An equilibrium analysis of search and breach of contract, I: steady states

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We study the steady-state equilibria of models where individuals meet pairwise in a costly stochastic search process and negotiate contracts to produce output. Different meetings yield different outputs, and so an individual in a contract may wish to continue search to find a better match. If he is successful, he will break his original contract. In anticipation of possible breaches, contracts may provide for compensation to be paid to the breached-against partner. We examine the effects that several alternative damage rules have on equilibrium search and breach behavior.

1. Introduction

■ The literature on markets where agents have imperfect information about their trading possibilities has been growing steadily.[1] Much of this analysis depends fundamentally on asymmetries: either buyers or sellers set prices but not both. In contrast, we consider a symmetric model where individuals meet pairwise and negotiate contracts. Assume that individuals find potential contracting partners in a costly, stochastic search process but that negotiation is costless and instantaneous. A contract is an agreement to carry out a single project.[2] The worth of a project depends on the quality of the match between the two individuals.[3] For simplicity, we assume precisely two qualities: good (project with large output) and poor (project with small output). An individual,

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Financial assistance from the NSF is gratefully acknowledged. We thank the referees and Alvin Klevorick for helpful suggestions.

[1] For examples, see the October, 1977, issue of the *Review of Economic Studies*, the discussion and reprints in Diamond and Rothschild (1978), and Ioannides (1975) whose model is the closest to the one analyzed here. For a correction of Ioannides' results, see Butters (1977).

[2] Seasonal opportunities generate many examples of markets with this single-project feature; e.g., summer house rentals. Other examples include hiring architects for home renovations or painters for portraits. In later work we plan to analyze the case of continuous production.

[3] Here we follow Satterthwaite (1977).

therefore, can be in any of three positions: without a partner, in a poor partnership, or in a good partnership.

Individuals can continue to search after joining a partnership. Therefore, one of the parties to a contract may later come upon a better match and desire to break his contract. In practice, the possibility that individuals may wish to break their current contracts to form better ones is recognized by provisions for payment of damages. Damages under common law are frequently *compensatory* in the sense that they exactly compensate for breach; i.e., they leave the breached-against partner in the same financial position as before the breach.[4] As an alternative to externally determined damages, parties to a contract may write damage rules into the contract itself.[5] Such provisions are called *liquidated* damage rules. We examine both compensatory and liquidated damages in this paper. We also consider efficiency: the search and breach behavior that maximizes net social output.

We are concerned with the equilibrium steady states[6] of a model where individuals are perfectly informed about the distribution of possible partners they might meet. We consider two distinct, simple meeting technologies. In one, the probability of an individual's meeting *any given potential partner* is independent of the number of other potential partners. In this case, the individual's probability of meeting someone at all rises linearly with the number of potential partners. The aggregate number of meetings (which we assume, by appeal to large numbers, to equal the expected number) increases with the square of the number of searchers. We refer to this technology as the *quadratic* case. It is a reasonable model of meeting for an economy with a low density of potential partners.[7] In this case an additional searcher raises the meeting probabilities of all his potential partners. Since meetings may be of value, a searcher thus creates a positive externality for other searchers. As the presence of this externality suggests, efficiency calls for more search than occurs in equilibrium with compensatory damages. Moreover, the externality may lead to multiple equilibria.

With a high density of potential partners, the quadratic technology seems a poor approximation. Therefore, we also examine the case where an individual's probability of meeting *someone at all* is independent of the number of potential partners when that number is positive. We refer to this technology as the *linear* case, since the aggregate number of matches increases linearly with the number of searchers. Adding potential partners to the search process does not alter the probability of an individual's meeting someone, but does affect his chances of meeting potential partners in different positions (i.e., with or without a current partner). When individuals who already have partners continue to search, they may impose a negative externality on potential partners; for, as we shall see below, an individual would generally prefer, for given quality of match, to be

[4] The law is not uniform in fixing the point before breach from which compensation is measured. A partner may be restored to the position he held either *before* or *after* he signed the contract which was breached. We shall confine our discussion of compensatory damages to post-signature compensation. For a discussion of the common law treatment of damages, see McCormick (1935).

[5] For a discussion of damages for individually optimal contracts, see Mortensen (1978). We build on Mortenson's analysis by considering equilibrium with many partnerships.

[6] In our companion piece (Diamond and Maskin, 1979) we substitute an evolving economy for a steady state.

[7] For discussions of allocations where individuals are assumed never to meet more than one potential partner, see Diamond and Mirrlees (1975) and Landes and Posner (1977).

paired with someone who does not already have a partner. The addition of searchers with partners reduces the proportion of partnerless searchers and therefore makes all other searchers worse off.[8] Again we expect a discrepancy between the efficient and compensatory damage rules. While such a discrepancy does not, in fact, arise in the simplest model we consider, it occurs in straightforward extensions (see Section 16).

With either technology there are search externalities. Thus, not only is the decision to search generally inefficient under compensatory damages, but so too is the decision whether to breach. We call the decision of two individuals, *both* of whom have contracts, to break those contracts to form a new partnership a *double breach* (as opposed to the case—called a *single breach*—where only one individual has a contract). With compensatory damages the incentives for two breaching parties coincide with efficiency for all four original partners (the two breaching individuals and their partners). That is, the two individuals find it in their interest to breach precisely when by so doing they increase the sum of the expected payoffs of these four partners. A double breach, however, affects individuals other than the principal four by altering the distribution of matches these others face when searching; the breach replaces four individuals with poor matches by two partnerless individuals and two with good matches. This alteration generally affects the value of search for others, but may not be adequately accounted for by compensatory damages. Once again, circumstances where compensatory damages[9] lead to inefficient breaching behavior cannot arise in our simplest model, but are considered in Section 16 and in the sequel to this article, where we note the tendency for too little breach.

When individuals choose their own damages (i.e., liquidated damages), they do not generally select compensatory damages. Parties to a contract must divide the product of that contract in some way. For symmetry, we assume that they split the surplus from the contract equally.[10] This *surplus* equals the excess of the sum of values to the individuals of having the new contract over the sum of values of their previous positions plus the damages they must pay. Because the surplus depends on damage payments, an individual who breaches can, in effect, get his new partner to share the burden of the damages he pays to his old partner. In this way, a pair of individuals in a contract exerts some monopoly power over potential partners, and we expect liquidated damages, *ceteris paribus*, to be higher than compensatory damages.[11]

These higher damages induce opposing effects on the incentives for both search and breach, sometimes making comparisons with compensatory damages difficult. On the one hand, search is encouraged (relative to compensatory damages) by the greater return higher damages yield when breach occurs. On the other hand, search is discouraged by the higher damages set by others, which diminish opportunities for breach. With only two qualities of match, however, liquidated damages necessarily induce at least as much search as do compensa-

[8] The addition of such searchers reduces the proportion of partnerless searchers with the quadratic technology also. Others are not worse off, however, because their chances of meeting any given partnerless individual are not reduced.

[9] Single breach also alters the search environment by removing individuals with poor matches from the market. As we shall see, however, this externality does not generate an inefficiency.

[10] While this assumption appears restrictive, the basic nature of the results carries over with a wide variety of other bargaining outcomes.

[11] For a more detailed discussion of liquidated damages, see Section 6.

tory. In Section 17 we briefly consider models with three qualities of match, where either effect predominates. With breach there are also offsetting effects. Breach is discouraged because higher damages reduce the range of profitable breaches. Breach is fostered, however, when the value of new contracts is increased as a result of higher damages. Only the first effect appears in models with two qualities of match. In such models there is at least as much breach in equilibrium under compensatory damages as under liquidated damages. In Section 17 we raise the possibility that either effect may be larger with three or more qualities of match.

After setting up the model (Sections 2–4), we begin with the analysis of equilibrium under compensatory damages for a quadratic meeting technology in Section 5, where we also examine efficient behavior. After considering the equilibrium with no damage payments (Section 6) and comparing compensatory with liquidated damages (Sections 7 and 8), we study equilibrium under liquidated damages (Sections 8 and 9), still for a quadratic technology. Section 10 discusses the possibility of damage rules which induce efficiency. We then repeat the entire equilibrium analysis, replacing the quadratic by a linear meeting process (Sections 11–14). In Section 15 we discuss damages when qualities of match are difficult to observe. In the models used through Section 15, either no individual with a partner searches or no poor project is carried out. This structure limits the range of inefficiencies which can occur. To capture the omitted inefficiencies, we change the model in Section 16 to require some poor contracts to be carried out even when individuals with partners search. Section 17 considers how results differ when more than two match qualities are possible. These two sections also summarize our principal findings. Section 18 contains a few concluding remarks.

2. Parameters

■ We consider a model with two types of individuals.[12] Individuals are distinguished by type only in that each partnership (contract) requires exactly one partner of each type. Individuals search for a partner (of the opposite type) with whom to undertake a single project. If partners are well matched, the project is worth $2X$. If they are not well matched, output is $2X'$. We assume $X > X' > 0$. After partners have stopped searching—and only then—the project corresponding to their partnership is completed. Individuals are risk neutral and are able to make side payments with no bankruptcy constraints. Each individual can engage in at most one project and belong to at most one partnership.

Individuals can meet new potential partners only if they search, and the cost of search is a flow, c, per unit time. Under the quadratic technology, a is the probability per unit time that any two searchers (of opposite types) meet. Under the linear technology, a is the probability that a given searcher meets someone at all, per unit time. We assume a is sufficiently small so that we can ignore the possibility that two partners who are both searching will simultaneously find new potential partners.[13] When two individuals meet, the probability of their matching poorly is p and the probability they are a good match is $1 - p$. There is an inflow per unit time of ab new individuals (of each type); individuals initially

[12] Think, for example, about buyers and sellers or lessors and lessees.

[13] We have implicitly modeled contracting as instantaneous. Without instantaneous contracting, there would be simultaneous meetings.

have no partners. All parameters are the same for individuals of both types, and so we shall refer to just one type.

3. Contracting with compensatory damages

■ Let M denote an individual who is partnerless, and let N denote one in a poor partnership. All individuals start as M's. Some then become N's, while others move directly to good partnerships. Having no possible gain, individuals with good matches have no reason to search. Thus we need not keep track of them. If two M's meet and make a good match, they form a partnership and divide the value of the project, $2X$, equally. If two M's meet and make a poor match, they form a partnership which calls for equal division if the project is completed and for damage payments if the contract is breached by one of the partners. Compensatory damages are those which exactly compensate the partner who is breached against. Thus if V_M is the (expected) value of being an M and V_N is the value of being an N, then the compensation paid to the partner of a breaching N is $V_N - V_M$. With symmetry, expected damage payments equal expected damage receipts. Thus, two M's are always willing to form a partnership.

Consider next two N's who meet and make a good match. They will breach their old contracts and form a new partnership if and only if the aggregate value of their positions increases by more than the damages that they have to pay. That is, if and only if

$$2X - 2V_N \geq 2D, \tag{1}$$

where D represents damages. Compensatory damages to each breached-against partner are $V_N - V_M$. Therefore, the surplus from the new match S equals

$$S = 2X - 4V_N + 2V_M \tag{2}$$

and is positive if (1) holds.[14] We postulate that output in the new contract is divided to split the surplus evenly. (Since, in this case, the partners enter the contract from equal positions, halving the surplus is equivalent to halving the product.) We note that with compensatory damages, the surplus is positive—i.e., breach by the N's is worthwhile—if and only if breach increases the sum of the positional values of the four parties to the original two contracts. Nevertheless, a breach when the surplus is positive need not increase the sum of all individuals' positional values.

The final meeting possibility of interest is a good match between an M and an N. Once again, breach is worthwhile if the contracting surplus, in this case $2X - V_M - V_N - D$, is positive. With compensatory damages, the surplus is $2X - 2V_N$, which is positive when search is costly. We again assume that the new contract (if signed) splits the surplus equally between the parties. This rule gives the M partner a positional value of $V_M + \frac{1}{2}(2X - 2V_N) = X - V_N + V_M$ and the N partner, $V_N + \frac{1}{2}(2X - 2V_N) = X$.[15]

From these calculations we see than an individual gains at least as much from meeting an M as from meeting an N. An M makes a better partner than an N because the resulting surplus is larger: an M requires a smaller share of output to place him in his precontract position, and he does not require damage payments.

[14] In a competitive equilibrium without search costs, there is no surplus in this sense, since a contract of the same quality can be costlessly arranged with someone else.

[15] Note that the M partner bears the full brunt of the damage payments to N's old partner.

4. Search: quadratic meeting technology

■ To avoid trivial equilibria, we consider, throughout this paper, only those parameter values for which a steady-state equilibrium exists where partnerless individuals find it worthwhile to search.[16] Thus those without partners always search, those with good matches never do. An N may or may not find search profitable (we assume that either both partners to a contract search or neither does). If in searching an N meets an M with whom he makes a good match, he will break his existing contract to form a new partnership. Two N's may or may not find it worthwhile to break their two contracts to form a new partnership when they are well matched. Their action depends on the sign of the surplus given by (2).

Summing up, there are three possible search/breach configurations. Configuration A obtains when N's search and break their contracts whenever they find a good match. Under configuration B, N's search, but only breach for a good match with an M. In configuration C, only M's search.

Search and breach both affect the numbers of M's and N's in the market. Since search and breach behavior differs among configurations, we must consider each configuration separately when deriving the equations governing the numbers of searchers.

We start with the simplest configuration, C, where only M's search. Let h_M be the number of M's (of each type) who are searching. New entrants add to this number at the rate ab. The formation of partnerships subtracts from the number. With h_M searchers, each searcher finds a partner with probability ah_M, so altogether ah_M^2 matches (and, hence, contracts) are made.[17] Denoting a time derivative by a dot over the variable concerned, we have

$$\dot{h}_M = ab - ah_M^2. \tag{3}$$

In a steady state, the number of M's searching does not change. Equating $\dot{h}_M$ with zero, we see that equilibrium with behavior in configuration C occurs with $b^{1/2}$ searchers:

$$h_M^C = b^{1/2}. \tag{4}$$

In configuration B, both M's and N's search. An N, however, breaches only when he makes a good match with an M. Such a breach produces one new M (the N's old partner becomes partnerless) and eliminates an M (N's new partner was formerly an M). Thus, in configuration B, search by N's does not affect the number of M's who search. That is, (3) continues to hold. The number of N's increases when two M's make a poor match. Since p is the probability that a given match is poor, the rate of formation of contracts with poor matches is aph_M^2. A poor contract is dissolved when an N and M make a good match. Such a match can be either between an N of the first type and an M of the second

[16] By this assumption and our focus on steady states, we rule out consideration of cyclical processes in which at first there is no search while the stock of partnerless individuals builds up, and then search and matching occur until too few agents are left to make further search worthwhile, at which point the process begins again.

Regardless of parameters, however, there is always an equilibrium with no search.

[17] We assume numbers sufficient to replace stochastic terms by their means. Then, the equations are correct for our continuous time model where negotiations take no time. The continuous time model can be considered the limit of either of two discrete time models where negotiation takes one time period. In one case, we decrease the meeting rate, holding the length of a period fixed. In the other, we decrease the length of a period, holding constant the number of meetings per time (but not per period).

TABLE 1

NUMBERS OF SEARCHERS, QUADRATIC TECHNOLOGY

ACTION	NEW ENTRANTS	POOR MATCH OF M's	GOOD MATCH OF M's	SINGLE BREACH	DOUBLE BREACH
RATE OF FLOW	ab	aph_M^2	$a(1-p)h_M^2$	$2a(1-p)h_Mh_N$	$a(1-p)h_N^2$
CHANGE IN INDIVIDUALS OF EACH TYPE:					
WITHOUT PARTNERS (h_M)	+1	−1	−1	0	+1
IN POOR MATCHES (h_N)	0	+1	0	−1	−2
IN GOOD MATCHES	0	0	+1	+1	+1

type or vice versa. Each possibility occurs at the rate $a(1-p)h_Mh_N$ and reduces the number of N's of each type by one. Hence,

$$\dot{h}_N = aph_M^2 - 2a(1-p)h_Mh_N. \tag{5}$$

See Table 1 for an account of the changes in the numbers of searchers. In a steady state $\dot{h}_M = \dot{h}_N = 0$, so that

$$h_M^B = b^{1/2}$$

$$h_N^B = \left(\frac{p}{2(1-p)}\right)b^{1/2}. \tag{6}$$

Under configuration A, unlike under B, double breaches occur at the rate $a(1-p)h_N^2$. A double breach adds one M (of each type) to the market and subtracts two N's (of each type). Therefore, the dynamics under configuration A are described by

$$\dot{h}_M = ab - ah_M^2 + a(1-p)h_N^2$$

$$\dot{h}_N = aph_M^2 - 2a(1-p)h_Mh_N - 2a(1-p)h_N^2. \tag{7}$$

In a steady state, each equation of (7) is set to zero, giving

$$\frac{h_M^A}{h_N^A} = \left(\frac{1-p}{p}\right)\left(1 + \left(\frac{1+p}{1-p}\right)^{1/2}\right)$$

$$h_M^A = \left(\frac{1}{1-(1-p)(h_N^A/h_M^A)^2}\right)^{1/2} b^{1/2}. \tag{8}$$

As the probability of a good match increases, the ratio of M's to N's also increases. For later use, we note that the equation governing the total number of searchers under configuration A is

$$\dot{h}_M + \dot{h}_N = ab - a(1-p)(h_M + h_N)^2 \tag{9}$$

and, in the steady state,

$$h_M^A + h_N^A = (1-p)^{-1/2}b^{1/2}. \tag{10}$$

5. Steady states: compensatory damages and efficiency (quadratic technology)

■ To see whether a steady state can occur for a particular configuration, we need to check that in equilibrium individuals would choose to follow the breach and search rules defined by that configuration.

□ **Configuration C.** By definition, only M's find search worthwhile in a steady state under configuration C. The M's sign contracts with whomever they first meet and then cease search. For someone following this behavior, the expected payoff when entering the process (i.e., the positional value of being an M) is his half of the expected project output, $pX' + (1 - p)X$, less the expected cost of finding a partner, which equals the search cost per unit time, c, times the mean expected time for a meeting. The meeting technology is a Poisson process with an arrival rate ah_M; thus the expected time until a meeting is $(ah_M)^{-1}$:

$$V_M^C = pX' + (1 - p)X - c(ah_M^C)^{-1}$$

$$= pX' + (1 - p)X - \frac{c}{ab^{1/2}} . \tag{11}$$

Because past search does not affect future meeting prospects in a steady state, the value of being an M who searches does not change over time. The first condition for a configuration C steady state is that V_M^C be nonnegative. We assume this requirement is automatically met.[18]

The second condition is that N's do not find continued search worthwhile. If they do not search (i.e., if they follow the behavior dictated by configuration C) their positional value is just their share of output, X'. If some partnership of N's does continue to search for time Δt, each partner incurs the cost $c\Delta t$ and has a probability $a(1 - p)h_M^C\Delta t$ of making a good match with an M, thereby increasing his positional value by $X - X'$, as noted in Section 3. Thus an N's expected net gain from continued search is

$$a(1 - p)h_M^C\Delta t(X - X') - c\Delta t \quad \text{or} \quad a(1 - p)b^{1/2}(X - X')\Delta t - c\Delta t. \tag{12}$$

Since damages are compensatory, breach by a partner does not affect one's positional value. The condition that continued search be unprofitable becomes

$$c/ab^{1/2}(X - X') \geq (1 - p). \tag{13}$$

Given nonnegative V_M^C, a configuration C steady state with compensatory damages can occur for any combination of parameter values satisfying (13). Let the set of such parameters be called the compensatory region C. Then, for given values of b, c, X, and X', this region is depicted in $a - p$ space in Figure 1.[19] All the equations we analyze for the quadratic technology take the form, $c/ab^{1/2}(X - X')$ equals some function of p. While we could consider the relationship between p and any of the parameters on the left-hand side, we restrict ourselves to considering the $a - p$ tradeoff.

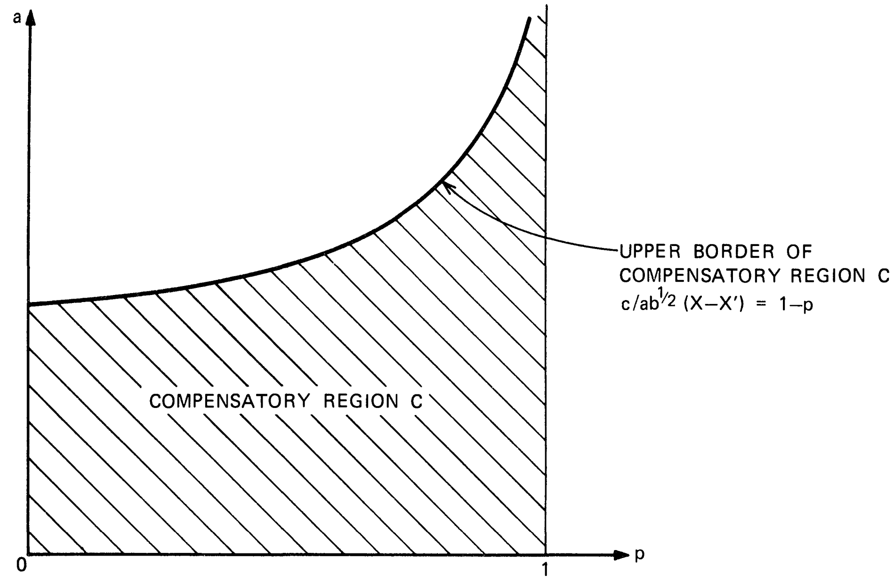
□ **Configurations A and B.** In a steady state with compensatory damages under configurations A or B, individuals continue to search until they find a good match. With this behavior the positional value of an N is no greater than that of an M; if one is planning to search until a good match is found, there is no advantage to finding a poor one. The compensatory damages corresponding to breach of a poor match are, therefore, zero, and an N will find it advantageous to breach for any good match. A compensatory equilibrium under configuration B is consequently impossible. We have a steady state under A if N's wish to continue searching. An individual (either M or N) who searches until finding a good match

[18] In the sequel, we examine this condition more closely.

[19] Figure 1 and subsequent figures fail to show the region of parameters for which no search is the only possible equilibrium.

FIGURE 1

CONFIGURATION C EQUILIBRIUM WITH COMPENSATORY DAMAGES AND QUADRATIC MEETING



has an expected payoff of output less expected search costs, $X - [c/a(1 - p) \times (h_M^A + h_N^A)]$. N's wish to continue searching provided this expected payoff is at least as large as X', the output available from a poor match. Compensatory region A, depicted in Figure 2, is defined by

$$c/a(X - X')b^{1/2} \leq (1 - p)^{1/2}. \tag{14}$$

Since $(1 - p)^{1/2} \geq (1 - p)$, with equality at 0 and 1, regions A and C overlap, as illustrated in Figure 2.

Thus, with compensatory damages and the quadratic technology, configuration B is not possible. For different parameter values we can have a configuration A steady state, a configuration C steady state, or both.

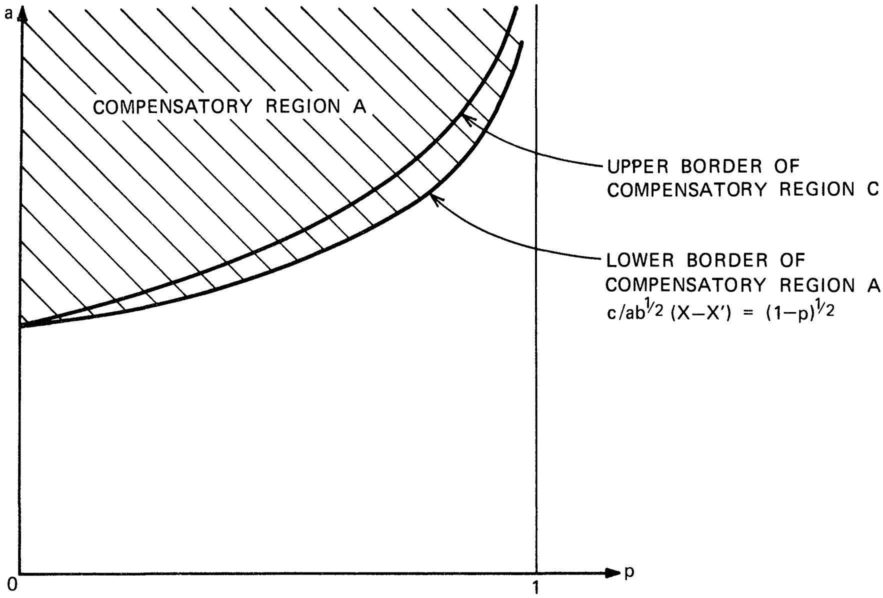
□ **Efficiency.** For any given combination of parameters, the *efficient* steady state is the configuration A, B, or C steady state which maximizes the aggregate net output flow.[20] Once again, we can rule out configuration B right away. Configurations A and B give rise to the same gross output flow, since the output per match is X and the rate of matching, which must equal the rate of new entrants in a steady state, is ab. Under A, however, search costs are lower because N's leave the market sooner (being willing to double breach). Since search continues until a good match is made and no poor contracts are carried out, there is no value, from the standpoint of efficiency, in passing up a good match between N's. Thus configuration A is always more efficient than B.[21]

[20] Because of increasing returns to numbers of searchers, a social planner could, in general, increase the flow of net product by calling for nonsteady state behavior. Rather than having individuals search continually, he could halt search to allow the stock of potential searchers to grow. During this time, of course, no search costs would be incurred. After the population grew to a sufficient size, search could resume with rapid meetings. To make comparisons easier, however, we consider only outputs in steady states.

[21] In more elaborate models, however, A's dominance over B disappears. See Section 16.

FIGURE 2

CONFIGURATION A EQUILIBRIUM WITH COMPENSATORY DAMAGES AND QUADRATIC MEETING



In configuration A the net output flow equals the rate of good matching times the output of a good match less the search cost times the number of searchers:

$$Q^A = abX - c(h_M^A + h_N^A)$$
$$= abX - cb^{1/2}(1 - p)^{1/2}. \tag{15}$$

In a steady state the project completion rate equals the entrance rate, ab. For configuration C equilibrium, therefore, net output flow is

$$Q^C = ab(pX' + (1 - p)X) - ch_M^C = ab(pX' + (1 - p)X) - cb^{1/2}, \tag{16}$$

where ab is the rate of project completion and $(pX' + (1 - p)X)$ is the expected output per completion.

From (15) and (16) the configuration A steady state is more efficient than that of configuration C if and only if

$$c/ab^{1/2}(X - X') \leq p((1 - p)^{-1/2} - 1)^{-1}. \tag{17}$$

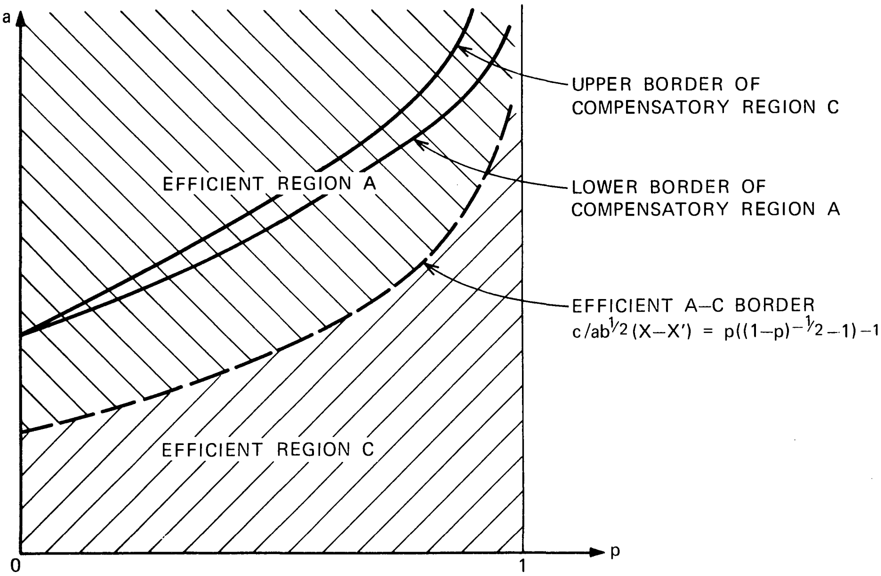
This locus is illustrated in Figure 3. Note that the efficiency border lies below both the lower border of compensatory region A and the upper border of compensatory region C. This alignment implies that a compensatory equilibrium never involves more than the efficient number of searching individuals. It shows, moreover, that there are parameter values for which some or all of the compensatory equilibria entail strictly too little search.[22]

The figure also shows that for some parameter values there is an equilibrium with the most efficient configuration. This feature depends critically on the dis-

[22] If, for a choice of parameters, there are multiple compensatory equilibria, the equilibrium with more search is more efficient. We have not analyzed the stability of equilibrium.

FIGURE 3

EFFICIENT BEHAVIOR WITH QUADRATIC MEETING



crete nature of the alternative qualities of match. When configuration A equilibrium is efficient, all N's search and there is no possibility of additional individuals' searching. When configuration C equilibrium is efficient, there are externalities which individuals ignore in their decision making, but they are too small to justify such a large change in behavior as continued search by all N's. To show that these externalities are always present, we briefly consider a model in Appendix 1 with a continuum of match qualities. There we show that starting at equilibrium, slightly increasing the highest quality of match in which an individual still continues to search always increases the net output flow. That is, the result demonstrates that there is a bias towards too little search (relative to efficiency) in compensatory equilibria with quadratic meeting. The bias derives from the fact that the decision by an individual to search makes potential partners better off and never harms individuals of the same type. This positive externality is simply not captured by compensatory damages, which concern only the immediate parties to a breach. Consequently, individuals do not have sufficient incentive to search.

We should emphasize that this unambiguous bias towards too little search depends crucially on the unambiguous positive externality of search under the quadratic technology with this structure of matching and output. Indeed we shall show below (see Section 16) that with a linear technology and a slightly more elaborate model, the results of this section can be reversed and that, for some parameter values, there can be too much search in a compensatory equilibrium. On the other hand, if we assume quadratic meeting, the bias we have demonstrated is by no means special to our simple model.

6. No damage payments

■ In some circumstances individuals do not use formal contracts to retain their partners while searching for better deals. Rather, they maintain their contacts while understanding that these potential partners may or may not be available at later times. In terms of the model described above, damage payments would always equal zero. Since compensatory damages were zero for all breaches that occurred ($V_N = V_M$ in region A and on the border of region C), the regions of different equilibria are the same with compensatory damages as with no damages.[23] This equivalence does not generally carry over once poor contracts may be completed by searching N's, as is shown in the sequel to this paper.

7. Compensatory and liquidated damages

■ The common law bases damage payments on the need to compensate for a breach of contract. In theory, courts allow the substitution of privately set damage levels only when these approximate a suitable level for compensation, although, in practice, there is some opportunity for divergence between privately contracted damages (liquidated damages) and perfect compensation. The legal doctrine against liquidated damages in excess of the level needed for compensation is essentially paternalistic.[24] Individuals are to be prevented from mistakenly promising large compensation, because they do not fully anticipate events which might make them unable or unwilling to carry out their contracts.

There are at least two other potential arguments in favor of compensatory damages. One is the assertion that they are efficient.[25] We saw above that, with compensatory damages, breach will occur if and only if there is an increase in the sum of the positional values of the principal parties to a breach—the breachers and those they breach against. In this sense compensatory damages are efficient. What this analysis leaves out, as we have already noted, is the external effect these parties have on the rest of the market. Thus, equilibrium with compensatory damages is not necessarily efficient.

Another argument in support of compensatory damages is the claim that they are identical to the damages that rational parties to a contract would themselves choose. It is useful to review this argument. Suppose that i and j are negotiating a contract that yields them positional values V^i and V^j, respectively, and suppose that, if either of them breaches, the payoff received in the new contract is independent of the damages set in the old. In such a case, i, say, will be willing to breach in order to sign a new contract with positional value $\bar{V}^i$ if and only if $\bar{V}^i - D^i > V^i$, where D^i is the damage payment that i makes to j. If damages are compensatory, $D^i = V^j - V_M$, where V_M is the value of being partnerless. Thus i will breach if and only if $\bar{V}^i + V_M > V^i + V^j$. An analogous condition holds for j. We see that, with compensatory damages, i and j will breach precisely in those cases where they can increase the sum of their positional values. Since advance side payments are possible, it is clearly in i's and j's joint interest to set damages at the compensatory level.[26]

This argument is correct under the assumption that the terms in new contracts are independent of the damage payments in existing ones. In many instances, however, this independence is implausible. If someone has to pay very high damages to form a new partnership (relative to those of his new partner), he can forcefully argue that he should receive a larger share of the product of the new contract in compensation. If new partners divide the surplus (as

[23] Moreover, the equivalence remains true under the linear technology.
[24] See McCormick (1935).
[25] See Posner (1972).
[26] See Mortenson (1978).

defined above) in some fixed proportion, a change in damage payments affects the division of output in the new partnership. Once shares in new deals become tied to previous damage payments, a pair of individuals in a contract has some monopoly power over potential partners. Raising damage payments by one dollar increases the payment to the previous partner by one dollar. But the burden of payment is shared by the new partner. Damages cannot be raised without limit because higher damages mean that breach is less likely and only when breach occurs can monopoly power be exerted. Still, they will be higher than compensatory damages.

One argument against this line of reasoning is that the nature of bargaining will be affected if it is believed that damage payments are raised precisely to exploit the new partner. This argument may reduce the importance of the phenomenon we are analyzing. It should not eliminate it completely, however, as long as liquidated damage payments are much more easily observed than positional values, as should generally be the case. That is, knowledge of positional values implies knowledge of the appropriate compensatory damages. This, in turn, implies that any attempt to increase damages to exploit monopoly power is transparent. However, once we consider a greater range in possible qualities of poor matches and the possible lack of experience of M's in evaluating the value of a match, the transparency of the attempt to exploit disappears. Required damage payments, being expressed in a written contract, may be accurately measured while the possibilities given up by a potential new partner can only be guessed. While the exploitation of monopoly power as a determinant of liquidated damages is probably less important quantitatively than other factors we have omitted from the analysis (e.g., risk sharing), it is useful to analyze how it affects equilibrium. In addition to the intrinsic interest of this question, the analysis will simplify later discussions of the effects of deliberate manipulation of damage rules by the courts (Section 10) and of contract formation with different bargaining assumptions (Section 15).

In game theoretic terms, a contract allows an established partnership to use a commitment strategy. By binding himself to pay damages, an individual puts himself in a position he cannot reverse when negotiating with a new potential partner. (Presumably the return for this binding commitment is a side payment or similar commitment by the original partner.) The possibility of renegotiating the original contract's damage rules limits the range of liquidated damages where the commitment is credible.

8. Contracting with liquidated damages

■ We assume that parties to a contract split the surplus equally. Thus, an individual's share in a new contract *does* depend on the damages set in his previous contract, and so monopoly extraction becomes possible. Such extraction alters positional values (except the value of a good contract) and thus alters search and breach strategies. This point is made rather starkly in the model: individuals make poor contracts but never complete them. The rationale for these contracts is solely to "milk" future partners for damage payments. While such contracts are artificial in this very simple setting—after all, everyone should be aware that no poor contract will be carried out—the artificiality disappears in somewhat more elaborate models where poor contracts *are* sometimes fulfilled. One such model is presented in Section 15. We saw in Section 5 that, because of externalities, compensatory damages provide too little incentive for search with quadratic meeting. We shall now see that the possibility of exercising monopoly power over potential partners may have a mitigating effect on these externalities and that, consequently, for some parameter values, a liquidated damage rule may be more efficient than compensatory damages.

We start with the case where both M's and N's are searching. The level of liquidated damages cannot be optimal if an increase in the level does not diminish the possibility of a new contract, since an increase raises the profit accruing to the original contracting pair from any new contract.[27] Thus, there are two possibilities. One is that damages are set precisely equal to the surplus that derives from a good match with an M. In this case equilibrium is under configuration B; there are single breaches (breaches where one of the new partners is an M) but no double breaches (breaches where both new partners are N's). The damages are:

$$D^B = 2X - V_M - V_N. \tag{18}$$

The second possibility is that the partners forego some of the profit from single breaches for the opportunity to make matches resulting in double breaches. In this case the steady state is under configuration A, and damages are set to exhaust the surplus from a double breach:

$$D^A = X - V_N. \tag{19}$$

If only M's search (configuration C), then a pair of N's contemplating search would use damages D^B.

We note that compensatory damages, $V_N - V_M$, are always less than chosen liquidated damages. When D^B is optimal, the inequality is clear because $2X - V_M - V_N > V_N - V_M$, provided $X - V_N > 0$, and the latter inequality holds since search is costly. When D^A is optimal, the gain to a pair of partners if one of them double breaches is $X + V_M - 2V_N$, which is nonnegative in configuration A. Thus $X - V_N \geq V_N - V_M$. If the gain from a double breach is zero, then D^B is more profitable than D^A: there is greater gain in each single breach with no lost profits from foregone double breaches.

9. Equilibrium with liquidated damages (quadratic technology)

■ As in the analysis of compensatory damages (Section 5), we proceed by checking the conditions for equilibrium under each configuration.

□ **Configuration C.** We have an equilibrium under C if search is worthwhile for an M but not for an N. We continue to assume that either both parties to a match search or neither does and that search is worthwhile for an M. With only M's searching, liquidated damages are set by N's at a level which makes the surplus from a new contract zero. The gain to a pair of N's from a good match with an M thus exceeds the gain occurring with compensatory damages. This gain to a pair of N's from a breach by one of them equals the (liquidated) damage payment minus compensatory damages. The compensatory damages

[27] We assume that the level of damages called for by the contract cannot depend on the size of the potential surplus. Here the size of potential surplus varies as the potential new partner is an M or an N. With more qualities of match, it would also vary with match quality.

measure the actual cost of dissolving the partnership; the liquidated damages measure the financial return, since the surplus in the new partnership is zero. Therefore, the upper boundary of the liquidated region C lies strictly below that of the compensatory region C. Equating per unit search cost with expected return, we obtain the equation for the boundary

$$c = (1 - p)ah_M(2X - V_M - V_N - (V_N - V_M))$$
$$= 2(1 - p)ab^{1/2}(X - X')$$

or

$$c/ab^{1/2}(X - X') = 2(1 - p). \tag{20}$$

□ **Configuration B.** We have an equilibrium under configuration B if two conditions are satisfied. One is that search is worthwhile for an N. The second is that N's prefer to set damages at D^B rather than D^A, so there are no double breaches. When all other pairs of N's set damages at D^B, the remaining pair cannot find a profitable double breach, even if they set damages at zero $(2X - 2V_N - (2X - V_N - V_M) < 0)$. Therefore, the second condition is always satisfied. Because N's make new contracts only with M's, the willingness of N's to search in configuration B is defined by the same expression, in terms of h_M, as the willingness of N's not to search in configuration C. Moreover, the number of M's is the same in the two regions, since the $\dot{h}_M = 0$ equations are the same for configurations B and C. Therefore, liquidated regions B and C partition $a - p$ space, as Figure 4 illustrates.

For later use we derive the formula for V_N^B. The value of a position now equals expected value after a brief time less search costs for that time. If an N finds a good match, he and his old partner extract all the surplus from the new contract and, between them, share a positional value of $2X$. Since either of them might find this good match, their prematch positions are the same. Thus,

$$V_N^B = -c\Delta t + 2ah_M^B\Delta t(1 - p)X + (1 - 2ah_M^B\Delta t(1 - p))V_N^B$$

or

$$V_N^B = X - \frac{c}{2ah_M^B(1 - p)} . \tag{21}$$

□ **Efficiency between B and C.** Above we determined the locus of parameters separating the regions where output is greater under configuration A than under C. We now compare output in B with C. Under configuration B, only good contracts are carried out and, in the steady state, the flow of completions equals the flow of new entrants. Therefore, using (6), we have

$$Q^B = abX - c(h_M^B + h_N^B)$$
$$= abX - cb^{1/2}\left(\frac{2 - p}{2 - 2p}\right) , \tag{22}$$

and using (16),

$$Q^B - Q^C = abp(X - X') - cb^{1/2}\left(\frac{p}{2 - 2p}\right) . \tag{23}$$

Setting $Q^B - Q^C$, as given by (23), equal to zero, we obtain the same equation as that defining the liquidated $B - C$ border, (20). Thus in the choice between configurations B and C, liquidated damages result in the efficient option. From

FIGURE 4

THE BORDER BETWEEN LIQUIDATED REGIONS B AND C WITH QUADRATIC MEETING

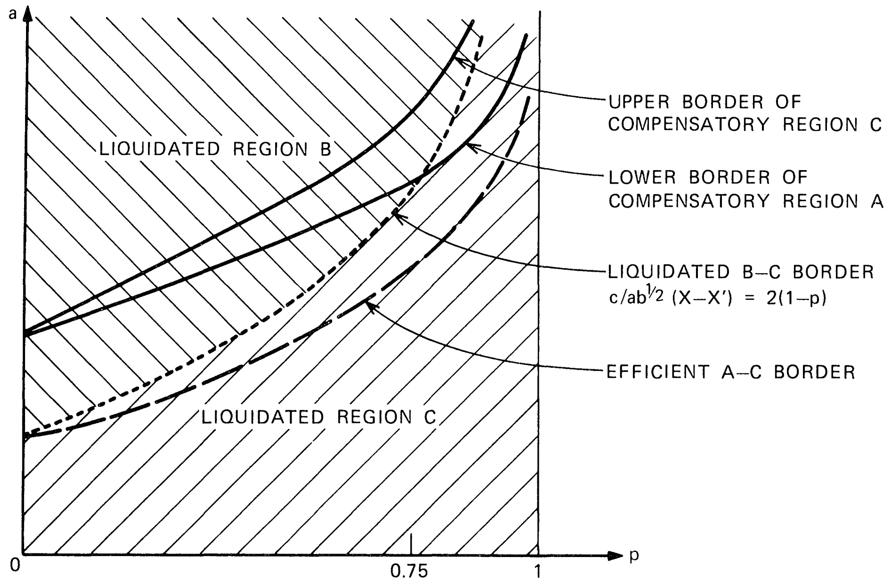


Figure 4 we see that for some parameter values B is more efficient than C, and B would occur with liquidated damages while C would occur with compensatory damages. Recall, however, that A is always more efficient than B.

□ **Configuration** A. The boundaries for steady states in configuration A are set by two conditions: the willingness of a pair of N's to continue searching and their preference for damages set at D^A rather than D^B so that double breaches are profitable. The latter condition requires the expected profit from a good match with damages set at D^A to exceed that of a good match with damages D^B, given that everyone else uses D^A. Given that a match is good, the probability is $h_M/h_M + h_N$ that it is with an M. The gain to the pair is one-half the surplus—$\frac{1}{2}(2X - V_M - V_N - D)$—plus the excess of liquidated over compensatory damages, $D - V_N + V_M$ or $\frac{1}{2}(2X - 3V_N + V_M + D)$. With damages set at D^A, the gain is $\frac{1}{2}(3X - 4V_N + V_M)$. With damages at D^B, it is $\frac{1}{2}(4X - 4V_N)$. With probability $h_N/h_M + h_N$, any good match is with an N. In this case, the gain to the pair is $X - 2V_N + V_M$ when damages are D^A. There is no surplus, and so no new match, if the damages are D^B. Thus, the condition for preferring D^A to D^B is

$$\tfrac{1}{2}h_M(X - V_M^A) \le h_N(X - 2V_N^A + V_M^A). \tag{24}$$

That is, the condition states that the expected gain from a match with an M that accrues from higher (i.e., D^B) damages be less than the expected foregone profits from double breaches.

To evaluate (24) we must determine the values, V_i^A. Because there is no discounting, the sum of entering M's positional values equals the net output flow. Hence

$$V_M^A = Q^A/ab = X - \frac{c}{ab^{1/2}(1-p)^{1/2}}, \tag{25}$$

from equation (15). The simplest way to proceed is to derive another expression for V_M^A by considering the possible positions an M could attain in a brief time Δt. He pays search cost $c\Delta t$ for this time and could meet another M to form a good or poor partnership, or could meet an N to make a good contract, or could form no new partnership at all. Thus,

$$V_M^A = -c\Delta t + ah_M\Delta t(1-p)X + ah_M\Delta tpV_N^A + \tfrac{1}{2}ah_N\Delta t(1-p)(X+V_M^A) + (1 - ah_M\Delta t - ah_N(1-p)\Delta t)V_M^A. \quad (26)$$

We can replace V_M^A by $X - (c/ab^{1/2}(1-p)^{1/2})$ in (26) and solve for V_N^A to obtain

$$V_N^A = X - \frac{c}{a(1-p)(h_M+h_N)}\left(1 - \frac{1-p}{2Rp}\right)$$
$$= X - \frac{c}{a(1-p)^{1/2}b^{1/2}}\left(1 - \frac{1-p}{2Rp}\right), \quad (27)$$

where

$$R = \frac{1-p}{p}\left[1 + \left(\frac{1+p}{1-p}\right)^{1/2}\right] = \frac{h_M^A}{h_N^A}. \quad \text{(See (8).)}$$

Returning to (24), the condition under which D^A is preferable to D^B is

$$\tfrac{1}{2}h_M\frac{c}{ab^{1/2}(1-p)^{1/2}} \le h_N\left(\frac{2c}{a(1-p)^{1/2}b^{1/2}}\left(1 - \frac{1-p}{2Rp}\right) - \frac{c}{ab^{1/2}(1-p)^{1/2}}\right)$$

or

$$\frac{R}{2} \le 1 - \frac{(1-p)}{Rp}. \quad (28)$$

Solving for p, we obtain

$$p \ge \frac{\sqrt{2}}{2}. \quad (29)$$

Thus, a liquidated damage equilibrium with configuration A becomes possible only for "high" values of p, the probability of a poor match. Although higher p makes a good match less likely, the choice between D^B and D^A depends on profitability of breach *assuming* a good match. As p rises, so does steady state h_N relative to h_M. (See (8). The relationship between h_M^A/h_N^A and p is shown in Figure 5.) Thus, from (24), the potential profit from double breaches becomes more important.

The second condition for a configuration A equilibrium is that V_N^A exceed X', so that search is worthwhile. With (25), this condition can be written

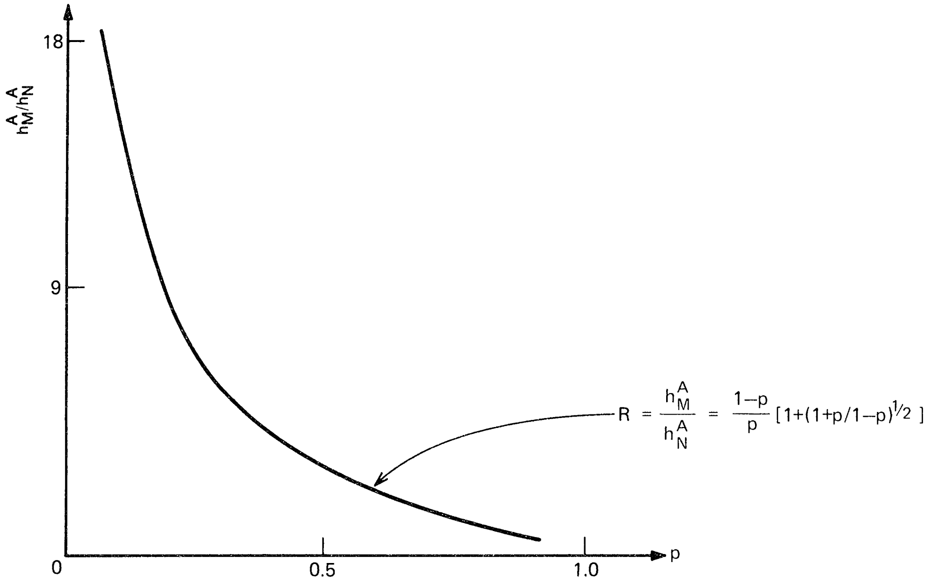
$$\frac{c}{a(X-X')b^{1/2}} \le (1-p)^{1/2}\left(1 - \frac{1-p}{2Rp}\right)^{-1}. \quad (30)$$

Region A is shown in Figure 6, where we have plotted the equation corresponding to (30) across the entire $a - p$ plane, although only a segment of this curve represents the liquidated border.

With compensatory damages we have noted that the amount of search in equilibrium is no greater than efficiency requires. Notice that the same bias holds for liquidated damages, even though they induce more search than do compensatory damages. This can be seen in Figure 6, where the $A - C$ efficiency border is uniformly below the lower borders of both liquidated regions A and B. Therefore, no equilibrium can involve more than the efficient level of search.

FIGURE 5

THE RATIO OF THE NUMBER OF M's TO N's

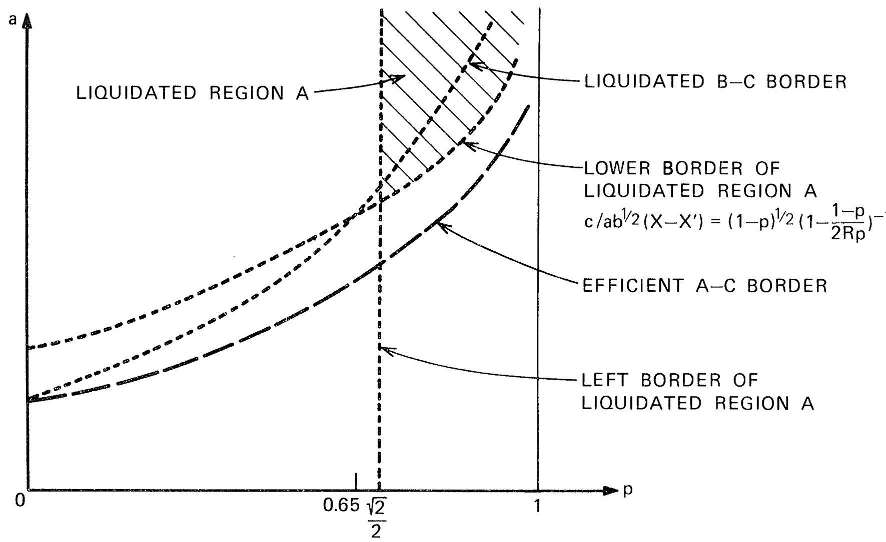


10. Setting damages for efficiency, a partial summary of the quadratic case

■ With a quadratic technology neither compensatory nor liquidated damages necessarily result in an efficient equilibrium. One may ask, therefore, whether some other damage rule does. The answer is no, but the absence of efficient damages should not, by itself, be terribly surprising. Damage rules affect both search and breach decisions. Only by happy coincidence could a single instrument induce the right decisions in both categories.

FIGURE 6

CONFIGURATION A EQUILIBRIUM FOR LIQUIDATED DAMAGES AND QUADRATIC MEETING



From Figure 3 we see that when configuration C is efficient, the use of compensatory damages results in an efficient equilibrium. Difficulties arise, however, when configuration A is efficient. Notice that for some parameter values where A behavior is efficient, compensatory damages lead only to an equilibrium in configuration C. When partners can set their own damages, they improve the profitability of breach, and hence of search, by increasing damages. Thus, if courts set damages above the compensatory level, they can improve the incentives for search and thereby enlarge the set of parameters for which an equilibrium in A can occur. Damages can be increased, however, only to the point where the surplus from a double breach becomes zero; i.e., where $D = D^A$. Beyond this point, double breaches become unprofitable, so that equilibrium in A is impossible. Thus, the set of parameter values for which configuration A is attainable consists exactly of those values above the extended lower border of liquidated damage region A. The constraint that D_A be more profitable than D_B, $p \geq \sqrt{2}/2$, is not relevant when courts are setting damages. As Figure 6 makes clear, there are choices of a and p for which configuration A is efficient but not sustainable as an equilibrium.[28]

11. Linear meeting technology

■ In markets where potential traders are hard to find, the quadratic technology is a plausible simple approximation to the process of traders' meeting. When there are many traders, however, an individual's problem is less one of finding a potential partner than of finding a partner who makes a good match. Such markets can be approximated by assuming that the rate of finding potential traders is independent of the number of potential traders searching. Then, additional searchers do not raise the probability of others' finding trading partners. Since searchers generally prefer to meet an M rather than an N, the distribution of searchers between M's and N's is important. Additional M's generally make search more valuable for potential partners, while additional N's tend to have the opposite effect.

To study the linear technology we follow the same procedure as before. We first consider equilibrium with compensatory damages. Since, in equilibrium, poor matches are never made, the issue of $M - N$ distribution does not arise; all searchers are M's. Therefore, searchers exert no externalities on others, and equilibrium is efficient. We then examine liquidated damages, where we demonstrate that the incentives for search and contract formation may be too great.

12. Dynamics of the linear technology

■ With the linear technology, a searcher has a probability $ah_M/(h_M + h_N)$ of finding a potential partner who is an M and a probability $ah_N/(h_M + h_N)$ of finding an N (assuming $h_M + h_N > 0$), where h_M and h_N, as before, denote the numbers of M's and N's among searchers. The analysis of the numbers of searchers closely parallels that of the quadratic technology, the only change

[28] Even if one ignored the desirability of double breaches, the ability to subsidize search of N's by higher damages is limited by the need to keep single breaches profitable; i.e., the need to have M's willing to enter such partnerships. The set of parameters for which search by N's can be induced, at all, is thus limited to the region of liquidated B equilibria.

being the substitution of $ah_i/(h_M + h_N)$ for ah_i (with $i = M$ or N) in the differential equations.

In configuration C only M's search. Their numbers increase by new entrants, ab, and decrease because of matches, each of the h_M searchers having a probability of meeting a. Thus,

$$\dot{h}_M = ab - ah_M. \tag{31}$$

The steady state thus occurs at

$$h_M^C = b. \tag{32}$$

In configuration B the rate of meetings between two M's is no longer independent of the number of N's. Rather it equals $ah_M^2/(h_M + h_N)$. New N's are created by poor matches and are eliminated by single breaches. Thus,

$$\dot{h}_M = ab - ah_M^2/(h_M + h_N)$$

and

$$\dot{h}_N = aph_M^2/(h_M + h_N) - 2a(1 - p)h_M h_N/(h_M + h_N). \tag{33}$$

Setting these two equations equal to zero, we have the steady state values

$$h_M^B = b\,\frac{2 - p}{2 - 2p}$$

$$h_N^B = b\,\frac{p(2 - p)}{4(1 - p)^2}\,. \tag{34}$$

Configuration A differs from B by the occurrence of double breaches, which add M's and subtract N's. Thus

$$\dot{h}_M = ab - ah_M^2/(h_M + h_N) + a(1 - p)h_N^2/(h_M + h_N)$$

$$\dot{h}_N = aph_M^2/(h_M + h_N) - 2a(1 - p)h_M h_N/(h_M + h_N) - 2a(1 - p)h_N^2/(h_M + h_N). \tag{35}$$

These give the steady equations

$$-(h_M^A)^2 + (1 - p)(h_N^A)^2 + b(h_M^A + h_N^A) = 0$$

$$p(h_M^A)^2 - 2(1 - p)h_M^A h_N^A - 2(1 - p)(h_N^A)^2 = 0. \tag{36}$$

For some calculations we are interested in the aggregate number of searchers, h. Under configuration A agents stop searching only after good matches; hence, the exit rate is $a(1 - p)h$. Thus

$$\dot{h} = ab - a(1 - p)h. \tag{37}$$

Thus we have the steady state value

$$h^A = \frac{b}{1 - p}\,. \tag{38}$$

Comparing these equations with those for the quadratic technology, we note that the ratios of searchers without and with partners, h_M^i/h_N^i, are the same under both technologies for $i = A$ or B.

13. Compensatory damages with a linear technology

■ As with the quadratic meeting process, the linear technology produces no steady state equilibria with compensatory damages under configuration B.

Under configuration C the value of search for an M is the expected output $pX' + (1-p)X$ less expected search costs c/a. (Again, we assume that this value is positive.) An N would gain $X - X'$ from further search and would expect to incur search costs $c/a(1-p)$. Thus, the condition for equilibrium under configuration C is

$$c/a(X - X') \geqq (1-p).$$

That is, the compensatory region C border is defined by the equation $c/a(X - X') = 1 - p$. We shall see that all the other border conditions also take the form $c/a(X - X')$ equal to some function of p. Notice that these equations are independent of b. This independence derives from a search technology where the number of potential partners does not affect the probability of meetings. Net output under configuration C equals the expected value of a match times the rate of matching, less the aggregate flow of search costs. It also equals the aggregate value of positions of new entrants:

$$\begin{aligned} Q^C &= ab(pX' + (1-p)X) - bc \\ &= abV_M^C. \end{aligned} \tag{40}$$

Under configuration A everyone searches until finding a good match. Thus, a poor match has the same positional value as no match at all. An N finds further search worthwhile if the expected gain, $X - X'$, exceeds expected search costs $c/a(1-p)$. This condition is the complement of that for equilibrium under C. Thus, as Figure 7 illustrates, compensatory regions A and C form a partition of $a - p$ space.

Calculating net output under configuration A, we obtain

$$Q^A = abX - \frac{cb}{1-p} = abV_M^A. \tag{41}$$

Comparing the equations for Q^A and Q^C with those defining regions A and C, we find that the efficiency border coincides with the compensatory $A - C$ border. That is, with the linear technology and compensatory damages the equilibrium configuration is efficient.

For later reference we note that aggregate net output under configuration B is given by

$$\begin{aligned} Q^B &= abX - c(h_1^B + h_2^B) \\ &= abX - c\left(\frac{2-p}{2(1-p)}\right)^2. \end{aligned} \tag{42}$$

Thus the efficient $B - C$ border is defined by

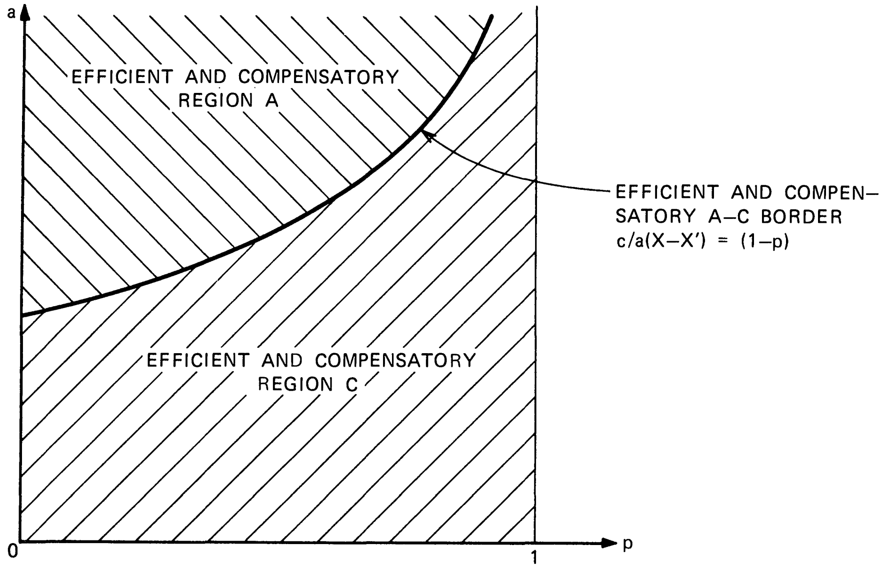
$$c/a(X - X') = \frac{4(1-p)^2}{4-3p}. \tag{43}$$

14. Liquidated damages with a linear meeting technology

■ As before, the use of liquidated damage rules may lead to the signing of a class of contracts which are never carried out. That they are never carried out and yet are valuable underscores the fact that with liquidated damages at least part of the value of contract is the profit extracted from a new partner if the contract is breached.

FIGURE 7

EFFICIENT BEHAVIOR AND EQUILIBRIUM WITH COMPENSATORY DAMAGES AND LINEAR MEETING



☐ **Configuration *C*.** For an equilibrium under configuration C, a pair must not find continued search profitable. If they do continue to search, they will set damages equal to $2X - V_N - V_M$ to extract all the surplus from a new match. The pair incurs costs $2c\Delta t$ to search for time Δt. Their gain from a good match is the excess of liquidated damages over compensatory damages (which are $V_N - V_M$). When search is just worthwhile, V_N equals X'. Thus the condition for an equilibrium is

$$c \geq 2a(1 - p)(X - X') \qquad \text{or} \qquad c/a(X - X') \geq 2(1 - p). \tag{44}$$

The border defining the liquidated region C lies below (see Figure 8) the compensatory $A - C$ border. Since for N's, search with liquidated damages is more valuable than search with compensatory damages, the liquidated region C is smaller than its compensatory counterpart.

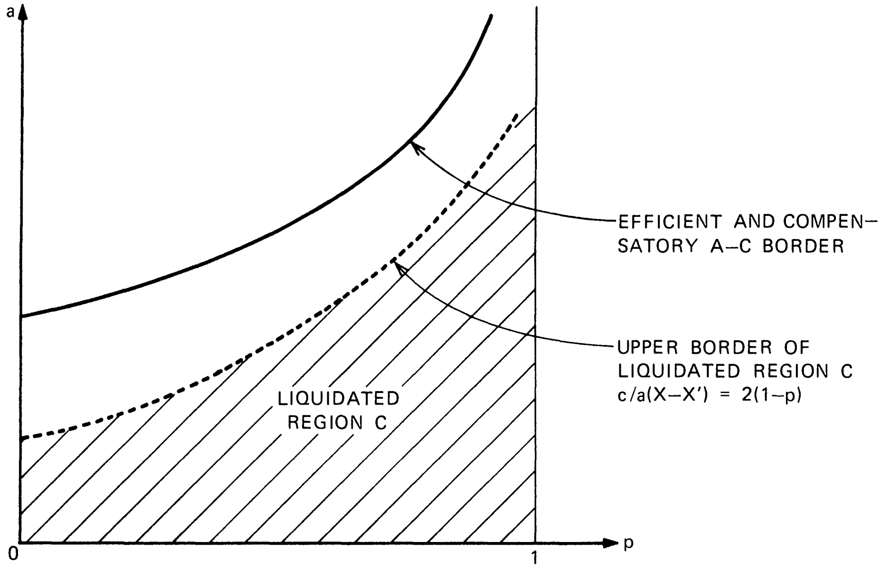
☐ **Configuration *B*.** For an equilibrium under configuration B, search must be worthwhile to an N whose contract sets damages at $2X - V_N - V_M$. Liquidated regions B and C are not contiguous for the linear technology, as they were for the quadratic technology. The gap between the regions derives from the fact that when other N's are searching (as in configuration B), search with a linear technology is less worthwhile than when they are not (as in configuration C). Under configuration B search by a pair of N's costs $2c$ per period and yields profits $(X - X')$ with probability $2a(1 - p)h_M^B(h_M^B + h_N^B)^{-1}$. Thus an equilibrium under configuration B requires[29]

$$\frac{c}{a(X - X')} \leq \frac{4(1 - p)^2}{2 - p} \,. \tag{45}$$

[29] As with the quadratic technology, D^B is preferable to D^A when all other N's are using D^B.

FIGURE 8

EQUILIBRIUM IN CONFIGURATION C WITH LIQUIDATED DAMAGES AND LINEAR MEETING



As Figure 9 shows, the efficient $B - C$ border lies above the liquidated borders of both regions B and C. Thus, a liquidated equilibrium under configuration B is sometimes possible where configuration C behavior is more efficient. That is, between configurations B and C, there is an unambiguous bias with liquidated damages in favor of B, implying too much search relative to efficiency.

□ **Configuration** A. For a steady state under configuration A two conditions must hold: damages $D^A = X - V_N$ must be preferable to $D^B = 2X - V_M - V_N$ and continued search by N's must be worthwhile. The first condition will determine the left border of liquidated region A, and the second, the lower border.

Rather than deriving these equations directly, we shall obtain them by relating the linear model to the quadratic. First, we notice that h_M^A/h_N^A is the same in both models. Thus, if the aggregate meeting rate in the quadratic model, $a^Q(h_M^{AQ} + h_N^{AQ})$ equals the linear meeting rate a^L, the consequences of search look the same to an individual under either technology. The equations for the positional value in the quadratic model with parameter a^Q are consequently the same as those in the linear model with parameter $a^L = a^Q(1-p)^{-1/2}b^{1/2}$ (using (10) for $h_M^{AQ} + h_N^{AQ}$). The equations for the region A borders in the linear model can be obtained from their counterparts in the quadratic model by substituting for a^Q. For the left border, no substitution is needed, and we again have $p = \sqrt{2}/2$. For the lower border we substitute into (30) to obtain

$$c/a(X - X') = (1-p)\left(1 - \frac{1-p}{2Rp}\right)^{-1}, \tag{46}$$

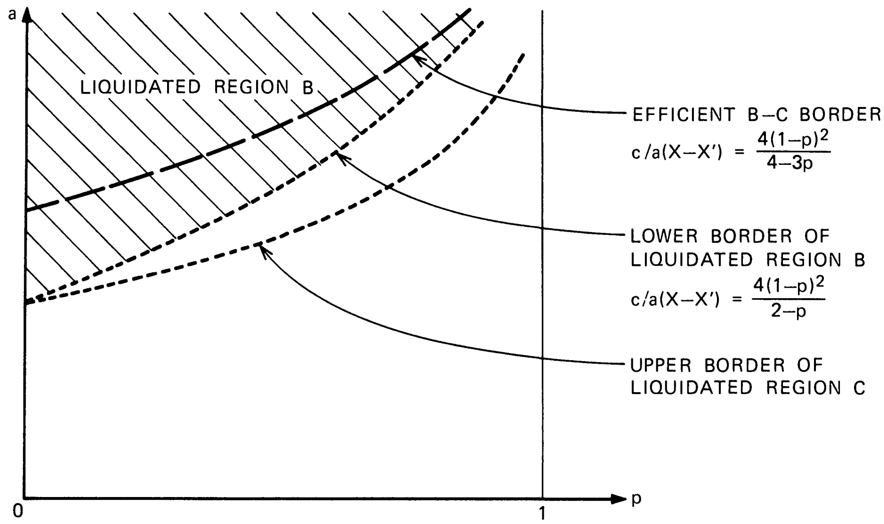
where

$$R = \frac{1-p}{p}\left(1 + \left(\frac{1+p}{1-p}\right)^{1/2}\right) = \frac{h_M^A}{h_N^A}.$$

Region A is shown in Figure 10.

FIGURE 9

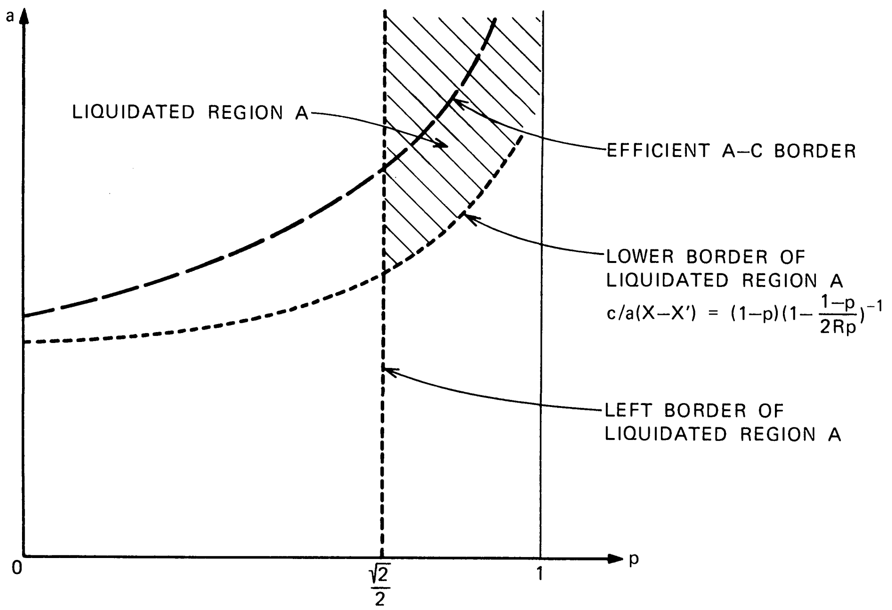
EQUILIBRIUM IN CONFIGURATION B WITH LIQUIDATED DAMAGES AND LINEAR MEETING



From the figure we see that configuration A can occur as a liquidated equilibrium both where it is efficient and where configuration C would be more efficient. That is, there is a tendency toward too much search. The fact that configuration B can arise as an equilibrium outcome, whereas B is never efficient, demonstrates that for liquidated damages in the linear model, as in the quadratic, there is a bias towards too little breach of contract. Unlike the quadratic case, however, there are parameters in the linear model for which no liquidated equi-

FIGURE 10

EQUILIBRIUM IN CONFIGURATION C WITH LIQUIDATED DAMAGES AND LINEAR MEETING



librium falls into configurations A, B, or C. We have not explored the situation where some N's search while others do not.

15. A model of exaggeration

In the model above, the decision to continue searching is made *jointly* by poorly matched partners, based on their *joint* financial interests. With compensatory damages this assumption is of no consequence: each person is indifferent about whether his partner searches; each would make the same individual decision as the joint decision. With liquidated damages, the situation is different. Some (configuration A) or all (configuration B) of the profit from continued search goes to one's current partner in the form of damage payments above the compensatory level. Thus, individual and joint decisions would often differ. Only by assuming that search behavior is observable can we postulate a jointly made search decision and joint maximization. Alternatively, we might assume an *ex post* side payment which returns to the breaching party the excess of liquidated over compensatory damages in return for the promise of similar treatment should roles be reversed. There would then be no concern over the observability of search behavior. Instead we would assume that the liquidated damage clause, but not the side payment agreement, is observable by potential new partners.

Even with compensatory damages, we have implicitly assumed that potential new partners can observe the damage payment that must be made. We have also assumed observability of the qualities of the existing matches, and so the values of the current positions of potential partners. Without these assumptions, we would need an alternative specification of the division of the output of a new contract.

Now let us consider the implications of dropping the assumption that the value of an existing match is observable. We continue to assume that everyone knows the value of search for an individual without a partner. For this analysis we assume compensatory damages.

We shall not attempt to introduce stochastic elements into the contracting process. Rather we shall examine the equilibrium where an individual with a partner is free to make any claim for the value of his existing match. These claims are treated as fully truthful in the negotiation process. We continue to assume that compensatory damages are based on the true value of an existing position. Of course, these assumptions are artificial in the extremely simple model we employ. They would be less artificial with a greater number of possible matches.

Let V_N be the value of the position of an individual in a poor match. We assume that an N claims a positional value of $V_N + E$, where E is the amount of exaggeration.[30] When a meeting takes place, the surplus from a potential new match is calculated by using $V_N + E$, rather than V_N. This determines both whether a new contract is made and the division of output if a new contract is undertaken. We assume that E is chosen before one knows the position of one's potential partner. Analytically, E serves the same role as the excess of liquidated over compensated damages.

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[30] In using the true value of V_N for damage payments, we are assuming that V_N incorporates the gains from future exaggeration.

To compare these two mechanisms, exaggeration and liquidated damages, let us consider a meeting between j and k, who make a good match. With liquidated damages, they will sign a new contract if there is a positive surplus, $2X \geq V^j + V^k - D^j - D^k$, where the V^i's are true values and the D^i's are chosen damages. The damage payment, D^i, is necessarily zero for a party without a partner. With exaggeration, a new contract is made if apparent surplus is positive, $2X \geq V^j + V^k + E^j + E^k - D^j - D^k$, where damages here are compensatory. If party i has no partner, both D^i and E^i are zero. Apparent surplus is divided equally between the new partners. Thus, with either model there is a gain of half a dollar for each dollar of augmentation of either value or damages, provided the deal still occurs. The only substantive difference between exaggerating value and exaggerating damages, therefore, is the distribution of the additional payments. By the assumptions about joint maximization, however, it makes no difference for individual behavior which partner receives the additional payments.

Imperfect negotiation is a real phenomenon. We have not gone very far toward modeling it. Where, as here, the imperfections do not affect the bargainers equally, the presence of imperfection will affect search incentives and thus equilibrium.

16. Efficiency and completion of poor contracts

■ In this section we review the efficiency characteristics of the models analyzed above, which we will refer to collectively as the basic model, and discuss their robustness when the model is extended so that poor contracts are occasionally completed when N's choose to search. These results are summarized in Table 2. In this extended model, each individual faces a probability, $aK\Delta t$ per time interval Δt, of being forced to leave the search market. An M who is unable to continue searching exits with zero output. If an N must leave, his partner can choose to join him to carry out their project or can reenter the search market as an M. Carrying out the project is mutually advantageous if the output of the project exceeds the value of continued search, $2X' \geq V_M$. Appendix 2 contains the equations for this extended model.[31]

□ **Double breaches.** In the basic model, behavior under configuration A is more efficient than that under configuration B for any set of parameter values. Since there are no compensatory equilibria with configuration B, the amount of breach is always correct, given the decision of N's to search. In contrast, the possibility of configuration B equilibria with liquidated damages indicates a tendency toward too little breach with these higher damages. These results hold with both linear and quadratic meeting technologies.

In the extended model there are parameter values for which configuration B is more efficient than configuration A (see Appendix 2, example 1). With A_1 search costs decrease from realizing good matches by double breaches rather

[31] Economies where market conditions change over time—either for individuals or in the aggregate—constitute another class of situations where search by N's and completion of poor contracts are simultaneously possible. An example of an individual charge is a nonconstant search cost. Changing aggregate conditions might be due to a nonconstant flow of new contracts or, as in the sequel, to a diminishing number of searchers.

TABLE 2

EFFICIENCY OF EQUILIBRIUM*

QUADRATIC MEETING		LINEAR MEETING	
COMPENSATORY DAMAGES	LIQUIDATED DAMAGES	COMPENSATORY DAMAGES	LIQUIDATED DAMAGES
TOO LITTLE SEARCH	TOO LITTLE SEARCH	EFFICIENT SEARCH (TOO MUCH SEARCH)	TOO MUCH SEARCH
EFFICIENT BREACH (TOO LITTLE DOUBLE BREACH)	TOO LITTLE DOUBLE BREACH	EFFICIENT BREACH (TOO LITTLE DOUBLE BREACH)	TOO LITTLE DOUBLE BREACH

*PARENTHETIC STATEMENTS REFER TO THE EXTENDED MODEL WHEN DIFFERENT FROM THE BASIC MODEL.

than waiting for single breaches. This may be less than the gain with B from forced departure by N's rather than M's. In addition, it is now possible to have equilibrium under configuration B with compensatory damages. For some parameter values such a B-equilibrium is efficient. For other parameter values net output is higher under configuration A than with the equilibrium behavior of configuration B (see Appendix 2, example 2). Thus, in the extended model there are parameter values for which there is a tendency toward too little breach, even with compensatory damages.[32]

To understand this bias consider a configuration B equilibrium with compensatory damages where double breach is just worthwhile; that is, where $X + V_M^B = 2V_N^B$. Let us analyze the effect of one double breach. For the four parties to the breach, there is, by assumption, no net gain or loss. Others, however, are affected by the altered search environment: two searching N's are replaced by one searching M. We shall argue that this alteration creates an external economy. It will suffice to consider the quadratic technology; the argument is even stronger for the linear case.

For the typical N there is no value to meeting another N when double breaches are just worthwhile. On the other hand, there is a positive gain ($(1 - p) \times (X - V_N)$) to meeting an M. Since a double breach increases the number of M's, the direct effect of a double breach is to increase V_N. For the typical M the net gain from meeting an N is $(1 - p)(X - V_N)$, while that from meeting an M is $(1 - p)(X - V_M) + p(V_N - V_M)$. The direct effect of a double breach on an M is therefore $(1 - p)(X - V_M) + p(V_N - V_M) - 2(1 - p)(X - V_N) = p(V_N - V_M) > 0$, using the fact $X - V_N = \frac{1}{2}(X - V_M)$ (since we are considering the situation where double breaches are just worthwhile). Thus, the direct effect of a double breach on search market possibilities improves the position of all other M's and N's.

This change in positional values for all searchers implies a change in the surplus from meetings, which in turn alters positional values. This is the indirect effect of double breach. With compensatory damages, alterations in an M's positional value do not affect the positional value of a potential partner who is an N. Thus, since under configuration B, N's engage only in single breaches,

[32] We believe that there do not exist equilibria in configuration A when configuration B is more efficient.

there are no indirect effects on N's. An M, however, is indirectly affected in two ways. His positional value is increased since, with probability $aph_M\Delta t$, he will become an N himself. If the increase in V_N is ΔV_N, the value of this effect is $aph_M\Delta V_N\Delta t$. The M's positional value is adversely affected by the diminished surplus from partnerships with N's. Since the surplus equals $2(X - V_N)$ and the probability of meeting an N and forming a good match is $a(1 - p)h_N\Delta t$, the adverse influence is $-a(1 - p)h_N\Delta V_N\Delta t$, and the total indirect effect is $(aph_M - a(1 - p)h_N)\Delta V_N\Delta t$. Since $ph_M^B > (1 - p)h_N^B$ (see Appendix 2), this effect is positive. Thus, double breaches induce external economies and we can expect a tendency toward too little breach in equilibrium.[33]

□ **Search behavior, quadratic technology.** With the quadratic technology additional searchers imply additional possibilities of meeting. When N's are willing to search, the gain from breaches is at least as large as the cost of search. With compensatory damages, the surplus from a single breach is shared with an M. Thus, additional searching by N's generates an external economy to M's and we expect a bias toward too little search. In other words, when deciding whether to search, N's consider their share of the surplus from single breaches rather than the full social value[34] of single breaches. Since the social value of a single breach exceeds the gain to N, an N's incentive to search is too small. Thus, we can have equilibria under configuration C when either A or B would be more efficient. The extension of the basic model we are considering does not alter this conclusion.

With liquidated damages the analysis is somewhat different. Under configuration B the gains to search are taken in the form of damages above the compensatory level and the surplus is set equal to zero. Thus there is no effect on M's of additional search by N's who are setting liquidated damages to follow configuration B rules. Indeed, for the quadratic technology the borderline between regions of B and C liquidated damages equilibria coincides with the border separating regions where configuration B behavior is more or less efficient than that of configuration C. Alternatively, if damages are set to exhaust the surplus from a double breach (configuration A behavior), there is a positive surplus from a single breach and the external economy argument applies once again.[35]

□ **Search behavior, linear technology.** With a linear technology we found that compensatory damages were efficient in the basic model. This result depends crucially on poor projects' never being completed (when N's search) and, therefore, does not carry over to the extended model. Instead, search by N's generates a negative externality.

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[33] The argument is virtually the same for the linear technology, only the direct effect on an M is even more clearly positive since increasing the number of M's and decreasing the number of N's both improve the distribution of potential partners.

[34] As modeled, a single breach alters the search environment by reducing the number of N's by one. This externality, in general, affects the remaining searchers, but does not generate an inefficiency. If it is socially advantageous that N's search, then it is socially desirable that they single breach because a single breach is always at least as socially profitable as a double breach. This result does not hold if we drop the assumption that all M's are alike. In that case some single breaches are desirable, while others may not be.

[35] Appendix 1 shows the insufficiency of search with compensatory damages in a model with a continuum of match qualities. The next section discusses the difficulties in analyzing search with liquidated damages with more than two qualities of match.

With the linear technology additional searchers do not increase the probabilities of meetings of existing searchers. Rather, search by N's substitutes the probability of meeting an N for some of the probability of meeting an M. In the basic model having a partner was of no value to searching N's: $V_N = V_M$. Thus M's are unaffected by a change from meeting an M to meeting an N. In the extended model, having a partner is worthwhile in the event one is forced to leave (provided $2X' > V_M$). Thus, $V_N > V_M$ and everyone prefers meeting an M to meeting an N. Thus, there is a consequent tendency to too much search. This is illustrated by example 3 in Appendix 2 where there is an equilibrium with configuration B where configuration C is more efficient.

In the basic model liquidated damages make a partner worth having, so that $V_N > V_M$. Thus there is a tendency toward too much search. The same effect exists in the extended model.[36]

Once one recognizes that searching for good deals is a costly activity, it is clear that the behavior which affects the expected cost of finding a good deal will generate externalities. When individuals offering below-average deals join the search process, their effect on others depends on the characteristics of the meeting technology. Where the dominant effect of these additional searchers is an increased opportunity for deals, they generate external economies. Where the dominant effect of these additional searchers is to make it more difficult to find better deals, they generate external diseconomies.

17. Comparison of liquidated and compensatory damages

■ In the basic model it is straightforward to consider the effects of increasing damages from their compensatory level to the level arising in equilibrium with liquidated damages. The increase in damages raises the return to search, since higher damages extract profits from M's. This, in turn, can lead to an increase in the amount of search in equilibrium—region C with compensatory damages is larger than region C with liquidated damages.[37] Furthermore, the increase in stated damages can make double breaches unprofitable. Thus with liquidated damages we have only a configuration B equilibrium for some of the parameters where the compensatory equilibrium falls under A.

These two results—more search and less breach with liquidated as compared with compensatory damages—do not carry over to more general models.[38] To see possible complications assume there are three qualities of match. We refer to partnerless individuals as M's, those in the poorest matches as N's, and those with intermediate matches as O's. Assume that there is an equilibrium with compensatory damages with all parties searching until they find the best match. If we change to liquidated damages, the N's may set damages at a level to rule out double breaches to produce new contracts both between N's and between an N and an O. This decreases the value of search for O's. The increase in profitability from single breaches raises the value of search for O's. Either of these conflicting effects can dominate. Thus, there may be less search in the liquidated equilibrium than in the compensa-

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[36] In the extended model two N's might exit when one must leave although it is socially advantageous to have one of them continue as an M. In this sense there may be too little search.

[37] Also the union of liquidated regions A and B is larger than compensatory region A.

[38] Examples confirming this statement are available from the authors.

tory. However, for those compensatory equilibria with no search by those with partners, the corresponding liquidated equilibria do involve search by some individuals with partners.

To see a possible complication in breach incentives, consider the three-quality model where there is a probability of forced end of search, as in the previous section. Assume that in the compensatory damage equilibrium, two N's double breach for the best match, but not for an intermediate match, while two O's never double breach. The switch to liquidated damages conceivably might raise the positional value of an O to the point where N's choose to set damages low enough (although still higher than compensatory) to make double breach by N's for an intermediate contract worthwhile. This implies an increase in double breach.

18. Concluding remarks

■ The basic results of the paper have been summarized in Sections 10, 16, and 17. We wish to close by addressing the issues that motivated our undertaking this study. That information about trading opportunities is limited is a pervasive fact of economic life outside some well-organized markets. Much formal economic analysis has tended to skirt around this limitation, however, because of modeling difficulties. Our analysis, we hope, offers to incorporate informational limitations explicitly. We mention several possibilities.

In a model of frictionless markets the implications of a contract breach are easily measured in terms of market prices. In addition, the altered willingness to trade with others which follows a contract breach is of little consequence to others who have access to markets, unless the breaching parties are large relative to the market. We have considered limited trading opportunities, complicating the evaluation of the effects of breach on the parties involved and also permitting their changed behavior to affect others significantly. Furthermore, ignorance, uncertainty, risk aversion, and production decisions before market prices become known seriously complicate the analysis of the efficiency effects of different rules for measuring damages. A proper consideration of these issues should be imbedded in a market structure that reflects the same facts of economic life. While we have not come to grips with these additional issues, our equilibrium approach to modeling contracting may prove useful in addressing them.

Regular trading habits, based in part on limited information about alternatives, are an important basis for the "goodwill" value of ongoing businesses. Many government actions, such as highway relocation, seriously affect these values. In an equilibrium model where these values are endogenously determined one may be able to examine their proper treatment by benefit-cost analysis.

The mathematical search literature has focused on two issues—the presence of a distribution of prices in the market for a homogeneous good and the tendency for prices to exceed marginal costs where limited information on trading opportunities generates monopoly power for the price setter. As a fairly tractable model of equilibrium, the framework we have developed is usable for further analyses of these questions as well as for more general consideration of efficiency with search. After the model has been altered to consider continuous production, it may prove interesting in explorations of the workings of the labor market.

Appendix 1

Continuum of qualities, compensatory damages

■ For the quadratic technology we briefly consider a model with a continuum of qualities in which equilibrium with compensatory damages is inefficient for any parameter values where search is worthwhile.[39] Let $F(X)$ be the distribution of quality of match with output $2X$, with density f. We assume that the distribution has no atoms and no gaps. We also assume that search is profitable.

Equilibrium is characterized by a quality X^0 such that individuals search until finding a match of quality at least X^0. Since in a steady state the search environment does not change, a contract that is never carried out has positional value no greater than no partnership at all and, consequently, entails no compensatory damages. At X^0, the value of continued search just equals the value of the current contract, X^0. The value of search, with a cutoff rule X^0 equals expected output $\int_{X^0}^{\infty} XdF(X)/(1 - F(X^0))$ less expected search costs $c/ah(1 - F(X^0))$, where h is the number of searchers

$$h = b^{1/2}(1 - F(X^0))^{-1/2}. \tag{A1}$$

Thus, at equilibrium X^0 satisfies[40]

$$X^0 = \int_{X^0}^{\infty} XdF(X)(1 - F(X^0))^{-1} - ca^{-1}b^{-1/2}(1 - F(X^0))^{-1/2}. \tag{A2}$$

At equilibrium the net output level equals the average quality of match times the number of matches (which equals the flow of new entrants) less search costs, ch:

$$Q^0 = ab \int_{X^0}^{\infty} XdF(X)(1 - F(X^0))^{-1} - cb^{1/2}(1 - F(X^0))^{-1/2}. \tag{A3}$$

Differentiating Q^0 with respect to X^0, we see that a small increase in the willingness to search always raises net output

$$\frac{dQ^0}{dX^0} = \tfrac{1}{2}abf(X^0)(1 - F(X^0))^{-2}\left(\int_{X^0}^{\infty} XdF - X^0(1 - F(X^0)\right) > 0. \tag{A4}$$

Appendix 2

A model with completion of poor contracts: examples of inefficiency

■ Section 16 argues that the models of sections 2–14 do not exhibit the full range of possible search and breach inefficiencies. To illustrate those omitted inefficiencies, we alter the model to assume that, with probability $aK\Delta t$ in time Δt, any given individual must leave the search market. If he is an M, he exits with zero payoff. If an N, he and his partner can both leave and carry out their project, each receiving payoff X'. For cases where $2X' > V_M$, both partners

[39] With the linear technology equilibrium would be efficient.

[40] There may be multiple equilibria. A sufficient condition for uniqueness is to have a single sign for $1 - F(X) + \tfrac{1}{2}f(X)(X - (1 - F(X))^{-1} \int_X^{\infty} XdF)$.

would choose to depart together if one were forced to leave. We shall only consider configurations with this property. Notice that because one partner's departure leads to the other's leaving too, the probability that an individual in a poor partnership exits is actually $2aK\,\Delta t$ in time Δt.

For the quadratic meeting technology, examples 1 and 2 show the existence of an equilibrium under configuration B. In the former example configuration B is most efficient; in the latter configuration A is most efficient. For the linear technology, example 3 shows equilibrium in configuration B when both configurations A and C are more efficient.

For the quadratic technology, the equations of motion are obtained by adding departure terms to equations (3), (5), and (7).

Hence, under configuration C,

$$\dot{h}_M = -ah_M^2 + ab - aKh_M. \tag{A5}$$

This equation yields the steady state

$$h_M^C = -\frac{K + (K^2 + 4b)^{1/2}}{2}. \tag{A6}$$

Aggregate net output is

$$Q^C = a(h_M^C)^2(1 - p)X + a(h_M^C)^2 pX' - ch_M^C. \tag{A7}$$

Under configuration B, we have:

$$\begin{aligned} \dot{h}_M &= -ah_M^2 + ab - aKh_M \\ \dot{h}_N &= aph_M^2 - 2ah_M h_N(1 - p) - 2aKh_N. \end{aligned} \tag{A8}$$

These equations yield the steady state values:

$$\begin{aligned} h_M^B &= -\frac{K + (K^2 + 4b)^{1/2}}{2} \\ h_N^B &= \frac{(h_M^B)^2 p}{2h_M^B(1 - p) + 2K}. \end{aligned} \tag{A9}$$

Aggregate net output is

$$Q^B = [2ah_M^B h_N^B(1 - p) + a(h_M^B)^2(1 - p)]X + 2aKh_N^B X' - c(h_M^B + h_N^B). \tag{A10}$$

Positional value of an N with compensatory damages is

$$V_N^B = \frac{h_M^B(1 - p)X + 2KX' - ca^{-1}}{h_M^B(1 - p) + 2K}. \tag{A11}$$

Under configuration A the equations of motion are:

$$\begin{aligned} \dot{h}_M &= -ah_M^2 + ah_N^2(1 - p) + ab - aKh_M \\ \dot{h}_N &= ah_M^2 p - 2ah_M h_N(1 - p) - 2ah_N^2(1 - p) - 2ah_N K. \end{aligned} \tag{A12}$$

Aggregate net output under configuration A is

$$\begin{aligned} Q^A = [&2ah_M^A h_N^A(1 - p) + a(h_M^A)^2(1 - p) \\ &+ a(h_N^A)^2(1 - p)]X + 2aKh_N^A X' - c(h_M^A + h_N^A). \end{aligned} \tag{A13}$$

Positional value of an N with compensatory damages is

$$V_N^A = \frac{(h_M^A + h_N^A)(1 - p)X + h_N^A(1 - p)V_M^A + 2KX' - ca^{-1}}{2h_N^A(1 - p) + h_M^A(1 - p) + 2K}. \qquad \text{(A14)}$$

Example 1: Equilibrium in B with B most efficient:

Consider the parameter values $a = b = c = 1$, $p = 0.7$, $K = 0.3$. From stationary points of the equations of motion we have

$$h_M^A = 0.8846, \qquad h_N^A = 0.4000, \qquad h_M^B = h_M^C = 0.8612, \qquad h_N^B = 0.4649.$$

Consider the values $X = 110$, $X' = 100$. Then we have equilibrium in B

$$V_N^B = 101.8 > X', \qquad V_M^B = 77.5, \qquad S = X + V_M^B - 2V_N^B = -16.$$

In addition, we have the highest value in B since

$$Q^B = 77.5, \qquad Q^A = 77.1, \qquad Q^C = 75.5.$$

Example 2: Equilibrium in B with A most efficient:

Consider the same parameter values for a, b, c, p, K, and X'. Let $X = 125$. Then there is an equilibrium in B, since

$$V_N^B = 106.4 > X', \qquad V_M^B = 84.4, \qquad S = -3.$$

But A is most efficient, since

$$Q^A = 84.5, \qquad Q^B = 84.4, \qquad Q^C = 78.9.$$

With linear meeting dynamics in configuration C are described by the equation

$$\dot{h}_M = -ah_M + ab - aKh_M. \qquad \text{(A15)}$$

In steady state equilibrium

$$h_M^C = \frac{b}{1 + K}. \qquad \text{(A16)}$$

Net output under C satisfies:

$$\begin{aligned} Q^C &= ah_M^C(pX' + (1 - p)X) - ch_M^C \\ &= \frac{ab}{1 + K}(pX' + (1 - p)X) - \frac{bc}{1 + K}. \end{aligned} \qquad \text{(A17)}$$

Under configuration B there are two equations of motion:

$$\begin{aligned} \dot{h}_M &= \frac{-ah_M^2}{h_M + h_N} + ab - aKh_M \\ \dot{h}_N &= \frac{aph_M^2}{h_M + h_N} - \frac{2ah_Mh_N(1 - p)}{h_M + h_N} - 2aKh_N. \end{aligned} \qquad \text{(A18)}$$

These equations yield the steady state values

$$\begin{aligned} \frac{h_M^B}{h_N^B} &= \frac{1 - p + K + ((1 - p + K)^2 + 2pK)^{1/2}}{p} \equiv R^B \\ h_M^B &= \frac{b(1 + R^B)}{R^B + KR^B + K}. \end{aligned} \qquad \text{(A19)}$$

Aggregate net output in B is given by

$$Q^B = a(1-p)X\left(\frac{2+R^B}{1+R^B}\right)\left(\frac{b(1+R^B)}{R^B+KR^B+K}\right) + 2aKX'\left(\frac{b(1+R)^B}{R^B(R^B+KR^B+K)}\right)$$
$$- c\left(\frac{1+R^B}{R^B}\right)\left(\frac{b(1+R^B)}{R^B+K+R^BK}\right). \quad \text{(A20)}$$

V_N^B with compensatory damages satisfies

$$V_N^B = -c + \frac{h_M^B}{h_M^B+h_N^B}a(1-p)X + 2aKX'$$
$$+ \left(1 - \frac{h_M^B a}{h_M^B+h_N^B}(1-p) - 2aK\right)V_N^B \quad \text{(A21)}$$

or

$$V_N^B = \frac{\dfrac{R^B}{1+R^B}(1-p)X + 2KX' - c/a}{\dfrac{R^B}{1+R^B}(1-p) + 2K}. \quad \text{(A22)}$$

For configuration A the equations of motion are:

$$\dot{h}_M = \frac{-ah_M^2}{h_M+h_N} + \frac{a(1-p)h_N^2}{h_M+h_N} + ab - aKh_M$$
$$\dot{h}_N = \frac{aph_M^2}{h_M+h_N} - \frac{2a(1-p)h_Mh_N}{h_M+h_N} - \frac{2a(1-p)h_N^2}{h_M+h_N} - 2aKh_N. \quad \text{(A23)}$$

These yield the steady state values:

$$\frac{h_M^A}{h_N^A} = \frac{1-p+K+((1-p+K)(1+p+K))^{1/2}}{p} \equiv R^A$$
$$h_N^A = \frac{b(1+R^A)}{(1+K)(R^A)^2 - 1 + p + K(R^A)}. \quad \text{(A24)}$$

Steady state output is then

$$Q^A = a(1-p)X(h_M^A + h_N^A) + 2aKh_N^AX' - c(h_M^A + h_N^A). \quad \text{(A25)}$$

Example 3: Equilibrium in B with A and C more efficient.

Consider the choice of parameter values $a = 1, b = 1, c = 1, K = 1/2, p = 1/2$, $X' = 20$, $X = 23$. These numbers give rise to $R^B = 4.45$, $Q^B = V_M^B = 12.82$, $V_N^B = 20.16, Q^A = 12.83, Q^C = 13.67$. Notice that $V_N^B \geq X'$ and $2V_N^B \geq X + V_M^B$, but that $Q^C > Q^A > Q^B$. Therefore, this is an example of a compensatory equilibrium in B where configurations C and A are more efficient. We note that with these parameter values there is no equilibrium in either A or C.

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