How Can Cooperative Game Theory Be Made More Relevant to Economics? : An Open Problem

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Abstract Game Theory pioneers J. von Neumann and O. Morgenstern gave most of their attention to the cooperative side of the subject. But cooperative game theory has had relatively little effect on economics. In this essay, I suggest why that might be and what is needed for cooperative theory to become more relevant to economics.

Cooperative game theory is the part of game theory that pertains when players can sign binding contracts determining their actions and payoffs. J. von Neumann and O. Morgenstern devoted most of their seminal book [6] to cooperative theory, with subsequent major contributions by Nash [4] and Shapley [5].

But despite its auspicious beginnings, cooperative game theory has been used far less than noncooperative theory as a predictive tool in economics. Indeed, inspection of the current leading game theory textbooks used in graduate economics programs reveals that the ratio of cooperative to noncooperative theory is remarkably low (in one such text, [1], the ratio is 0). And all Nobel Memorial Prizes awarded for game theory to date have recognized work exclusively on the noncooperative side.

This imbalance may seem strange. Cooperative theory seems to offer the important advantage of giving insight into how coalitions behave, i.e., how subsets of players bargain over which actions are played. Such bargaining seems basic to many aspects of economic and political life from the European Union, to the Paris climate change agreement, to the OPEC cartel. Moreover, on the face of it, cooperative theory appears to be far less dependent on particular details about strategies—and, therefore, more robust and general—than noncooperative theory.

To understand the sense in which noncooperative is more detail dependent, let us briefly go over the basic noncooperative and cooperative models. In a noncooperative game, each player $i, i = 1, \ldots, n$, chooses a strategy s_i from a strategy set S_i, and the payoffs of the game are given by the mapping

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$$g : S_1 \times \cdots \times S_n \to \mathbb{R}^n ,$$

where $g_i(s_1, \ldots, s_n)$ is player i's payoff if strategies $(s_1, \ldots, s_n)$ are played. The standard prediction for what will happen in game g is that players will choose *Nash equilibrium* strategies [3]. Strategies $(s_1^*, \ldots, s_n^*)$ constitute a Nash equilibrium if

$$g_i(s_1^*, \ldots, s_n^*) \geq g_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*), \text{ for all } i \text{ and all } s_i \in S_i . \qquad (1)$$

As formula (1) makes clear, Nash equilibrium depends crucially on what strategies are or are not in each player's strategy set; for example, adding a single strategy s_i' to S_i can destroy $(s_1^*, \ldots, s_n^*)$ as an equilibrium and change the predicted outcome of the game discontinuously, even if s_i' generates payoffs quite similar to those of s_i^*.

By contrast, cooperative games are typically described by a characteristic function v. Given a coalition of players $S \subseteq \{1, \ldots, n\}$, $v(S)$ is the sum of payoffs that the members of S can get on their own. An often-used predictive concept in cooperative game theory is the *Shapley value* (unlike Nash equilibrium in noncooperative theory, the Shapley value has serious competition as a predictive concept; there are some other leading notions, such as the core and the bargaining set). Given characteristic function v, player i's Shapley value payoff is

$$\sum_{S \subseteq \{1,\ldots,i-1,i+1,\ldots,n\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S)) ,$$

i.e., player i gets his expected marginal contribution to coalitions, where the expectation is taken over all possible coalitions that he might join.

Notice that in the cooperative-game setting, players' strategies no longer are modeled explicitly—only the resulting payoffs matter. Thus, many different noncooperative games can be associated with the same characteristic function. In that sense, the characteristic function approach is more general than the noncooperative model. Moreover, in cooperative games, the discontinuities that arise in noncooperative games no longer occur: the characteristic function and Shapley value vary continuously with the payoff possibilities. In that sense, cooperative games are more robust than noncooperative games.

So why, despite these advantages, is cooperative game theory currently dominated by noncooperative theory as applied to economics? Perhaps one answer is that the characteristic function, by assumption, rules out *externalities*—situations in which a coalition's payoff depends on what other coalitions are doing. Yet, interactions between coalitions are at the very heart of economics, e.g., bargaining between unions and management, competition between companies, and trade between nations. Moreover, even in the (relatively small) cooperative literature that *does* accommodate externalities (the partition-function approach; see [2]), extensions of the Shapley value and of other leading cooperative concepts do not predict competition between coalitions; instead, they assume as a matter of definition that the grand coalition—the coalition of all players—always forms. Of course this flies in the face of *reality*, where, in most settings, we don't typically see just a single big coalition, but rather several smaller coalitions. Furthermore, there is a good *theoretical* reason why, in a model with externalities, we should *not* expect the grand coalition to form.

To illustrate this point, let us consider the following three-player game, in which coalitions can produce public goods. The coalition of players 1 and 2—$\{1, 2\}$—can produce a total payoff of 12 for itself, $\{1, 3\}$ can produce 13, and $\{2, 3\}$ can produce 14. The grand coalition $\{1, 2, 3\}$ can produce 24. A player can produce nothing on his own. However, if the other two players form a coalition, he can free-ride on the public good they produce and enjoy a payoff of 9 (which is the externality that the coalition confers on him).

I claim that, we should not expect the grand coalition to form in this game. To see why not, imagine that all bargaining is conducted at a particular site and player 1 arrives there first, followed by 2, and finally by 3. When player 2 arrives, player 1 can make him offer to join 1 in a coalition. Let us explore what 2 must be offered to be willing to join. Notice that if he does not join with 1, he will be in competition with 1 for signing up 3. In this competition, 1 will be willing to bid 13 (the gross value of the coalition with 3) minus 9 (which he would get as a free-rider if 3 signed up with 2), i.e., 4. Similarly, 2 will be willing to bid $14-9 = 5$. Hence, 2 will win the bidding war for 3 and will pay 4 (notice that because, in this thought experiment, 1 and 2 don't form a coalition, 3 has no possibility of free-riding and so will be willing to accept 4). Hence 2's payoff if he refuses to join with 1 is $14 - 4 = 10$. Thus, player 1 must offer him 10 in order to sign him up.

Assuming 2 is signed up, 1 must then offer 3 a payoff of 9 to attract him to coalition $\{1, 2\}$ (because 3 has the option to free-ride on $\{1, 2\}$ and get 9 that way). Hence, altogether player 1 must pay $10 + 9 = 19$ in order to form the grand coalition. But this leaves only $24 - 19 = 5$ for himself. Clearly, he would be better off refraining from signing up 2—in which case, as analyzed above, 2 will form a coalition with 3. And 1 obtains a free-riding payoff of 9.

I conclude that with arrival order 1, 2, 3, two separate coalitions will form: $\{2, 3\}$ and $\{1\}$. A similar conclusion follows for the five other possible arrival orders.

Unfortunately, cooperative game theory in its current state does not allow for such a two-coalition outcome. In my view, it remains an open problem—perhaps the most important open problem in cooperative theory—to develop an approach that properly accommodates the formation of multiple coalitions. Only by solving this problem can we make cooperative game theory relevant to economics.

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