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Roy Radner and incentive theory

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1 Introduction

How to provide agents with sufficient motivation to do what society – or its proxy in the guise of a center or social planner – wants them to do is the subject of incentive theory. The theory has normative force whenever, metaphorically speaking, the invisible hand of the market fails to provide such motivation automatically. Market failure can come about either because markets are imperfect in some way or because they do not exist at all. Indeed, a leading example of a non-market environment is the internal organization of a large corporation. Alfred Chandler (1977) made just this point when he gave his study of the modern American enterprise the title *The Visible Hand*.

The study of these large enterprises – which itself is an enterprise that is currently blossoming – has drawn on incentive theory in a fundamental way. And it is not surprising that Roy Radner – who had a long-standing fascination and more than casual personal acquaintance with large organizations – should have been inspired to make important contributions to this theory.

In this essay, I shall provide an outline of some of the major results in incentive theory with particular attention to Radner's work on the subject.

2 A simple model of a team

Let me begin with one of the first formal attempts to model organizations, viz, team theory, whose creation is due to Marschak and Radner (1972). Let Θ be the set of possible states of the world, and for each $\theta \in \Theta$, let $p(\theta)$ be the prior probability of θ. There are n agents, indexed by $i = 1, \ldots, n$. Each agent i has an action space A_i and a private signal space S_i. Both of these may in part be

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exogenous and in part the choice of the team (or team "designer"). S_i can be thought of as a partition of Θ. That is, each signal $s_i \in S_i$ corresponds to a subset of Θ. Given the vector of signals $s = (s_1, ..., s_n)$, let $\pi(\theta \mid s)$ be the distribution of θ (derived from $p(\theta)$ using Bayes' rule) conditional on s. In addition to the n agents, the center (or CEO, social planner, etc.), whom we shall designate as agent 0, may be an active participant, in which case he has an action space A_0 (for simplicity, let us assume, however, that he observes no private signals, so that we can dispense with S_0).

The team is interested in implementing a *collective choice rule,* that is, a rule that specifies all agents' actions as a function of the available information s. Thus a collective choice rule f is a mapping

$$f : S_1 \times ... \times S_n \to A_0 \times ... \times A_n.$$

Much of team theory concerns the question of what constitutes the best way for agents to communicate with one another in order to implement the desired collective choice rule, assuming that communication is costly. A common simplifying assumption in this theory is that all agents and the center share the same objectives. Incentive theory, however, gains much of its interest from the presumption the agents have *different* preferences. Let us suppose that agent i's preferences can be represented by the (von Neumann-Morgenstern) utility function

$$u_i(a, \theta),$$

where $a = (a_0, ..., a_n)$. The fact agent i's payoff depends on other agents' actions embodies the idea that there may be externalities to actions. Similarly, the center has utility function

$$u_0(a, \theta),$$

which, if the center is just a surrogate for the group of agents as a whole, may take the form

$$\sum_{i=1}^{n} \lambda_i u_i(a, \theta).$$

Here λ_i is the "welfare weight" for agent i. Usually, in both team and incentive theory, the function f is chosen to maximize the expectation of u_0, i.e.,

$$f(s) \in \arg\max_a \sum_{\theta \in \Theta} u_0(a, \theta)\pi(\theta \mid s). \tag{1}$$

3 Adverse selection

For the time being, let us drop the actions $a_1, ..., a_n$ (but not a_0). Then the incentive problem is how to ensure that the center's action a_0 properly reflects agents' information s (in the sense of satisfying (1)), in view of the fact that the signals are private information. Models like this, where the major substantive difficulty is the private nature of information, are often called problems of *adverse selection* (or hidden information).

The solution to an adverse selection problem is normally formulated as an *incentive mechanism* (also variously called a "game form," "outcome function," "contract," or "constitution"). Suppose that each agent i is allocated a "message" space M_i. A message $m_i \in M_i$ can be thought of as agent i's announcement about his signal s_i (but this interpretation is not necessary). Then an incentive mechanism g is a function

$$g : M_1 \times ... \times M_n \to A_0.$$

We interpret this mechanism as specifying that the center will take action $g(m) \in A_0$ if the messages are $m = (m_1, ..., m_n)$. Thus $g(m)$ is called the *outcome* of the mechanism. For each vector of signals s, there will be a corresponding equilibrium (perhaps more than one) of the incentive mechanism (where each agent i evaluates the outcome $g(m)$ using his utility function u_i). Of course, exactly what an equilibrium is will depend on the solution concept that pertains. For a given solution concept, let $E_g(s)$ be the equilibrium outcome (for simplicity, we suppose that the equilibrium outcome is unique). If, for all s,

$$f(s) = E_g(s), \tag{2}$$

we say that g *implements* f (or that f is implemented by g) with respect to the solution concept. Much of the incentive literature consists of characterizing which social choice rules are implementable in this sense, with respect to particular solution concepts.

4 Adverse selection with dominant strategies

By far the simplest (and strongest) solution concept is equilibrium in dominant strategies. Agent i with signal s_i has a *dominant strategy* $m_i(s_i)$ for mechanism g if $m_i(s_i)$ solves

$$\max_{m_i} \sum_{\theta} u_i(g(m_i, m_{-i}), \theta)\pi_i(\theta \mid s_i) \quad \text{for all } m_{-i},$$

where $\pi_i(\theta \mid s_i)$ is the distribution of θ conditional on s_i and m_{-i} is the vector of other agents' messages.

Having a dominant strategy makes life easy for agent i because it obviates the need for him to form beliefs about what other players know and how they behave. Clearly, requiring that an equilibrium be independent of beliefs is demanding. Nevertheless, Groves (1973) showed that, in a special but important case of the Marschak-Radner framework, there is a large class of collective choice rules that are implementable. Specifically, suppose that the center's action a_0 takes the form

$$a_0 = (x, y_1, ..., y_n),$$

where x can be interpreted as the choice of a public good and the y_i's (which are scalars) are transfers of a private good (or money). Assume, moreover, that each agent i's utility takes the form

$$u_i(a_0, \theta) = v_i(x, s_i) + y_i. \tag{3}$$

That is, utility is *quasi-linear*. Then, as Groves demonstrated, any $f(s) = (x(s), y_1(s), \ldots, y_n(s))$ for which

$$x(s) \in \arg\max_x \sum_{i=1}^{n} v_i(x, s_i) \quad \text{for all } s \tag{4}$$

is implementable in dominant strategies provided that each $y_i(s)$ takes the form

$$y_i(s) \equiv \sum_{j \neq i} v_j(x(s), s_j) + k_i(s_{-i}), \tag{5}$$

where s_{-i} is the vector of signals excluding that of agent i and $k_i(\cdot)$ is an arbitrary function of s_{-i}. (Notice that (4) is the requirement that the public good be chosen to maximize social surplus.)

To see this, suppose that agents are confronted with a mechanism in which each agent i is asked to report a signal value $\hat{s}_i \in S_i$, and the outcome, given reports $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_n)$, is $(x(\hat{s}), y_1(\hat{s}), \ldots, y_n(\hat{s}))$, where $x(\cdot)$ and $(y_1(\cdot), \ldots, y_n(\cdot))$ satisfy (4) and (5) respectively. Then, given (3), agent i's maximization problem is

$$\max_{\hat{s}_i} \left[v_i(x(\hat{s}_i, \hat{s}_{-i}), s_i) + \sum_{j \neq i} v_j(x(\hat{s}_i, \hat{s}_{-i}), \hat{s}_j) + k_i(\hat{s}_{-i}) \right]. \tag{6}$$

By varying $\hat{s}_i$, agent i can vary $x(\hat{s}_i, \hat{s}_{-i})$. But, by definition of $x(s)$,

$$x(s_i, \hat{s}_{-i}) = \arg\max_x \left[v_i(x, s_i) + \sum_{j \neq i} v_j(x, \hat{s}_j) \right].$$

Hence $\hat{s}_i = s_i$ solves (6). That is, it is a dominant strategy for agent i to tell the truth, establishing that $(x(s), y_1(s), \ldots, y_n(s))$ is implementable.

Green and Laffont (1979) showed, in fact, that any implementable social choice rule satisfying (4) *must* satisfy (5). To understand why this is so,[1] notice first that if mechanism $g : M_1 \times \ldots \times M_n \to A_0$ implements a collective choice rule f in dominant strategies and if, for all i and all s_i, $m_i(s_i)$ is agent i's dominant strategy when his signal is s_i, then g^* where

$$g^*(s_1, \ldots, s_n) \equiv g(m_1(s_1), \ldots, m_n(s_n))$$

also implements f.[2] Observe that g^* is a "direct revelation" mechanism in the sense that strategies consist of announcing a signal, and it is a dominant strategy

[1] The following argument is drawn from Laffont and Maskin (1980).

[2] Actually, it is conceivable that, in going from g to g^*, we might introduce additional, non-optimal equilibria. Although this is potentially a serious problem, we shall ignore it here (but see Dasgupta et al. (1979)).

for agents to announce signals truthfully. Thus, it suffices to restrict attention to direct revelation mechanisms when searching for mechanisms that implement a collective choice rule. Now, suppose that $f(s) = (x(s), y_1(s), \ldots, y_n(s))$ satisfies (4) and is implementable in dominant strategies. Suppose that, for all i, S_i is an open interval of real numbers, $v_i(\cdot,\cdot)$ is a twice differentiable function of x and s_i (with $\frac{\partial v_i}{\partial x} > 0$, $\frac{\partial^2 v_i}{\partial x^2} < 0$, and $\frac{\partial^2 v_i}{\partial x \partial s_i} > 0$), and $x(\cdot)$ and $y_i(\cdot)$ are differentiable functions of s_i. Since we can restrict attention to direct revelation mechanisms, the fact that f is implementable implies that, for all s_i and s_{-i},

$$s_i \in \arg\max_{\hat{s}_i} \left[v_i(x(\hat{s}_i, s_{-i}), s_i) + y_i(\hat{s}_i, s_{-i}) \right].$$

Hence

$$\frac{\partial v_i}{\partial x}\left(x(s_i, s_{-i}), s_i\right)\frac{\partial x}{\partial s_i}(s_i, s_{-i}) + \frac{\partial y_i}{\partial s_i}(s_i, s_{-i}) = 0. \tag{7}$$

Now from the above analysis, we know that one solution to the differential equation (7) is $y_i(s) = \sum_{j \neq i} v_j(x(s), s_j)$. Moreover, from the theory of differential equations, we know that all solutions differ by a constant $k_i(s_{-i})$. Hence, we can conclude that (5) holds.

The form (3) embodies the assumption of *private values:* agent i's payoff depends on θ only through his signal s_i, i.e., in particular, his payoff does not depend on s_{-i}. If we relax this assumption and allow s_{-i} to affect v_i, we are in the realm of *common values*. Radner and Williams (1988) showed that $f(s) = (x(s), y_1(s), \ldots, y_n(s))$ can be implemented in dominant strategies even when there are common values, if v_i takes the form

$$v_i(x, s) = w_i(x, s_i) + z_i(s). \tag{8}$$

To see this, observe that when (8) holds and agent i is confronted with the direct revelation mechanism $(x(\hat{s}), y_1(\hat{s}), \ldots, \hat{y}_n(\hat{s}))$ satisfying (4) and

$$y_\ell(\hat{s}) = \sum_{j \neq \ell} w_j\left(x(\hat{s}_j, \hat{s}_{-j}), \hat{s}_j\right) + k_\ell\left(\hat{s}_{-\ell}\right) \quad \text{for all } \ell,$$

his maximization problem is

$$\max_{\hat{s}_i}\left[w_i(x(\hat{s}_i, \hat{s}_{-i}), s_i) + z_i(s_i, s_{-i}) + \sum_{j \neq i} w_j(x(\hat{s}_i, \hat{s}_{-i}), \hat{s}_j) + k_i(\hat{s}_{-i}) \right]. \tag{9}$$

But because $z_i(s_i, s_{-i})$ does not depend on $\hat{s}_i$, $\hat{s}_i = s_i$ solves (9), establishing that it is a dominant strategy for i to tell the truth.

Radner and Williams went a step further, in fact, and showed that, with common values, (8) *must* hold for a collective choice rule $f(s) = (x(s), y_1(s), \ldots, y_n(s))$ satisfying (4) to be implementable. To see this, let us make the same differentiability assumptions as before (with the additional assumption that $\frac{\partial^2 v_j}{\partial x \partial s_i} \geq 0$ for $j \neq i$). Suppose that f is implementable by direct revelation mechanism $(x(\hat{s}), y_1(\hat{s}), \ldots, y_n(\hat{s}))$. Then, analogous to (7), we obtain the following first-order condition for agent i:

$$\frac{\partial v_i}{\partial x}\left(x(s_i,\hat{s}_{-i}),s_i,s_{-i}\right)\frac{\partial x}{\partial s_i}(s_i,\hat{s}_{-i})+\frac{\partial y_i}{\partial s_i}(s_i,\hat{s}_{-i}) \tag{10}$$
$$=0 \quad \text{for all } s_i, s_{-i}, \quad \text{and } \hat{s}_{-i}.$$

Because $\frac{\partial^2 v_i}{\partial x \partial s_i} > 0$, $\frac{\partial^2 v_i}{\partial x^2} < 0$, and $\frac{\partial^2 v_j}{\partial x \partial s_i} \geq 0$, we have $\frac{\partial x}{\partial s_i} > 0$.[3] Hence if, given s_i and $\hat{s}_{-i}$, (10) is to hold for all s_{-i}, we must have $\frac{\partial^2 v_i}{\partial x \partial s_{-i}} = 0$. Hence, v_i must be additively separable between x and s_{-i}, i.e., it takes the form (8).

5 Adverse selection: Other solution concepts

The positive results for dominant strategies in the case of quasi-linear preferences do not readily generalize to significantly broader environments, as the results of Gibbard (1973), Hurwicz (1972), and Satterthwaite (1975) make clear. Accordingly, a large literature has developed in which various species of Nash equilibrium (see Moore 1992 for a recent survey) or Bayesian equilibrium (see Palfrey 1992) are appealed to instead.

One principle that this literature makes clear is that typically the more that Nash or Bayesian equilibrium is refined – i.e., the more restrictive the definition of equilibrium – the *bigger* the class of implementable collective choice rules becomes. At first this principle may seem at odds with the foregoing discussion. After all, it was precisely because insufficiently many collective choice rules were implementable in dominant strategies that the solution concept was relaxed. The paradox is resolved, however, when one notices that, in order to satisfy equation (2), not only must there be an equilibrium (a requirement which is hard to satisfy when dominant strategy equilibrium is the solution concept) but there must be no equilibrium outcomes *other* than $f(s)$ (a requirement which is more problematic for Nash and Bayesian equilibrium). By refining the Nash and Bayesian concepts (for which the existence of equilibrium is usually not a problem), there is, therefore, hope of eliminating the unwanted equilibria.

6 Moral hazard

We temporarily left actions $(a_1, \ldots, a_n)$ out of the model above in order to concentrate on pure adverse selection, but we can readily restore them to that model if these actions are *perfectly observable* to the center. Indeed, in that case we can regard $(a_1, \ldots, a_n)$ as part of the center's choice a_0, since he can simply "order"

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[3] To see this, note that the first-order condition determining $x(s)$ is $\sum_{j=1}^{n} \frac{\partial v_j}{\partial x}(x(s), s) = 0$. Differentiating this identity with respect to s_i, we obtain

$$\sum_{j=1}^{n} \frac{\partial^2 v_j}{\partial x^2}\frac{\partial x}{\partial s_i} + \sum_{j=1}^{n} \frac{\partial^2 v_j}{\partial x \partial s_i} = 0,$$

from which the conclusion follows.

agents to choose the desired actions. The more difficult problem arises when the a_i's are only imperfectly observable – the case of *moral hazard*.

Assume, therefore, that the center cannot observe $(a_1, ..., a_n)$ but only a noisy signal $z \in Z$. Let $q(z \mid a_1, ..., a_n)$ be the distribution of z conditional on $(a_1, ..., a_n)$. We will think of the center as choosing a_0 contingent on the realization of z. Hence, it will be convenient to suppose that the agents first (and simultaneously) choose their actions, and that then, after z is realized, the center chooses a_0. Because I wish to focus on the case of "pure" moral hazard, I will drop the signals $s = (s_1, ..., s_n)$. Hence, for $i = 0, 1, ..., n$, we can write agent i's utility as

$$\phi_i(a_0(\cdot), a_1, ..., a_n) = \sum_{z \in Z} u_i(a_0(z), a_1, ..., a_n) q(z \mid a_1, ..., a_n).$$

7 The principal-agent relationship

Suppose that $n = 1$ (so that there is just one agent) and that the center's payoff depends on a_1 only through z:

$$\phi_0(a_0(\cdot), a_1) = \sum_{z \in Z} r_0(a_0(z), z) q(z \mid a_1),$$

where $r_0(\cdot, \cdot)$ is a function of a_0 and z. Assume, finally, that A_0 consists of the real numbers and that A_1 is a set of nonnegative numbers (we can think of a_0 as a monetary transfer and a_1 as an effort level). Then we are in the standard principal-agent framework (the center is the principal).

Let us first consider the case in which the principal's and agent's payoffs are *linear* in a_0. Specifically suppose that

$$r_0(a_0, z) = z - a_0$$

and

$$u_1(a_0, a_1) = a_0 - \tfrac{1}{8} a_1,$$

where z is the output produced by the agent (and which accrues to the principal), and $a_1 \in \{0, 1\}$ (i.e., the agent can either "work" and set $a_1 = 1$, or "shirk" and set $a_1 = 0$). Let us suppose that if the agent works, there is an equal chance of high ($z = 2$) or low ($z = 0$) output. But if he shirks, output is low for sure. That is,

$$\Pr\{z = 2 \mid a_1 = 1\} = \Pr\{z = 0 \mid a_1 = 1\} = \frac{1}{2} \text{ and } \Pr\{z = 0 \mid a_1 = 0\} = 1.$$

Because expected net surplus from the agent's working $\left(\frac{1}{2} \cdot 2 - \frac{1}{8}\right)$ is positive, it is efficient for the agent to work (i.e., set $a_1 = 1$). Thus, because payoffs are linear in a_0, the Pareto frontier (the locus of Pareto optimal payoffs) is the straight line $v_0 + v_1 = \frac{7}{8}$, where v_0 and v_1 are the principal's and agent's payoffs, respectively. Now for the agent to be induced to work, his monetary payments when output is high ($a_0(2)$) or low ($a_0(0)$) must be such that

$$\frac{1}{2}a_0(2) + \frac{1}{2}a_0(0) - \frac{1}{8} \geq a_0(0). \qquad (11)$$

Hence, in particular, the principal and agent can sustain the expected payoffs $\left(\frac{1}{2}, \frac{3}{8}\right)$ on the Pareto frontier by agreeing on monetary payments $a_0(2) = \frac{3}{4}$ and $a_0(0) = \frac{1}{4}$.

Next, suppose instead that the Pareto frontier is *nonlinear*. Specifically, assume that

$$\begin{aligned} r_0(a_0, z) &= z - a_0 \\ u_1(a_0, a_1) &= a_0 - \tfrac{1}{4}a_0^2 - \tfrac{1}{8}a_1, \end{aligned}$$

but that the model is otherwise the same as before. Notice that the agent is now risk-averse with respect to his monetary payment.

Observe that if $a_1 = 0$ (i.e., the agent shirks), then either the principal's or the agent's payoff must be non-positive. Hence, assuming that a player has the option not to participate if his payoff is negative, it remains efficient for the agent to work, i.e., to choose $a_1 = 1$. To derive the Pareto frontier, take $v_0 \equiv r_0(a_0, 1) = 1 - a_0$ and $v_1 \equiv u_1(a_0, 1) = a_0 - \frac{1}{4}a_0^2 - \frac{1}{8}$. Replacing a_0 by $1 - v_0$ (using the first equation) in the second equation, we obtain

$$v_1 = \frac{7}{8} - v_0 - \frac{1}{4}(1 - v_0)^2.$$

Because this curve is strictly concave, convex combination of points on the frontier lie strictly below. This implies that points on the frontier can no longer be sustained since in order to induce the agent to work it must be the case that $a_0(2) > a_0(0)$; i.e., the agent's payoff is a convex combination of two different points.

Suppose, however, that this principal-agent model is repeated infinitely many times and that players maximize their discounted sum of payoffs using discount factor δ. Then, the principal maximizes

$$E\sum_{t=0}^{\infty}\delta^t(z^t - a_0^t)$$

and the agent maximizes

$$E\sum_{t=0}^{\infty}\delta^t\left(a_0^t - (a_0^t)^2 - \frac{1}{8}a_1^t\right),$$

where, for each t, z^t is the period-t realization of z and a_0^t and a_1^t are the choices of a_0 and a_1 in period t.

Even in the repeated game, Pareto optimal points are unattainable as equilibria. To see this, note that if (v_0, v_1) are the average payoffs[4] of a Pareto optimal

[4] If $\left(v_0', v_1'\right)$ are a pair of total payoffs in the repeated game, then the corresponding average payoffs (v_0, v_1) are those that would on average have to accure every period to sum to $\left(v_0', v_1'\right)$, i.e.,

$$\left(v_0', v_1'\right) = \frac{(v_0, v_1)}{1 - \delta}.$$

perfect Bayesian equilibrium of the repeated game, then

$$(v_0, v_1) = (1-\delta)\left(v_0^1, v_1^1\right) + \delta\left(\frac{1}{2}(v_0(2), v_1(2)) + \frac{1}{2}(v_0(0), v_1(0))\right), \qquad (12)$$

where (v_0^1, v_1^1) are the first period equilibrium payoffs and, for $z = 0, 2$, $(v_0(z), v_1(z))$ are the average continuation equilibrium payoffs (i.e., the average equilibrium payoffs starting in period 2) following the realization of output z in the first period. Now if (v_0, v_1) correspond to a Pareto optimum, then from (12), so must $\left(v_0^1, v_1^1\right)$and $\frac{1}{2}(v_0(2), v_1(2)) + \frac{1}{2}(v_0(0), v_1(0))$. But since Pareto optimality requires that the agent be induced to work in the first period, we must have $v_1(2) > v_1(0)$. Moreover, because the Pareto frontier is strictly concave, this implies that $\frac{1}{2}(v_0(2), v_1(2)) + \frac{1}{2}(v_0(0), v_1(0))$ cannot be Pareto optimal, and so neither can (v_0, v_1).

Nevertheless, as Radner (1981) and (1985) showed, any point in the *interior* of the utility possibility set (the UPS is the set of payoffs that are feasible—including those obtained by randomization—in the one-shot model), no matter how close to the Pareto frontier, can be attained as the average payoffs of a perfect Bayesian equilibrium (PBE) of the repeated game, provided that δ is near enough 1. To see this, choose small $\varepsilon > 0$ and consider the interior point $\left(\frac{1}{2}-\varepsilon, \frac{5}{16}-\varepsilon\right)$ near the Pareto optimal point $\left(\frac{1}{2}, \frac{5}{16}\right)$. Let B be the ball of radius ε around the point $\left(\frac{1}{2}-\varepsilon, \frac{5}{16}-\varepsilon\right)$. I will argue that, for δ near enough 1, any point (v_0, v_1) in B can be "decomposed" in the sense that there exist a_0^1 and

$$(v_0(2), v_1(2)), (v_0(0), v_1(0)) \in B, \qquad (13)$$

such that

$$\begin{aligned}(v_0, v_1) \quad = \quad & (1-\delta)\left(1 - a_0^1, a_0^1 - \frac{1}{4}\left(a_0^1\right)^2 - \frac{1}{8}\right) \\ & +\delta\left(\frac{1}{2}(v_0(2), v_1(2)) + \frac{1}{2}(v_0(0), v_1(0))\right) \qquad (14)\end{aligned}$$

and

$$\begin{aligned}(1-\delta)\left(a_0^1 - \frac{1}{4}\left(a_0^1\right)^2 - \frac{1}{8}\right) + \delta\left(\frac{1}{2}v_1(2) + \frac{1}{2}v_1(0)\right) \\ \geq (1-\delta)\left(a_0^1 - \frac{1}{4}\left(a_0^1\right)^2\right) + \delta v_1(0). \qquad (15)\end{aligned}$$

Establishing that, for given δ, all points in B can be decomposed according to (13)-(15) allows us to conclude that all points in B correspond to PBE's for discount factor δ. Indeed, we can iteratively *construct* the PBE corresponding to (v_0, v_1). Specifically, choose $a_0 = a_0^1$ and $a_1 = 1$ as the first-period actions. Let $(v_0(2), v_1(2))$ and $(v_0(0), v_1(0))$ be the continuation payoffs after high and low output respectively. Because (15) holds, the agent does not have the incentive to deviate from working. Hence, the first-period behavior is consistent with equilibrium. But from (13), both $(v_0(2), v_1(2))$ and $(v_0(0), v_1(0))$ can be decomposed à la (13)-(15). These decompositions will determine the equilibrium second-period behavior following high and low output. Continuing in the same way, we can derive the equilibrium behavior for all subsequent periods, thereby completing the construction.

It remains, therefore, only to show that we can actually perform the decomposition. Consider the point $\left(\frac{1}{2}, \frac{5}{16} - \varepsilon\right)$ in B. Let

$$a_0^1 = 2 - \frac{\sqrt{9 + 16\varepsilon}}{2} \tag{16}$$

and

$$(v_0(2), v_1(2)) = \left(\left(\frac{3}{2} - \delta - \frac{(1-\delta)\sqrt{9+16\varepsilon}}{2}\right)/\delta, \frac{5}{16} - \varepsilon + \frac{1-\delta}{8\delta}\right) \tag{17}$$

$$(v_0(0), v_1(0)) = \left(\left(\frac{3}{2} - \delta - \frac{(1-\delta)\sqrt{9+16\varepsilon}}{2}\right)/\delta, \frac{5}{16} - \varepsilon - \frac{1-\delta}{8\delta}\right). \tag{18}$$

Simple substitution verifies that (14) and (15) hold when the values given by (16)-(18) are used. As for (13), note that, because B is a ball, the vertical distance from the point $p = \left(\left(\frac{3}{2} - \delta - \frac{(1-\delta)\sqrt{9+16\varepsilon}}{2}\right)/\delta, \frac{5}{16} - \varepsilon\right)$ to B's boundary is on the order of the *square root* of the horizontal distance x from p to $\left(\frac{1}{2}, \frac{5}{16} - \varepsilon\right)$, if x is small (see Fig. 1). But $x = \frac{1-\delta}{2\delta}\left(\sqrt{9+16\varepsilon} - 3\right)$ and so, as δ tends to 1, x does indeed become small. Furthermore, the vertical distance from $(v_0(2), v_1(2))$ or $(v_0(0), v_1(0))$ to p is $\frac{1-\delta}{8\delta}$, which (for δ near 1) is of the same order as x, and hence less than $\sqrt{x}$. Hence for δ near 1, $(v_0(2), v_1(2))$ and $(v_0(0), v_1(0))$ lie in B.

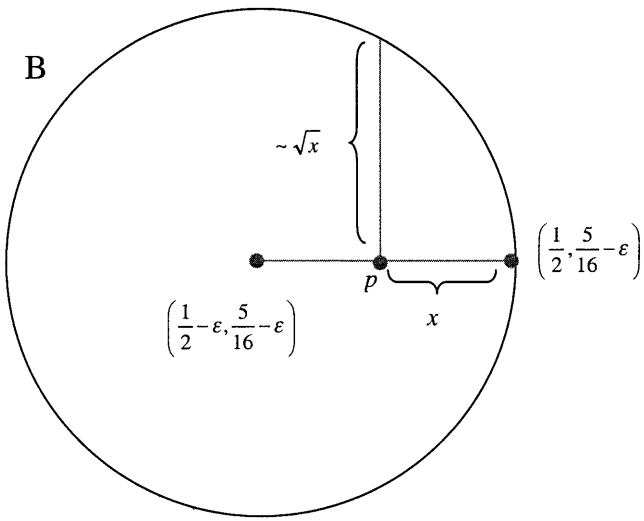

Fig. 1. Decomposition of $\left(\frac{1}{2}, \frac{5}{16} - \epsilon\right)$

We have shown, therefore, that $\left(\frac{1}{2}, \frac{5}{16} - \varepsilon\right)$ can indeed be decomposed for δ near 1. The argument is similar for the other points of B. Hence repetition permits points that are nearly efficient to be attained as equilibria.

To summarize, for the agent to be induced to work in a one-shot principal-agent relationship, his monetary payment contingent on output must be variable. This variability has no adverse consequences if the agent is risk-neutral, but interferes with Pareto optimality if he is risk-averse. Once the relationship is repeated, the agent's monetary payment no longer need be made variable; the agent can be "punished" or "rewarded" through variations in his continuation payoff. Furthermore, if δ is near 1, not much variation in these payoffs is required to provide adequate incentive —so the equilibrium shortfall from Pareto optimality is correspondingly small. That is, repetition allows us to exploit the fact that the Pareto frontier is *locally* linear.

8 Partnerships

Next let $n = 2$ but eliminate agent 0 (the center), so that we are now in a partnership (double moral hazard) framework, i.e., neither agent can observe the other's action. In this setting, an efficient outcome may be impossible to implement even if the Pareto frontier is linear. Specifically, consider the following model based on an example in Fudenberg et al. (1994). Suppose that each player's action a_i can equal w ("work") or s ("shirk"). Working imposes a disutility of 3, whereas shirking is costless. There are two possible output levels, $z = 0$ and $z = 12$. If both players work, the probability that $z = 12$ is $\frac{2}{3}$; if only one works the probability is $\frac{1}{3}$; and if neither works it is 0. Output is divisible and can be allocated in any way between the two agents. Agent i's utility is

$$z_i - d_i(a_i),$$

where z_i is his share of total output and $d_i(a_i)$ is his disutility from action a_i (i.e., either 0 or 3).

It is easy to verify that it is efficient for both agents to work and that the Pareto frontier is the straight line $v_1 + v_2 = 2$. Despite the linear preferences, however, no point on the frontier is implementable. To see this, note that to induce player i to work (given that the other player is working), his shares – $z_i(12)$ and $z_i(0)$ – of the output when $z = 12$ and $z = 0$ must satisfy

$$\frac{2}{3}z_i(12) + \frac{1}{3}z_i(0) - 3 \geq \frac{1}{3}z_i(12) + \frac{2}{3}z_i(0).$$

Hence

$$z_i(12) - z_i(0) \geq 9 \qquad i = 1, 2. \tag{19}$$

Adding the two inequalities (19) together, we obtain

$$12 \geq 18,$$

a contradiction. Thus efficiency is not implementable.

Informally, to induce an agent to work, the difference between the outputs allocated to him in the high and low states must be sufficiently big (9, to be precise, and therefore 18 if we add the two agents' differences together). But the difference between high and low total output is only 12. So to get both agents to work, output has to be "thrown away" in the low state, i.e., output must be reduced to -6, which is inefficient. An alternative to throwing away output is to resurrect the center (agent 0). Imagine that this agent chips in two units of output in the high state and takes away 4 in the low state. Then, agents 1 and 2 can be induced to work without any expected efficiency loss (the center breaks even on average: $\frac{2}{3}2 - \frac{1}{3}4 = 0$). This is basically Holmström's (1982) interpretation of the Alchian and Demsetz (1972) rationale for separation of ownership and management in corporations: the owner can serve as a "budget-breaker" in setting up an efficient incentive scheme for managers.

In studying the principal-agent model above, we noted that the value of repeating the game was to exploit the fact that a concave frontier is still locally linear. In our partnership example, however, efficiency is not implementable even when the frontier is linear. Consequently, it should not be surprising that repetition does not help to restore efficiency. Indeed, the partnership example is closely related to one used by Radner et al. (1986) to illustrate the potential inefficiency of repeated game equilibria when there is double moral hazard.

The inefficiency in our partnership example, however, turns out to depend crucially on the fact that there are only two possible observable outcomes (this is true as well of the Radner-Myerson-Maskin example). Indeed let us now modify the model so that there are three possible output levels, $z = 12, 8, 0$. If both agents work, the probability distribution over these levels is $\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{6}\right)$. If agent 1 shirks and 2 works, the distribution is $\left(\frac{1}{3}, 0, \frac{2}{3}\right)$. If 1 works and 2 shirks, it is $\left(0, \frac{1}{2}, \frac{1}{2}\right)$, and if both shirk it is $(0, 0, 1)$.

Once again, it is efficient for both agents to work, and the Pareto frontier is described by $v_1 + v_2 = 2$. In this case, however, it is possible to implement any point on the frontier. Specifically, suppose we let $z_2(12) = 12$, $z_1(8) = 8$, and set all the other allocations equal to zero. That is, we give agent 2 all the output when $z = 12$ and agent 1 all the output when $z = 8$. It is straightforward to verify that (w, w), i.e. both agents working, is an equilibrium:

$$\frac{1}{3}0 + \frac{1}{2}8 + \frac{1}{6}0 - 3 \geq \frac{1}{3}0 + \frac{2}{3}0 \tag{20}$$

and

$$\frac{1}{3}12 + \frac{1}{2}0 + \frac{1}{6}0 - 3 \geq \frac{1}{2}0 + \frac{1}{2}0. \tag{21}$$

Intuitively, it makes sense to allocate agent 1 all the output when $z = 8$: if he had shirked, such an output level would not have been possible; and so the allocation serves as an effective inducement for his working; similarly, assigning agent 2 all the output when $z = 12$ is a good way to reward *him* for working.

Mathematically, the virtue of having sufficiently many output levels (in this case, 3) is that we can satisfy incentive constraints (20) and (21) together with the efficiency conditions

$$z_1(0) + z_2(0) = 0$$
$$z_1(8) + z_2(8) = 8$$
$$z_1(12) + z_2(12) = 12$$

simultaneously. More generally, Radner and Williams (1988) and Legros (1988) showed that, as long as agents utilities are linear in output, then for generic partnership games where the number of output levels is at least m_1+m_2-1 (where m_i is the number of actions in A_i), efficiency is implementable. As Fudenberg et al. (1994) showed, a similar result obtains for a repeated partnership (with δ near 1) without the hypothesis that the Pareto frontier is linear.

9 Conclusion

Roy Radner once expressed the wish that a book as elegant as Debreu's (1957) analysis of competitive markets might one day be written about nonmarket institutions (specifically, the large firm). His own work on teams and incentives (not to mention his many contributions to our understanding of information, and organizational structure) constitutes a good start toward making that wish come true.

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