Econometrica, Vol. 56, No. 3 (May, 1988), 549–569

A THEORY OF DYNAMIC OLIGOPOLY, I: OVERVIEW AND QUANTITY COMPETITION WITH LARGE FIXED COSTS

BY ERIC MASKIN AND JEAN TIROLE[1]

The paper introduces a class of alternating-move infinite-horizon models of duopoly. The timing is meant to capture the presence of short-run commitments. Markov perfect equilibrium (MPE) in this context requires strategies to depend only on the action to which one's opponent is currently committed. The dynamic programming equations for an MPE are derived.

The first application of the model is to a natural monopoly, in which fixed costs are so large that at most one firm can make a profit. The firms install short-run capacity. In the unique symmetric MPE, only one firm is active and practices the quantity analogue of limit pricing. For commitments of brief duration, the market is almost contestable. We conclude with a discussion of more general models in which the alternating timing is derived rather than imposed.

Our companion paper applies the model to price competition and provides equilibrium foundations for kinked demand curves and Edgeworth cycles.

KEYWORDS: Markov perfect equilibrium, short-run commitment, reaction, natural monopoly, contestability, endogenous timing.

INTRODUCTION

IN THIS PAIR of papers, we present a theory of how oligopolistic firms behave over time. One of our goals is to study certain well-known concepts, such as contestability and the kinked demand curve, that are implicitly dynamic but have usually been discussed in static models. The main ingredient of our study is the idea of reactions based on short-run commitments.

When we say that firm 1 is *committed* to a particular action in the short-run—whether a quantity or a price—we mean that it cannot change that action for a finite (although possibly brief) period, during which time other firms might act. By firm 2's *reaction* to 1 we mean the response it makes, possibly after some lag, to 1's chosen action. Short-run commitment ensures that, by the time firm 2 reacts, firm 1 will not already have changed its action.[2]

To formalize the idea of reaction based on commitment, we introduce a class of infinite-horizon sequential duopoly games. In the simplest version of these games (the exogenous timing framework), the two firms move alternatingly. Firms maximize their discounted sum of single-period profits, and our goal is to characterize the perfect equilibria. The fact that, once it has moved, a firm cannot move again for two periods implies a degree of commitment.

[1] This work was supported by grants from the NSF and the Sloan Foundation. We thank Peter Diamond, David Kreps, Reinhard Selten, Robert Wilson, two referees, and especially John Moore for helpful discussions and comments.

[2] It is useless to respond (in our sense of the term) to another's action if, by the time one has done so, the other firm has already moved again. That is why our conception of reaction is intimately tied to commitment. It may be worthwhile reacting to a firm's move because the firm is committed to that move, at least for a time.

We have in mind primarily exogenous or technological reasons for commitment, e.g., installed capital that has little scrap value, or lags in producing and disseminating price lists. Alternatively, short-term contracts might serve to bind the firm temporarily.[3]

We suppose that each firm uses a strategy that makes its move in a given period a function only of the other firm's most recent move. Our primary justification for this simplifying assumption—we provide a lengthier discussion below—is that it makes strategies dependent only on the *physical* state of the system, those variables that are directly payoff-relevant. Consequently, we can speak legitimately of a firm's reaction to another's action, rather than to an entire history of actions by both firms.

Section 2 derives the dynamic programming equations associated with an equilibrium in strategies of this sort, i.e., with a "Markov perfect equilibrium" (MPE). Then, in Section 3, we begin the formal analysis of our project with a study of models where firms compete in capacities (quantities) and in which fixed costs are so large that only one firm can make a profit. For the exogenous timing version of the model, we show that there exists a unique symmetric MPE. In this equilibrium, only one firm produces (thus the model may be considered more an example of monopoly than duopoly) and, furthermore, for discount factors that are not too low, operates above the pure monopoly level in order to deter entry. Such behavior can be thought of as the quantity-analogue of limit pricing behavior (see Gaskins (1971), Kamien and Schwartz (1971), and Pyatt (1971)). Moreover, as the discount factor tends to one, so that future profits become increasingly important, the entry-deterring quantity approaches the competitive (i.e., zero-profit level) quantity, a result much in the spirit of the recent contestability literature (see, for example, Baumol, Panzar, and Willig (1982)).

Of course, the assumption that firms' moves necessarily alternate is artificial; one might wonder why we did not suppose that moves are simultaneous. To provide proper foundations for our alternating move hypothesis, therefore, we consider a more elaborate class of models where firms can, in principle, move at any time they choose (Section 4). Yet, as before, once a firm selects a move, it remains committed to that action for a finite length of time. When we restrict attention to strategies that are functions only of the physical state, we find that, in a number of cases of interest (in particular, the models studied in these two papers) the equilibrium behavior in endogenous timing models closely parallels that in the games where alternation is imposed.

[3] Here, however, we are on weaker ground theoretically because, once we admit the possibility of contractual commitment, we have to explain why commitments of indefinite duration are impossible or too costly, a knotty question. One explanation may be transaction costs—the expense of drawing up a complex contract of infinite length. A second (related) reason is the difficulty of foreseeing all possible later contingencies that might arise. A third is the possibility of renegotiation; the contracting parties' ability to replace their original contract with a new one at a later date limits the commitment value of the former. See also the discussion in Section 9 of our companion paper, which suggests that in some settings, oligopolists would not opt for lengthy contracts even if these were not subject to the difficulties we have mentioned.

Our companion paper (Maskin–Tirole (1988)) studies models of price competition in markets with undifferentiated commodities. We show that two classical phenomena, the kinked demand curve equilibrium and the Edgeworth cycle, arise naturally as equilibria of our models.

The third paper in this series (Maskin–Tirole (1987)) considers competition in the absence of fixed costs (Cournot competition), and develops differential methods for studying equilibrium (such techniques do not apply in the first two papers, where the models are highly discontinuous).

2. THE GENERAL MODEL WITH FIXED TIMING

We next present the basic features of the simpler (exogenous timing) class of models that we analyze below.

A. The Model

We consider a duopoly; the model can be generalized to more than two firms but at the expense of simplicity. Each firm i ($i=1,2$) chooses actions a^i from a bounded action space A (we assume that the action space is bounded so that dynamic programming is applicable). Depending on the interpretation of the model the variable a^i could represent the choice of a price, quantity, location, etc. It could even represent a vector of choices. Firms act in discrete time, and the horizon is infinite. Periods are indexed by t ($t=0,1,\dots$) and T is the time between two consecutive periods. At time t firm i's instantaneous profit π^i is a function of the current actions of the two firms but not of time:

$$\pi^i = \pi^i\left(a_t^1, a_t^2\right).$$

Firms discount future profits with the same interest rate r. Thus their discount factor is

$$\delta = \exp(-rT).$$

Firm i's intertemporal profit can then be written

$$\Pi^i = \sum_{t=0}^{\infty} \delta^t \pi^i\left(a_t^1, a_t^2\right).$$

As mentioned in the introduction, we wish to model the ideas that (a) firms are committed to their actions for a finite length of time, during which time other firms might move, and that (b) they react to the current actions of other firms. The simplest way of accomplishing both objectives is to assume, following Cyert and DeGroot (1970), that firms move sequentially. In odd-numbered periods ($t=1,3,5,\dots$) firm 1 chooses an action to which it is committed for two periods. That is, $a_{2k+2}^1 = a_{2k+1}^1$ for all k. Similarly, firm 2 moves in even-numbered periods ($t=0,2,4,\ldots$) and $a^2_{2k+1}=a^2_{2k}$. Thus there is a lag T between a firm's actions and its rival's reaction.[4]

The model ignores the issue of who moves first (we might suppose for completeness that the first mover is determined by historical accident). Instead, we are interested in the *long-run* properties of the model, those that are independent of what happens at the beginning of the game. Indeed, in the applications of our model provided in these two papers, steady-state is always reached regardless of initial conditions. Moreover, the nature of steady-state is independent of the initial conditions (modulo possibly relabeling the firms).

We require equilibrium of this model to be perfect. That is, starting from any point in the game tree, the firm to move selects the action that maximizes its intertemporal profit given the subsequent strategies of its rival and itself. We do not accept any perfect equilibrium, however, but just those whose strategies depend only on the "payoff-relevant" history. Specifically, at time $t=2k$, the only aspect of history that has any "direct" bearing on current or future payoffs is the value of a^1_{2k-1}, for only this variable, among all those before time $2k$, enters any instantaneous profit function from time $2k$ on. Thus, if the equilibrium is to depend only on payoff-relevant history, firm 2's strategy at time $2k$ must depend only on a^1_{2k-1}. That is,

$$a^2_{2k}=R^2_{2k}\left(a^1_{2k-1}\right).$$

Moreover, because the future appears the same starting from any time period, time *itself* is not a payoff-relevant variable, and so above we can drop the subscript "$2k$" from R. Thus, we can represent the firms' behavior—their strategies—by a pair of *dynamic reaction functions*:[5]

$$R^1\colon A\to A$$

and

$$R^2\colon A\to A.$$

Actually, although it will not play a major role in this paper, we must allow for the possibility that R^1 and R^2 are *random* functions, so that $R^1(a^2)$ and $R^2(a^1)$ are, in general, random variables.

Because dynamic reaction functions depend only on the payoff-relevant state of the system, they might alternatively be called "Markov strategies." A pair of reaction functions (R^1, R^2) forms a *Markov perfect equilibrium* (MPE) if and only if (i) $a^2_{2k}=R^2(a^1_{2k-1})$ maximizes firm 2's intertemporal profit at any time $2k$, given a^1_{2k-1} and assuming that henceforth each firm i will move according to R^i; and (ii) the analogous condition holds for firm 1. Of course, if R^1 and R^2 are

[4] Notice that we are supposing that firms' actions are equally spaced. Although this assumption does not affect the most salient qualitative features of equilibrium in Section 3 and the models of our companion piece, it does considerably simplify the analysis of equilibrium.

[5] We use the modifier "dynamic" to distinguish this concept from the "reaction functions" of static Cournot analysis.

random functions we must replace (i) by the statement that each possible realization of $R^2(a^1_{2k-1})$ maximizes firm 2's *expected* intertemporal profit (we assume risk neutrality). The following proposition is a simple consequence of the theory of dynamic programming.

PROPOSITION 1: *A Markov perfect equilibrium is a perfect equilibrium. That is, given that its rival ignores all but the payoff-relevant history, a firm can just as well do the same.*

We have several reasons for restricting our attention to Markov strategies. Their most obvious appeal is their simplicity. Firms' strategies depend on as little as possible while still being consistent with rationality.

More relevant from our perspective is that Markov strategies seem at times to accord better with the customary conception of a reaction in the informal industrial organization literature than do, say, the reactions emphasized in the repeated game (or "supergame") tradition, the best-established formal treatment of dynamic oligopoly to date. In supergames, reactions are, typically, threats made to dissuade the rival firm from selecting certain actions. The idea that reacting is following through on a threat is very different from the reasoning behind, say, the kinked-demand curve story. In the kinked-demand curve world, cutting one's own price in response to another firm's price cut is not carrying out a threat at all. It is merely an act of self-defense, an attempt to regain lost customers. Put another way, the reaction is a response only to the other firm's price cut and not to earlier history or to one's own past prices.

In our companion piece we discuss some well-known methodological difficulties with the supergame approach (e.g., the large number of equilibria, and the nonrobustness of equilibrium to the horizon). Because we do not know how successfully our alternative framework of short-run commitments and Markov strategies overcomes these problems in general, we limit our comparison of the two approaches to the simple price and quantity settings that have been the source of most applications of supergames to industrial organization.

The reader may wonder whether focussing attention on the payoff-relevant states buys us anything in more general models. After all, most past actions are likely to have at least some influence on current and future payoffs. Yet, if we make strategies contingent on all past actions, the Markov restriction has no bite. This is certainly an apt criticism of the formal Markov assumption. But it neglects our preoccupation with *short-run* commitment. Such commitment implies that recent actions have a stronger bearing on current and future payoffs than those of the more distant past. A natural hypothesis posits that past actions having only a small influence on *payoffs* have a correspondingly circumscribed effect on current *behavior*. The Markov assumption captures this hypothesis, albeit through the crude device of supposing literally *no* impact on payoffs by actions before the recent past.

B. *Markov Perfection and Dynamic Programming*

We can solve for a Markov perfect equilibrium by invoking the game theoretic analogue of dynamic programming. To this end, we define four value functions. Given an equilibrium pair of Markov strategies (R^1, R^2) let $V^1(a^2)$ be the present discounted value of firm 1's profits given that last period firm 2 played a^2 and that henceforth both firms play optimally, i.e., according to their Markov strategies, and let $W^1(a^1)$ be the present discounted value of firm 1's profits given that last period firm 1 played a^1 and that henceforth both firms play optimally. $V^2(a^1)$ and $W^2(a^2)$ are defined symmetrically.

These value functions must be consistent with the reaction functions. Specifically, given that firm i's choice of a^i is restricted to a bounded set, the following are necessary and sufficient conditions for the reaction and value functions to correspond to an equilibrium:

$$V^1(a^2) = \max_{a^1} \{\pi^1(a^1, a^2) + \delta W^1(a^1)\}$$
$$= \pi^1(R^1(a^2), a^2) + \delta W^1(R^1(a^2))$$

and

$$W^1(a^1) = \pi^1(a^1, R^2(a^1)) + \delta V^1(R^2(a^1)),$$

(with analogous equations for V^2 and W^2), where expectation operators should appear before the expressions on the right-hand side if R^1 and R^2 are random functions.

3. *Quantity Competition with Large Fixed Costs: Fixed Timing*

We turn next to a specific application of our general model, the analysis of markets with large fixed costs. For this purpose, we shall take quantities to be firms' strategy variables. One should interpret a choice of quantity as that of a scale of operation or capacity. We shall express profit as a direct function of quantities/capacities. Our profit function is, therefore, a reduced form, which subsumes instantaneous price competition.

The industrial organization literature has traditionally distinguished among three types of costs of production. Variable costs are incurred only during the period of production and are directly related to the level of operation. Fixed costs (measured as a flow) persist only as long as production continues, but are, strictly speaking, independent of scale. Pure sunk costs (again, considered as a flow) continue as a liability forever. That is, they are incurred with or without production.

Both fixed and sunk costs have been regarded as barriers to entry. The entry-deterring role of sunk costs is not controversial. When sunk costs take the form of an irreversible investment in nondepreciable capital, a firm's variable cost curves may be forever changed, giving it a permanent advantage over potential entrants or later rivals. This effect has been studied by Spence (1977, 1979), Dixit (1979), and Fudenberg and Tirole (1983). Even when capital is not infinitely durable it may still deter entry as argued by Eaton and Lipsey (1980).[6]

The deterrent that fixed costs create is one of the subjects of the "natural barriers to entry" literature (see Scherer (1980) for a survey). A firm in an oligopolistic industry (one with large fixed costs) can, by virtue of its incumbency, deter entry since the revenue available to a potential entrant does not outweigh the high fixed costs it has to bear. This view has recently been challenged by Grossman (1981) and Baumol, Panzar, and Willig (1982), who maintain that incumbency gives a firm no privileged position *per se* if its costs are merely fixed rather than sunk. Such a firm ought not be able to earn substantial monopoly profit while its potential competitors earn nothing. These authors feel that the threat of entry should drive the profit of the incumbent to zero, the "competitive" level. We shall attempt in this section and the next to reconcile these conflicting views.

Returning to the model of Section 2, we shall suppose that two identical firms move alternately and choose nonnegative quantities (more accurately, capacities), q. They maximize the discounted sum of instantaneous profits, with discount factor δ. If q is chosen to be strictly positive, we shall assume that the firm incurs a fixed cost F. We shall suppose that this cost is incurred up-front. But, since the firm is committed to the capacity q for two periods, we can think of $f = F/(1+\delta)$ as the per-period or flow equivalent of F. Viewed this way, the fixed cost can be thought of as a "short-term" sunk cost. To simplify matters, we assume that variable costs are linear: variable cost of $q = cq$; and that demand is also linear: price $= 1 - (q^1 + q^2)$, where q^i is firm i's choice[7] of q. Thus, firm 1's instantaneous profit is

$$\pi^1(q^1, q^2) = \begin{cases} q^1(1 - q^1 - q^2) - cq^1 - f, & \text{if} \quad q^1 > 0, \\ 0, \quad \text{if} \quad q^1 = 0, \end{cases} \tag{1}$$

and firm 2's profit is symmetric.

We shall assume that fixed costs are so large that one but not two firms can operate profitably. Specifically, let $\pi^m = d^2/4$, where $d = 1 - c$ (π^m is just monopoly profit gross of fixed costs). Then our profitability assumption requires

$$2f > \pi^m > f. \tag{2}$$

(Actually, as a referee pointed out, we could probably replace the left inequality with the assumption that f exceeds Cournot profit.)

For comparison, we first consider what these demand and cost assumptions imply about equilibrium in the traditional static Cournot model. In that model, a pair of quantities $(\bar{q}^1, \bar{q}^2)$ is an equilibrium if, for each firm i, $q^i = \bar{q}^i$ maximizes

[6] Although our model shares with that of Eaton and Lipsey the property that capital is not infinitely durable, our conclusions about the nature of equilibrium under the threat of entry differ markedly from theirs. See the discussion following Proposition 2.

[7] To ensure the applicability of dynamic programming we shall restrict q^1 to a large but bounded set (see Section 2).

π^i given $\bar{q}^j$ $(j \neq i)$. One can easily verify that, given our demand and cost assumptions, there are three equilibria: $(q^m, 0)$, $(0, q^m)$ (where q^m denotes the monopoly level $d/2$), and a mixed strategy equilibrium in which each firm sets $q = \sqrt{f}$ with probability α and with probability $1-\alpha$ produces nothing, where $\alpha = (d/\sqrt{f}) - 2$.

None of these three equilibria really models the idea that the threat of entry should drive an incumbent's profit to zero. The two monopolistic equilibria obviously do not: the presence of a second firm has no effect at all. One can maintain that such equilibria are unconvincing because, were the other firm to enter, the incumbent would not keep q at the monopolistic level. But such dynamic considerations are attacks not so much against the *equilibria* but rather against the static nature of the game itself. There is simply no opportunity in a one-shot, simultaneous move game to react.

The mixed strategy equilibrium perhaps comes closer to capturing the zero-profit story. At least the two identical firms are treated symmetrically and earn zero profits on average. Of course, the equilibrium also has the unfortunate property that, with positive probability, *neither* firm or *both* firms operate.

In view of the shortcomings of the static quantity model, we turn to an analysis of the equilibrium of our dynamic model. Throughout we make the cost and demand assumptions (1) and (2). Our main goal is to exhibit, for each possible value of the discount factor δ, the unique symmetric Markov perfect equilibrium, i.e., the unique MPE such that $R^1 = R^2$. Our emphasis on *symmetric* equilibrium is meant to underscore the idea that the firms are inherently identical, so that, placed in the same circumstances, they should behave the same way (i.e., the firms will react identically to a given quantity level q of the other firm). Given the large fixed costs, of course, only one firm will end up operating, but, in symmetric equilibrium, that firm will be determined by historical accident (e.g., it was lucky enough to get there first) rather than by basic strategic differences between firms.

Here is an outline of the steps leading to the characterization theorem, Proposition 2. We first show (Lemma 1) that equilibrium reaction functions are downward sloping, as in the static Cournot model. We then demonstrate (Lemma 4) that, in a symmetric equilibrium, there exists a deterrence level $\bar{q}$, i.e., a level of operation above which one's rival is deterred from entering and below which the rival *will* enter with positive probability. To do so, we establish (Lemma 3) that if a firm reacts to q by operating at a positive level, then that level must exceed q. If instead the firm chose a level $r < q$, it would induce its rival to produce a level $\hat{r}$ greater than r, which in turn would lead the firm to operate below r. Continuing iteratively, we find that in every period the firm produces less than its rival, implying that its profit must be negative, an impossibility.

These results straightforwardly imply that if a firm operates at all, it does so at or above the deterrence level (Lemma 5). Thus in equilibrium a firm either drops out of the market forever or induces the other firm to do so.

LEMMA 1: *Equilibrium dynamic reaction functions R^i are nonincreasing. That is, if $q > \hat{q}$ and r and $\hat{r}$ are realizations of $R^i(q)$ and $R^i(\hat{q})$ respectively, then $r \leqslant \hat{r}$.*

REMARK: Lemma 1, which does not assume symmetry, is a result that obtains much more generally than in this specific model. The only property of π^i it requires is that the cross partial derivative π^i_{12} be nonpositive and, for $q^i > 0$, strictly negative. See Section 8 of our companion article for a discussion of the role the cross partial assumption plays in our results.

PROOF: Suppose that, contrary to our assertion, $q > \hat{q}$ but $r > \hat{r}$, where, r and $\hat{r}$ are realizations of, say, $R^2(q)$ and $R^2(\hat{q})$ (recall that the R^i's may be random functions). By definition of R^2, r is a best response to q. Thus

$$(3) \qquad \pi^2(q, r) + \delta W^2(r) \geqslant \pi^2(q, \hat{r}) + \delta W^2(\hat{r}).$$

Similarly,

$$(4) \qquad \pi^2(\hat{q}, \hat{r}) + \delta W^2(\hat{r}) \geqslant \pi^2(\hat{q}, r) + \delta W^2(r).$$

Adding (4) to (3), we obtain

$$\pi^2(q, r) - \pi^2(\hat{q}, r) - \pi^2(q, \hat{r}) + \pi^2(\hat{q}, \hat{r}) \geqslant 0,$$

which can be rewritten as

$$(5) \qquad \int_{\hat{q}}^{q} \int_{\hat{r}}^{r} \pi^2_{12}(x, y)\, dy\, dx \geqslant 0.$$

But because $\pi^2_{12}(x, y)$ is nonpositive and, for $y > 0$, strictly negative, inequality (5) is impossible. *Q.E.D.*

By "dropping out of the market" we mean choosing $q = 0$. We next show that if firm 1 drops out of the market with positive probability in response to a (positive) move by firm 2 that was, in turn, an optimal reaction to a previous move by firm 1, then firm 1 in fact drops out of the market with probability 1.

LEMMA 2: *In any Markov perfect equilibrium, if 0 is a realization of $R^1(q)$ and $q > 0$ is a realization of $R^2(\hat{q})$ for some $\hat{q}$, then $R^1(q) = 0$.*

PROOF: Because reaction functions are nonincreasing and 0 is a realization of $R^1(q)$, firm 1 must react to any quantity above q by setting 0 with probability 1. Thus, $R^1(q + \Delta) = 0$ for any $\Delta > 0$. Now if $R^1(q) > 0$ with positive probability, $W^2(q + \Delta) > W^2(q)$ for sufficiently small Δ because $q + \Delta$ induces firm 1 to drop out with probability 1, whereas q does not. Thus for sufficiently small Δ, playing q earns firm 2 a strictly lower payoff than $q + \Delta$, a contradiction of the optimality of q. *Q.E.D.*

Henceforth we shall confine our attention to symmetric equilibrium (ones where $R^1 = R^2$). We first establish Lemma 3.

LEMMA 3: *In a symmetric MPE, if r is a positive realization of $R(q)$ (we can drop the superscripts from reaction functions because of symmetry) $r > q$.*

PROOF: Suppose first that $0 < r < q$. From Lemma 1, $R(r) \geqslant r$. Moreover, for any realization of $\hat{r}$ of $R(r)$, there exists a realization $\hat{\hat{r}}$ of $R(\hat{r})$ such that $\hat{\hat{r}} \leqslant R(r)$. Continuing iteratively we find that the firm who responds to q can continue to act optimally in such a way that it always produces no more than the other firm. Thus, in any period where it produces positively, it must lose money —in particular, when it produces r. Since it can ensure itself zero profit by dropping out, operating at a positive level cannot be optimal. Hence $r < q$ is impossible.

Next suppose that $r = q$. If 0 is a realization of $R(r)$, then from Lemma 2, $R(r) = 0$, an impossibility since $R(r) = R(q)$. Thus all realizations of $R(r)$ must be positive. From the preceding paragraph, $r \leqslant R(r)$. Thus, repeating the argument of that paragraph, we can once again show that the firm that responds to q can always act optimally in ways such that it produces no more than the other firm, which gives us the same contradiction as before. *Q.E.D.*

We next show that in a symmetric equilibrium, there exists a deterrence level.

LEMMA 4: *In a symmetric MPE there exists $\bar{q} > 0$ such that, for all $q > \bar{q}$, $R(q) = 0$, and, for all $q < \bar{q}$, there exists a positive realization of $R(q)$.*

PROOF: Consider a sequence $\{q_n\}$ tending monotonically to infinity. Suppose that for all n there exists a positive realization r_n of $R(q_n)$. From the definition of π^i, $\{r_n\}$ must be bounded, otherwise instantaneous payoffs become unboundedly negative.[8] Hence for sufficiently large n, $q_n > r_n$, a contradiction of Lemma 3. There consequently exists $\hat{q} > 0$ such that for all $q > \hat{q}$, $R(q) = 0$. Let $\bar{q}$ be the infimum of all such $\hat{q}$. Then for all $q > \bar{q}$, $R(q) = 0$ and, for all $q < \bar{q}$, there exists a positive realization of $R(q)$. It remains to show that $\bar{q} > 0$. Assume therefore that $\bar{q} = 0$. Choose $\varepsilon > 0$ so small that $\pi^1(q^m, \varepsilon) > 0$, where q^m is the monopoly quantity. Because $\bar{q} = 0$, then $R(\varepsilon) = 0$, and so firm 1 earns zero profit the first period after firm 2 has played ε. Moreover, firm 1 can earn no more than monopoly profit (the theoretical maximum) in any subsequent period. However if firm 1 responds to ε by playing q^m, it earns positive profit the first period, and, if it continues to play q^m, monopoly profit thereafter. Hence $R(\varepsilon) \neq 0$, and so $\bar{q} > 0$. *Q.E.D.*

A firm "takes over the market" if it operates at a level that induces the other firm to drop out. We next demonstrate that, in response to $q > 0$, a firm either takes over or drops out of the market.

LEMMA 5: *In a symmetric MPE, for all q and all positive realizations r of $R(q)$, $R(r) = 0$.*

[8] This argument may seem to rely on prices becoming negative. However, as long as the marginal cost c is positive, profit is unbounded from below even if the price is bounded below by zero. Firm i's set of possible quantities must be big enough to include all the r_n's.

PROOF: Suppose that, contrary to the Lemma, there exists a positive realization $\hat{r}$ of $R(r)$. From Lemma 3, $r > q$ and $\hat{r} > r$, a contradiction of Lemma 1. *Q.E.D.*

We are nearly ready to establish our main proposition, which asserts that, for any $\delta > 0$, there exists a unique symmetric MPE and exhibits that equilibrium explicitly. To state the proposition, we consider the equations

$$\pi(q,q) + \frac{\delta}{1-\delta}\pi(q,0) = 0, \tag{6}$$

$$T(q) = \arg\max_{\tilde{q}} \{\pi(\tilde{q},q) + \delta\pi(\tilde{q},0)\}, \tag{7}$$

$$\pi(q,q) + \delta\pi(q,0) + \frac{\delta^2}{1-\delta}(\pi^m - f) = 0, \tag{8}$$

$$\pi(T(q),q) + \delta\pi(T(q),0) + \frac{\delta^2}{1-\delta}(\pi^m - f) = 0, \tag{9}$$

where $\pi(x,y) = \pi^1(x,y)$ and π^m is $d^2/4$.

PROPOSITION 2: *There exist numbers δ_1, $\delta_2 \in (0,1)$ such that, if δ is the firms' discount factor, the unique symmetric MPE of the game with instantaneous profit given by* (1) *and* (2) *is*

$$R(q) = \begin{cases} 0, & q \geqslant q^* \\ q^*, & q < q^* \end{cases} \quad \text{if} \quad \delta_1 \leqslant \delta < 1, \tag{10}$$

$$R(q) = \begin{cases} 0, & q \geqslant q^{**} \\ q^{**}, & \underline{q} \leqslant q < q^{**} \\ T(q), & q < \underline{q} \end{cases} \quad \text{if} \quad \delta_2 \leqslant \delta < \delta_1, \tag{11}$$

and

$$R(q) = \begin{cases} 0, & q \geqslant q^{***} \\ T(q), & q < q^{***} \end{cases} \quad \text{if} \quad 0 < \delta < \delta_2, \tag{12}$$

where q^* q^{**} and q^{***} are the largest of the roots of (6), (8), *and* (9) *respectively, and $\underline{q}$ solves $T(\underline{q}) = q^{**}$.*

PROOF: Let $\bar{q}$ be the deterrence level of Lemma 4.

CASE I: $\bar{q} \geqslant q^m$. If $q < \bar{q}$ then there exists a positive realization r of $R(q)$. From Lemma 5, $R(r) = 0$. Hence from the definition of $\bar{q}$, $r \geqslant \bar{q}$. We have

$$V(q) = \pi(r,q) + \delta W(r).$$

If $r > \bar{q}$, suppose that a firm responds to q with $(r+\bar{q})/2$ rather than r. Since $\bar{q} \geqslant q^m$, $\pi((r+\bar{q})/2, q) > \pi(r, q)$. Furthermore, since $(r+\bar{q})/2 > \bar{q}$, $R((r+\bar{q})/2) = 0$, and so $W((r+\bar{q})/2) > W(r)$. Therefore $(r+\bar{q})/2$ generates higher profit than r, a contradiction. We conclude that for $q < \bar{q}$, the only positive realization of $R(q)$ is $\bar{q}$. Hence, from Lemma 1, $R(q) = \bar{q}$ for all $q < \bar{q}$. From Lemma 5, $R(\bar{q}) = 0$. Therefore, $V(\bar{q}) = 0$. Now for $q < \bar{q}$,

$$V(q) = \pi(\bar{q}, q) + \frac{\delta}{1-\delta}\pi(\bar{q}, 0).$$

Because $\pi(\bar{q}, q)$ is decreasing in q, we have $V(q) > 0$ for all $q < \bar{q}$. Furthermore, for all $q > \bar{q}$ we must have

$$\pi(\bar{q}, q) + \frac{\delta}{1-\delta}\pi(\bar{q}, 0) < 0,$$

otherwise $R(q) \neq 0$. Hence $\bar{q}$ must equal q^*, the greatest root of (6). From (6)

$$q^* = \frac{d + \sqrt{d^2 - 4(2-\delta)f}}{2(2-\delta)}. \tag{13}$$

Thus because $\bar{q} \geqslant q^m = d/2$, (13) implies

$$\delta^2 d^2 - (2d^2 + 4f)\delta + 8f \leqslant 0. \tag{14}$$

Notice that because $d^2 > 4f$, (14) holds for $\delta = 1$. Since it clearly does not hold for $\delta = 0$, there exists $\delta_1 \in (0,1)$ such that it holds if and only if $\delta \in [\delta_1, 1)$. Thus $\bar{q} \geqslant q^m$ implies that $\delta \in [\delta_1, 1)$ and that (10) holds. Furthermore it is clear that for $\delta_1 \leqslant \delta < 1$, (10) defines an MPE.

CASE II: $\bar{q} < q^m$. By the same argument as in case I, $R(q) \geqslant \bar{q}$ for all $q < \bar{q}$. In particular, since monopoly profit is the highest conceivable profit level per period, $R(0) = q^m$.

Now suppose that for $q < \bar{q}$, r is a realization of $R(q)$ but $r \neq \max\{T(q), \bar{q}\}$, where $T(q)$ is given by (7). Then $r \neq T(q)$, because $r \geqslant \bar{q}$. But since

$$V(q) = \pi(r, q) + \delta\pi(r, 0) + \frac{\delta^2}{1-\delta}(\pi^m - f),$$

it is clear that discounted profit could be raised by choosing $\hat{r}(\geqslant \bar{q})$ equal to $\bar{q}$ or $T(q)$. We conclude that

$$R(q) = \max\{\bar{q}, T(q)\}$$

for $q < \bar{q}$.

SUBCASE A: $\bar{q} \geqslant T(\bar{q})$. Then, for q less than $\bar{q}$,

$$\pi(\bar{q}, q) + \delta\pi(\bar{q}, 0) + \frac{\delta^2}{1-\delta}(\pi^m - f) > 0. \tag{15}$$

The inequality reverses for $q > \bar{q}$. Hence $\bar{q} = q^{**}$, where q^{**} is the larger root of

(8). From (8),

$$(16) \qquad q^{**} = \frac{(1+\delta)d + \sqrt{(1+\delta)^2 d^2 - \dfrac{4(2+\delta)}{1-\delta}\left(f - \dfrac{\delta^2 d^2}{4}\right)}}{2(2+\delta)}.$$

Because $\bar{q} < q^m$, we know from case I that $\delta < \delta_1$. But from (16) we know that there exists $\delta_2 \in (0, \delta_1)$ such that

$$q^{**} \geqslant T(q^{**}) = \frac{d}{2} - \frac{q^{**}}{2(1+\delta)}$$

holds if and only if $\delta \in [\delta_2, \delta_1)$. Thus $\bar{q} < q^m$ and $\bar{q} \geqslant T(\bar{q})$ imply that $\delta \in [\delta_2, \delta_1)$, that $\bar{q} = q^{**}$, and that $R(q) = \max\{q^{**}, T(q)\}$ for $q < q^{**}$. Now for $\delta \in (\delta_2, \delta_1)$, $q^{**} > T(q^{**})$, and there exists $\underline{q} < q^{**}$ such that $q > T(q)$ if and only if $q > \underline{q}$. Hence $R(q)$ takes the form (11). Furthermore, if $\delta \in [\delta_2, \delta_1)$ and $R(q)$ is defined by (11) it is straightforward to verify that (R, R) constitutes an MPE.

SUBCASE B: $\bar{q} < T(\bar{q})$. Then, for $q < \bar{q}$,

$$\pi(T(q), q) + \delta\pi(T(q), 0) + \frac{\delta^2}{1-\delta}(\pi^m - f) > 0,$$

with the inequality reversed for $q > \bar{q}$. Hence $\bar{q} = q^{***}$, the larger root of (9). By elimination, we conclude that if $\bar{q} < T(\bar{q})$, then $R(q)$ is defined by (12) and $\delta < \delta_2$. Conversely, one can easily check that for $\delta < \delta_2$, (R, R) with R defined by (12), constitutes a symmetric MPE. *Q.E.D.*

Proposition 2 shows that, regardless of the discount factor, equilibrium takes a simple form. Namely, there is a deterrence level $\bar{q}$, such that if a firm's rival is currently operating at or above this level, the firm will produce nothing. However, if the rival falls short of $\bar{q}$, the firm will operate at least at the level $\bar{q}$. Thus, there is a unique steady-state outcome wherein the single firm in the market operates at the level $\max\{\bar{q}, q^m\}$. Moreover, starting from any other position, that steady-state is reached in a maximum of three periods.

The deterrence level $\bar{q}$ monotonically increases in the discount factor δ (and decreases in the fixed cost f). When δ is comparatively high (greater than δ_1), $\bar{q}$ is above the monopoly quantity q^m (see Figure 1). That is, to drive out its rival or deter it from entering, a firm must operate above the monopoly level. If the firm actually uses all the capacity it has installed,[9] it, therefore, charges less than the monopoly price. Given these restrictions, the firm will produce exactly $\bar{q}$. This is a result reminiscent of the limit pricing literature (see Gaskins (1971), Kamien and Schwartz (1971), and Pyatt (1971)): an incumbent firm sells at a price sufficiently low that the immediate short-run losses of entry outweigh the longer run gains.

[9] This will be the case, for instance, if the marginal cost c reflects primarily installation rather than operating expense.

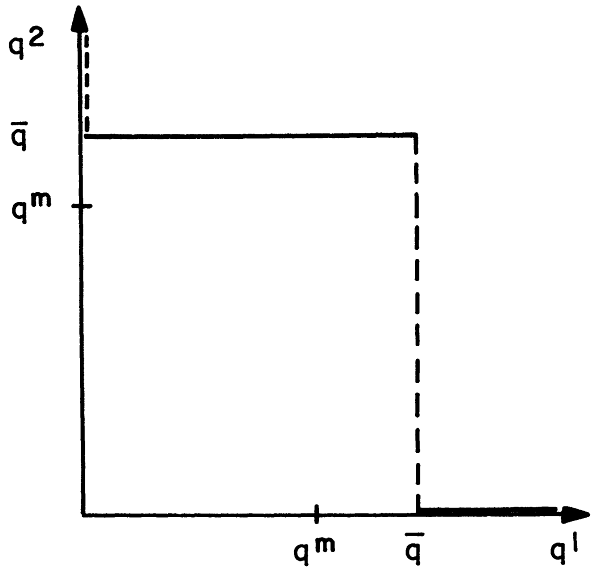

FIGURE 1. $\delta_1 \leqslant \delta < 1$.
(Dotted lines denote firm 1's reaction function; solid lines denote firm 2's reaction function.)

Since $\bar{q}$ must satisfy (6) notice that, as δ tends to 1, $\pi^1(\bar{q}, 0)$ tends to zero. That is, instantaneous profit is driven down to the competitive level. Hence our model confirms the heuristic stories of Grossman (1981) and Baumol, Panzar, and Willig (1982) if firms place sufficient weight on future profits.

Our conclusion differs from that of Eaton and Lipsey (1980), although those authors' model shares with ours the property that instantaneous profit tends to zero as the length of commitment shrinks. Eaton and Lipsey allow for only one level of capital and do not obtain our "contestability" conclusion that instantaneous output/capacity tends toward the socially optimal level as the threat of entry increases. Indeed, in their model, profit is driven to zero because of the accumulation of socially useless capital. (Of course, our result relies on the "exact-Cournot form." That is, the firms are supposed to choose prices that clear the market given their capacity. As we note in footnote 9, this property holds if the marginal cost of investment in capacities is sufficiently large. For smaller investment costs, results intermediate between those of Eaton-Lipsey and ours would hold.)

When the discount factor is less than δ_1, the deterrence level is below the monopoly level (see Figures 2 and 3). Hence, the steady-state quantity is the monopoly level itself, a result in keeping with the barriers to entry tradition. *How* a firm takes over the market from its rival depends on the discount factor and the rival's quantity, q. The firm could always drive the rival out by choosing $\bar{q}$. However, for moderate discount factors ($\delta_2 < \delta < \delta_1$) and low values of q or for low discount factors ($\delta < \delta_1$) and *any* q (less than $\bar{q}$), the firm prefers to operate *above* $\bar{q}$, namely at $T(q)$. $T(q)$, defined by (7), can be thought of as the optimal

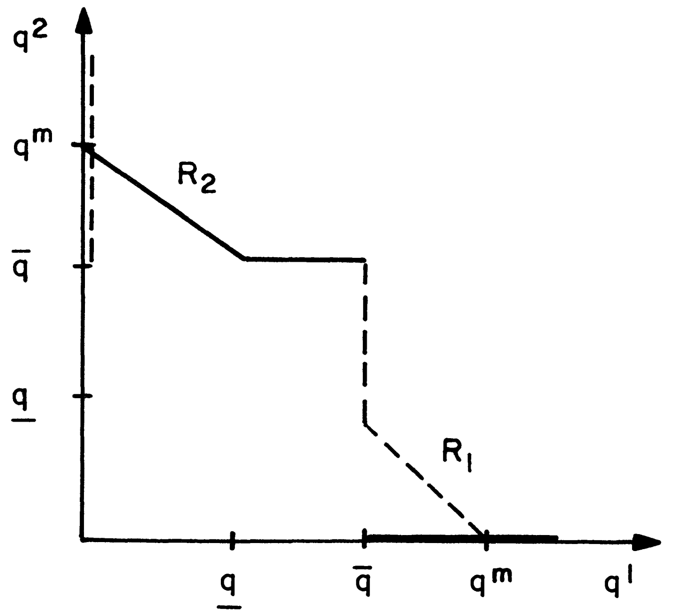

FIGURE 2. $\delta_2 \leqslant \delta < \delta_1$.

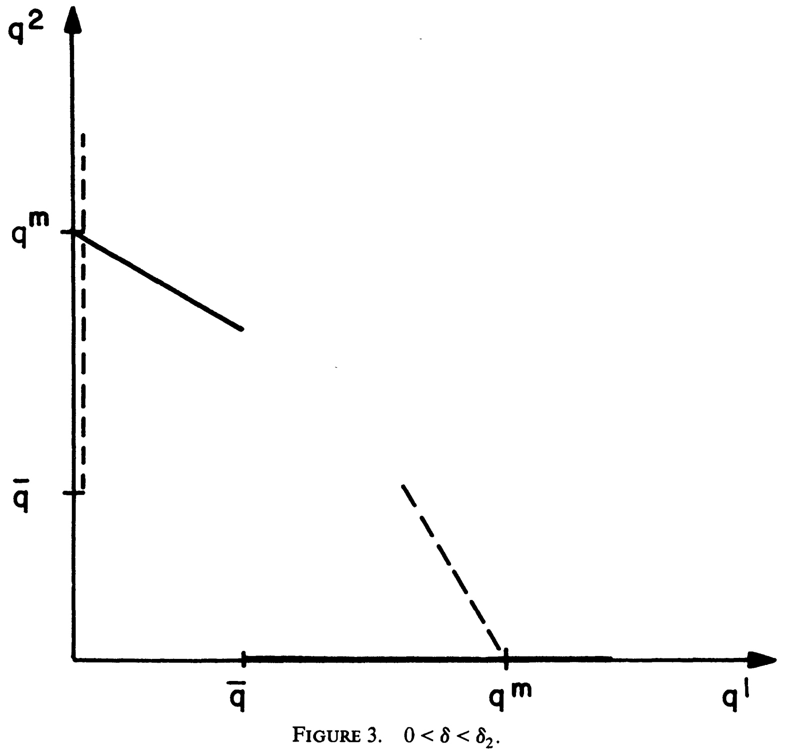

FIGURE 3. $0 < \delta < \delta_2$.

"two-period reaction" function. It is a firm's best response to q in a game with a two-period horizon, given that the other firm does not produce in the second period.

We ought to mention that although Proposition 2 exhibits the unique *symmetric* MPE, there are also, for sufficiently large fixed costs and discount factors, exactly two other, highly asymmetric MPE's. Specifically, for such costs and discount factors, if firm 1 always uses its two-period reaction function, then firm 2 will always stay out of the market. Conversely, if firm 2 never enters, the two-period reaction function is optimal for firm 1. Thus this pair of strategies is an MPE, and so is the pair with the roles of the players interchanged.

PROPOSITION 3: *There exist* $\underline{\delta} \in (0,1)$ *and* $\underline{f} \leqslant \pi^m$ *such that if* $\underline{\delta} \leqslant \delta < 1$ *and* $\underline{f} \leqslant f < \pi^m$, *there are exactly two asymmetric equilibria:*

$$(R^1, R^2) = (T, 0)$$

and

$$(R^1, R^2) = (0, T),$$

where T *satisfies* (7).

PROOF: See the Appendix.

Notice that in these asymmetric equilibria, the firm remaining in the market ultimately operates at the monopoly level even if δ is near 1. Thus, if one does not accept our above justification for emphasizing the symmetric equilibrium, one may place less weight on our contestability conclusions.

Our uniqueness result depends, of course, on the Markov assumption. For the usual "Folk Theorem" reasons, there are many symmetric perfect equilibria in this model that are not Markovian. For example, there is one in which the firms take turns operating at the monopoly level. Despite our defense of Markov strategies, therefore, one might wonder why the firms do not "agree" to adopt this more profitable equilibrium in preference to the Markov equilibrium.

One answer might be that tacit collusion between duopolists arises in industries where each firm expects the other to remain in the market for a long time. But if only one firm ends up operating in the long run, the opportunity for collusive behavior may be smaller (admittedly, this is an informal argument that awaits rigorous treatment).[10] Another possible explanation is that alternating monopoly is disadvantageous because of the cost of entering and reentering the market. This line is pursued in the following section. Finally, one might interpret our infinite-horizon model as the limit of a sequence of finite-horizon models as the horizon grows (we stress this interpretation in Maskin-Tirole (1987)). We

[10] This response does not apply to the price-setting model of our companion paper, where both duopolists are present throughout. As we shall see, however, Markov equilibria themselves are collusive in that model.

conjecture that the unique limit of the finite-horizon equilibria is our symmetric MPE (this is certainly the case in the small fixed cost model of our (1987) paper).

4. ENDOGENOUS TIMING

We admitted in the introduction that the imposition of alternating moves is artificial. There seems no reason why, in principle, firms could not move simultaneously.[11] In this section we extend the alternating moves model to allow the relative timing of firms to be endogeneously determined. There is a variety of alternative ways such an extension might be made, depending on the particular technological or contractual reasons why firms are committed in the short-run. Here we discuss two possible endogenous timing models. Although they are highly stylized, they suggest that our results may be robust to more satisfactory constructs.

In our first pass at endogenous timing, we abandon the assumption that firms alternate. We will continue, however, to suppose that time is measured discretely, and so the intertemporal profit functions are the same as before. Firm 1 (firm 2) is no longer constrained to move only in odd- (even-)numbered periods. Nonetheless, when a firm *does* act, it remains committed to that action for two periods.[12] If in any period a firm does not have a commitment pending it is free to move. Failure to do so amounts to being out of the market for a period (and therefore corresponds to zero profit). Thus at any time where it is uncommitted, the firm can either move or select the "null action."

From the point of view of a firm about to act, the payoff-relevant information is whether (i) the other firm is currently committed, and (ii) if so, at what level. We continue to require that strategies be Markov, i.e., dependent only on payoff-relevant information. Thus a Markov strategy for firm i can be described by the pair $\{R^i(\cdot), S^i\}$, where $R^i(\cdot)$, as before, describes how firm i reacts to the other firm's current action and S^i prescribes its move when the other firm is not currently committed. Both S^i and $R^i(a^j)$, for any action a^j, are random variables that take their values in the union of the action space with the null action.

Notice that if, along the equilibrium path, a firm chooses prices according to $R^i(\cdot)$, the firms alternate in their moves (alternating mode). By contrast, when S^i dictates i's equilibrium behavior, firms act at the same time (simultaneous mode).

Markov strategies and equilibria are now more complicated than in the fixed-timing model. Nevertheless, in the two cases we consider in this pair of papers—the quantity model of this paper and the price model of Part II—steady-state equilibrium behavior remains essentially the same as before. This fact is established for the price model in our companion paper. The

[11] Of course, literal simultaneity is unlikely. However, that firms act in ignorance of other firms' moves is all that is needed for *de facto* simultaneity.

[12] More generally, we might imagine that the firm is committed for m periods. What is important is that m be greater than 1, i.e., the period of commitment should exceed the basic decision period. This is certainly true of firms constrained by contracts for labor, machinery leasing, or franchising. It is also likely to hold for firms that compete in prices by mailing price lists or who are pressured by their retailers not to change prices too quickly.

following observations for the quantity model with large fixed costs are proved in Maskin-Tirole (1982).

First, for δ near 1, there exists a simultaneous mode strategy S such that $\{R(\cdot), S\}$ defines a symmetric equilibrium, where $R(\cdot)$ is given by (10) in Proposition 2, and S is a random quantity choice (S has no arguments because, in the simultaneous mode, there is no payoff relevant variable). Starting at any initial configuration of actions, the firms switch to the alternating mode in finite time with probability one, and stay in that mode forever. The steady state output is $\bar{q}$ as defined in (6). Second, if we introduce an entry/reentry cost exceeding $\pi^m(1+\delta)$ and the fixed cost is "sufficiently large,"[13] (a slightly modified version of) R describes the long-run behavior of the system in the *unique* symmetric MPE of the endogeneous timing game.

Let us now turn to a second way of endogenizing the timing. This model is highly special, but its simplicity allows us to *derive* the exogenous timing two-period commitment framework directly (not only as an equilibrium outcome as in the previous model). Specifically, let us now suppose that time is continuous and discounted at rate r. Instantaneous profit $\pi^i(a^1, a^2)$ now represents a flow per unit of time. When a firm chooses an action, its period of commitment to that action is stochastic. We shall assume, in fact, that commitment lengths are governed by a Poisson process.[14] Thus, in the time interval Δt, the probability that the commitment will lapse is $\lambda\,\Delta t$, where λ is the Poisson parameter.

Random commitment may arise when capital has an uncertain working lifetime (here, as in the fixed-timing model, we assume that the fixed cost of capital is incurred up-front). Of course, the Poisson property—the assumption that the probability the machine will give out in the next instant is independent of its current age—is extreme. Its primary virtue is its simplicity.[15]

In this model, the physical state of the system from the point of view of a firm about to choose an action is exactly as in the discrete framework. Thanks to our Poisson assumption, the length of time the other firm has been committed to this action is not relevant; only the action itself is. Hence, a Markov strategy is exactly the same as before. Indeed, formally, our continuous time model reduces to the two-period commitment, discrete-time framework of Section 2. To see this more clearly, note that the dynamic programming equations describing Markov perfect equilibrium have the same form in both cases. For instance, if firm 2 is currently committed to action a^2 and firm 1 is about to act, the present discounted value of 1's profit in the continuous-time model is

$$V^1(a^2) = \max_{a^1}\left\{\pi^1(a^1, a^2)\,\Delta t + \lambda\,\Delta t W^1(a^1)e^{-r\Delta t} + (1-\lambda\,\Delta t)V^1(a^2)e^{-r\Delta t}\right\},$$

[13] It is sufficient that $2\pi^m/3 < f$. The entry cost rules out equilibria in which firms take turns being monopolists.

[14] We could introduce the possibility of a null action without appreciable change.

[15] If we abandoned the Poisson assumption, a firm's reaction would also depend on the age of the other firm's machine. We believe, however, that the qualitative results of Section 3 would not be significantly altered in this more elaborate model.

where $W^1(a^1)$ is firm 1's present discounted profit assuming that it is currently committed to a^1 and firm 2 is about to act. But this equation can be rewritten as

$$V^1(a^2) = \max_{a^1}\left\{\frac{\pi^1(a^1, a^2)}{\lambda + r} + \frac{\lambda}{\lambda + r} W^1(a^1)\right\},$$

which is the exact analogue of the equation we obtained in Section 2B.

Department of Economics, Harvard University, Cambridge, MA 02138, U.S.A.
and
Department of Economics, MIT, Cambridge, MA 02139, U.S.A.

Manuscript received June, 1985; final revision received April, 1987.

APPENDIX

PROPOSITION 3: *There exist $\underline{\delta} \in (0,1)$ and $\underline{f} < \pi^m$ such that if $\underline{\delta} \leqslant \delta < 1$ and $\underline{f} \leqslant f < \pi^m$, there are exactly two asymmetric equilibria*:

$$(R^1, R^2) = (T, 0)$$

and

$$(R^1, R^2) = (0, T),$$

where T satisfies (7).

PROOF: We begin by observing that, for δ sufficiently high and $f \geqslant (5/7)\pi^m$, firm 1 must lose money over the two periods if it plays positively the first period and firm 2 responds with its "two-period reaction function." That is,

$$\text{(A1)} \qquad \sup_{q>0}\left\{\left(\pi^1(q,0)\right) + \delta\left(\pi^1(q, T(q))\right)\right\} < 0.$$

Substituting for π^1 in the left-hand side of (A1) and using

$$T(q) = \frac{d}{2} - \frac{q}{2(1+\delta)},$$

we obtain

$$\max\left\{q(d-q) - f + \delta\left(q\left(\frac{d}{2} - q\right) + \frac{q^2}{2(1+\delta)} - f\right)\right\},$$

which for $\delta = 1$ is negative if

$$\text{(A2)} \qquad f > \frac{9}{56}d^2 = \frac{9}{14}\pi^m.$$

Hence, by continuity, (A1) holds for δ in a neighborhood of 1 if $f > (5/7)\pi^m$.

We next assert that, for sufficiently high δ and f, the counterpart of Lemma 5 for asymmetric equilibria holds (the proof can be found in our (1982) working paper).

CLAIM: For sufficiently high $\delta < 1$ and $f < \pi^m$, $R^2(r) = 0$ for all positive realizations r of $R^1(q)$ and all q, if R^1 and R^2 are equilibrium dynamic reaction functions.

Consider an asymmetric Markov perfect equilibrium. For each i, let $\bar{q}^i = \inf\{q \mid R^j(q) = 0\}$ (conceivably $\bar{q}^i$ could equal 0 or ∞), $j \neq i$. Suppose first that $0 < \bar{q}^1$, $\bar{q}^2 < \infty$. Let

$$\tilde{q}^1 = \limsup_{\varepsilon \to 0} \left\{ r \mid r \in R^1(\bar{q}^2 - \varepsilon) \right\}.$$

By definition of $\bar{q}^2$, $r^1(\varepsilon) = \sup\{r \mid r \in R^1(\bar{q}^2 - \varepsilon)\} > 0$ for any $\varepsilon > 0$. Hence from the above claim and Lemma 1, $R^2(r^1(\varepsilon)) = 0$, and so $r^1(\varepsilon) \geqslant \bar{q}^1$. We conclude that $\tilde{q}^1 \geqslant \bar{q}^1$. If $\bar{q}^1 > q^m$, then $\tilde{q}^1 = \bar{q}^1$, otherwise for small ε, firm 1 could reduce its output, raise its short-period profit and still deter entry in response $\bar{q}^2 - \varepsilon$. If $\tilde{q}^1 \leqslant q^m$, then $\bar{q}^1 \leqslant q^m$, and so $\tilde{q}^1 = \max\{\bar{q}^1, T(\bar{q}^2)\}$.

There are therefore three possibilities:

(i) $\tilde{q}^1 = \bar{q}^1 > q^m$,

(ii) $q^m \geqslant \tilde{q}^1 = \bar{q}^1 \geqslant T(\bar{q}^2)$,

or

(iii) $q^m \geqslant \tilde{q}^1 = T(\bar{q}^2) > \bar{q}^1$.

We first rule out cases (ii) and (iii). If, to the contrary, either case holds, then if firm 1 plays $q^m + \varepsilon$, firm 2 will not produce. Hence firm 1 can earn essentially monopoly profit indefinitely. Thus for high discount factors $\bar{q}^2 > q^m$. If $\bar{q}^2 = \infty$, then

$$\lim_{q \to \infty} \left[(d - q - \tilde{q}^1)\tilde{q}^1 + \delta(d - \tilde{q}^1)\tilde{q}^1 + \frac{\delta^2}{1-\delta}(\pi^m - f) \right] \geqslant 0$$

which is plainly false. Hence $\bar{q}^2 < \infty$. Then

$$(d - \bar{q}^2 - \tilde{q}^1)\tilde{q}^1 + \delta(d - \tilde{q}^1)\tilde{q}^1 + \frac{\delta^2}{1-\delta}(\pi^m - f) = 0. \tag{A3}$$

Similarly,

$$(d - \tilde{q}^1 - \bar{q}^2)\bar{q}^2 + \frac{\delta}{1-\delta}(d - \bar{q}^2)\bar{q}^2 - \frac{f}{1-\delta} = 0. \tag{A4}$$

Now as δ tends to 1, $\bar{q}^2$ must grow indefinitely if (A3) is to hold. But (A4) clearly cannot hold for arbitrarily large $\bar{q}^2$, and so, for large δ, cases (ii) and (iii) are impossible.

We conclude that either

(iv) $\bar{q}^i > q^m, \quad i = 1,2,$

or

(v) $\bar{q}^1 = 0 \quad \text{or} \quad \bar{q}^2 = 0.$

If (iv) holds then

$$\begin{aligned} (d - \bar{q}^1 - \bar{q}^2)\bar{q}^1 + \frac{\delta}{1-\delta}(d - \bar{q}^1)\bar{q}^1 - \frac{f}{1-\delta} &\\ = (d - \bar{q}^1 - \bar{q}^2)\bar{q}^2 + \frac{\delta}{1-\delta}(d - \bar{q}^2)\bar{q}^2 - \frac{f}{1-\delta} &\\ = 0, & \end{aligned} \tag{A5}$$

which implies $\bar{q}^1 = \bar{q}^2$, violating asymmetry.

Thus (v) must hold. If $\bar{q}^2 = 0$, then $R^2(q) = T(q)$. From (A1), we deduce that $R^1(q) = 0$. Similarly $\bar{q}^1 = 0$ implies that $(R^1, R^2) = (T, 0)$. *Q.E.D.*

REFERENCES

BAUMOL, W., J. PANZAR, AND R. WILLIG (1982): *Contestable Markets and the Theory of Industrial Structure*. New York: Harcourt, Brace, Jovanovich.

CYERT, R., AND M. DEGROOT (1970): "Multiperiod Decision Models with Alternating Choice as the Solution of the Duopoly Problem," *Quarterly Journal of Economics*, 84, 419–429.

DIXIT, A. (1979): "A Model of Duopoly Suggesting a Theory of Entry Barriers," *Bell Journal of Economics*, 10, 20–32.

EATON, B., AND R. LIPSEY (1980): "Exit Barriers are Entry Barriers: The Durability of Capital as a Barrier to Entry," *Bell Journal of Economics*, 11, 721–729.

FUDENBERG, D., AND J. TIROLE (1983): "Capital as a Commitment: Strategic Investment in Continuous Time," *Journal of Economic Theory*, 31, 227–256.

GASKINS, D. (1971): "Dynamic Limit Pricing: Optimal Pricing under Threat of Entry," *Journal of Economic Theory*, 2, 306–322.

GROSSMAN, S. (1981): "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs," *Econometrica*, 49, 1149–1172.

KAMIEN, M., AND N. SCHWARTZ (1971): "Limit Pricing and Uncertain Entry," *Econometrica*, 39, 441–454.

MASKIN, E., AND J. TIROLE (1982): "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," MIT Working Paper #320.

——— (1987): "A Theory of Dynamic Oligopoly, III: Cournot Competition," *European Economic Review*, 31, 947–968.

——— (1988): "A Theory of Dynamic Oligopoly II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles," *Econometrica*, 56, 571–599.

PYATT, G. (1971): "Profit Maximization and the Threat of New Entry," *Economic Journal*, 81, 242–255.

SCHERER, F. (1980): *Industrial Market Structure and Economic Performance*. New York: Rand McNally.

SPENCE, M. (1977): "Entry, Capacity, Investment, and Oligopolistic Pricing," *Bell Journal of Economics*, 8, 534–544.

——— (1979): "Investment Strategy and Growth in a New Market," *Bell Journal of Economics*, 10, 1–19.