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Correlated Equilibria and Sunspots*

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We examine when "sunspots" (uncertainty that has no influence on endowments, preferences, or technology) can affect equilibrium in a simple two-period, two-commodity, two-class economy. We find that such an effect is possibly only if the signals (random variables) that different agents observe are *imperfectly* correlated (neither perfectly correlated nor independent) and at least one commodity is a Giffen good. For two special cases we characterize the set of equilibria due to sunspots. We conclude by showing the intimate connection between the sunspot equilibria of our finite horizon model and those of the overlapping generations literature. *Journal of Economic Literature* Classification Numbers 021, 026. © 1987 Academic Press, Inc.

1. INTRODUCTION

In this note we study a simple finite horizon economy with only *extrinsic* uncertainty. That is, the randomness (e.g., sunspots) in no way influences endowments, preferences, or other economic data. Nonetheless, it *can* affect the nature of equilibrium if (and only if) agents observe its realization differentially. In our model there are one or more "certainty" equilibria—equilibria that would prevail were there no randomness at all. When we say that uncertainty "affects" equilibrium, we mean that it gives rise to prices and allocations distinct from those in any certainty equilibrium. Although

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differential information is essential for the creation of such new equilibria, our model is closely analogous to recent treatments of symmetric information sunspot equilibria in infinite horizon, overlapping generations models (cf. Shell [8], Azariadis [2], and Azariadis and Guesnerie [3]).

We introduce a simple two-period, two-good, two-class model in Section 2. In the first period, agents observe the realizations of signals (random variables) and choose production (investment) levels before learning market prices. They trade their produced good for others' goods in the second period. Extrinsic uncertainty can affect equilibrium only if there is imperfect correlation (but neither perfect correlation nor complete independence) among the signals different agents observe. Moreover, at least one of the produced commodities must be a Giffen good. In Sections 3 and 4 we consider two special cases of the model. In the first example, only one class of agents observes a signal ("sunspots"). We examine the nature of the "sunspot" equilibria, those that differ from the unique certainty equilibrium (the equilibrium that arises if agents ignore their signals). In the second example (Section 4) both types receive imperfectly correlated signals. In this case, we completely characterize the set of sunspot equilibria. Section 5 shows that there is a close connection between our finite-horizon sunspot equilibria and those treated in the overlapping generations literature.

2. The Model

There are two equal-sized classes of consumers. Each consumer is endowed with a finite amount of leisure. Type 1 consumers can sacrifice leisure to produce good 1. Similarly, type 2 consumers produce good 2. The marginal product of labor is 1. Consumers of both types consume leisure and the good produced by the *other* type. They do not consume the good they themselves produce. Thus consumer i's utility is $U^i(l_i, c_j)$, where l_i is his labor and c_j is his consumption of good j. Utility functions are concave and twice continuously differentiable.

Consumers first decide how much labor to allocate to production, and then there is trade in the produced goods. Trade is competitive. If x_i is the per capita quantity of good i, the price of good 2 (with good 1 the numeraire) is

$$p = x_1/x_2.$$

In equilibrium, output, labor, and consumption are all equal:

$$x_i = l_i = c_i, \qquad i = 1, 2.$$

Type i consumers receive a signal $\tilde{s}_i$ in some finite set S_i before deciding how much to produce. The signal has no effect on the economic data; it is purely extrinsic. The pair $(\tilde{s}_1, \tilde{s}_2)$ is drawn from some joint distribution. Let $l_i(\tilde{s}_i)$ denote the labor supply as a function of the signal received. The equilibrium price in our model is $l_1(s_1)/l_2(s_2) = x_1(s_1)/x_2(s_2)$. Taking the price as given, a type i consumer chooses $l_i = l_i(s_i)$, where

$$l_i(s_i) = \arg\max_{l_i} E_{s_j}\left[U^i\left(l_i, l_i \frac{x_j(s_j)}{x_i(s_i)}\right) \Big| \, s_i\right]. \tag{1}$$

In equilibrium, $l_i(s_i)$ must equal $x_i(s_i)$. Thus, the first order condition for the consumer's maximization can be expressed as

$$E_{s_j}\left[U_1^i(x_i(s_i), x_j(s_j)) + U_2^i(x_i(s_i), x_j(s_j)) \frac{x_j(s_j)}{x_i(s_i)} \Big| \, s_i\right] = 0, \tag{2}$$

where U_k^i denotes the partial derivative of U^i with respect to its kth argument (given concavity, the second order conditions are satisfied automatically).

DEFINITION 1. An *equilibrium* relative to signals $\{\tilde{s}_1, \tilde{s}_2\}$ is a pair of functions $\{x_1(\tilde{s}_1), x_2(\tilde{s}_2)\}$ satisfying (2).

We shall classify equilibria according to their dependence on signals.

DEFINITION 2. An equilibrium pair $\{x_1(\tilde{s}_1), x_2(\tilde{s}_2)\}$ is a *certainty equilibrium* if $x_i(\tilde{s}_i)$ does not depend on $\tilde{s}_i$. A noncertainty equilibrium $\{x_1(\tilde{s}_1), x_2(\tilde{s}_2)\}$ is *perfectly correlated* if $\tilde{s}_1$ and $\tilde{s}_2$ are so correlated; otherwise, $\{x_1(\tilde{s}_1), x_2(\tilde{x}_2)\}$ is called an *imperfectly correlated* equilibrium.[1]

In our model it is clear that a perfectly correlated equilibrium $\{x_1(\tilde{s}_1), x_2(\tilde{s}_2)\}$ is just a randomization over certainty equilibria. That is, for each possible realization (s_1, s_2), $\{x_1(s_1), x_2(s_2)\}$ is a certainty equilibrium. Thus, although signals can affect the allocation of resources in a perfectly correlated equilibrium, such an equilibrium is *ex-post* observationally indistinguishable from one where there are no signals at all. We conclude that signals "matter" in an equilibrium only if they are *imperfectly correlated*.[2]

[1] Note that $\tilde{s}_1$ and $\tilde{s}_2$ need not be correlated themselves for $\{x_1(\tilde{s}_1), x_2(\tilde{s}_2)\}$ to be deemed an imperfectly correlated equilibrium. The correlation resides in the fact that all members of a given class observe the game signal.

[2] Our concept of a correlated equilibrium is closely related to the game theoretic notion of the same name (see Aumann [1]). The only difference is that here we are concerned with a competitive economy rather than a well-defined game. Our observation that signals "matter" only if they are imperfectly correlated corresponds to the game theoretic principle that perfectly correlated equilibrium payoff vectors lie in the convex hull of the ordinary Nash equilibrium payoffs, but imperfectly correlated equilibrium payoffs need not.

We now derive necessary conditions for the existence of a nondegenerate imperfectly correlated equilibrium (one that is not a certainty equilibrium). Existence is intimately tied to the presence of Giffen goods. A commodity is a Giffen good if its consumption increases with the price of the good for some range of prices. And, by a slight abuse of terminology, leisure is a Giffen good if the amount of labor supplied decreases with the price of the good it produces. It is easily checked that, when consumption is a Giffen good, so is leisure.

PROPOSITION. *Consider a nondegenerate imperfectly correlated equilibrium. Leisure must be a Giffen good for at least one class of consumers. Furthermore, if the signals s_1 and s_2 are independent, consumption must be a Giffen good for at least one class of consumers.*

Proof. Let us define the signals that can be observed by each class: $s_1 \in \{s_1^1, ..., s_1^m\}$ for class 1; $s_2 \in \{s_2^1, ..., s_2^n\}$ for class 2. Rank the signals so that in the imperfectly correlated equilibrium:

$$x_1^1 \leqslant \cdots \leqslant x_1^k \leqslant \cdots \leqslant x_1^m$$

and

$$x_2^1 \leqslant \cdots \leqslant x_2^h \leqslant \cdots \leqslant x_2^n,$$

with at least one strict inequality, where x_j^i is the production level of class j when it observes signal $s_j = s_j^i$. Without loss of generality one can assume that $x_1^1/x_2^1 \leqslant x_1^m/x_2^n$. In this case,

$$\frac{x_1^1}{x_2^n} \leqslant \cdots \leqslant \frac{x_1^1}{x_2^h} \leqslant \cdots \leqslant \frac{x_1^1}{x_2^1} \leqslant \frac{x_1^m}{x_2^n} \leqslant \cdots \leqslant \frac{x_1^m}{x_2^h} \leqslant \cdots \leqslant \frac{x_1^m}{x_2^1},$$

with at least one strict inequality. Thus, the distribution of prices (with good 1 as numeraire) that type 1 consumers face when their signal is s_1^m fully dominates that when the signal is s_1^1. This implies that, whatever the signal received by class 2, class 1 faces better terms of trade if its own signal is s_1^1 rather than s_1^m. Because, by assumption, type 1 consumers produce more for signal s_1^m than for s_1^1, leisure must be a Giffen good.

Assume next that the signals s_1 and s_2 are not correlated. Then the expectation in (2) does not depend on the signal s_i. For a nondegenerate equilibrium, the equation

$$E_{x_j}[U_1^i(x_i, x_j)\, x_i + U_2^i(x_i, x_j)\, x_j] = 0 \tag{3}$$

must have at least two distinct solutions x_i for some class i. The ratio x_i/x_j

is the price of i's consumption (produced by j) in terms of labor. Thus, if consumption is not a Giffen good for class i, we have

$$U_1^i + U_{11}^i x_i + U_{12}^i x_j < 0.$$

But this last inequality implies that the bracketed expression in (3) is monotonic in x_i for any x_j. Thus, (3) cannot have two solutions. Q.E.D.

3. Example 1

Consider the offer curves for the two classes represented in Fig. 1. Consumption and, therefore, leisure are Giffen goods for class 1 (but not for class 2). There exists a unique certainty (and, hence unique perfectly correlated) equilibrium A. To construct an imperfectly correlated equilibrium, choose $\bar{x}_2$ such that there are several values of x_1 that are optimal for class 1 when the terms of trade are $x_1/\bar{x}_2$. Let x_1^1 and x_1^2 denote two such values. Assume that the signal s_1 takes on the values $\{s_1^1, s_1^2\}$, and define $x_1^1 = x_1(s_1^1)$ and $x_1^2 = x_1(s_1^2)$. We suppose that s_1 is observed by class 1 only; class 2 observes no signal. Under certainty (i.e., if it observed s_1), class 2 would produce $y_2 = x_2(s_1^1)$ or $z_2 = x_2(s_1^2)$. But because class 2 does not observe s_1, it produces some quantity between y_2 and z_2. By continuity one can find a number α in (0, 1) such that when (s_1^1, s_1^2) have respective probabilities $(\alpha, 1-\alpha)$, class 2 offers $\bar{x}_2$ (i.e., the equilibrium conditions are satisfied).

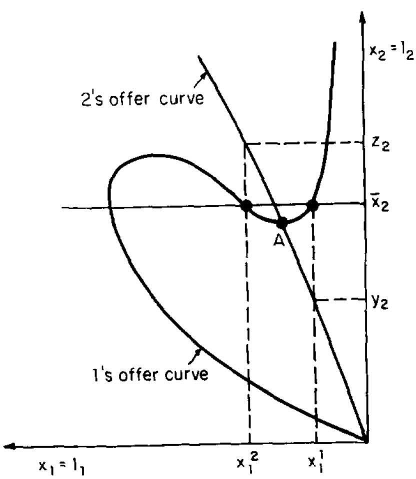

FIG. 1. Point A corresponds to the unique certainty equilibrium of the economy. The points $(x_1^1, \bar{x}_2)$ and $(x_1^2, \bar{x}_2)$ correspond to an imperfectly correlated equilibrium.

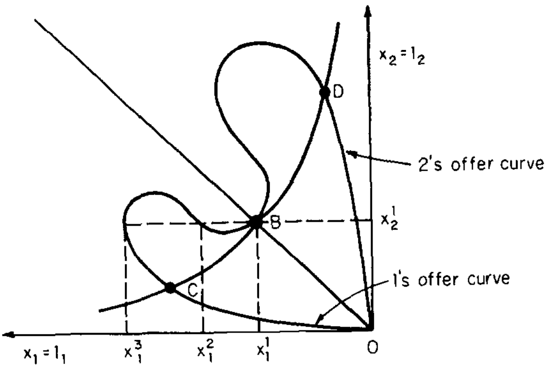

FIG. 2. Points B, C, and D are certainty equilibria. Points (x_1^1, x_2^1), (x_1^2, x_2^1), and (x_1^3, x_2^1) correspond to an imperfectly correlated equilibrium.

4. EXAMPLE 2

Consider the following symmetric offer curves depicted in Fig. 2. There are three certainty equilibria, B, C, D. Let $B = (x_1^1, x_2^1)$. Choose $\{x_1^k\}$ such that[3]

(a) $x_1^h < x_1^k$ for $k > h$, and

(b) (x_1^k, x_2^1) belongs to 1's offer curve.

Choose $\{x_2^k\}$ analogously

Assume that:

(i) each class i receives a signal $s_i \in \{s_i^k\}$.

(ii) these signals are not perfectly correlated; in particular, the probability of s_i^1 is positive whatever the signal received by j.

(iii) the agents are infinitely risk averse.

The offers $\{x_i^k(s_i^k)\}$ form an imperfectly correlated equilibrium. Because of the extreme risk aversion, each class i behaves as if the other class j always offered the lowest value x_j^1. Are there other imperfectly correlated equilibria? To see that there are not, consider a set of equilibrium offers $\{\{x_i^k\}, \{x_2^k\}\}$. Because of infinite risk aversion, (x_1^1, x_2^1) must form a certainty equilibrium if s_j^1 has positive conditional probability given any s_i^k. But C and D cannot be candidates for this equilibrium. In particular, at C, leisure is locally not a Giffen good. Thus, given the corresponding labor

[3] As illustrated in Fig. 2, k assumes three values for class 1 and one for class 2. More generally for these offer curves, k might take on any number of values up to three for either class.

supply by class 2, there exists a unique point on the offer curve for class 1. Similarly, we can rule out D. Hence, all the imperfectly correlated equilibria are as described above.[4]

5. Discussion

This note emphasizes the role of *imperfectly correlated* signals in generating equilibria that could not occur without extrinsic uncertainty. Both the correlation and its imperfect nature are important. On the one hand, if signals were independently distributed across agents, then, with large numbers, the sample distribution would be known by everyone, and so only certainty equilibria would be possible. On the other hand, if, in our model, all agents observed the same signal, all equilibria would be randomizations among certainty equilibria.

This last feature distinguishes our approach from that of Cass and Shell [4] and Peck and Shell [6]. In the models of the latter two papers, all agents observe the same signal. However, beforehand, they trade securities that are contingent on the realization of this signal. Thus the signal induces wealth effects that can give rise to new equilibria. This is true regardless of the set of securities that are available. Cass and Shell avoid the implications of the First Fundamental Welfare Theorem by supposing that some agents are born only after the securities market closes, whereas Peck and Shell posit non-Walrasian trade in the post-sunspot market. In our model, by contrast, there are no securities and hence no wealth effects. Thus a signal observed by everyone can create no equilibrium outside the set of certainty equilibria.

The reader may be disturbed by our assumption that a large class of consumers observe exactly the same signal and yet the signal is unobservable to others. Why, for example, could a member of the informed class not be persuaded (or bribed) to divulge his information? There are at least two ways around this difficulty.

First, one could construct a model in which the members of a given class do not observe the same thing, but rather private signals that are only imperfectly correlated with each other (determined by a idiosynchratic as well as an aggregate shock). Equilibrium in such a model with large numbers would be much like that in the present formulation. However, the

[4] We have assumed infinite risk aversion only so that we can characterize *all* imperfectly correlated equilibria. Using the equivalence result proved in the next section, we could readily derive equilibria with less extreme risk aversion. For example, we could consider the equilibria described by Azariadis [2] or Azariadis and Guesnerie [3] and transpose these into our two-period framework.

acquisition of information by outsiders would be more difficult, since learning the value of any single agent's signal would be of little benefit.[5,6]

Alternatively, one could consider a model in which, instead of the large number of traders we have assumed in each class, there are only a few, i.e., a model of monopolistic competition. In such a model, bribery might be difficult since large agents would be likely to take into account the effect on equilibrium of selling information. We can suppose that, after observing their signals, agents choose production levels in Cournot fashion. Then, since, formally speaking, the agents are playing a game, the Cournot outcome is merely a correlated equilibrium in the sense of Aumann [1].[7]

As we mentioned in the Introduction, there is a close analogy between our finite-horizon model and those of the literature on sunspots in overlapping-generations economies. In the overlapping-generations literature, agents live for two periods, and, at any given time, old and young generations coexist. It is assumed that sunspot activity (the signal) is observed by everybody, but there is a sense in which information is nonetheless asymmetric. In any period, the old generation trades a paper asset against consumption. The price of the paper asset depends on the labor supply by the young generation, which in turn is influenced by current sunspot activity. The crucial assumption is that, when they buy the asset, members of old generation do not known what the sunspot activity will be. Thus they choose their labor supply decision without knowing the sunspot conditions on which the young generation's labor supply is based. In this sense, the young generation has private information.

We now develop this point of view more formally. Consider the simplest overlapping generations model, studied, for example, in Azariadis and Guesnerie [2] and Grandmont [5]. There is a single consumption good and paper asset. A typical consumer lives for two periods. He supplies labor (l) when young, and consumes (c) only when old. His utility function is $U(l, c)$. He produces one unit of output with one unit of labor. Thus, the competitive wage is one (where current consumption is the numeraire). If he is born at time t, the consumer buys the paper asset at t at price p_t and

[5] With such imperfect correlation it could remain true that, in a large economy, one could learn the nature of the aggregate shock by sampling only a small *proportion* of agents. But the *absolute* size of the same might have to be very large.

[6] If it were prohibitively costly for an individual agent to sample numerous signal values, we might suppose that a firm could be set up to collect and sell the information to a large number of agents. Of course, the costs of collection might still outweigh the benefits. Moreover, such a market in information could well be subject to serious free rider problems (agents would attempt to acquire the information for free from other agents; the firm might be tempted to sell phony information to avoid collection costs).

[7] Moreover, by increasing the number of agents in each class, one can show, a la Novshek and Sonnenschein [7], that the Cournot equilibrium converges to the Walrasian equilibrium that we considered in Sections 2–4.

sells it in period $t+1$ at price p_{t+1}. If M_t is the quantity of asset he purchases, his budget constraints are $l = p_t M_t$ and $c = p_{t+1} M_t$, i.e.,

$$c = (p_{t+1}/p_t)\, l. \tag{4}$$

Now consider a signal $s_t \in \{s^1, ..., s^n\}$, interpreted as the level of sunspot activity. The evolution of activity is governed by a Markov process with transition matrix $T = \{t_{ij}\}$, where $t_{ij} = \Pr\{s_{t+1} = s^j \mid s_t = s^i\}$.

DEFINITION 3. A *stationary sunspot equilibrium* is a price function $p(s)$ and a labor supply function $x(s)$ such that

$$x(s) = \arg\max_{l} E_{\hat{s}}\left[U\left(l, l\,\frac{p(\hat{s})}{p(s)}\right) \Big|\, s\right] \tag{5}$$

and

$$p(s)\, M = x(s), \tag{6}$$

where M is the money supply (the total quantity of the paper asset). Equation (6) defines equilibrium in the paper asset market, whereas (5) determines the optimum labor supply. Substituting (5) into (6), we can identify a stationary sunspot equilibrium with a labor supply function $x(\cdot)$ satisfying

$$x(s) \text{ maximizes } E_{\hat{s}}\left[U\left(l, l\,\frac{x(\hat{s})}{x(s)}\right) \Big|\, s\right]. \tag{7}$$

Comparing (7) and (1) we see that a stationary sunspot equilibrium is nothing but a symmetric correlated equilibrium in which both classes receive signals in the same signal space. In particular, a deterministic cycle (as in Grandmont [5]) corresponds to a perfectly correlated equilibrium.

Thus, despite the difference in interpretation, there is a strong connection between sunspot and correlated equilibria. To develop this conntection further, consider a signal space $\{s^1, ..., s^n\}$ and an $n \times n$ *symmetric* matrix $C = \{c_{ij}\}$, where for all i, j, $c_{ij} \in [0, 1]$ and $\sum_i \sum_j c_{ij} = 1$. C is a correlation matrix; it gives the joint probability of signals received by two classes of traders in a correlated equilibrium. Because C is symmetric, there is a corresponding unique matrix $T = \{t_{ij}\}$ of conditional probabilities of one class's signal given that of the other. Thus, one can construct a stationary sunspot equilibrium from a correlated equilibrium associated with a symmetric correlation matrix C. Conversely, by deriving the matrix C from the Markov transition matrix T, one can a construct a correlated equilibrium from a stationary sunspot equilibrium.

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