

In Privatization (ed. by H. Siebert), Institut für Weltwirtschaft an der Universität Kiel, 1992.

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Auctions and Privatization

1. Introduction

Auctions provide a familiar and simple method for reallocating resources from sellers to buyers. Their attractive properties have been proven not only in theory but by long experience. They are particularly appealing in the case in which a seller is uncertain how much each buyer values the resources being sold (when the goods are capital — the main focus of the paper — the buyer's valuation corresponds to how productive this capital is in his hands): Rather than forcing the seller to set a sales price — a difficult task in view of the incompleteness of information — auctions permit the terms of trade to arise endogenously. Moreover, they perform quite well with respect to the objectives that (1) the resources get into the hands of those who value them the most (i.e., use them the most profitably) and (2) these recipients pay the seller as much as possible for them. Indeed, by inducing buyers to compete against each other, auctions tend to fulfill these two objectives better than do the most common alternatives to auctions: price-setting by the seller, negotiation between the seller and individual buyers, and, as has sometimes been proposed for the countries of Eastern Europe, simply giving the assets away.¹

Privatizing productive assets in formerly centralized economies² is a task to which auctions seem especially well-suited. A serious problem besetting privatization is how to determine, after many years of socialism, what the most efficient private uses for these assets are. The difficulty is compounded since the institutions that typically perform this crucial allocative role in decentralized economies — e.g., the stock market and the market for corporate control — are largely absent. Indeed, of the two objectives mentioned, I believe that it is fair to say that the first — that

Remark: I should like to thank the conference participants and, in particular, my discussants, Philippe Aghion and Roy Smith, for their comments. The National Science Foundation provided research support.

1 Actually, auction theory pertains to these other methods as well as to selling procedures more conventionally labeled "auctions." I refer to conventional auctions, however, when I say that they perform well with respect to the objectives (1) and (2).

2 In a number of Eastern European countries a considerable degree of decentralization occurred in recent years under socialist regimes. In these countries, renationalization may first be needed before thorough-going privatization is possible [see Hinds, 1990].

of allocative efficiency — is generally the more urgent in the countries of Eastern Europe. Raising revenue through the sale of capital is, of course, a nonnegligible consideration, but there are other ways of generating revenue. And to the extent that assets are sold to one's own citizens (and capital seems likely, if only for political reasons, to remain primarily in domestic hands) the price they pay for these assets may be regarded as merely a transfer payment that "washes out" in the calculation of social surplus. (See Section 2 for a further discussion of why revenue-generation perhaps should not be the primary goal.)

In any case, I shall be concerned in this paper primarily with the question of which forms of auctions best promote efficiency.¹ (See, however, the discussion in Section 2 of some of the other goals of privatization.) This is in contrast to the bulk of the recent theoretical literature on alternative auction institutions, which primarily emphasizes their revenue-producing properties (see McAfee and McMillan [1987], Bulow and Roberts [1989], Milgrom [1987], Maskin and Riley [1985], and Wilson [1990] for surveys). Happily, however, the auctions that are most efficient often turn out to do a good job of remunerating the seller as well (not surprisingly, since it should be easier to extract high payments from those who place high value on the resources being sold).

For the purpose of measuring economic efficiency, I shall assume that the social value of a unit of capital is equal to the maximum of the potential buyers' private valuations of the item. This assumption is justified if, for example, the winning buyer sells his output in a competitive market (including foreign competitors, if appropriate) and his other inputs (e.g., labor) are also supplied competitively. (If the output market is competitive, then the winner's marginal contribution to consumer surplus is zero; similarly, pure competition implies that he has no effect on the other factor markets. Hence, his profit is the correct measure of how much he adds to social surplus.)² Of course, it may be harder to maintain this assumption in the case of imperfect competition (see Section 8).

In Section 2, I shall briefly review some of the major objectives that the process of privatization is supposed to achieve. But I shall reiterate my contention that it may be reasonable to give efficiency the most weight.

In Sections 3 and 4, I present the main theoretical results. From the work of Vickrey [1961], it is known that the second-bid and English auctions are efficient under the assumption of *private values*, the case in which no buyer's information

1 If there were an adequate capital market, the question of which auction is most efficient would not matter much, since the market could correct any misallocations. Moreover, if there were large numbers of bidders, the standard forms of auctions would all approximate full efficiency [see Wilson, 1977, and Milgrom, 1979]. It is precisely the absence of these features in Eastern Europe that makes the choice of auction important.

2 This argument ignores risk-aversion and financial constraints, which will be considered in Sections 5 and 6.

affects any other buyer's valuation (Proposition 1). The high-bid auction, however, is not efficient except under strong symmetry and informational assumptions (Proposition 2). The English auction (but not the other two) remains efficient under *common values* (in which buyers' valuations may depend on others' private information), provided that each buyer's information can be represented by a one-dimensional parameter (Proposition 4). When information is multidimensional, no auction can be fully efficient (Proposition 6), but the second-bid and English auctions tend to be more efficient than the high-bid auction (Proposition 8).

Sections 5–7 qualify the results of Sections 3 and 4 by taking up, in succession, risk aversion, financial constraints (including a discussion of “voucher” systems), and costly information acquisition. The paper concludes (Section 8) by extending some of the results to the case in which complementary items are auctioned off simultaneously.

2. Objectives

There are at least five goals that privatization is often considered to advance: (1) efficiency — that is, getting capital into the hands of the most productive entrepreneurs; (2) competition — ensuring that the industries that result from privatization are not too highly concentrated; (3) revenue-generation — to be used either for public projects or for redistribution; (4) proper allocation of risks across members of the economy; and (5) income redistribution. There is also a sixth goal that is mentioned: the desirability of privatizing for its own sake, the idea that it is politically and morally preferable to have capital under private rather than state control.

We need not dwell on this last objective, since it will be attained automatically if one privatizes for any of the other reasons. As for income distribution, taxation is, I would maintain, a more direct and effective method than the reallocation of assets of highly uncertain value.¹ In any case, any scheme that badly compromises efficiency will render the issue of redistribution academic; there will be little to divide up anyway.

More generally, I am skeptical about the idea of using privatization to further goals not directly related to the first four mentioned above. The German Treuhandanstalt — the state agency charged with transferring East German firms into private

1 Given the assets' uncertainty, their reallocation would make the relative income distribution itself a random variable unless everyone were allotted the same portfolio. But such a uniform portfolio might interfere with effective control (see below). Moreover, the allocation of assets to attain distributional goals may compromise efficiency. No such objections, however, can be made against using the revenue from the *sale* of assets for redistribution.

property — has at times required potential buyers to submit employment and regional investment plans as well as bids. This means that there are multiple criteria by which the winning buyer is determined. The Treuhandanstalt therefore has considerable allocative power — both in establishing these other criteria and in deciding how much weight to give fairly incommensurable goals. To invest the agency with such power is first of all contrary to the underlying philosophy of privatization: that allocative decisions will be made more efficiently through private competition than by state agencies. It also opens the door to the risk of bureaucratic corruption and regulatory capture. But even granting that the Treuhandanstalt has the information, competence, and will, to balance these criteria, that it *should* be performing this balancing act remains questionable. After all, unemployment is again a problem that can be dealt with directly — through unemployment insurance — rather than by the inefficient device of requiring firms to employ a certain number of workers. It can always be alleged that imperfections in the political process require second-best measures such as employment quotas. But such allegations should be viewed with some suspicion.

I suggested two reasons in the introduction for down-playing revenue-generation as a goal. First, as long as buyers are domestic their payments contribute a pure transfer from the standpoint of net social welfare.¹ (If in the rather unlikely event there were a substantial fraction of foreign buyers, however, it would, of course, be socially desirable to extract as much money as possible from them.) Second, revenue-generation will be well-served by auctions that promote efficiency (see Sections 3 and 4). Finally, in most Eastern European countries the current condition of many enterprises seems so poor and fraught with uncertainty that the possibility of raising significant revenue from their sale is doubtful.²

Competition among firms is desirable, of course, to prevent them from exercising monopoly power and to discipline management [see Hart, 1983]. Indeed, as we saw in the introduction, competition is needed simply to guarantee that awarding capital to the buyer with the highest valuation is the socially efficient thing to do. Yet, competition need not require an unconcentrated industry; foreign trade³ (or potential entry) may work effectively as well. Thus, our emphasis on efficiency should be thought of as applying particularly to domestic industries with sufficient outside

1 Measured as the sum of consumer and producer surplus.

2 This does not imply, however, that the issue of efficiency is similarly reduced in importance. Although these enterprises may not be worth much today, they could well, when suitably reorganized, become quite valuable and so the long-run stakes are high.

3 As Murphy and Shleifer [1991] point out, however, the scope for trade between Eastern Europe and the West seems to be limited for now by the low quality of Eastern European products.

competitors.¹

The goals of efficiency and proper risk allocation are, of course, highly interrelated. It is nonetheless useful to try to make a conceptual — and perhaps practical — distinction between them. I conceive of “efficiency” as pertaining to the *control* of resources: the decision about how they will be used, who will manage them, when management will be replaced, etc. By contrast, “risk-allocation” concerns the more passive issue of investors’ portfolio adjustment.

Now, in principle, a properly designed auction — in which enterprises are sold off in small bits (see Section 8) — will solve the control and risk allocation problems simultaneously. There is concern, however, that given the poor information and lack of experience of potential investors in Eastern Europe, it may be too risky a strategy to rely on such an auction. In particular, it is feared that such a scheme may lead to the ownership structure of firms being too diffuse for effective control [see Borensztein and Kumar, 1990]. This has led some to propose *holding companies* as a way of fostering more effective control [see Blanchard et al., 1990, and Tirole, 1991]. Others have suggested *two-part auctions*, in which large pieces of firms (shares on the order of twenty or thirty percent, allowing for effective control) are sold separately from much smaller pieces, which investors presumably buy to balance their portfolios.

In this paper, I am concerned primarily with control rather than portfolio adjustment. Accordingly, one can interpret the auctions analyzed as corresponding to the “control part” of a two-part auction. (Alternatively, if holding companies are used, one can think of them as the sell-off mechanisms following the holding company phase.)

3. Private Values

I suppose that there are n potential buyers for an indivisible unit of capital (see Section 8 for the extension to multiple units). Buyer i ’s valuation v_i of the capital is the monetary return he would derive from employing it in the most efficient way available to him. Thus, if he wins the auction, his payoff is $u_i(v_i - b)$, where b is his payment and u_i is his von Neumann-Morgenstern utility function. For now I assume that buyers face no financial constraints (see Section 6 for a discussion of such constraints). That is, buyer i is able to pay up to v_i . I will say that an auction is *efficient* if, in equilibrium, the winner is the buyer with the highest valuation.

I assume that the value of v_i is private information to buyer i . I shall sometimes require that everyone believes that v_1, \dots, v_n are jointly distributed according to the

¹ Unfortunately, in the absence of such competitors efficient auctions may actually work against competition — see Section 8.

cumulative distribution function $F(v_1, \dots, v_n)$ and that these beliefs are common knowledge.

This formulation does not demand that buyer i knows the precise return he would earn from the capital — v_i can represent the buyer's expected return if he is risk-neutral (see Section 5 for an analysis of risk aversion when the return on capital is a random variable). It implies, however, that learning other buyers' valuations does not cause buyer i to revise his estimate of his own return (this is the *private values*¹ assumption), and that buyer i is the best judge of what that return is. This latter assumption means, in turn, that we can decentralize decision-making: once it is decided that i should be awarded the capital, it can be left to him to decide what to do with it. If, in contrast, a buyer's judgment were in question, then it might be desirable to have him submit his investment plans for scrutiny as well as his bid.

In the standard high-bid (or "first-price") auction, buyers simultaneously submit bids (proposed monetary payments) to the auctioneer. The winner (the recipient of the capital) is the high bidder (ties are broken by some stochastic device such as flipping a coin), and he pays his bid; the losers all pay nothing.² The second-bid auction (also called the "Vickrey auction," since it was proposed and analyzed by Vickrey [1961]) has the same rules, except that the winner, instead of paying his own bid pays only the second highest bid. Finally, in the open or English auction, buyers call out bids publicly, with the stipulation that each successive bid should be higher than the previous one. The auction ends when no one wishes to raise the bidding further, the winner is the last buyer to bid, and he pays that bid; losers again pay nothing. In a slight variant on the English auction called the open-exit auction [Milgrom and Weber, 1982; Bikhchandani and Riley, 1991], the auctioneer continuously raises the asking price, starting from zero. At any time, a bidder has the option of withdrawing (publicly) from the auction, but once out he cannot reenter. The winner is the last bidder to remain in the auction, and he pays the price that prevailed when the penultimate bidder exited.³

Notice that in the second-bid auction, it is a dominant strategy for buyer i to bid his valuation v_i . (A bidding strategy s is *dominant* for a buyer if, for any other strategy s' , s is at least as good as s' regardless of the other buyers' behavior, and for at least one possible behavior pattern by other buyers, s is strictly better than s' . With this

- 1 Private values do not prevent the capital's worth to be, in part, determined by exogenous non-idiosyncratic factors. It implies, however, that no buyer has private information about such factors. Moreover, it ensures that if and when the winning buyer resells the capital, his valuation embodies no information of use to potential purchasers (otherwise, his bid might be a pertinent signal to those purchasers).
- 2 This auction is strategically equivalent to a "Dutch" auction, in which the auctioneer begins with a high price and lowers it continuously until some buyer accepts.
- 3 The open-exit version of the English auction is particularly simple technically — it avoids the issue of one buyer attempting to outbid another by an infinitesimal and also makes a bidder's exit unambiguous. Thus, I will adopt it for analysis.

definition, a buyer can have at most one dominant strategy): submitting a bid $b_i < v_i$ rather than v_i changes the outcome only if the highest bid by other buyers falls in the interval (b_i, v_i) . But in that case, buyer i would be strictly better off bidding v_i . Similarly, bidding $b_i > v_i$ rather than v_i affects the result only if some other buyer bids in the interval (v_i, b_i) , in which case buyer i would, again, be better off bidding v_i .¹

For basically the same reasons, the strategy consisting of exiting when the prevailing price reaches one's reservation price is a dominant strategy in the English auction.² Thus, in the case of private values, the English and second-bid auctions are essentially equivalent. And since buyers bid their reservation prices in these auctions, they are efficient. Summarizing, I can state:

Proposition 1 (Vickrey): In the case of private values, the second-bid and English auctions are efficient.

Equilibrium behavior in the high-bid auction is more complex. It is clearly not optimal to bid one's reservation price since then one gains nothing from winning. Thus, equilibrium will involve buyers "shading" their bids somewhat, that is, bidding somewhat less than their valuations. But the degree to which one buyer shades will clearly depend on how much he believes other buyers are shading. Thus, there exist no dominant strategies in the high-bid auction.

In particular, attitudes toward risk may affect buyers' behavior. Notice that in the discussion of English and second-bid auctions, risk attitudes did not figure; bidding one's valuation was dominant regardless. But in a high-bid auction, the more risk-averse a buyer is, the more he wishes to insure himself against the eventuality of losing, that is, the less he will shade his bid. This phenomenon tends to work against efficiency — it means that a buyer may win the auction not because his valuation is highest but because he is especially risk-averse. To ensure an efficient outcome, therefore, there should be sufficient homogeneity of risk preferences.

Significant differences in the way buyers' valuations are distributed can also compromise efficiency. Suppose, for example, that there are two (risk-neutral) buyers, and that it is commonly believed that buyer 1's valuation is drawn from the uniform distribution on $[0,1]$ and that buyer 2's valuation is drawn (independently of buyer 1's) from the uniform distribution on $[0,10]$. Presumably, this discrepancy

1 Because bidding one's valuation is a dominant strategy, buyers have a strong impetus to do so. Although Nash equilibria entailing other strategies are possible, they seem quite implausible, since these other strategies are (weakly) dominated. Indeed, all but the dominant-strategy equilibrium are eliminated by the common refinements of Nash equilibrium such as trembling-hand perfect equilibrium.

2 Again, other equilibria are possible, but none is plausible.

in distributions reflects known ex ante differences between the buyers. I claim that, in the equilibrium of the high-bid auction,¹ buyer 1 will make the same bid if his valuation is 1 as buyer 2 makes when his valuation is 10. This implies that the equilibrium is inefficient since buyers with valuations near 1 may well win over those with valuations near 10.

To see why this claim holds, let $b_1(1)$ and $b_2(10)$ be the equilibrium bids by buyer 1 with valuation 1 and buyer 2 with valuation 10, respectively. Because buyers' equilibrium bids are nondecreasing in their valuations,² buyer 1 never bids above $b_1(1)$ and buyer 2 never bids above $b_2(10)$. If $b_1(1) < b_2(10)$, then buyer 2 can reduce his bid below $b_2(10)$ and still win with probability 1, which contradicts the fact that $b_2(10)$ is optimal. Similarly, $b_1(1) > b_2(10)$ leads to contradiction.

To summarize, I can state

Proposition 2: With private values, if (a) F is a symmetric distribution,³ (b) it is common knowledge that buyers believe that (v_1, \dots, v_n) is distributed according to F , and (c) buyers share the same utility function (i.e., $u_i = u$ for all i), then the high-bid auction is efficient. If, however, any of hypotheses (a)–(c) is dropped, it is, in general, inefficient.⁴

1 An equilibrium always exists in the high-bid auction provided that valuations are continuously distributed and affiliated [see Maskin and Riley, 1991].

2 To understand this intuitively plausible property, suppose that $v > v'$. If b and b' are the bids that the buyer makes when his valuation is v and v' respectively, then

$$[*] \quad q(v-b) \geq q'(v-b') \quad \text{and}$$

$$[**] \quad q'(v'-b') \geq q(v'-b),$$

where q and q' are the buyer's probabilities of winning corresponding to b and b' respectively. Subtracting [*] from [**), we obtain

$$q(v-v') \geq q'(v-v'),$$

and so $q \geq q'$. But this implies that $b \geq b'$ (provided that $q > 0$), otherwise the buyer would never use b' .

3 As a technical requirement I also posit henceforth that F be continuous and exhibit affiliation (see footnote 1 above).

4 To understand Proposition 2 a bit more formally, note that equilibrium in the high-bid auction consists of a vector of bidding functions $(b_1(\cdot), \dots, b_n(\cdot))$, where $b_i(v_i)$ is the bid that buyer i makes if his valuation is v_i . Under hypotheses (a)–(c) there exists a symmetric equilibrium, i.e., for all i , $b_i(\cdot) = b(\cdot)$, where $b(\cdot)$ is strictly increasing (see footnote 2 above for why $b(\cdot)$ is increasing). Hence, the high bidder is the buyer with the highest valuation. If, however, any of (a)–(c) is relaxed, there is no longer any reason for equilibrium to be symmetric, and so efficiency is lost. Observe, in particular, that (b) requires not only that buyers believe that the v_i 's are distributed symmetrically but that these beliefs be *common knowledge*, i.e., each buyer knows that other buyers have these beliefs, he knows that other buyers know that he has these beliefs, etc.

It has been pointed out by Graham and Marshall [1987], Robinson [1985], and Alexander [1991] that the second-bid and English auctions are more susceptible to collusion among buyers than is the high-bid auction. In the second-bid auction, a coalition of buyers might agree that only one of them will submit a serious bid (presumably the buyer with the maximum valuation in the coalition). Should that bid win, this arrangement serves to lower the expected payment that the winner has to make, and the rest of the coalition can then share this benefit through transfer payments. None of the other members of the coalition has the incentive to “cheat,” since no one gains by outbidding the buyer with the highest valuation. The reasoning is much the same in the English auction. By contrast, any agreement to bid low in the high-bid auction is vulnerable to the incentive each buyer has to raise his bid slightly above the agreed-upon level.

Notice, however, that although the incentive to collude may affect the *revenues* that the second-bid and English auctions generate, it does not affect their *efficiency*. As long as a coalition submits as its bid the maximum valuation of its members — and to do so is its dominant strategy — the winner will still be the buyer with the highest valuation.

4. Common Values

I now modify the model of Section 3 by reinterpreting v_i as buyer i 's private *signal* about the capital's value to him. His actual valuation φ_i may then depend not only on his own signal but on others' as well — that is, $\varphi_i = \varphi_i(v_1, \dots, v_n)$, where $\varphi_i(\cdot)$ is assumed to be differentiable — and his payoff if he wins the auction and pays b is $u_i(\varphi_i(v_1, \dots, v_n) - b)$. We thus have a model of *common values*.¹ In this setting an auction is efficient if, whenever the winner is buyer i , $\varphi_i(v_1, \dots, v_n) \geq \varphi_j(v_1, \dots, v_n)$ for all j .

An example of common values much discussed in the literature [see Milgrom and Weber, 1982] is that of mineral rights. Suppose that the item to be auctioned is the right to drill for oil on a particular piece of land. Buyer 1 may have acquired seismic information about the land in question, whereas buyer 2 may have drilled some test holes. Clearly, we would expect each buyer's private information to be valuable to the other, so that common values obtain.²

1 This is a weaker definition of common values than is sometimes used in the auctions literature. The stronger definition requires that buyers who have the same information share the same valuation, i.e., that all the functions φ_i be the same.

2 In this oil example, buyers are likely to agree on whether a given signal is favorable or not, e.g., an empty test hole would be bad news for everyone. But examples of disagreement are quite possible, too. Imagine, for instance, that buyers 1 and 2 are proposing to manufacture two very different sorts of cars — say, that buyer 1 wants to build an electric

In the private values model, v_i is, of course, a scalar (it is just buyer i 's valuation). But in the common values setting, where v_i has no such specific interpretation, it can well be multidimensional — a buyer might receive several different signals. It turns out that the efficiency of auctions depends crucially on whether v_i is one- or multidimensional, and so I shall distinguish between those cases.

a. One-Dimensional Signals

Let us consider the one-dimensional case first. Without loss of generality, we can parameterize the signals so that φ_i is increasing in v_i , i.e.,

$$[1] \quad \partial \varphi_i / \partial v_i > 0.$$

More substantively, I shall assume

$$[2] \quad \partial \varphi_i / \partial v_i \geq \partial \varphi_j / \partial v_i \text{ for all } i \text{ and } j.$$

Condition [2] asserts that a marginal change in buyer i 's information affects his valuation at least as much as it does that of any other buyer. To see that it is satisfied in quite natural circumstances, consider the following example. Suppose that two companies are vying for the right to construct a train line. Company 1 proposes to build the train near town A, whereas company 2 intends to build near town B. It is known that the residents of each town are more likely to use the train if it is near their town. What is not known is how many people in each town are likely to take the train at all (profitability is directly related to the number of passengers). Assume that company 1 does market research in town A, estimating how many people would use its train line. Because these people are less likely to use company 2's train, condition [2] is therefore satisfied. In any case, if [2] fails to hold, we shall see that existence of equilibrium in the high-bid, second-bid, and English auctions may be problematic [see Maskin, 1992]. Even more seriously, *no* kind of auction is likely to be efficient (see below).

Proposition 3: Suppose that [1] and [2] hold in the one-dimensional common values case. If the valuation functions are symmetric¹ and hypotheses (a)–(c) of Proposi-

car, whereas buyer 2 is planning one that runs on gasoline — and that they are in competition for some crucial piece of capital equipment. Private information that a better storage battery is available will tend to raise φ_1 and lower φ_2 , whereas the discovery of new petroleum reserves in Alaska does just the opposite.

1 The valuation functions are symmetric provided that, for all i and any permutation π of $\{1, \dots, n\}$, $\varphi_i(v_1, \dots, v_n) = \varphi_{\pi(i)}(v_{\pi(1)}, \dots, v_{\pi(n)})$.

tion 2 hold, the high-bid, second-bid, and English auctions are efficient, provided equilibria exist.¹

An argument similar to that of footnote 2 (p. 122) implies that buyers' equilibrium bids are increasing functions of their signals, and symmetry ensures that each buyer uses the same strategy. Hence, the winner is the buyer with the highest signal.²

For the same reasons as with private values, the high-bid auction fails to be efficient when any of the symmetry hypotheses are dropped. As for the high-bid and English auctions, we saw that they were equivalent and efficient in the private values case. With common values, they remain equivalent³ and efficient⁴ when

1 When $n = 2$, [1] and [2] suffice for existence, but stronger conditions are needed when $n > 2$. The fact that all three auctions are efficient does not mean that they are equivalent in terms of the winner's expected revenue. For such equivalence, the Revenue Equivalence Theorem [Myerson, 1981; and Riley and Samuelson, 1981] demands in addition that the v_i 's be independently distributed and all buyers be risk-neutral.

2 In the case of private values, the fact that the winner has the highest v_i means that, by definition, the auction is efficient. In the common values case a bit more argument is needed. Specifically, we need to show that if $v_i > v_j$ then $\varphi_i(v_1, \dots, v_n) \geq \varphi_j(v_1, \dots, v_n)$. Now, by symmetry of the valuation functions

$$(*) \quad \varphi_i(v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_{j-1}, v, v_{j+1}, \dots, v_n) = \\ \varphi_j(v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_{j-1}, v, v_{j+1}, \dots, v_n).$$

From [1] and [2], (*) implies that

$$(**) \quad \varphi_i(v_1, \dots, v_{i-1}, v', v_{i+1}, \dots, v_{j-1}, v, v_{j+1}, \dots, v_n) \geq \\ \varphi_j(v_1, \dots, v_{i-1}, v', v_{i+1}, \dots, v_{j-1}, v, v_{j+1}, \dots, v_n)$$

if $v' > v$. Hence, the result follows from (**) if we take $v' = v_i$ and $v = v_j$.

3 To see that the English and second-bid auctions are equivalent when $n = 2$, suppose that $(b_1(\cdot), b_2(\cdot))$ is an equilibrium of the English auction, i.e., buyer i drops out when the asking price reaches $b_i(v_i)$. Now suppose the price has reached p without either buyer dropping out. If buyer 1 decides to stay in until the price reaches $p + \Delta p$ (for Δp small), he wins only if buyer 2's signal lies between $b_2^{-1}(p)$ and $b_2^{-1}(p + \Delta p)$. Thus, his valuation if he wins is (approximately) $\varphi_1(v_1, b_2^{-1}(p))$. He will therefore stay in as long as $\varphi_1(v_1, b_2^{-1}(p)) \geq p$. In other words, $b_1(v_1) = \varphi_1(v_1, b_2^{-1}(b_1(v_1)))$. Now suppose that buyer 2 bids according to $b_2(\cdot)$ in the second-bid auction. The marginal benefit to buyer 1 of bidding $p + \Delta p$ rather than p is the probability that buyer 2's signal falls between $b_2^{-1}(p)$ and $b_2^{-1}(p + \Delta p)$ times the quantity $\varphi_1(v_1, b_2^{-1}(p + \Delta p)) - (p + \Delta p)$. Hence, again it is optimal for buyer 1 to bid $b_1(v_1) = \varphi_1(v_1, b_2^{-1}(b_1(v_1)))$. An equilibrium of the English auction thus corresponds to an equilibrium of the second-bid auction. The converse follows similarly.

4 To see that the English auction is efficient, consider signal values v_1 and v_2 such that $b_1(v_1) \geq b_2(v_2)$. We must show that $\varphi_1(v_1, v_2) \geq \varphi_2(v_1, v_2)$. Choose v'_1 and v'_2 such that

$$(*) \quad v'_1 \leq v_1 \text{ and } v'_2 \geq v_2,$$

and $b_1(v'_1) = b_2(v'_2)$ (this is possible since $b_1(\cdot)$ and $b_2(\cdot)$ are increasing). From the preceding footnote, $b_1(v'_1) = \varphi_1(v'_1, b_2^{-1}(b_1(v'_1))) = \varphi_1(v'_1, v'_2)$ and similarly $b_2(v'_2) = \varphi_2(v'_1, v'_2)$. Hence,

there are only two bidders. (Moreover, although buyers no longer have a dominant strategy, equilibrium behavior remains robust in the sense that it does not depend on buyers' beliefs about the distribution of signals nor on their attitudes toward risk, i.e., we see from footnote 3 (p. 125) that buyer 1's optimal bid depends neither on how he believes v_2 is distributed nor on u_1 .) However, equivalence fails for more than two bidders. Specifically, the English auction is efficient, but the second-bid auction is not.

To illustrate, suppose that $n = 3$ and that

$$\varphi_1(v_1, v_2, v_3) = v_1 + 1/2 v_2 + 1/6 v_3$$

$$\varphi_2(v_1, v_2, v_3) = v_2 + 1/2 v_1 + 1/2 v_3$$

$$\varphi_3(v_1, v_2, v_3) = v_3.$$

Now, in the second-bid auction, buyer i 's bid can be a function only of v_i because that is the only information he has. But if, say, $v_1 = 2$, $v_2 = 1$, and $v_3 \approx 3/2$, then $\varphi_1 \approx \varphi_2 > \varphi_3$, and so whether it is more efficient for buyer 1 or buyer 2 to win turns on whether v_3 is slightly more or slightly less than $3/2$. Since this information about v_3 cannot be incorporated in the bids of buyers 1 and 2, therefore, the second-bid auction cannot ensure efficiency in this example. The English auction, by contrast, enables buyers to make inferences when others drop out. Specifically, in this example buyer 3 will drop out first for the parameter values we have been discussing. Since $b_3(\cdot)$ is increasing (indeed $b_3(v_3) = v_3$), buyers 1 and 2 can infer the exact value of v_3 from noting the price when buyer 3 exits. Thus, the auction reduces to two buyers and the argument of footnote 4 (p. 125) implies that equilibrium is efficient.

This argument generalizes and I can state

Proposition 4: Consider one-dimensional common values and assume that [1] and [2] hold. Then, provided that an equilibrium exists, the English auction is efficient and its equilibrium behavior is robust in the sense that buyers' strategies depend neither on their beliefs about the distribution of signals nor on their attitudes toward risk.¹ By contrast, the second-bid auction is efficient or robust in general only when $n = 2$.²

$$(**) \quad \varphi_1(v'_1, v'_2) = \varphi_2(v'_1, v'_2).$$

But [1] and [2] together with (*) and (**) imply that $\varphi_1(v_1, v_2) \geq \varphi_2(v_1, v_2)$, as required.

- 1 In the case of private values, an equilibrium that is robust in this sense is equivalent to a dominant strategy equilibrium [see Dasgupta et al., 1979, or Ledyard, 1978]. For common values, however, a robust equilibrium is weaker than one in dominant strategies.
- 2 We have already seen (in footnote 4, p. 125) that the English auction is efficient when $n = 2$. Consider the case $n = 3$ (the extension to $n > 3$ is a bit more elaborate — see Maskin [1992] — although it is based on the same ideas). Suppose that $(b_1(\cdot), b_2(\cdot), b_3(\cdot))$ describes the equilibrium exiting behavior of the buyers, assuming that no one has previously exited.

Conditions [1] and [2] do not suffice to ensure the existence of equilibrium in the English auction when $n > 2$.¹ However, there exists a modification of the second-bid auction that is efficient and for which [1] and [2] *do* suffice. Specifically, for each vector $v_{-i} = (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$, define $v_i^*(v_{-i})$ so that $\varphi_i(v_i^*(v_{-i}), v_{-i}) = \max_{j \neq i} \varphi_j(v_i^*(v_{-j}), v_{-j})$.²

Proposition 5: Suppose that [1] and [2] hold in the one-dimensional common values case. Consider the “modified” second-bid auction in which each buyer i submits a “bid” \hat{v}_i , the winner is the bidder for whom $\varphi_i(\hat{v}_1, \dots, \hat{v}_n)$ is maximal, and he pays $\max_{j \neq i} \varphi_j(v_i^*(\hat{v}_{-j}), \hat{v}_{-j})$. Then it is an equilibrium for each buyer to take $\hat{v}_i = v_i$, and so the auction is efficient.³

I mentioned earlier that efficiency may well be impossible to attain when [2] fails. To see what can go wrong, imagine that there are two potential oil drillers — driller A with fixed cost 1 and marginal cost 2, driller B with fixed cost 2 and

That is, buyer i exits when the price reaches $b_i(v_i)$, as long as no other buyer dropped out earlier. Without loss of generality, suppose, for some v_1, v_2, v_3 , that

$$(*) \quad b_1(v_1) \leq b_2(v_2) \leq b_3(v_3).$$

Then buyer 1 drops out first. It will suffice to show that

$$(**) \quad \varphi_1(v_1, v_2, v_3) \leq \max \{ \varphi_2(v_1, v_2, v_3), \varphi_3(v_1, v_2, v_3) \},$$

since, after buyer 1 drops out, the auction reduces to two buyers and so we can apply the argument of footnote 4 (p. 125) to conclude that the auction is efficient. Now just as we derived formula (**) of footnote 4 (p. 125), one can show that

$$(***) \quad \varphi_1(b_1^{-1}(b_2(v_2)), v_2, b_3^{-1}(b_2(v_2))) = \varphi_2(b_1^{-1}(b_2(v_2)), v_2, b_3^{-1}(b_2(v_2))) \\ = \varphi_3(b_1^{-1}(b_2(v_2)), v_2, b_3^{-1}(b_2(v_2))).$$

Because (*) implies that $v_1 \leq b_1^{-1}(b_2(v_1))$, we conclude that [1], [2] and (***) imply that

$$(***) \quad \varphi_1(v_1, v_2, b_3^{-1}(b_2(v_2))) \leq \varphi_3(b_1^{-1}(b_2(v_2)), v_2, b_3^{-1}(b_2(v_2))).$$

But, because $b_3^{-1}(b_2(v_2)) \leq v_3$, (***) together with [1] and [2] implies that

$$\varphi_1(v_1, v_2, v_3) \leq \varphi_3(v_1, v_2, v_3),$$

completing the argument.

- 1 For example, there is no equilibrium when $\varphi_1(v_1, v_2, v_3) = v_1$, $\varphi_2(v_1, v_2, v_3) = v_2$, $\varphi_3(v_1, v_2, v_3) = 2/3 v_1 + 2/3 v_2 + v_3$, and the v_i 's are distributed independently and uniformly on $[0, 1]$.
- 2 This is a bit of an oversimplification because this equation may not be possible to satisfy; see Maskin [1992].
- 3 A proof of Proposition 5 can be found in Maskin [1992]. This modified second-bid auction has the advantages over the English auction that (i) an equilibrium exists in a broader range of cases and (ii) equilibrium behavior is particularly simple. Notice, however, the rules of the auction are *defined* in terms of the functions $\varphi_i(\cdot)$. That is, the auction designer must know these functional forms, a demanding requirement. By contrast, the designer can be ignorant of the forms if he uses an English auction.

marginal cost 1 — who are competing for drilling rights. Oil can be sold at a price of 4. Driller A does some (private) tests and discovers that the reserve to be auctioned contains v units. We have $\varphi_A(v) = 2v - 1$ and $\varphi_B(v) = 3v - 2$. Notice that $d(2v - 1)/dv < d(3v - 2)/dv$, and so hypothesis (iii) is violated. Moreover, I claim that there is no way to induce driller A to reveal his information while maintaining efficiency (assuming that v can never be measured directly). Efficiency requires that driller B get the drilling rights if $v > 1$ and that A get them if $1/2 < v < 1$. Suppose that driller A is given a reward $R(\hat{v})$ if he claims that there are \hat{v} units of oil. Then, if $v > 1 > v'$, incentive compatibility requires

$$[3] \quad R(v) \geq 2v - 1 + R(v'), \quad \text{and}$$

$$[4] \quad 2v' - 1 + R(v') \geq R(v).$$

Subtracting [3] from [4] we obtain

$$2(v' - v) \geq 0,$$

a contradiction. Hence, efficiency is impossible.

This is, of course, no great surprise. Conditions [1] and [2] in effect require that if a buyer receives good news (in the sense that his valuation rises) this should not decrease his chances of winning the auction if he reveals the information. If, perversely, his chances fall — as in the driller example — one would hardly expect him to reveal the news.

b. Multidimensional Signals

I now turn to the case in which v_i is a vector. Specifically, suppose that the components of v_i vary in the unit interval. Unfortunately, efficiency is now unattainable.

Proposition 6: Suppose that each v_i is multidimensional and (v_1, \dots, v_n) is continuously distributed. If the valuation functions are twice differentiable and, for each i , there exist parameter values for which it is efficient for buyer i to be the winner, there exists no efficient auction.

To get a feeling for why Proposition 6 holds, let us suppose that $n = 2$ and $\varphi_1(x_1, y_1, x_2, y_2) = 2x_1 + y_1 + x_2$ and $\varphi_2(x_1, y_1, x_2, y_2) = 2x_2 + y_2 + x_1$. (For the general proof, see Maskin [1992].) Suppose that, contrary to the Proposition, there exists an efficient auction and that, moreover, equilibrium behavior is robust. (It is not necessary to assume robustness; see Maskin [1992].) From the Revelation Principle

[see Dasgupta et al., 1979, or Myerson, 1979] we can assume that the auction is a “direct revelation” mechanism in which buyers’ bids are announcements of their signal values and, in equilibrium, these announcements are truthful. Let $b_1(\hat{x}_1, \hat{y}_1, \hat{x}_2, \hat{y}_2)$ be buyer 1’s payment if he bids (\hat{x}_1, \hat{y}_1) and buyer 2 bids (\hat{x}_2, \hat{y}_2) . Fix buyer 2’s signal values at $(x_2, y_2) = (x_2^*, y_2^*)$. From efficiency, buyer 1 should win the auction if $x_1 + y_1 \geq x_2^* + y_2^*$ ($2x_1 + y_1 + x_2^* \geq 2x_2^* + y_2^* + x_1$). Hence, in particular, buyer 1 should win if he announces parameter values in the locus $L = \{(\hat{x}_\lambda, \hat{y}_\lambda) \mid \hat{x}_1 + \hat{y}_1 = x_2^* + y_2^*\}$. Thus, because equilibrium is assumed to be robust, $b_1(x_1, y_1, x_2^*, y_2^*)$ must be a constant along locus L . (Robustness implies that buyer 1 should be willing to announce his signal values truthfully even if he knows that $(x_2, y_2) = (x_2^*, y_2^*)$. If, however, $b_1(\cdot)$ varies along L , then he will not be willing to announce values for which it is larger since he would still win the auction if he announced parameter values for which it is smaller.) But buyer 1 must be indifferent between winning and losing if (x_1, y_1) belongs to L (since it is equally efficient to have buyer 1 or buyer 2 win). Thus, since $b_1(\cdot)$ is constant on L , so must be $2x_1 + y_1 + x_2^*$ (since buyer 1’s payoff is $2x_1 + y_1 + x_2^* - b_1(x_1, y_1)$), which is clearly untrue. Thus, such an auction cannot exist. Intuitively, this is so because a one-dimensional payment b_1 is not sufficient to elicit two-dimensional information (x_1, y_1) .

Just as the high-bid, second-bid, and English auctions perform equally efficiently in the one-dimensional case provided that there is sufficient symmetry, so the same thing is true in the multidimensional case, at least if each buyer i ’s information v_i can be *summarized* in the sense that there exist a real-valued function χ_i of v_i and a real-valued function γ_i such that $\varphi_i(v_1, \dots, v_n) = \gamma_i(v_1, \dots, v_{i-1}, \chi_i(v_i), v_{i+1}, \dots, v_n)$. That v_i can be summarized means that, for the purpose of ascertaining his own valuation, buyer i needs to keep track just of the single number $\chi_i(v_i)$. (Thus, information can be summarized, for example, if a valuation is a linear function of buyers’ information parameters.)

Proposition 7: Suppose that (1) valuation functions are symmetric (in the sense of footnote 1, p. 124), (2) hypotheses (a)–(c) of Proposition 2 hold, and (3) the distribution of v_i conditional on v_{-i} is the same as that conditional on $\chi_i(v_i)$. Then, the high-bid, second-bid, and English auctions are equally efficient, in the sense that, in equilibrium, the same buyer wins in each auction for any given values of (v_1, \dots, v_n) .¹

More research remains to be done on the multidimensional case for asymmetric buyers. It can be shown, however, that, at least when the asymmetry is not too great, the English and second-bid auctions perform better than the high-bid auction.

1 Of course, in view of Proposition 6, the three auctions — although equally efficient — will all fail to be fully efficient.

Proposition 8: Suppose that hypotheses (2)–(3) of Proposition 7 hold. Then the English and second-bid auctions are more efficient than the high-bid auction (in terms of expected social surplus) provided that the asymmetries across buyers (as measured by differences in their valuation functions) are not too great.

5. Risk

So far, the only risk I have considered explicitly is that of losing the auction. But where a large capital item is concerned, the uncertainty associated with its return may be considerable. If buyers are significantly risk-averse, furthermore, this uncertainty may interfere with efficient allocation.

Specifically, suppose that buyers 1 and 2 are risk-averse and that their valuations \tilde{v}_1 and \tilde{v}_2 are random variables. (Let us assume private values throughout this section.) Assume that \tilde{v}_1 is much riskier than \tilde{v}_2 — so that, given his risk-aversion, buyer 1 bids lower than buyer 2 in a second-bid auction — but that the expectation of \tilde{v}_1 exceeds that of \tilde{v}_2 . If society is collectively less risk-averse than buyer 1, then the fact that buyer 2 beats out buyer 1 in the auction may well constitute a deviation from efficiency.

One common way to try to correct this distortion is through risk-sharing — so that buyer 1 is assigned ownership of only a fraction of the asset (such a scheme, however, may interfere with effective control). Another way is to reduce the uncertainty through taxes and subsidies. If the tax authority knew the distribution \tilde{v}_1 , it could clearly tax and subsidize in a way that replaced \tilde{v}_1 by its mean — thereby removing the distortion. But, as I have emphasized, a major reason for holding an auction in the first place is that the distribution \tilde{v}_i is private information to buyer i . Therefore, suppose that instead the tax authority announces that it will tax away a fraction α of the winning buyer's *realized* return, whether 1 or 2 wins. (Let us suppose that although the distribution of \tilde{v}_i is private information, the tax authority can observe its realization *ex post*.) This has the effect of making buyer i 's return $(1 - \alpha)\tilde{v}_i$. (Note that a negative return is also taxed, in which case the buyer is subsidized.) Now, if $\alpha = 1$, the buyer's return is always 0, and so a 100 percent tax will certainly not eliminate the distortion. However, we can establish

Proposition 9: There exists $\alpha^* < 1$ such that if the tax rate is α , where $1 > \alpha > \alpha^*$, the winner of the second-bid auction is the buyer for whom the expectation of \tilde{v}_i is highest. Thus, if society is risk-neutral, efficiency obtains.

Proof: If the tax rate is α , then in the second-bid auction buyer i will bid $b_i(\alpha)$ such that

$$E u_i((1-\alpha)\tilde{v}_i - b_i(\alpha)) = 0,$$

where u_i is buyer i 's utility function. Differentiating with respect to α , we obtain

$$- E [(\tilde{v}_i + b'_i(\alpha)) u'_i((1-\alpha)\tilde{v}_i - b_i(\alpha))] = 0.$$

Thus, $b'_i(1) = - E \tilde{v}_i$. But, for all i , $b_i(1) = 0$. Hence, for α near 1, $b_i(\alpha)$ is biggest for the buyer i for whom $E \tilde{v}_i$ is biggest.

Q.E.D.

Notice that this argument also establishes that the tax scheme not only reduces distortions but increases the total revenue collected from the winning buyer. Revenue rises for two reasons: (1) the tax scheme reduces the "risk premium" that a buyer subtracts from his bid, and (2) profit is taxed directly.

To work perfectly, this tax scheme requires that tax authority be able to monitor the winning buyer's net return from capital, that is, the return after the cost of *all* other inputs has been subtracted. If instead some inputs, for example, effort, are unobservable, then the scheme may create a moral hazard (because of the insurance, the buyer may no longer provide so much of the unobservable input) whose distortionary effects have to be weighed against the distortion of risk aversion.

It may happen that the tax authority cannot monitor the realization of \tilde{v}_i itself, but only some random variable \tilde{x}_i that is positively correlated with \tilde{v}_i . In this case, distortion may still be reducible through a tax scheme, provided that the correlation is strong enough.

Note that if much of the uncertainty concerning \tilde{v}_i is due to exogenous factors that are expected to be resolved in a relatively short period of time, there is a case for the state retaining temporary ownership (either partial or exclusive). Not only may waiting to privatize improve the efficiency of the ultimate allocation, but it may also increase the revenue generated.

6. Financial Constraints

Up to now, I have assumed that the winning buyer faces no financial constraints: he can pay for the capital that he has won and, if it turns out to generate a loss, he can absorb that, too (either directly or through loans). This may not be a satisfactory assumption, however, when applied to economies without a well-functioning capital market.

Financial constraints create at least two possible problems. First, the buyer with the highest valuation may not be able to pay the winning bid, in which case capital may not be allocated to its most efficient use. One possible way around this problem

is for the seller himself to loan the money, which will then be repaid out of the buyer's return on capital.

This solution, however, may run into the second difficulty that financial constraints create, namely, that the return may be insufficient to repay the loan — that is, the buyer may go bankrupt. The possibility of bankruptcy creates a distortion because it induces buyers to overbid in the auction; in effect, it implies that, from the buyer's point of view, the realization of \tilde{v}_i cannot fall below a certain level.

This tendency to promote overbidding may not be altogether a bad thing — to some extent it may counteract the tendency to *underbid* discussed in the section on risk aversion. But, of course, there is no reason why the two effects should cancel each other out. Still, it may be possible to attack the two problems simultaneously. The sort of tax/subsidy scheme described in the preceding section has the effect of reducing the variability of the return on capital, and hence also the risk of bankruptcy. Such a scheme may face other problems, such as the creation of moral hazard, but these can be balanced against the overbidding that bankruptcy generates.

Various “voucher” schemes have been proposed as a way around financial constraints. In these schemes potential buyers are issued vouchers, which are used in place of money when bidding for assets. Observe that if vouchers are nontransferable — i.e., if there is no voucher market — then such a scheme may well interfere with efficiency: for instance, it may be efficient for a given investor to own all the capital, but if he is allocated only a fraction of the vouchers this cannot happen. On the other hand, if they *are* transferable, then they are equivalent to money, in which case the scheme is simply a way of redistributing wealth (and perhaps not a very effective way, in view of the low values of Eastern Europe's capital values).

There is, however, at least one theoretical argument that may favor keeping vouchers nontransferable. As we shall see in Section 7, there may be a tendency for buyers to overinvest (relative to social efficiency) in information about the value of the capital being sold. If this is the case, then restricting their investment choices — as nontransferability does — may help reduce their expenditure on information.

7. Costly Information

In Sections 3 and 4, I implicitly assumed that buyer i 's information v_i about his valuation is given to him exogenously. It would be more realistic to suppose, however, that this information is the outcome of a costly investment by the buyer.

Let us restrict attention to one-dimensional signals. Once we allow for endogenous information, strong efficiency results such as Proposition 4 may no longer obtain. Specifically, we have

Proposition 10: Suppose that [1] and [2] hold. If

$$[5] \quad \frac{\partial \varphi_i}{\partial v_i} \geq \frac{\partial \varphi_j}{\partial v_i} > 0 \text{ for all } j \neq i,$$

then, given the other buyers' investments, buyer i will invest too much in information relative to social efficiency in equilibrium of the English auction. (If the second inequality in [5] holds instead with equality for all j , then buyer i 's investment is efficient.) If, on the other hand,

$$[6] \quad \frac{\partial \varphi_i}{\partial v_i} > 0 > \frac{\partial \varphi_j}{\partial v_i} \text{ for all } j \neq i,$$

then buyer i underinvests in equilibrium.

Proposition 10 implies that, except in the case of private values (for which $\partial \varphi_j / \partial v_i = 0$ when $j \neq i$), investment in information is likely to be inefficient. Specifically, in the plausible circumstance that all buyers agree that an increase in v_i raises their valuations, buyer i will overinvest. This is why I suggested in Section 6 that nontransferable vouchers may have the advantage of inducing buyers to limit their information-gathering.

I establish Proposition 10 formally in Maskin [1992], but the idea is easy to explain informally. The private gain to investing in information about v_i is proportional to $\partial \varphi_i / \partial v_i$.¹ The social gain, however, is proportional to $(\partial \varphi_i / \partial v_i) - \max_{j \neq i} (\partial \varphi_j / \partial v_i)$.² Hence if [5] holds, the private gain exceeds the social gain and overinvestment results — similarly if [6] holds.

8. Complementarities

Often a production process entails different sorts of capital that are mutually complementary. The production of vases, for example, requires both kilns and potting wheels. Such complementarities can create inefficiencies if capital is not

1 Suppose that, if he obtains no additional information, buyer i 's signal value is v_i and he is just indifferent between winning and losing the auction. If he obtains some extra information, he will learn that actually his signal value is either $v_i + \Delta v_i$ or $v_i - \Delta v_i$ with equal probability. Hence the value of this information is $1/2 (\partial \varphi_i / \partial v_i) \Delta v_i$.

2 The net social gain to awarding the capital to buyer i equals the difference between i 's valuation and the next highest valuation.

auctioned properly.¹ Even a highly efficient vase maker may be reluctant to enter a strong bid for the kiln if he thinks he may not win the potting wheel.

This suggests that items that are likely to be complementary should be auctioned simultaneously, even though it may turn out that they are not awarded to the same bidder. I shall argue that there, in fact, exists a simple modification of the second-bid auction that attains efficiency in the face of complementarities. (I shall assume private values throughout this section.)

Suppose that goods A and B are likely to be complementary. There are two buyers, 1 and 2. Each bidder i is asked to submit *three* bids: his professed valuation $\hat{v}_i(1,0)$ if he just gets good A ; his professed valuation $\hat{v}_i(0,1)$ if he just gets good B ; and his professed valuation $\hat{v}_i(1,1)$ if he gets both. Let \hat{A}_1 and \hat{B}_1 be the amounts of A and B that buyer 1 is awarded by the auction. Then (\hat{A}_1, \hat{B}_1) solves

$$[7] \quad \max_{A_1, B_1} \hat{v}_1(A_1, B_1) + \hat{v}_2(1 - A_1, 1 - B_1).$$

Buyer 1 pays $\hat{v}_2(1,1) - \hat{v}_2(1 - \hat{A}_1, 1 - \hat{B}_1)$ and buyer 2 pays $\hat{v}_1(1,1) - \hat{v}_1(\hat{A}_1, \hat{B}_1)$. Note that buyer 1's payoff (ignoring the constant $\hat{v}_2(1,1)$) is

$$v_1(A_1, B_1) + \hat{v}_2(1 - A_1, 1 - B_1).$$

Hence, because (\hat{A}_1, \hat{B}_1) solves [7], it is a dominant strategy for him to bid truthfully, and similarly for buyer 2. In view of [7] the outcome is efficient because bidders are truthful.

This modification is nothing other than the Groves [1973]/Clarke [1971] procedure applied to multi-item auctions. Thus, it extends to any number of goods and buyers. Such schemes, however, may compromise effective competition. Imagine, for example, that there are two factories for manufacturing a certain kind of machinery. If the same buyer is allowed to purchase both factories — and these schemes would make that likely as long as monopoly profit exceeded the sum of duopoly profits — then he would monopolize the industry.²

1 For that matter, different shares of the same firm can be regarded as complementary if holding them all improves control.

2 That is why in Section 2, I emphasized the need for “outside” competitors — either from other industries or other countries.

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