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OPTIMAL BAYESIAN MECHANISMS

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Abstract

Arrow [1979] and D'Aspremont and Gerard-Varet [1979] showed that when agents' preference parameters are independently distributed there exist public decision-making mechanisms that take optimal decisions in Bayesian equilibrium and balance the budget. We show that this result extends to the case where the parameters are correlated.

Optimal Bayesian Mechanisms

Introduction

Suppose that a population of agents $i = 1,\ldots,n$ have von Neumann-Morgenstern utility functions

$$v_i(d,\theta_i) + t_i,$$

where d is a public decision and t_i is a transfer of a private good to agent i. The parameter θ_i is private information for agent i and the joint distribution of $(\theta_1,\ldots,\theta_n)$ is given by the c. d. f. $F(\theta_1,\ldots,\theta_n)$, which is common knowledge. Suppose that $d^*(\theta_1,\ldots,\theta_n)$ solves the problem

$$(0) \qquad \max_{d \varepsilon D} \sum_{i=1}^{n} v_i(d,\theta_i),$$

where D is the space of possible public decisions (assume a solution to (0) exists). We shall refer to $d^*(\theta_1,\ldots,\theta_n)$ as the optimal public decision given $\theta_1,\ldots,\theta_n$.

A <u>mechanism</u> is a public decision function $d(\theta_1,\ldots\theta_n)$ and a set of transfers $\{t_i(\theta,\ldots,\theta_n)\}$. The interpretation of such a mechanism is that agents announce the values of their parameters (possibly untruthfully) and, on the basis of the announcements $(\overset{o}{\theta}_1,\ldots,\overset{o}{\theta}_n)$, the public decision $d(\overset{o}{\theta}_1,\ldots,\overset{o}{\theta}_n)$ is taken and agent i receives transfer $t_i(\overset{o}{\theta}_1,\ldots,\overset{o}{\theta}_n)$. In an <u>optimal Bayesian mechanism</u>, for all $(\theta_1,\ldots,\theta_n)$,

$$(1) \qquad d(\theta_1,\ldots,\theta_n) = d^*(\theta_1,\ldots,\theta_n)$$

$$(2) \qquad \overset{o}{\theta}_i \text{ maximizes } E_{\theta_{-i}|\theta_i}[v_i(d^*(\overset{o}{\theta}_i,\theta_{-i}),\theta_i) + t_i(\overset{o}{\theta}_i,\theta_{-i})]$$ [1]

and

$$(3) \qquad \sum t_i(\theta_1,\ldots,\theta_n) = 0,$$

where $E_{\theta_{-i}|\theta_i}$ denotes the expectation operator with respect to θ_{-i} conditional on

θ_i. That is, an optimal Bayesian mechanism (a) induces each agent to tell the truth about his parameter, assuming that other agents are truthful, (b) chooses the optimal public decision and (c) balances the budget.

Arrow [1979] and D'Aspremont and Gerard-Varet [1979] (ADG) showed that optimal Bayesian mechanisms exist when the θ_i's are independently distributed. The transfers in these mechanisms take the form

$$(4) \qquad \overset{o}{t}_i(\theta_1,\ldots,\theta_n) = \overset{o}{t}_{ii}(\theta_i) - \frac{1}{n-1}\sum_{j\neq i}\overset{o}{t}_{jj}(\theta_j),$$

where

$$(5) \qquad \overset{o}{t}_{ii}(\theta_i) = E_{\theta_{-i}}\sum_{j\neq i} v_j(\overset{o}{d^*}(\theta_i,\theta_{-i}),\theta_j).^2$$

Laffont and Maskin [1979] and Riordan [1984] showed that the independence hypothesis in the ADG proposition can be weakened. Making stronger assumptions about the v_i's and θ_i's, they demonstrated that when, roughly speaking, the θ_i's are nonpositively correlated, optimal Bayesian mechanisms can be found where transfers take the additively separable form (4), although t_{ii} is no longer given by (5).[3]

In this paper we present two results. We first provide a proposition in the spirit of the Laffont-Maskin-Riordan results on additively separable transfers (Theorem 1). We then argue (Theorem 2) that if one does not impose the additively separable form, one can find optimal Bayesian mechanisms <u>regardless</u> of the nature of the correlation across θ_i's.

1. Additively Separable Transfers

For the purposes of this section <u>only</u>, assume that (a) the space of public decisions D is (0,1); (b) agent i's parameter θ_i is a number in (0,1) and that agent i has a differentiable conditional density function $f_i(\theta_{-i}|\theta_i)$ corresponding to $F(\theta_1,\ldots,\theta_n)$; (c) the function $v_i(\cdot,\cdot)$ is strictly concave in its first argument, twice continuously differentiable, and satisfies

$$\frac{\partial^2 v_i}{\partial d \partial \theta_i} > 0. \tag{6}$$

Suppose first that we attempt to mimic the ADG solution (5). That is, suppose we take

$$\begin{aligned} t_{ii}(\overset{\circ}{\theta}_i) &= E_{\theta_{-i}|\overset{\circ}{\theta}_i} \sum_{j\neq i} v_j(d^*(\overset{\circ}{\theta}_i,\theta_{-i}),\theta_j) \\ &= \int \sum_{j\neq i} v_j(d^*(\overset{\circ}{\theta}_i,\theta_{-i}),\theta_j) f(\theta_{-i}|\overset{\circ}{\theta}_i)\, d\theta_{-i} \end{aligned} \tag{7}$$

and then define t_i by (4). Then, assuming that other agents are truthful, agent i maximizes

$$\int v_i(d^*(\overset{\circ}{\theta}_i,\theta_i),\theta_i) f(\theta_{-i}|\theta_i)\, d\theta_{-i} + t_{ii}(\overset{\circ}{\theta}_i), \tag{8}$$

since $t_{jj}(\overset{\circ}{\theta}_j)$, $j \neq i$, does not depend on $\overset{\circ}{\theta}_i$. The first derivative of (7) is

$$\begin{aligned} &\int \Big[\frac{\partial v_i}{\partial d}(d^*(\overset{\circ}{\theta}_i,\theta_{-i}),\theta_i)\frac{\partial d^*}{\partial \overset{\circ}{\theta}_i}(\overset{\circ}{\theta}_i,\theta_{-i}) f(\theta_{-i}|\theta_i) \\ &+ \sum_{j\neq i} \frac{\partial v_j}{\partial d}(d^*(\overset{\circ}{\theta}_i,\theta_{-i}),\overset{\circ}{\theta}_i)\frac{\partial d^*}{\partial \overset{\circ}{\theta}_i}(\overset{\circ}{\theta}_i,\theta_{-i}) f(\theta_{-i}|\overset{\circ}{\theta}_i) \\ &+ \sum_{j\neq i} v_j(d^*(\overset{\circ}{\theta}_i,\theta_{-i}),\theta_i)\frac{\partial f_i}{\partial \overset{\circ}{\theta}_i}(\theta_{-i}|\overset{\circ}{\theta}_i)\Big]\, d\theta_{-i}. \end{aligned} \tag{9}$$ [4]

Now, because $d^*(\theta_i,\ldots,\theta_n)$ solves (1)

$$\sum_{j=1}^{n} \frac{\partial v_j}{\partial d}(d^*(\theta_i,\ldots,\theta_n),\theta_j) = 0. \tag{10}$$

Therefore, when $\overset{\circ}{\theta}_i = \theta_i$, (9) becomes

$$\int \sum_{j\neq i} v_j(d^*(\overset{\circ}{\theta}_i,\theta_{-i}),\theta_i)\frac{\partial f_i}{\partial \theta_i}(\theta_{-i}|\theta_i)\, d\theta_{-i}. \tag{11}$$

But unless $\frac{\partial f_i}{\partial \theta_i} = 0$, there is no reason why (11) should vanish. Hence, although truthtelling satisfies the first-order conditions for a maximum when the θ_i's are independent, truthtelling is not optimal when the θ_i's are correlated. That is why the ADG procedure requires independence.

However, suppose instead that we <u>define</u> $t_{ii}(\overset{o}{\theta}_i)$ so that it satisfies the first order conditions for a maximum at $\overset{o}{\theta}_i = \theta_i$. Then, for all θ_i,

$$(12) \qquad -\int \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\theta_i)\frac{\partial d^*}{\partial \theta_i}(\overset{o}{\theta}_i,\theta_{-i})f_i(\theta_{-i}|\overset{o}{\theta}_i)d\theta_{-i} = \frac{\partial t_{ii}}{\partial \theta_i}(\overset{o}{\theta}_i).$$

With $t_{ii}(\cdot)$ defined by (12), the first derivative of agent i's maximand becomes

$$(13) \qquad \begin{aligned} &\int [\frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\theta_i)f_i(\theta_{-i}|\theta_i) \\ &- \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\overset{o}{\theta}_i)f_i(\theta_{-i}|\overset{o}{\theta}_i)]\frac{\partial d}{\partial \theta_i}(\overset{o}{\theta}_i,\theta_{-i})d\theta_{-i}. \end{aligned}$$

Clearly, (13) vanishes at $\overset{o}{\theta}_i = \theta_i$. Also, in view of (6), $\frac{\partial d^*}{\partial \theta_i}(\overset{o}{\theta}_i,\theta_{-i}) > 0$. Therefore, to establish that truth-telling is optimal, it suffices to show that (13) is nonpositive for $\overset{o}{\theta}_i > \theta_i$ and nonnegative for $\overset{o}{\theta}_i < \theta_i$. Now these second-order conditions will not, in general, be satisfied. To ensure that they hold, we must impose stronger conditions on f_i. For any vector θ_{-i}, let $\theta_i^*(\theta_{-i})$ be a value of agent i's parameter such that

$$(14) \qquad \frac{\partial v_i}{\partial d}(d^*(\theta_i^*(\theta_{-i}),\theta_{-i}),\theta_i^*(\theta_{-i})) = 0.$$

If $\theta_i^*(\theta_{-i})$ exists, it is unique. To see this, suppose that $\theta_i = \theta_i^*$ satisfies

$$\text{(15)} \qquad \frac{\partial v_i}{\partial d}(d^*(\theta_i^*,\theta_i),\theta_i^*) = 0.$$

Differentiating $\frac{\partial v_i}{\partial d}(d^*(\theta_i',\theta_{-i}),\theta_i)$ with respect to θ_i we obtain

$$\text{(16)} \qquad \frac{\partial^2 v_i}{\partial d^2}(d^*(\theta_i',\theta_{-i}),\theta_i)\frac{\partial d^*}{\partial \theta_i}(\theta_i,\theta_{-i}) + \frac{\partial^2 v_i}{\partial d \partial \theta_i}(d^*(\theta_i',\theta_{-i}),\theta_i).$$

From (0),

$$\text{(17)} \qquad \frac{\partial d^*}{\partial \theta_i} = \frac{-\partial^2 v_i}{\partial d \partial \theta_i} \Big/ \sum_{j=1}^{n} \frac{\partial^2 v_i}{\partial d \partial \theta_i}.$$

Substituting (17) into (16) we obtain

$$\text{(18)} \qquad \frac{\sum_{j\neq i} \frac{\partial^2 v_j}{\partial d^2}}{\sum_{j=1}^{n} \frac{\partial^2 v_j}{\partial d^2}} \; \frac{\partial^2 v_i}{\partial d \partial \theta_i},$$

which is positive. Hence,

$$\text{(19)} \qquad \frac{\partial v_i}{\partial d}(d^*(\theta_i,\theta_{-i}),\theta_i) \begin{cases} > 0, \text{ if } \theta_i > \theta_i^* \\ < 0, \text{ of } \theta_i < \theta_i^* \end{cases},$$

establishing the uniqueness of $\theta_i^*(\theta_{-i})$. If $\theta_i^*(\theta_{-i})$ fails to exist, set it equal to 1 if $\frac{\partial v_i}{\partial d}(d^*(\theta_i,\theta_{-i}),\theta_i)$ is positive for all θ_i', and equal to zero, otherwise.

From the definition of d^*,

$$\sum_{j=1}^{n} \frac{\partial v_j}{\partial d}(d^*(\theta_i^*,\theta_{-i}),\theta_{-i}) = 0.$$

Thus, $\theta_i^*(\theta_{-i})$ is that value of θ_i which makes the social and individual marginal products of d both zero. In that case, $\theta_i^*(\theta_{-i})$ is the <u>representative</u> or <u>average</u> value of θ_{-i}.

One way of formalizing the idea that θ_i and θ_{-i} are nonpositively correlated is to suppose that as θ_i moves away from the average value of θ_{-i} the conditional density $f_i(\theta_{-i}|\theta_i)$ does not fall. That is,

$$(20)\qquad \frac{\partial f_i}{\partial \theta_i}(\theta_{-i}|\theta_i) \begin{cases} \geq 0, \text{ if } \theta_i \geq \theta_i^*(\theta_{-i}) \\ \leq 0, \text{ if } \theta_i \leq \theta_i^*(\theta_{-i}) \end{cases}.$$

This is the condition we need to establish that truthtelling is optimal.

Theorem 1: Under the assumptions we have made about v_i and θ_i in this section, an optimal Bayesian mechanism exists if, for all i, f_i satisfies (20).

Proof: We need only show that the bracketed expression of (13) is nonpositive (nonnegative) for $\overset{o}{\theta}_i$ greater (less) than θ_i. Consider $\overset{o}{\theta}_i > \theta_i$.

Suppose first that

$$(21)\qquad \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\theta_i) > 0.$$

Then, because d^* is increasing in θ_i and v_i is concave in d,

$$(22)\qquad \frac{\partial v_i}{\partial d}(d^*(\theta_i,\theta_{-i}),\theta_i) > 0.$$

Furthermore, from (6)

$$(23)\qquad \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\overset{o}{\theta}_i) > \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\theta_i) > 0$$

From (19), (22) and (23) we have

$$\theta_i^*(\theta_{-i}) < \theta_i < \overset{o}{\theta}_i.$$

Therefore, from (20),

$$(24)\qquad f(\theta_{-i}|\theta_i) \leq f(\theta_{-i}|\overset{o}{\theta}_i).$$

But (23) and (24) together imply that the bracketed expression in (13) is nonpositive.

Suppose next that

$$(25) \qquad \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\theta_i) \leq 0.$$

Then, the bracketed expression is nonpositive unless

$$(26) \qquad \frac{\partial v_i}{\partial d}(d^*(\overset{o}{\theta}_i,\theta_{-i}),\overset{o}{\theta}_i) < 0.$$

But (26) implies that

$$\theta_i < \overset{o}{\theta}_i < \theta_i^*(\theta_{-i}), \text{ which in turn means that}$$

$$(27) \qquad f(\theta_{-i}|\theta_i) \geq f(\theta_{-i}|\overset{o}{\theta}_i).$$

Combining (25) - (27), we conclude once again that the bracketed expression is nonpositive. The argument for $\theta_i < \overset{o}{\theta}_i$ is similar.

Q.E.D.

If the inequalities in (20) are reversed, then one cannot find a transfer function $t_{ii}(\overset{o}{\theta}_i)$ that induces agent i to tell the truth. Hence, with positive correlation, there does not exist an optimal Bayesian mechanism with transfer functions of the form (4).

The conclusion that negative rather than positive correlation makes incentive requirements easier to fulfill accords well with intuition, as Laffont and Maskin [1979] point out. Positive correlation aggravates the free rider problem. If an agent believes his tastes are similar to those of others, he can relatively safely leave provision of a public good in their hands.

2. Nonseparable Transfers

One limitation of using an additively separable transfer function is that it does not exploit the differences in beliefs corresponding to different values of θ_i. If $f_{-i}(\theta_{-i}|\theta_i)$ depends on the value of θ_i, then an agent of type $\bar{\theta}_i$ will view a transfer depending on $\overset{o}{\theta}_{-i}$ as a different gamble than will an agent of type $\bar{\bar{\theta}}_i$. But those different views are irrelevant to i's maximization problem if the terms in $\overset{o}{\theta}_i$ are separable from the terms in $\overset{o}{\theta}_{-i}$. Thus additively separable transfer functions reduce our ability to discriminate among types. This assertion is confirmed by our next result, which illustrates the power of more general transfers. Henceforth, we drop the special assumptions of section 1 and revert to the less structured model of the introduction.

Theorem 2: In the model of section 1, there exists an optimal Bayesian mechanism.

Proof: Becasue the proof is virtually entirely algebraic manipulation, it will be helpful to consider the simplest possible case to illustrate the ideas as clearly as possible. Accordingly, suppose that there are just two agents and that θ_i can assume just two values: θ^1 and θ^2. After we go through the argument for this case, it should be apparent how the proof generalizes.

For any i, j, k, ε {1,2} let

$$v_h^{ijk} = v_h(d^*(\theta^i,\theta^j),\theta^k)$$

and let p^{ij} be the joint probability that $\theta_1 = \theta^i$ and $\theta_2 = \theta^j$. We shall take t^{ij} to be the transfer to agent 1 if $\overset{o}{\theta}_1 = \theta^i$ and $\overset{o}{\theta}_2 = \theta^j$. Hence, for balance, the transfer to agent 2 in that event is $-t^{ij}$. Our problem is to find numbers t^{11},

t^{12}, t^{21}, and t^{22} such that

(28) $$\sum_j p^{1j}(v_1^{1j1} + t^{1j}) \geq \sum_j p^{1j}(v_1^{2j1} + t^{2j})$$

(29) $$\sum_j p^{2j}(v_1^{2j2} + t^{2j}) \geq \sum_i p^{2j}(v_1^{1j2} + t^{1j})$$

(30) $$\sum_i p^{i1}(v_2^{i11} - t^{i1}) \geq \sum_i p^{i1}(v_2^{i21} - t^{i2})$$

(31) $$\sum p^{i2}(v_2^{i22} - t^{i2}) \geq \sum_i p^{i2}(v_2^{i12} - t^{i1}).$$

It clearly suffices to find t^{ij}'s such that

(32) $$v_1^{iji} + t^{ij} \geq v_1^{kji} + t^{kj}$$

and

(33) $$v_2^{ijj} - t^{ij} \geq v_2^{ikj} - t^{ik},$$

for all i, j, and k. To see that we can find such t^{ij}'s, first set

(34) $$t^{11} = 0.$$

From (32), we must have

$$v_1^{111} \geq v_1^{211} + t^{21}.$$

Hence,

(35) $$t^{21} = v_1^{111} - v_1^{211} - \alpha,$$

where

(36) $$\alpha \geq 0.$$

From (32),

(37) $$v_1^{212} + t^{21} \geq v_1^{112}.$$

From (35) and (37) we have

$$(38) \qquad v_1^{212} - v_1^{112} + v_1^{111} - v_1^{211} \geq \alpha.$$

We must show that the left hand side of (38) is nonnegative. But for any i, j, and k

$$v_1^{kjk} + v_2^{kjj} \geq v_1^{ijk} + v_2^{ijj},$$

that is,

$$(39) \qquad v_2^{kjj} - v_2^{ijj} \geq v_1^{ijk} - v_1^{kjk}.$$

Similarly, we have

$$(40) \qquad v_2^{ijj} - v_2^{kjj} \geq v_1^{kji} - v_1^{iji}.$$

Adding (39) and (40), we obtain

$$(41) \qquad 0 \geq v_1^{ijk} - v_1^{kjk} + v_1^{kji} - v_1^{iji}.$$

Therefore, the left hand side of (38) is nonnegative after all. Similarly, we have

$$(42) \qquad 0 \geq v_2^{ikj} - v_2^{ijj} + v_2^{ijk} - v_2^{ikk},$$

for all j and k. Now from (33) and (35)

$$v_2^{211} - v_1^{111} + v_1^{211} + \alpha \geq v_2^{221} - t^{22}.$$

Therefore,

$$(43) \qquad t^{22} = v_2^{221} - v_2^{211} + v_1^{111} - v_1^{211} - \alpha + \beta,$$

where

$$(44) \qquad \beta \geq 0.$$

From (33) we require

$$(45) \qquad v_2^{222} - t^{22} \geq v_2^{212} - t^{21}.$$

Substituting from (35) and (43), we can rewrite (45) as

(46) $$v_2^{222} - v_2^{212} + v_2^{211} - v_2^{221} \geq \beta.$$

From (42) the left hand side of (46) is nonnegative.

Now, from (32),

(47) $$v_1^{222} + t^{22} \geq v_1^{122} + t^{12}.$$

Using (43) we can rewrite (47) as

(48) $$t^{12} = v_1^{222} - v_1^{122} + v_2^{221} - v_2^{211} + v_1^{111} - v_1^{211} - \alpha + \beta - \gamma,$$

where

(49) $$\gamma \geq 0.$$

From

$$v_1^{121} + t^{12} \geq v_1^{221} + t^{22},$$

we deduce that

(50) $$v_1^{121} - v_1^{221} + v_1^{222} - v_1^{122} \geq \gamma,$$

which is possible in view of (41).

We have shown that when we define the t^{ij}'s by (34), (35), (43), and (48), all but two of the eight incentive constraints given by (32) and (33) are satisfied if α, β and γ are nonnegative and satisfy (38), (46), and (50). It remains to show that we can choose α, β, and γ within this range so that the last two incentive constraints,

(51) $$v_2^{111} - t^{11} \geq v_2^{121} - t^{12}$$

and

(52) $$v_2^{122} - t^{12} \geq v_2^{112} - t^{11}$$

also hold.

From (48), (51) and (52) can be reexpressed as

$$(53)\quad (v_2^{122}-v_2^{112}) - (v_1^{222}-v_1^{122}) - (v_2^{221}-v_2^{211}) - (v_1^{111}-v_1^{211})$$
$$\geq -\alpha+\beta-\gamma \geq (v_2^{121}-v_2^{111}) - (v_1^{222}-v_1^{122}) - (v_2^{221}-v_2^{211}) - (v_1^{111}-v_1^{211}).$$

Now, it is possible to satisfy (53) as long as (i) the maximum value of $-\alpha+\beta-\gamma$ is not less than the rightmost expression in (53), and (ii) the minimum value of $-\alpha+\beta-\gamma$ is not greater than the leftmost expression in (53).

The maximum value of $-\alpha+\beta-\gamma$ occurs when $\alpha = \gamma = 0$ and $\beta = v_2^{222}-v_2^{212}+v_2^{211}-v_2^{221}$. Therefore, to establish (i) amounts to showing that

$$v_2^{222}- v_2^{212}+v_2^{211}-v_2^{221} \geq v_2^{121}-v_2^{111}-(v_1^{222}-v_1^{122})-(v_2^{221}-v_2^{211})-(v_1^{111}-v_1^{211}),$$

which reduces to

$$(54)\quad (v_2^{222}-v_2^{212}) + (v_2^{111}-v_2^{121}) + (v_1^{222}-v_1^{122}) + (v_1^{111}-v_1^{211}) \geq 0.$$

Now,

$$v_1^{121} + v_2^{122} \geq v_1^{211} + v_2^{212} = v_1^{111} + (v_1^{211}-v_1^{111}) + v_2^{222} + (v_2^{212}- v_2^{222}).$$

That is,

$$(55)\quad 0 \geq (v_1^{111}-v_1^{121}) + (v_2^{222}-v_2^{122}) + (v_1^{211}-v_1^{111}) + (v_2^{212}-v_2^{222}).$$

Furthermore, from (40) we have

$$(56)\quad v_1^{111} - v_1^{121} \geq v_2^{121} - v_2^{111},$$

and

$$(57)\quad v_2^{222} - v_2^{122} \geq v_1^{122} - v_1^{222}.$$

Substituting (56) and (57) in (55) we obtain

$$(58)\quad 0 \geq (v_1^{122} - v_1^{222}) + (v_2^{121} - v_2^{111}) + (v_1^{211} - v_1^{111}) + (v_2^{212} - v_2^{222}).$$

Similarly, by permuting indices, we get

$$(59) \qquad 0 \geq (v_1^{112}-v_1^{212}) + (v_2^{112}- v_2^{122}) + (v_1^{221}-v_1^{121}) + (v_2^{221}-v_2^{211}).$$

Now, (58) is just (54), establishing (i).

The minimum value of $-\alpha+\beta-\gamma$ occurs when $\beta = 0$, $\alpha = v_1^{212}-v_1^{112}+v_1^{111}-v_1^{211}$, and $\gamma = v_1^{121}-v_1^{221}+v_1^{222}-v_1^{122}$. Therefore, establishing (ii) amounts to showing that

$$(55) \qquad \begin{aligned} &(v_2^{122}-v_2^{112}) - (v_1^{222}-v_1^{122}) - (v_2^{221}-v_2^{211}) - (v_1^{111}-v_1^{211}) \\ &\geq -[(v_1^{212}-v_1^{112}) + (v_1^{111}-v_1^{211}) +(v_1^{121}-v_1^{221}) + (v_1^{222}-v_1^{122})] \end{aligned}$$

After cancellation and rearrangement, (55) becomes

$$(v_2^{122}-v_2^{112}) + (v_2^{211}-v_2^{221}) + (v_1^{212}-v_1^{112}) +(v_1^{121}-v_1^{221}) \geq 0,$$

which is just (59). Hence (ii) is established.

Q.E.D.

Notice that because the transfers t^{ij} do not depend on the probabilities, the theorem and method of proof continue to hold when agents have different beliefs about the joint distribution of the Θ_i's (as long as those beliefs are common knowledge).

Footnotes

[1] If f is a function of $\theta_1,\ldots,\theta_n$, the notation $f(\overset{\circ}{\theta}_i,\theta_{-i})$ is shorthand for

$$f(\theta_1,\ldots,\theta_{i-1},\overset{\circ}{\theta}_i,\theta_{i+1},\ldots,\theta_n).$$

[2] Because the θ_i's are presumed to be independent, the expectation in (5) is unconditional.

[3] See also D'Aspremont and Gerard-Varet [1982] for results on the correlated case.

[4] d^* is differentiable because it solves (1) and because v is strictly concave and twice continuously differentiable.

[5] Laffont and Maskin [1979] show that, under the hypotheses of Theorem 1, $\overset{\circ}{\theta}_i=\theta_i$ is a <u>local</u> maximum when t_{ii} is defined by (12). Riordan [1984] establishes a result related to the Theorem when n=2. Although his result is couched in terms of <u>positive</u> correlation, he correlates costs and benefits. Therefore, his positive correlation of costs and benefits amounts to negative correlation of benefits.

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