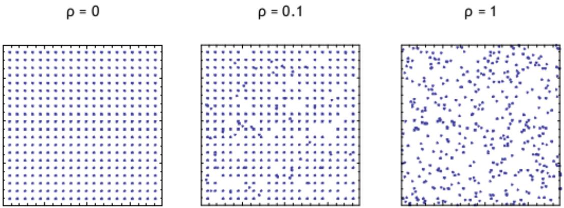


M. Mattheakis *et. al.*, "Linear and nonlinear photonic rogue waves in complex transparent media" (*working paper)*

Outline

Optical fiber lattices

- Fiber networks consist of 400 fibers in a square lattice
- Fibers extend in z axis, which in paraxial approximation plays the role of time
- Fibers are coupled through interfiber interaction (evanescent coupling)
- A disorder parameter ρ is introduced to control the randomness level



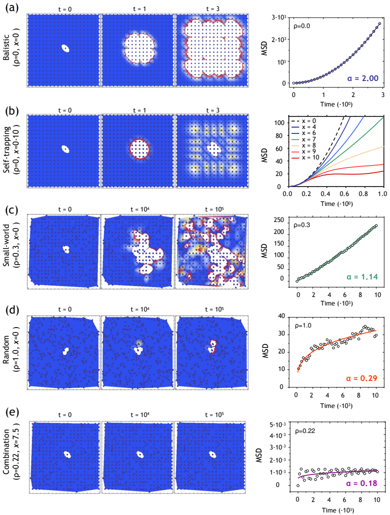
F. Perakis, M. Mattheakis, G.P. Tsironis, "Small-world networks of optical fiber lattices", *J. Opt.* **16** *(2014) 102003*

Wave-packet propagation

- Discrete Nonlinear Schrödinger Equation (DNLS) is chosen for investigating transport properties

$$i\frac{d\psi_n}{dt} = \sum_m V_{n,m}\psi_m - \chi|\psi_n|^2\psi_n$$

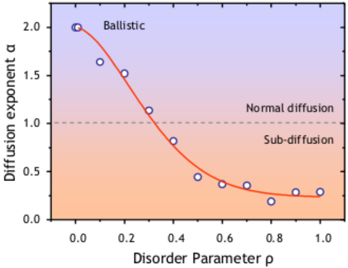
- DNLS allows to explore both the tunneling effect and the influence of nonlinearity
- A wave-packet is placed in the central fiber (excitation of a single fiber)
- Investigate the diffusion exponent by plotting the Mean Square Displacement (MSD) as function of time t
- No Anderson localization takes place since the excitation is initially localized

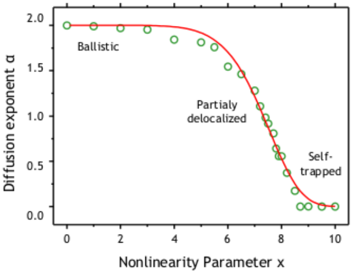


F. Perakis, M. Mattheakis, G.P. Tsironis, “Small-world networks of optical fiber lattices”, *J. Opt.* **16** *(2014) 102003*

Diffusion exponent

- Passage from ballistic to sub-diffusion due to structural disordered
- Transition from ballistic to sub-diffusion due to nonlinearity
- The combination of structural disordered with nonlinearity leads to almost complete localization



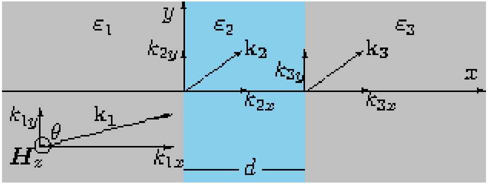


F. Perakis, M. Mattheakis, G.P. Tsironis, “Small-world networks of optical fiber lattices”, *J. Opt.* **16** *(2014) 102003*

Outline

Surface Plasmon Polaritons SPPs

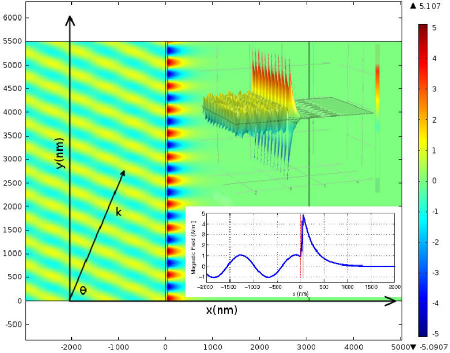
- Quasi particles formed by coupling of an EM wave with metal's free electron-oscillation field.
- Surface waves with evanescent decaying EM field in transverse axis
- Due to ohmic loss the SPPs decays exponentially also in propagation axis
- Maxwell equations support SPPs solutions for
  - Transverse Magnetic Polarization (TM)
  - Interface between metal ($Re[\varepsilon_m] < 0$) and dielectric ($Re[\varepsilon_d] > 0$)



Parameter	Value
ε_1	2.25
ε_2	$-15.13 - 0.93i$
ε_3	1.69
d	50 nm
θ	66.74^o

SPPs excitation (COMSOL simulation)

- A monochromatic EM wave is used for SPPs excitation in a dielectric-metal-dielectric configuration



C. Athanasopoulos, M. Mattheakis, G.P. Tsironis, "Enhanced surface plasmon polariton propagation induced by active dielectrics", *Excerpt from the Proceedings of the 2014 COMSOL Conference.*

SPP characteristics

- Dispersion relation β $(k_p = \omega_p / c)$

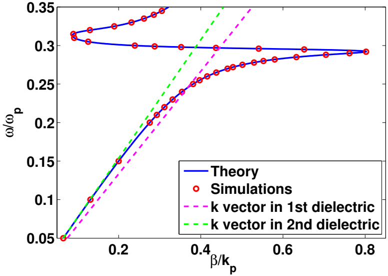
$$\beta = k_0 \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}$$

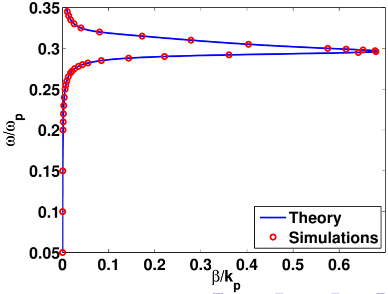
- Propagation length L

$$L = \frac{1}{2 Im[\beta]}$$

- Penetration length $t_{d(m)}$

$$t_{d(m)} = \frac{1}{k_0} Re \left[\sqrt{\frac{\varepsilon_d + \varepsilon_m}{-\varepsilon_{d(m)}^2}} \right]$$



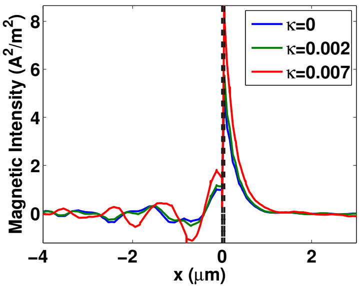


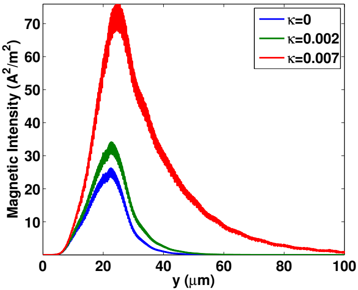
Active dielectrics

- Dielectrics with complex permittivity and refractive index

$$\varepsilon = \varepsilon' + i\varepsilon'' = (n_R + i\kappa)^2 \quad (\varepsilon'' \text{ and } \kappa \text{ account for gain})$$

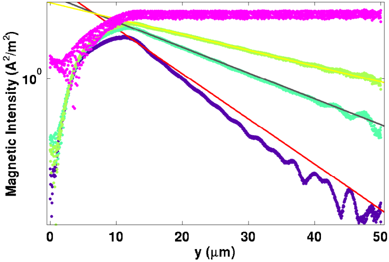
- Gain counterbalance the ohmic loss of metal
- Enhanced propagation and penetration length
- Enhanced the SPP intensity

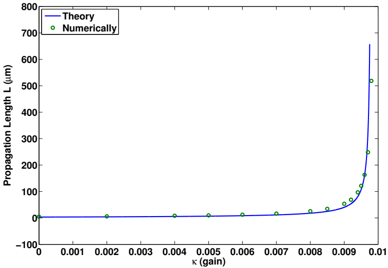




Infinite propagation Length

- There is a critical value of gain for which $Im[\beta] = 0$ resulting to lossless SSPs propagation ($L \rightarrow \infty$)





C. Athanasopoulos, M. Mattheakis, G.P. Tsironis, "Enhanced surface plasmon polariton propagation induced by active dielectrics", *Excerpt from the Proceedings of the 2014 COMSOL Conference.*

Outline

Conclusion

- Four methods for studying light propagation are developed
- GRIN lenses can form waveguides and beamsplitters
- Caustic and rogue waves formation arise in EM propagation through random GRIN networks
- Wavepacket sub-diffuses in fiber lattices due to randomness and nonlinearity
- Active (gain) dielectrics enhance SPP propagation and penetration length

Acknowledgements

Thanks for your attention

I would like to deeply thank ...

- Prof. Giorgos Tsironis
- Prof. Stelios Tzortzakis
- Dr. Giorgos Neofotistos
- Dr. Nikos Lazaridis
- Dr. Thomas Oikonomou
- Dr. Patrick Navez

CAPACITIES

European Union
European Social Fund
OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION, LIFELONG LEARNING AND RELIGIOUS AFFAIRS
MANAGING AUTHORITY
Co- financed by Greece and the European Union
NSRF 2007-2013
programme for development
EUROPEAN SOCIAL FUND

Publications I

- M. Mattheakis, G. P. Tsironis
  Extreme waves and branching flow in optical media.
  Chapter in a volume of Springer Series on Material Science (2014)
- M. Mattheakis, G. P. Tsironis, V. I. Kovanis
  Luneburg lens waveguide networks
  J. Opt. **14** *(2012) 114005 (8pp)* (invited issue)
- F. Perakis, M. Mattheakis, G. P. Tsironis
  Small-world networks of optical fiber lattices
  J. Opt. **16** *(2014) 102003 (7pp)* (invited issue)

Publications II

- C. Athanasopoulos, M. Mattheakis, G.P.Tsironis
  Enhanced surface plasmon polariton propagation induced by active dielectrics
  (Excerpt from the Proceedings of the 2014 COMSOL Conference in Cambridge (2014))
- M. Mattheakis, J.J. Metzger, G.P. Tsironis, R. Fleischmann
  Branched flow through optical complex systems
  (working paper)
- M. Mattheakis, I. J. Pitsios, G. P. Tsironis, S. Tzortzakis
  Linear and nonlinear photonic rogue waves in complex transparent media
  (working paper)