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Bargaining, Production, and Monotonicity in Economic Environments[1]

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The axiomatic bargaining theory of J. Nash (1950, *Econometrica* **18**, 155–162) presumes that only status quo utilities and the shape of the utility possibilities set are relevant to the bargaining outcome. Here we consider a class of economic problems for which bargaining solutions may depend on more than just utility information. (A fifty-fifty split of a single good between two bargainers is one example of such a solution.) It is shown that the requirements of *Pareto efficiency*, *weak symmetry*, and *technological monotonicity* (i.e., bargainers should gain from technological improvement) combine to characterize welfare egalitarianism. *Journal of Economic Literature* Classification Number: C78. © 1999 Academic Press

1. INTRODUCTION

Standard axiomatic bargaining theory a la Nash [3] requires that a solution depend only on bargainers' status quo utilities and the shape of the utility possibilities set. Consequently, two situations giving rise to the same status quo and utility possibilities must lead to the same utility outcome, regardless of whether the situations differ radically in terms of bargainers' preferences, endowments, or productive capabilities. This assumption of *welfarism* (that only bargainers' welfares should matter and not the

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circumstances giving rise to those welfares) has been embraced by most researchers in the field, and it figures prominently in most existing characterization results. However, welfarism is very strong (see Roemer [4, 5, 6]); indeed, it rules out what is perhaps the most common solution for dividing a single good between two bargainers, namely, splitting the good fifty–fifty.

Once welfarism is dropped, then it becomes natural to study solutions that directly map specifications of economic data to feasible economic outcomes. For example, Roemer [5, 6] considers a domain of pure-exchange environments and establishes the following result: if a bargaining solution satisfies Pareto efficiency, symmetry, continuity, resource monotonicity (the requirement that when the aggregate endowment rises, neither bargainer's utility falls), and consistency of resource allocation across dimension (CONRAD), then it must be *egalitarian* in the sense that it efficiently equates bargainers' utilities. This result constitutes a strengthening of Kalai's [1] characterization of egalitarianism within the classical welfarist framework.

The CONRAD axiom essentially requires that if there are some goods that only one bargainer likes and these goods are allocated first, then the outcome for the remaining (lower-dimensional) bargaining problem should be the same as if all goods had been allocated simultaneously. This might seem like a reasonable requirement, for not only is it logically weaker than welfarism, it is also perhaps more ethically defensible. However, the CONRAD axiom actually *implies* welfarism on the domain of problems Roemer considers, and so as a practical matter there would appear to be little difference between the two assumptions.

In this paper, we endorse the view that it may be important to allow bargaining solutions to depend directly on economic data. But rather than impose CONRAD, we enrich the economic environment with the possibility of production. Specifically, we study environments in which agents can use their share of the bargained-over endowment to produce output individually. For this class of problems it turns out that *Pareto efficiency*, *weak symmetry*, and *technological monotonicity*—which requires that when technology improves, neither agent's utility should fall—suffice by themselves to characterize welfare egalitarianism.

2. THE MODEL AND AXIOMS

We shall suppose that two agents are to divide a fixed aggregate endowment, $\omega \in \Re^m_{++}$ ($\Re^m_{++}$ is the set of m-dimensional vectors with strictly positive components, whereas $\Re^m_+$ consists of vectors with nonnegative components), of m goods where $m \geqslant 1$. Each agent i will use his share of the endowment goods as inputs to a continuous, nondecreasing production function f_i. It is assumed that while agents ultimately care about output, they bargain over *inputs*. (We have in mind that agents produce privately and hence that output may not even be verifiable.) Thus, the utility $u_i(x_i)$ that agent i can obtain from an amount x_i of inputs will depend directly on the quality of his production technology.

A *bargaining problem* is a pair (u_1, u_2) where u_i belongs to the set $\mathscr{U}$ of all continuous, nondecreasing functions $v: \mathfrak{R}^m_+ \to \mathfrak{R}_+$ such that $v(\omega) > 0$ and $v(0) = 0$. Disagreement should be thought of as the instance in which neither bargainer gets any of the inputs. Let $X = \{(x_1, x_2) \in \mathfrak{R}^m_+ \times \mathfrak{R}^m_+ : x_1 + x_2 \leqslant \omega\}$,[2] and define the *utility basis set* for (u_1, u_2) to be the set $U(u_1, u_2) = \{(v_1, v_2) \in \mathfrak{R}^2_+ : \exists (x_1, x_2) \in X$ with $u_i(x_i) = v_i,\ i = 1, 2\}$. Because of nonconvexities, agents may wish to randomize over feasible divisions of the endowment. Let $\mathscr{L}$ denote the set of all such lotteries. The *utility possibilities set* (UPS) $U^*(u_1, u_2)$ for (u_1, u_2) is defined to be the convex hull of $U(u_1, u_2)$. Agent i's utility from a lottery $l = (l_1, l_2) \in \mathscr{L}$ is simply the expectation of $u_i(\tilde{x}_i)$, where $\tilde{x}_i$ is distributed according to l_i. For convenience, this expected utility will be denoted $u_i(l_i)$.

Define the *Pareto frontier* of a problem (u_1, u_2) to be the set $P(U^*(u_1, u_2)) = \{(v_1, v_2) \in U^*(u_1, u_2)$: $\nexists (v_1', v_2') \in U^*(u_1, u_2)$ with $(v_1', v_2') > (v_1, v_2)\}$. We will be concerned in this paper with the case where $P(U^*(u_1, u_2))$ is *strictly decreasing*, i.e., where if (v_1, v_2) and (v_1', v_2') are two distinct points in $P(U^*(u_1, u_2))$, then either $v_1 > v_1'$ and $v_2 < v_2'$ or $v_1 < v_1'$ and $v_2 > v_2'$. Denote by Γ the set of all bargaining problems with strictly decreasing Pareto frontiers.

A *bargaining solution* (on Γ) is a correspondence $F: \Gamma \to \mathscr{L}$. In other words, a bargaining solution assigns to each bargaining problem in Γ a nonempty subset of the feasible lotteries. For expositional convenience, we will assume throughout that, for any solution F and problem (u_1, u_2), if $F(u_1, u_2)$ contains lottery l, then $F(u_1, u_2)$ also contains all other lotteries l' such that $u_i(l_i') = u_i(l_i)$ for $i = 1, 2$.

Our main interest is in the *Egalitarian solution*, F_e, which selects those Pareto efficient lotteries that equalize the two bargainers' utilities:

$$F_e(u_1, u_2) = \{l \in \mathscr{L} : u_1(l_1) = u_2(l_2) = v, \text{ where } (v, v) \in P(U^*(u_1, u_2))\}.$$

The following trio of axioms, stated with respect to a solution F, will serve to characterize the Egalitarian solution.

Pareto Efficiency (*PE*): For all $(u_1, u_2) \in \Gamma$ and for all $l \in F(u_1, u_2)$, $(u_1(l_1), u_2(l_2)) \in P(U^*(u_1, u_2))$.

[2] Throughout this paper, we will use the following notation with respect to given vectors $x, y \in \mathfrak{R}^m_+ : x > y$ if and only if $x_i > y_i$ for $i = 1, ..., m$; and $x \geqslant y$ if and only if $x_i \geqslant y_i$ for $i = 1, ..., m$.

Weak Symmetry (*WS*): For all $(u_1, u_2) \in \Gamma$ such that $u_1 = u_2$, $l \in F(u_1, u_2)$ implies that $u_1(l_1) = u_2(l_2)$.

Technological Monotonicity (*TM*): For any $(u_1, u_2), (u'_1, u'_2) \in \Gamma$, if $u_i \geqslant u'_i$ for $i = 1, 2$, then $u_i(l_i) \geqslant u'_i(l'_i)$ for any $l \in F(u_1, u_2)$ and $l' \in F(u'_1, u'_2)$.

PE is standard and requires no discussion.

The WS axiom is a much weakened version of the classical axiom of *symmetry* (cf. Nash [3]). Whereas classical symmetry demands that agents receive the same utility whenever the UPS is symmetric, WS requires equal utilities only when agents' utility functions are identical. Note that the "fifty–fifty split" solution in which ω is divided into two equal halves is ruled out by the stronger form of symmetry,[3] but it is clearly not ruled out by WS.

TM states that when agents' utility functions rise because of technological improvement, neither agent should be made worse off as a result. Like the monotonicity axiom from the standard bargaining literature (see Kalai [1]), this axiom can be justified both normatively and positively. For example, if agents are involved in some joint enterprise, then fairness may dictate that each should share in any benefits resulting from technological improvements. From a positive viewpoint, if agents have the opportunity to interfere with or damage one another's production function, then TM might be needed to eliminate their incentives to act on that opportunity. Notice that TM is considerably weaker than classical monotonicity—the former is satisfied by the fifty–fifty split solution, whereas the latter is not.[4]

[3] Consider a situation in which there is one unit of a single good to divide and in which agent's derived utility functions are

$$u_1(x) = \begin{cases} 2x & \text{if } 0 \leqslant x \leqslant \frac{1}{4} \\ \frac{2}{3}x + \frac{1}{3} & \text{if } x > \frac{1}{4} \end{cases} \qquad u_2(x) = \begin{cases} \frac{2}{3}x & \text{if } 0 \leqslant x \leqslant \frac{3}{4} \\ 2x - 1 & \text{if } x > \frac{3}{4} \end{cases}$$

It is easy to verify that the utility possibilities set is the symmetric set $\{(v_1, v_2) \in \mathfrak{R}^2_+ : v_1 + v_2 \leqslant 1\}$, and so a fifty–fifty split, which yields the unequal utilities, $\frac{2}{3}$ and $\frac{1}{3}$, is ruled out by classical symmetry.

[4] For example, suppose that there is one unit of a single good to divide and suppose agents' derived utility functions over the good are

$$u_1(x) = x, \qquad u_2(x) = \begin{cases} 2x & \text{if } 0 \leqslant x \leqslant \frac{1}{3} \\ \frac{1}{2}x + \frac{1}{2} & \text{if } x > \frac{1}{3}. \end{cases}$$

A fifty–fifty split gives agent 2 a utility of $\frac{3}{4}$. Now suppose that agent 1's utility function shifts up and agent 2's shifts down:

$$\hat{u}_1(x) = 2x, \qquad \hat{u}_2(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1 \\ \frac{1}{2}x + \frac{1}{2} & \text{if } x > 1. \end{cases}$$

As can be readily verified, the utility possibilities set here is larger than in the first situation. Thus, adherence to classical monotonicity would require both agents' utilities to rise. But a fifty-fifty split gives agent 2 a utility of only $\frac{1}{2}$.

Notice too that any solution F satisfying TM has the following property: for a given problem (u_1, u_2), all lotteries in $F(u_1, u_2)$ give rise to the same pair of utilities.[5]

Since a fifty–fifty split fails only to satisfy PE among our three axioms (e.g., increasing returns may imply that randomization is better than splitting the inputs in half), one might expect that a modification of this solution would satisfy all three axioms. For instance, bargainers could split the inputs in half and then share equally the utility gain of moving from that point to the frontier. That such a modification does not satisfy all of our axioms, however, will become evident from the characterization theorem given below.

3. THE CHARACTERIZATION THEOREM

We begin by proving the following lemma, which gives a sufficient condition for a bargaining problem to have a strictly decreasing Pareto frontier.

LEMMA 3.1. *Let (u_1, u_2) be a bargaining problem. If for $i = 1, 2$ $\nexists k_i \in \prod_{j=1}^{m} [0, \omega_j] \backslash \{\omega\}$ such that $u_i(x)$ is constant for $k_i \leqslant x \leqslant \omega$, then $(u_1, u_2) \in \Gamma$.*

Proof. Suppose that (u_1, u_2) is a problem for which the hypothesis holds but that (u_1, u_2) does not have a strictly decreasing Pareto frontier. Then, without loss of generality, we can consider $(a_1, a_2), (b_1, b_2) \in P(U^*(u_1, u_2))$ such that $a_1 = b_1$, $a_2 > b_2$. By convexity of $U^*(u_1, u_2)$, we have that $a_1 = u_1(\omega)$. Now, let (l_1, l_2) denote the lottery that achieves (a_1, a_2). Since $a_2 > b_2 \geqslant 0$, the support of l_2 is not $\{0\}$. But then feasibility implies that the support of l_1 is not $\{\omega\}$. So there exists $k_1 \in \prod_{j=1}^{m} [0, \omega_j] \backslash \{\omega\}$ such that $u_1(k_1) < u_1(\omega)$, as required. Q.E.D

We can now state and prove the following result:

THEOREM 3.2. *A solution F satisfies PE, WS, and TM if and only if $F = F_e$.*

Proof. Since it is obvious that F_e satisfies PE, WS, and TM, we shall prove only the converse implication. Let F be any solution satisfying the three axioms and consider a $(u_1, u_2) \in \Gamma$. The key is to construct from (u_1, u_2) a sequence of problems culminating in a symmetric problem.

[5] Let $(u'_1, u'_2) = (u_1, u_2)$ and consider $l \in F(u_1, u_2)$ and $l' \in F(u'_1, u'_2) = F(u_1, u_2)$. Then, because (trivially) $u_i \geqslant u'_i$ for $i = 1, 2$, we must have $u_i(l_i) \geqslant u'_i(l'_i) = u_i(l'_i)$. By a similar argument, we have $u_i(l'_i) \geqslant u_i(l_i)$, and so $u_i(l_i) = u_i(l'_i)$.

Because utility profiles are monotonically related at each step of the construction, the axioms ultimately dictate that F must choose an egalitarian allocation for (u_1, u_2).

Let v^* be the number for which $(v^*, v^*) \in P(U^*(u_1, u_2))$. First we construct from (u_1, u_2) a "better" problem $(\bar{u}_1, \bar{u}_2)$ in which (v^*, v^*) is still Pareto efficient but is attainable via a deterministic feasible division. To this end, note that because $P(U^*(u_1, u_2))$ is strictly decreasing, there exists a continuous, strictly decreasing function $z\colon [0, u_1(\omega)] \to \Re_+$ such that for all $x \in \prod_{j=1}^m [0, \omega_j]$, $(u_1(x), z(u_1(x))) \in P(U^*(u_1, u_2))$. Then define $\bar{u}_1 = u_1$ and $\bar{u}_2(x) = \max[u_2(x), \; z(u_1(\max\{\omega_1 - x_1, 0\}, ..., \; \max\{\omega_m - x_m, 0\}))]$. Notice that $P(U^*(u_1, u_2)) = P(U^*(\bar{u}_1, \bar{u}_2))$.

Next, observe that since $\bar{u}_1$ is continuous and nondecreasing, there exists $\alpha \in \prod_{j=1}^m (0, \omega_j)$ such that $\bar{u}_1(\alpha) = v^*$. By definition of $\bar{u}_2$, $\bar{u}_2(\beta) = v^*$ where $\beta = \omega - \alpha$. Let $\gamma = (\min\{\alpha_1, \beta_1\}, ..., \min\{\alpha_m, \beta_m\})$ and $\delta = (\max\{\alpha_1, \beta_1\}, ..., \max\{\alpha_m, \beta_m\})$.

Define the following functions from $\prod_{j=1}^m [0, \infty)$ to $\Re_+$:

$$b(x) = \begin{cases} \min\limits_{j \in \{1, ..., m\}} \left(\dfrac{x_j}{\gamma_j}\right) v^* & x \ngeq \gamma \\ v^* & x \geqslant \gamma \text{ and } x \ngeq \delta \\ v^* + \min\limits_{j \in \{1, ..., m\}} \left(\dfrac{x_j - \delta_j}{\omega_j - \delta_j}\right) (\min\{\bar{u}_1(\omega), \bar{u}_2(\omega), 2v^*\} - v^*) & x \geqslant \delta \end{cases}$$

$$\hat{u}_i(x) = \begin{cases} \min\{\bar{u}_i(x), b(x)\} & x \ngeq \delta \\ \min\{\bar{u}_1(x), \bar{u}_2(x), b(x)\} & x \geqslant \delta \end{cases} \quad \text{for} \quad i = 1, 2$$

$$u^*(x) = \begin{cases} b(x) & x \ngeq \delta \\ \min\{\bar{u}_1(x), \bar{u}_2(x), b(x)\} & x \geqslant \delta. \end{cases}$$

It is straightforward to verify that b is continuous and nondecreasing and that $(\hat{u}_1, \hat{u}_2)$ and (u^*, u^*) are well-defined bargaining problems. Observe now that since $b(x)$ is strictly less than $\min\{\bar{u}_1(\omega), \bar{u}_2(\omega)\}$ for all $x \in \prod_{j=1}^m [0, \omega_j] \backslash \{\omega\}$ (this follows because $(u_1, u_2) \in \Gamma$, implying that $v^* < \min\{\bar{u}_1(\omega), \bar{u}_2(\omega)\}$), we have by Lemma 3.1 that $(\hat{u}_1, \hat{u}_2)$ and (u^*, u^*) belong to Γ. Next, observe that (v^*, v^*) is attainable in both problems (e.g., with the feasible division (α, β)). Finally, note that $b(x) + b(\omega - x) \leqslant 2v^*$ for all $x \in \prod_{j=1}^m [0, \omega_j]$,[6] and so we have $u^*(x) + u^*(\omega - x) \leqslant 2v^*$ for all $x \in \prod_{j=1}^m [0, \omega_j]$. Thus (v^*, v^*) is efficient for (u^*, u^*) and hence efficient for $(\hat{u}_1, \hat{u}_2)$. The foregoing facts imply by PE and WS that if

[6] To see why this is true, note that, for any $j \in \{1, ..., m\}$, $\gamma_j + \delta_j = \omega_j$, and so, for $x \ngeq \gamma$, $b(x) + b(\omega - x) \leqslant (x_j/\gamma_j)\, v^* + (1 + (\omega_j - x_j - (\omega_j - \gamma_j))/(\omega_j - (\omega_j - \gamma_j)))\, v^* = 2v^*$. Similar reasoning for other values of x establishes that $b(x) + b(\omega - x) \leqslant 2v^*$ for all $x \in \prod_{j=1}^m [0, \omega_j]$.

$l \in F(u^*, u^*)$ then $(u^*(l_1), u^*(l_2)) = (v^*, v^*)$. Therefore, by TM and PE, F yields (v^*, v^*) for $(\hat{u}_1, \hat{u}_2)$. But $\bar{u}_i \geqslant \hat{u}_i$ for $i = 1, 2$, and so by TM, F must yield (v^*, v^*) for $(\bar{u}_1, \bar{u}_2)$. A final application of TM and PE implies that F chooses (v^*, v^*) for (u_1, u_2). Q.E.D

4. DISCUSSION

Nonconcave utility functions, such as might arise from production technologies with increasing returns, play a crucial role in our characterization result.[7] We note, however, that it is still possible to characterize egalitarianism with our axioms when utility functions are restricted to be quasiconcave. One need only modify the construction of $\bar{u}_1$ and $\bar{u}_2$ so that these functions are quasiconcave. Then, from the quasiconcavity of b, one can show that the existing construction of u^*, $\hat{u}_1$, and $\hat{u}_2$ endows these functions with the requisite property.

The issue of whether or not Theorem 3.2 generalizes to the case of n players appears to be more complicated. The primary reason for this is that the Pareto frontier of an n-dimensional utility possibilities set may include a level segment on which no single player attains maximum feasible utility. As a result, the natural generalization of the condition in Lemma 3.1 does not suffice to ensure that a feasible set's Pareto frontier is strictly decreasing.

Finally, we remark that while our axioms are incompatible with the scale invariance requirement typically appealed to in axiomatizations of the Nash [3] or Kalai–Smorodinsky [2] bargaining solutions, welfare egalitarianism on our class of problems need not require explicit interpersonal comparisons of utility. Suppose, for example, that each bargainer can conceive of a best possible outcome for himself or herself over all possible technologies. Then, utility scales could be chosen so that each bargainer's best possible outcome yields to him a utility of 1. Under this scenario, no judgments of the form "Bargainer 1 is better off than Bargainer 2" or "Bargainer 1 loses more than Bargainer 2" would be needed to equate utilities.

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[7] Indeed, it is a simple matter to verify that, for one-good problems with concave utility functions, a fifty–fifty split satisfies all of our axioms.

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