Appendix

A Calibration Details

A.1 Summary of External Calibration

	Description	Value	Target / Source
Preferences			
γ	Risk Aversion	2	Literature
ϕ	Procrastination Decay Rate	$-\log(0.5)$	Andersen et al. (2020)
Income			
y_t	Transitory Income	$\{0.75, 0.98, 1.28\}$	Guerrieri and Lorenzoni (2017)
$\mathbf{A}^y$	Income Transition Matrix	(see text)	Guerrieri and Lorenzoni (2017)
Interest Rates			
r_t	Short Rate	$\{-1\%, 0\%, 1\%, 2\%\}$	10-Year TIPS
$\mathbf{A}^r$	Short Rate Transition Matrix	(see text)	10-Year TIPS
ω^{cc}	Credit Card Wedge	10.3%	Credit Card - 10-Yr Treasury Spread
ω^m	Mortgage Wedge	1.7%	30-Yr FRM - 10-Yr Treasury Spread
Assets and Liabilities			
h	House Value	3.1	2016 SCF
θ	Max LTV	0.8	Greenwald (2018)
ξ	Mortgage Paydown	0.035	20 Year Half-Life
κ^{prepay}	Prepayment Fixed Cost	0.002	Numerical Stability
κ^{refi}	Refinancing Fixed Cost	0.05	FRB Documentation
$\underline{b}$	Credit Limit	$-\frac{1}{3}$	2016 SCF
Other Structural Assumptions			
λ^F	Rate of Forced Adjustment	$\frac{1}{15}$	2016 CPS Avg. Moving Rate
λ^R	Retirement Rate	$\frac{1}{30}$	Average Working Life
y^R	Retirement Fixed Income	y_L	Retirement Replacement Rate
-	Birth Distribution	$m_0 = \theta h, b_0 \sim U(0, y_L)$	Lifecycle Dynamics

Table 5: Externally Calibrated Parameters.

Notes: This table presents the model's externally calibrated parameters. See Section 4.1 for details.

A.2 SCF Details

Many of our calibrated parameters rely on data from the 2016 SCF. To construct a sample of households that is consistent with our model we impose the following data filters. The head of house must be in the labor force and aged 25-66. The household must own a home (with weakly positive home equity), possess a credit card, earn no income from Social

Security nor retirement accounts, and have after-tax permanent income between the 1st and 99th percentile. On this sample, we then condition on households with a home value to permanent income ratio between the 25th and 75th percentile.

All of our variables are scaled relative to permanent income. Following Kennickell (1995), Kennickell and Lusardi (2004), and Fulford (2015) we use the SCF's question about "normal income" to measure each household's permanent income.[63] Though this is an imperfect proxy for the household's permanent income, it has the benefit of being both straightforward and respecting the household's information set. We adjust each household's normal income for 2015 federal taxes, and deduct an additional 5% for state taxes.

We use the 2016 SCF to estimate six moments that are used in our calibration: (i) permanent income; (ii) average home value to permanent income; (iii) average LTV; (iv) average credit card debt to permanent income; (v) share of households with revolving credit card debt; and (vi) average credit limit to permanent income. Moments (ii) – (v) are reported in the main text. The average after-tax permanent income for our sample of homeowners is \$95,918. The average credit limit to permanent income is 0.36.

A.3 Estimation and Discretization of Ornstein-Uhlenbeck Processes

To calibrate our income and interest rate processes, we assume that these processes are discretized versions of continuous-time Ornstein-Uhlenbeck (OU) processes.

Consider a generic mean-zero OU process $u(t) = \int_0^t e^{-\kappa(t-s)}\sigma dZ_s$. Process $u(t)$ has the conditional distribution $u(t+\tau)|u(t) = \mathcal{N}\left(u(t)e^{-\kappa\tau}, \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\tau})\right)$.

Assume that $u(t)$ is only observed in discrete snapshots every Δ years. Let $d_s = u(s\Delta)$ denote the s'th snapshot of process $u(t)$. The discrete process d_s can be modeled as an AR(1) process:

$$
\begin{aligned}
d_{s+1} &= \rho d_s + \sigma_d \varepsilon_{s+1}, \text{ where} \\
\rho &= e^{-\kappa\Delta} \\
\sigma_d^2 &= \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\Delta}).
\end{aligned}
$$

Given any discrete-time AR(1) estimate, we can use the above formulas to back out the parameters of the underlying OU process: κ and σ. We discretize the OU process using standard finite difference methods. For details, see the numerical Appendix of Achdou et al. (2021).

[63]SCF respondents are asked whether or not their 2015 income was normal. If not, they are asked to report what their total income would be if it had been normal.

B Naive Present Bias: Passing to Continuous Time

Here we present a heuristic derivation of naive IG preferences as the continuous-time limit of a model where some of the decisions are made discretely. This heuristic approach is designed to capture the intuition of the more rigorous derivation in Harris and Laibson (2013). We begin by assuming a constant effort cost, as in Sections 2.1 and 2.2. The full setup with a stochastic effort cost, as introduced in Section 2.3, is presented in Appendix B.3.

B.1 Naive IG Current-Value Function

Assume that the current self lives for a discrete length of time, denoted Δ. After this time has elapsed, starting with the next self, time progresses continuously again.[64] Since the naive present-biased household incorrectly perceives that all future selves will discount exponentially, continuation-value function $v(x)$ characterizes the equilibrium starting with the next self at time Δ. The current self discounts all future selves by β, so the current-value function for the naive present-biased household is given by:

$$
\begin{aligned}
w(x) &= \max\left\{\max_{c}\ u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}[v(x_\Delta)|x],\ w^*(x) - \bar{\varepsilon}\right\} \quad \text{with} \\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\} \\
w^{prepay}(x) &= \max_{b',m'} w(b',m',y,r^m,r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'} w(b',m',y,r+\omega^m,r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
\tag{14}
$$

and where x_Δ denotes the vector of household states after time interval Δ has elapsed, for example $b_\Delta = b + (y + rb + \omega^{cc}b^- - (r^m + \xi)m - c)\Delta$.

Equation (14) captures the consumption/adjustment decisions made by the current self. In the left branch of the first line the household does not adjust, and chooses consumption rate c over the next Δ units of time to maximize the current-value function. In the right branch of the first line the household pays effort cost $\bar{\varepsilon}$ and fixed monetary cost κ^i to discretely adjust its mortgage. Importantly, this discrete-time value function is written such that there is no delay to refinancing (i.e., the current self benefits from refinancing).[65] Though this is unrealistic – there are time delays in refinancing – we write the Bellman equation in this

[64] This mixed discrete- and continuous-time setup is of course slightly non-standard. Alternatively, we could have assumed that future selves also make decisions in discrete time, as done in Laibson and Maxted (2020). In this case the continuation-value function $v(x)$ would be the discrete-time analogue of the continuous-time $v(x)$ that we use below.

[65] To see how the value function is written in this way, note that refinancing gives the current self the current-value function of w^*. As the first line of equation (14) shows, this value function consists of an undiscounted utility flow earned for the current self, $u(c)\Delta$.

way to emphasize that our results do not rely on assumptions about temporal delays.

Discrete-time Bellman equation (14) can be used to derive the current-value function in continuous time. Taking the time-step Δ to its continuous-time limit, we see that the term $u(c)\Delta$ drops out of the current-value function, leaving:

$$w(x) = \max\left\{\beta v(x),\ w^*(x) - \bar{\varepsilon}\right\}.$$

This recovers equation (9) in the main text.

B.2 Continuous Control: Consumption (Proof of Lemma 1)

We now derive the continuous-time first-order condition for consumption stated in Lemma 1. As shown in equation (14), the household makes a consumption choice in every period. For the consumption decision, equation (14) implies that consumption is given by the following first-order condition:[66]

$$u'(c(x)) = \beta e^{-\rho\Delta}\ \frac{\partial}{\partial b}\mathbb{E}[v(x_\Delta)|x].$$

Taking $\Delta \to 0$ yields

$$u'(c(x)) = \beta\frac{\partial v(x)}{\partial b},$$

which is equation (11) in Lemma 1. This derivation continues to hold in the full setup with a stochastic effort cost presented in Appendix B.3 below.

B.3 Full Setup with a Stochastic Effort Cost (Section 2.3)

Here we briefly spell out the full set of equations for the generalization with a stochastic effort cost that evolves according to the two-state process in Assumption 1. In what follows, we will denote value and policy functions in the normal high-cost state by the same functions as in the baseline model with a constant effort cost, e.g. $v(x)$ or $\mathfrak{R}(x)$. Alternatively, we will denote the corresponding value and policy functions in the temporary low-cost state with underlines, e.g. $\underline{v}(x)$ or $\underline{\mathfrak{R}}(x)$.

We first show how to generalize equation (8′), the HJBQVI equation for the value function

[66] We ignore difficulties such as the kink in the budget constraint when taking this first-order condition.

$v(x)$ of a $\beta = 1$ household:

$$\rho v(x) = \max \Bigg\{ \max_c \Big\{ u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c) \Big\} \tag{8''}$$
$$- \frac{\partial v(x)}{\partial m}(\xi m)$$
$$+ \sum_{y' \neq y} \lambda^{y \to y'} \big[v(b, m, y', r^m, r) - v(b, m, y, r^m, r) \big]$$
$$+ \sum_{r' \neq r} \lambda^{r \to r'} \big[v(b, m, y, r^m, r') - v(b, m, y, r^m, r) \big]$$
$$+ \lambda^R \big[v^R(x) - v(x) \big]$$
$$+ \lambda^F \big[v^*(x) - (v(x) - \bar{\varepsilon}) \big]$$
$$+ \phi \big[\underline{v}(x) - v(x) \big],$$
$$\rho(v^*(x) - \bar{\varepsilon}) \Bigg\}.$$

Relative to (8′), there is a new entry $\phi\big[\underline{v}(x) - v(x)\big]$. Parameter ϕ is the arrival rate of the low-effort-cost state, and $\underline{v}(x)$ is the household's value in this state. This value is given by

$$\underline{v}(x) = \max\{v(x), v^*(x) - \underline{\varepsilon}\}. \tag{15}$$

Intuitively, since the low-cost state only lasts for an instant (Assumption 1), the household either takes advantage of refinancing at the lower effort cost $\underline{\varepsilon}$ or it loses the opportunity in the next instant in which case its value reverts back to $v(x)$.

Instead of asserting equation (15) and justifying it with economic reasoning as we just did, we can also derive it from a full HJBQVI equation for $\underline{v}(x)$ that is symmetric to (8″) and in which the effort cost switches from $\underline{\varepsilon}$ to $\bar{\varepsilon}$ at a Poisson rate $\underline{\phi}$; then take $\underline{\phi} \to \infty$ (Assumption 1).

We next show how to generalize (9), the equation for the current-value function $w(x)$:

$$\begin{aligned}
w(x) &= \max \Big\{ \beta v(x),\ w^*(x) - \bar{\varepsilon} \Big\} \quad \text{and} \\
\underline{w}(x) &= \max \Big\{ \beta v(x),\ w^*(x) - \underline{\varepsilon} \Big\} \quad \text{with} \\
w^*(x) &= \max \big\{ w^{prepay}(x), w^{refi}(x) \big\} \\
w^{prepay}(x) &= \max_{b', m'} \ w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b', m'} \ w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned} \tag{16}$$

Relative to (9), there is a new line $\underline{w}(x) = \max\left\{\beta v(x),\ w^*(x) - \underline{\varepsilon}\right\}$ that captures the current-value of a household that has the opportunity to refinance at the low-effort-cost $\underline{\varepsilon}$.

Like in Appendix B.1, the current-value function in (16) can be derived from a setup in which the current self lives for a discrete length of time Δ:

$$
\begin{aligned}
w(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\phi\Delta}\mathbb{E}[v(x_\Delta)|x] + \left(1 - e^{-\phi\Delta}\right)\mathbb{E}[\underline{v}(x_\Delta)|x]\right],\ w^*(x) - \bar{\varepsilon}\right\}, \\
\underline{w}(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\underline{\phi}\Delta}\mathbb{E}[\underline{v}(x_\Delta)|x] + \left(1 - e^{-\underline{\phi}\Delta}\right)\mathbb{E}[v(x_\Delta)|x]\right],\ \underline{w}^*(x) - \underline{\varepsilon}\right\}, \\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\} \\
w^{prepay}(x) &= \max_{b',m'} w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'} w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds} \\
\underline{w}^*(x) &= \max\left\{\underline{w}^{prepay}(x), \underline{w}^{refi}(x)\right\} \\
\underline{w}^{prepay}(x) &= \max_{b',m'} \underline{w}(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
\underline{w}^{refi}(x) &= \max_{b',m'} \underline{w}(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
$$

where ϕ and $\underline{\phi}$ denote the Poisson switching rates between the two effort-cost states. As stated in Assumption 1 we assume that $\underline{\phi} \to \infty$. Therefore $e^{-\underline{\phi}\Delta} \to 0$ and

$$
\begin{aligned}
w(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\phi\Delta}\mathbb{E}[v(x_\Delta)|x] + \left(1 - e^{-\phi\Delta}\right)\mathbb{E}[\underline{v}(x_\Delta)|x]\right],\ w^*(x) - \bar{\varepsilon}\right\}, \\
\underline{w}(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}[v(x_\Delta)|x],\ \underline{w}^*(x) - \underline{\varepsilon}\right\}, \\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\} \\
w^{prepay}(x) &= \max_{b',m'} w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'} w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds} \\
\underline{w}^*(x) &= \max\left\{\underline{w}^{prepay}(x), \underline{w}^{refi}(x)\right\} \\
\underline{w}^{prepay}(x) &= \max_{b',m'} \underline{w}(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
\underline{w}^{refi}(x) &= \max_{b',m'} \underline{w}(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}
$$

Finally, we take the limit as $\Delta \to 0$. Using the property that $\underline{w}^*(x) = w^*(x)$ in the limit as $\Delta \to 0$ – which one can see by inspection since the left branch of the first line converges to the left branch of the second line – we recover equation (16).

C Proofs

C.1 Proof of Corollary 1

Recall that, with naiveté, the perceived continuation-value function of a $\beta < 1$ household equals the value function of an exponential $\beta = 1$ household and solves (8″). Assume that the household does not refinance at time t so that the perceived continuation-value function $v(x_t)$ is characterized by a standard HJB equation. This HJB equation is given by the left branch of (8″), which we write here as

$$\rho v(x) = \max_{c} \; u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^{-} - (r^m + \xi)m - c) + (\mathcal{B}v)(x) \tag{17}$$

where the operator $(\mathcal{B}v)(x)$ is short-hand notation for lines two to seven of (8″). Recall that we use hat-notation to denote the policy functions that naive households perceive for future selves, and denote by $\widehat{c}(x)$ and $\widehat{s}(x) = (y + rb + \omega^{cc} b^{-} - (r^m + \xi)m - \widehat{c}(x))$ the corresponding perceived consumption and liquid saving policy functions. In contrast, denote by $c(x)$ (from Proposition 1) and $s(x) = (y + rb + \omega^{cc} b^{-} - (r^m + \xi)m - c(x))$ the *actual* policy functions.

The following observation is important in the proof below: the HJB equation for the perceived continuation-value function (17) features the *perceived* policy functions $\widehat{c}(x), \widehat{s}(x)$, rather than the *actual* policy functions. But what determines the evolution of liquid wealth b are the *actual* policy functions.

Differentiate (17) with respect to b and use the envelope theorem:

$$(\rho - r(b))\frac{\partial v(x)}{\partial b} = \frac{\partial^2 v(x)}{\partial b^2}\widehat{s}(x) + \frac{\partial}{\partial b}(\mathcal{B}v)(x). \tag{18}$$

Define the marginal continuation-value of wealth $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$. From (18) it satisfies

$$(\rho - r(b))\,\eta(x) = \frac{\partial \eta(x)}{\partial b}\widehat{s}(x) + (\mathcal{B}\eta)(x). \tag{19}$$

If $\beta = 1$, from Itô's formula, the right-hand side of (19) also governs the expected change in the marginal value of wealth: $\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}\widehat{s}(x_t) + (\mathcal{B}\eta)(x_t)\right] dt$. But with $\beta < 1$ this is no longer true: the evolution of b is governed by the *actual* drift $s(x)$ rather than the *perceived* drift $\widehat{s}(x)$ and so

$$\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}s(x_t) + (\mathcal{B}\eta)(x_t)\right] dt. \tag{20}$$

Therefore, evaluating (19) along a particular trajectory x_t, we have

$$(\rho - r(b_t))\,\eta(x_t) = \frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] - \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)).$$

Rearranging

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] &= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)) \\
&= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(\widehat{c}(x_t) - c(x_t)) \\
&= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t)
\end{aligned}$$

Finally, recalling that $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$, the first-order condition is $u'(c(x)) = \beta\eta(x)$ and therefore

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}_t[du'(c(x_t))] &= (\rho - r(b_t))\,u'(c(x_t)) + \frac{\partial u'(c(x_t))}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t) \\
&= (\rho - r(b_t))\,u'(c(x_t)) - u''(c(x_t))c(x_t)\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} \\
&= \left[\rho + \gamma\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} - r(b_t)\right] u'(c(x_t)),
\end{aligned}$$

where going from the second line to the third line uses that, with CRRA utility, the coefficient of relative risk aversion is $\gamma = \frac{-u''(c(x_t))c(x_t)}{u'(c(x_t))}$. Dividing by $u'(c(x_t))$, we have (12). ■

C.2 Proof of Proposition 2

When proving Proposition 2, we refer to Appendix B.3 which spells out the full set of equations for the model with a stochastic effort cost satisfying Assumption 1.

We also note that clauses 1, 2a, and 2b do not rely on the infinite Poisson switching rate used in Assumption 1. The purpose of Assumption 1 is to create the sorts of deadlines that incentivize present-biased agents to complete effortful tasks (clause 2c).

C.2.1 Proof of Proposition 2, Clause 1

The proof of clause 1 follows from equation (16). Equation (16) shows that we can rewrite w^{prepay} and w^{refi} as:

$$\begin{aligned}
w^{prepay}(x) &= \max_{b',m'} \ \beta v(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'} \ \beta v(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}$$

These are exactly the same formulas as for v^{prepay} and v^{refi} in (7), except that there is an additional β discount factor. Since the additional β discount factor has no effect on the optimal choice of (b', m'), we recover clause 1 of Proposition 2 — the choice of (b', m') is independent of β. ■

Since $v^*(x) = \max\left\{v^{prepay}(x), v^{refi}(x)\right\}$ and $w^*(x) = \max\left\{w^{prepay}(x), w^{refi}(x)\right\}$, the above proof also implies that:

$$w^*(x) = \beta v^*(x). \tag{21}$$

This property will be used in the proof of clause 2 of Proposition 2.

C.2.2 Proof of Proposition 2, Clause 2

For clause 2a, when $\beta = 1$ the assumption that $\bar{\varepsilon}$ and $\underline{\varepsilon}$ are vanishingly small (Assumption 2) implies that $v(x)$ is arbitrarily close to $\underline{v}(x)$. Accordingly, policy function $\mathfrak{R}(x)$ converges pointwise to $\underline{\mathfrak{R}}(x)$ as the effort cost vanishes.

To prove clause 2b (procrastination when $\beta < 1$ and $\varepsilon = \bar{\varepsilon}$), consider the self in control at point x in the state space. Recall from (16) that the current-value function is given by $w(x) = \max\{\beta v(x), w^*(x) - \bar{\varepsilon}\}$. Therefore the current self will not adjust their mortgage when the value from not adjusting, $\beta v(x)$, is larger than the value from adjusting, $w^*(x) - \bar{\varepsilon}$.

The value of not adjusting is given by

$$\beta v(x) \geq \beta(v^*(x) - \bar{\varepsilon}), \tag{22}$$

where the inequality $v(x) \geq v^*(x) - \bar{\varepsilon}$ follows directly from equation (8″).

Alternatively, adjusting requires the household to incur the effort cost $\bar{\varepsilon}$ in the current period and the value of adjusting is given by

$$w^*(x) - \bar{\varepsilon} = \beta v^*(x) - \bar{\varepsilon}, \tag{23}$$

where the equality follows from equation (21).

Comparing the two alternatives (22) and (23) shows that the $\beta < 1$ household will always prefer to procrastinate whenever $\varepsilon_t = \bar{\varepsilon}$, since

$$\beta(v^*(x) - \bar{\varepsilon}) > \beta v^*(x) - \bar{\varepsilon}.$$

Procrastination enables the effort cost $\bar{\varepsilon}$ to be discounted by β, while there is at most an infinitesimal cost to delaying refinancing for an instant.

To prove clause 2c (no procrastination when $\beta < 1$ and $\varepsilon = \underline{\varepsilon}$), consider the self in control at point x in the state space. Following the second line of equation (16), it will be (weakly) optimal for the current self to adjust their mortgage if and only if:

$$w^*(x) - \underline{\varepsilon} \geq \beta v(x).$$

Above, the left-hand side is the current-value from refinancing at effort cost $\underline{\varepsilon}$, and the right-hand side is the current-value from not refinancing and having the effort cost reset immediately to $\bar{\varepsilon}$. Since $w^*(x) = \beta v^*(x)$ (see equation (21)), this can be rewritten as

$$\beta v^*(x) - \underline{\varepsilon} \geq \beta v(x). \tag{24}$$

First, consider the case in which the next self is expected to adjust the mortgage if the current self procrastinates.[67] Since the next self is expected to have $\beta = 1$, this means $\widehat{\mathfrak{R}}(x) > 0$. In this case, equation (8″) implies that $v(x) = v^*(x) - \bar{\varepsilon}$. Plugging this into (24) shows that the current self will adjust their mortgage whenever $\beta v^*(x) - \underline{\varepsilon} \geq \beta v^*(x) - \beta\bar{\varepsilon}$ or

$$\underline{\varepsilon} \leq \beta\bar{\varepsilon},$$

which is satisfied because Assumption 1 imposes that $\underline{\varepsilon} < \beta\bar{\varepsilon}$. Intuitively, this says that the current self will adjust their mortgage now if the cost of doing so, $\underline{\varepsilon}$, is less than the discounted cost of adjusting next period, $\beta\bar{\varepsilon}$. Thus, if $\widehat{\mathfrak{R}}(x) > 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\mathfrak{R}}(x)$, meaning that the household does not procrastinate.

Next, consider the case in which a $\beta = 1$ household would not refinance at point x, even in the low-effort-cost state $\varepsilon_t = \underline{\varepsilon}$, i.e., $\widehat{\underline{\mathfrak{R}}}(x) = 0$. In that case, equation (15) implies that $v(x) \geq v^*(x) - \underline{\varepsilon}$. Multiplying by β, this also implies that $\beta v(x) \geq \beta v^*(x) - \beta\underline{\varepsilon}$, and therefore

$$\beta v(x) > \beta v^*(x) - \underline{\varepsilon}.$$

Comparing this to equation (24) shows that it will not be optimal for the naive present-biased household to refinance. This is intuitive — if it is not optimal for a $\beta = 1$ household to refinance, there is no reason for it to be optimal for a naive $\beta < 1$ household to refinance. Thus, if $\widehat{\underline{\mathfrak{R}}}(x) = 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\underline{\mathfrak{R}}}(x)$.

Tying these two cases together, we have shown that:

1. If $\widehat{\mathfrak{R}}(x) > 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\mathfrak{R}}(x)$
2. If $\widehat{\underline{\mathfrak{R}}}(x) = 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\underline{\mathfrak{R}}}(x)$

Since clause 2a of Proposition 2 implies that $\widehat{\underline{\mathfrak{R}}}(x)$ converges pointwise to $\widehat{\mathfrak{R}}(x)$ as the effort

[67] Note that the next self will face the high-effort-cost $\bar{\varepsilon}$ if the current self procrastinates when $\varepsilon_t = \underline{\varepsilon}$.

cost vanishes, the first bullet above can be rewritten as: if $\widehat{\underline{\mathfrak{R}}}(x) > 0$ then $\underline{\mathfrak{R}}(x)$ converges pointwise to $\widehat{\underline{\mathfrak{R}}}(x)$. This completes the proof of clause 2c of Proposition 2. ■

D Supplements to Sections 4 and 5

D.1 MPCs and MPXs out of Discrete Wealth Shocks

In Section 4 the MPC and the MPX are defined over infinitesimal wealth shocks. Following Achdou et al. (2021), this section extends these definitions to discrete wealth shocks.

Let $C_\tau(x) = \mathbb{E}\left[\int_0^\tau c(x_t)dt \mid x_0 = x\right]$ denote total expected consumption from time 0 to time τ. Recall that $x = (b, m, y, r^m, r)$. Let $x + \chi$ be shorthand for the vector $(b + \chi, m, y, r^m, r)$, i.e. $x + \chi$ is point x plus a liquid wealth shock of size χ.

For a discrete liquidity shock of size χ the MPC is defined as:

$$MPC_\tau^\chi(x) = \frac{C_\tau(x+\chi) - C_\tau(x)}{\chi}.$$

The MPX is defined as (see Laibson et al. (2021) for details):

$$MPX_\tau^\chi(x) = MPC_\tau^\chi(x) + \frac{s}{\nu + r_0}\left(\frac{\mathbb{E}\left[c(x_\tau) \mid x_0 = x + \chi\right] - \mathbb{E}\left[c(x_\tau) \mid x_0 = x\right]}{\chi}\right).$$

Total consumption $C_\tau(x)$, which is used in the MPC calculation, can be calculated numerically using a Feynman-Kac formula (see Lemma 2 of Achdou et al. (2021) for details). To calculate the MPX we also need to solve for the expected consumption rate at time τ, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$. Again, a Feynman-Kac formula can be used to solve for this directly.[68] Numerically, we solve the Feynman-Kac formula for the sample path $r_t = 1\%$ for all t (i.e., no aggregate interest rate shocks) since these calculations are conducted in the steady state.

D.2 Naive Present Bias in Continuous Time: A Cake-Eating Model

A key motivation for this paper is to understand how present bias interacts with the complexities of household balance sheets. For comparison, this section eliminates those complexities and calculates the effect of naive present bias in a simplified "Eat-the-Cake" model.

In this textbook model, households are infinitely lived, have deterministic income $\bar{y}$, and have access to a single liquid asset with return r. Relative to the model in the main text, this stripped-down model eliminates income uncertainty, hard and soft borrowing constraints, multiple assets of varying liquidity, monetary policy, and retirement.

[68] The Feynman-Kac formula for $C_\tau(x)$ is provided in Achdou et al. (2021). The Feynman-Kac formula for $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is specified slightly differently. Here, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is given by $\Gamma(x, 0)$, where $\Gamma(x, 0)$ satisfies the PDE $0 = (\mathcal{A}\Gamma)(x, t)$ subject to the terminal condition $\Gamma(x, \tau) = c(x)$.

We normalize $\bar{y} = 0$. In this simple model, the consumption function is given by:

$$c(b) = \beta^{-\frac{1}{\gamma}} \left(r - \frac{r - \rho}{\gamma} \right) \times b.$$

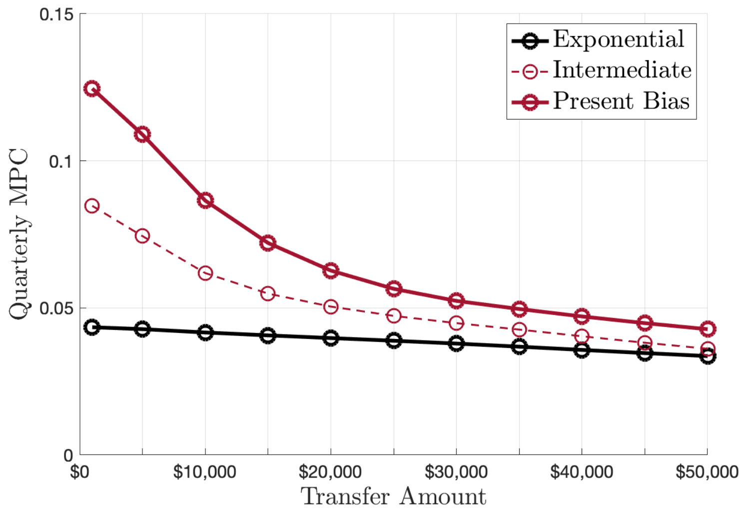
The term in parentheses is the consumption function for an exponential household (see e.g. Fagereng et al. (2019b)), and multiplicative factor $\beta^{-\frac{1}{\gamma}}$ adjusts for present bias (see Proposition 1).

For our Present-Bias Benchmark calibration the consumption rate is approximately 1%, which corresponds to an annual MPC of 1% and an annual MPX of less than 2%. These should be compared to our main model, which produces an annual MPC of 28% and an annual MPX of 37%. Our Exponential Benchmark calibration also produces a similar annual MPC and MPX in this stripped-down model, but should be compared to an annual MPC and MPX in the main model of only 15% and 22%, respectively. This highlights that present bias can amplify the effect of certain balance sheet complexities on household consumption-saving decisions.

E Additional Results

E.1 Model Solution Details: MPCs

(a) Quarterly MPCs Across Transfer Amounts



(b) Present-Bias Benchmark: MPCs over Liquid Wealth

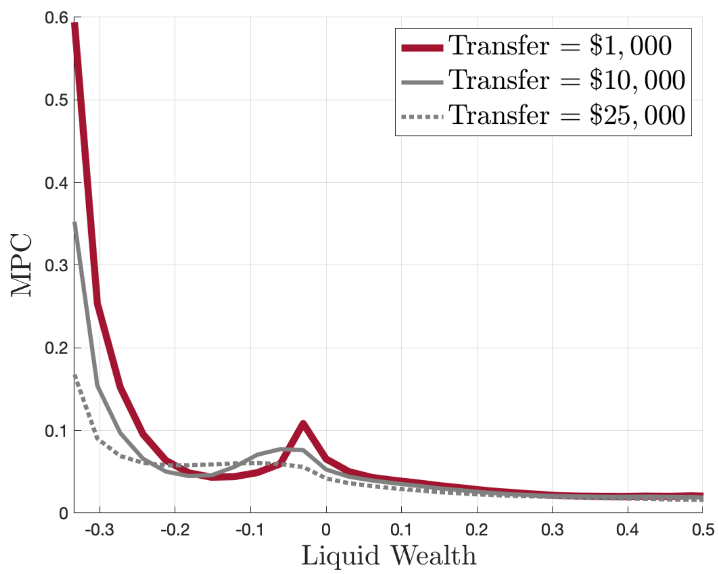


Figure 10: MPCs Across Transfer Amounts.
Notes: For the three calibration cases, the top panel plots quarterly MPCs out of transfers ranging from \$1,000 to \$50,000. The bottom panel replicates the MPC analysis in Figure 3 for the Present-Bias Benchmark calibration across transfer amounts of \$1,000 (benchmark), \$10,000, and \$25,000.

E.2 Model Solution Details: Steady State Distributions

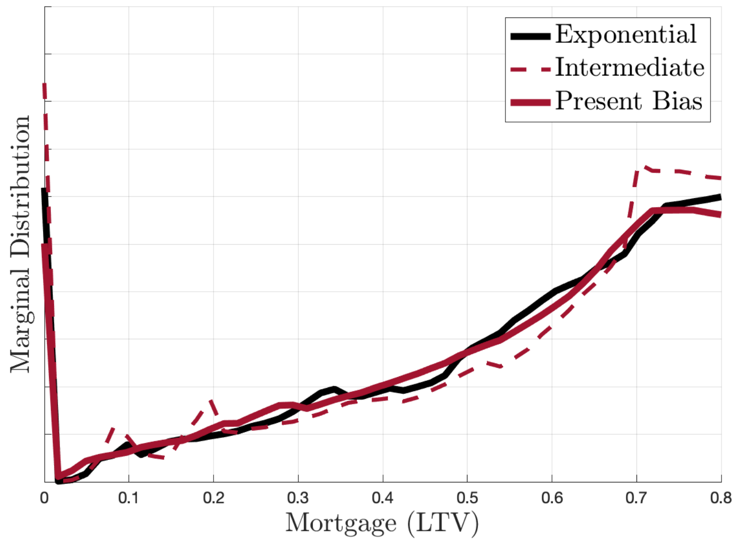


Figure 11: LTV Distribution.
Notes: This figure shows the steady state distribution of households over the LTV ratio.

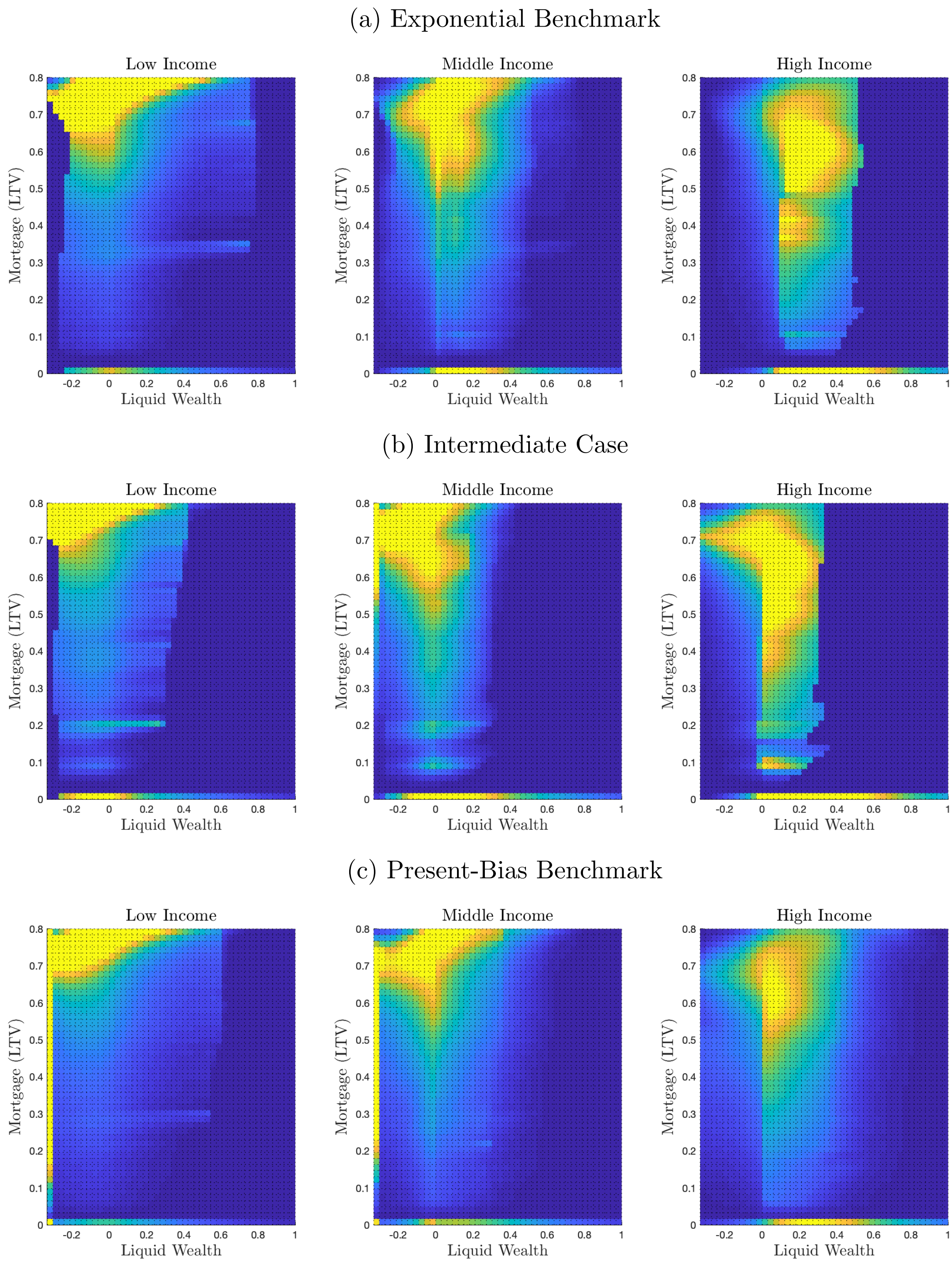


Figure 12: Steady State Distribution.

Notes: For the three calibration cases, this figure presents the full steady state distribution over income, liquid wealth, and mortgage debt. Dark blue regions are rarely encountered, while light yellow regions feature large masses of households.

E.3 Monetary Policy: Refinancing Dynamics

Figure 13 plots the adjustment regions following an interest rate cut from 1% to 0%. This figure replicates the phase diagrams in Figure 1, but now for the case of $r_t = 0\%$ and $r_t^m = 1\% + \omega^m$. Thus, Figure 13 plots the adjustment regions for households with a mortgage rate that is above the rate they can refinance into.

As in the main text, the red regions mark where households take a cash-out refinance and the blue regions mark where households prepay their mortgage. The gray regions indicate where households conduct a rate refinance, defined as the household increasing its mortgage balance by less than 5% during the refinance.

Relative to the steady state adjustment regions, the interest rate cut causes the red/gray refinancing regions to expand drastically. In particular, households with larger LTVs are more likely to refinance, since households with larger mortgages have more to gain by reducing their mortgage interest payments.

Table 6 presents details of the refinancing decision. The first row lists the share of households who find themselves in a refinancing region at the time of the interest rate cut. Conditional on refinancing, the second row lists the share of households who extract equity when refinancing. The next four rows list the share of households who have actually refinanced within 1 quarter, 1 year, 2 years, and 3 years following the interest rate cut. While refinancing is instant in the Exponential Benchmark and the Intermediate Case, procrastination means that refinancing occurs slowly in the Present-Bias Benchmark.

	Exponential	Intermediate	Present Bias
Share Refi Region (On Impact)	73.1%	68.5%	74.9%
(Share Cash Out)	81.0%	66.8%	77.3%
$\frac{1}{4}$ Year Realized Refi	75.2%	71.0%	13.6%
1 Year Realized Refi	80.0%	76.5%	42.0%
2 Year Realized Refi	84.5%	81.2%	62.7%
3 Year Realized Refi	87.8%	84.6%	74.3%
Average Refi Amount	0.31	0.17	0.29

Table 6: Refinancing Details.

Notes: For the three calibration cases, this table summarizes details of household refinancing decisions following an interest rate cut from 1% to 0%.

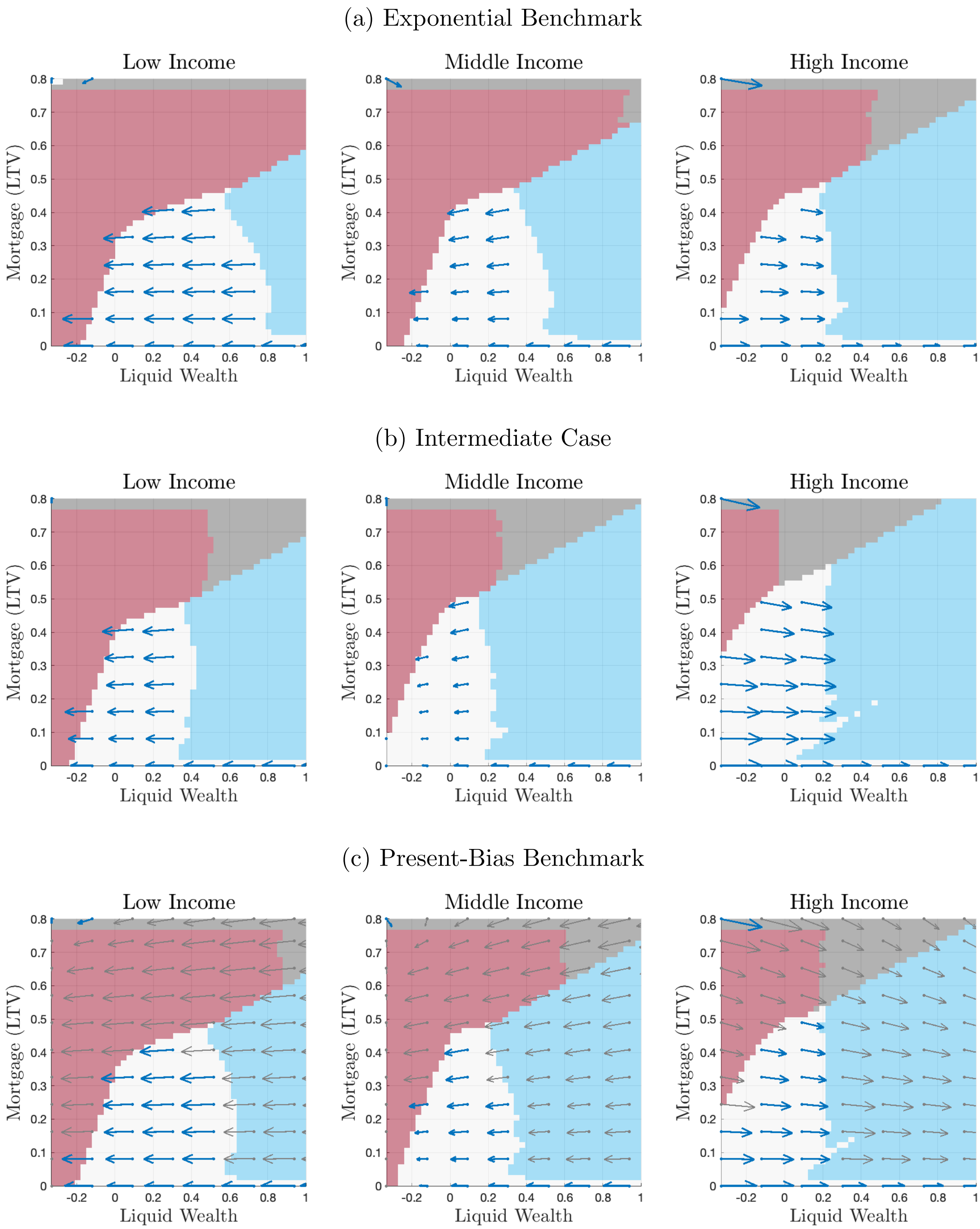


Figure 13: Rate-Cut Phase Diagrams.

Notes: For the three calibration cases, this figure presents the phase diagrams for households who can refinance into a lower mortgage rate following an interest rate cut from 1% to 0% (see Figure 1 for phase diagram details).

E.4 Monetary Policy and Refinancing Procrastination

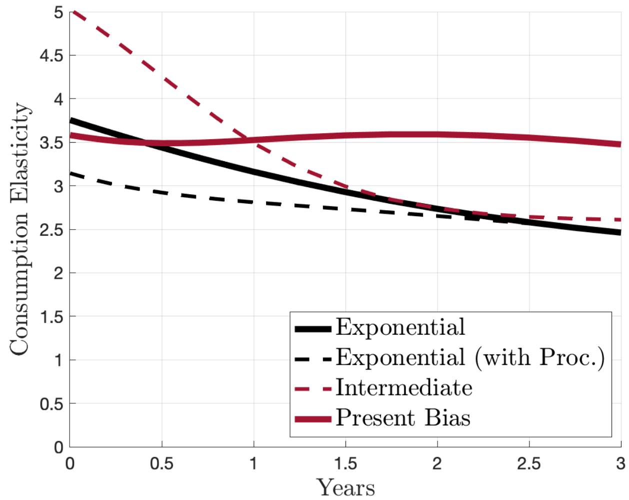


Figure 14: Consumption Response to Monetary Policy: Procrastination Sensitivity.
Notes: This figure adds a fourth case to the benchmark monetary policy analysis (Figure 6). This fourth case augments the Exponential Benchmark case with refinancing procrastination. The dashed black presents the consumption response to monetary policy in this exponential calibration with refinancing procrastination.

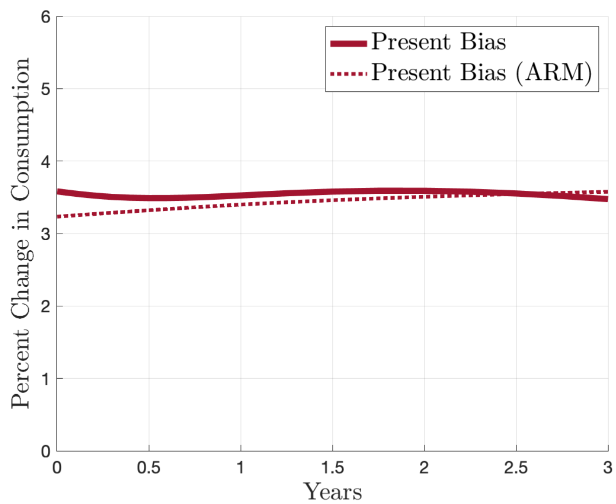


Figure 15: Monetary Policy Under FRMs Versus ARMs.
Notes: For the Present-Bias Benchmark calibration, this figure compares the consumption response to monetary policy under FRMs (solid line) versus ARMs (dotted line). The interest rate is cut by 2% in the ARM experiment, compared to 1% in the FRM experiment, since monetary policy produces larger movements in ARM rates than long-duration FRM rates.

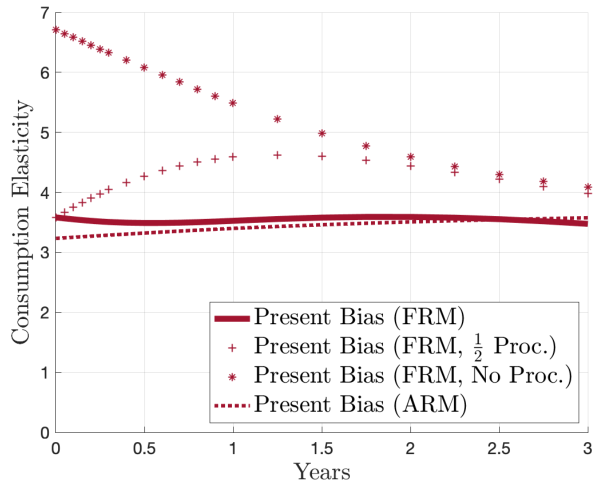


Figure 16: Monetary Policy with Procrastination Reduction.
Notes: This figure presents the consumption response to monetary policy in the Present-Bias Benchmark across varying levels of refinancing procrastination. The +'s assume that policymakers are able to halve the expected duration of procrastination at the time of the rate cut. The $*$'s make refinancing immediate at the time of the rate cut. The baseline consumption response under FRMs (solid red line) and ARMs (dotted red line) are presented for comparison.

E.5 Details on Aggregate House Price and Income Shocks

This section provides additional results for the analysis in Section 6.

House Price Shocks. Section 6.1 of the main text discusses the sensitivity of monetary policy to house price shocks. Here we provide further details on the fiscal policy experiment. Figure 17 plots the consumption response to fiscal stimulus in the negative (left) and positive (right) shock case. The corresponding MPCs are reported below in Table 7.

As the left panel of Figure 17 illustrates, the negative house price shock weakens the consumption response in the Present-Bias Benchmark over the first quarter, but strengthens the consumption response thereafter. The right panel of Figure 17 shows that the opposite is true of fiscal policy following a positive house price shock. In both cases, present bias strongly amplifies the consumption response to fiscal policy.

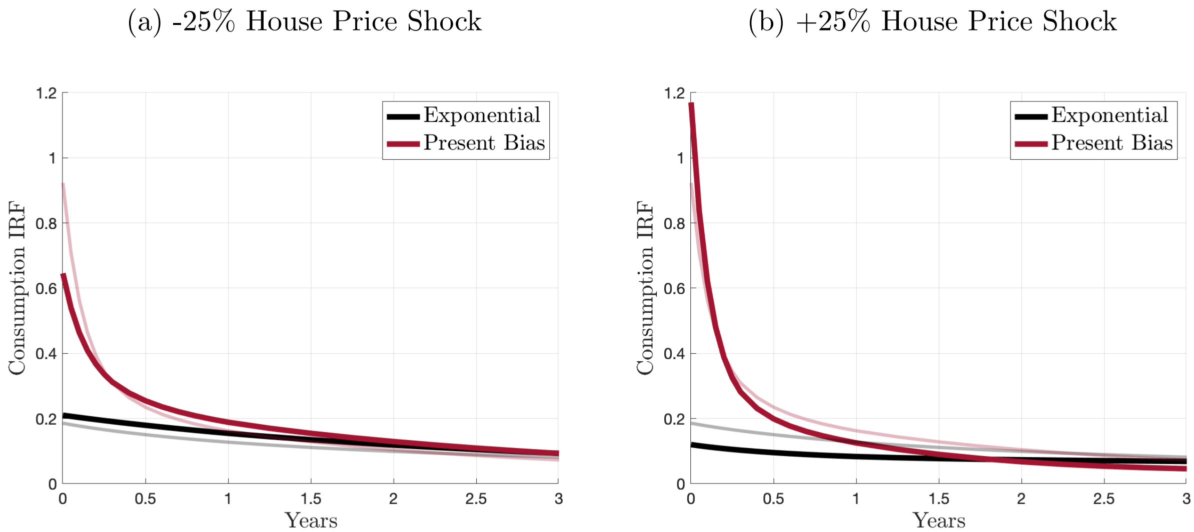


Figure 17: Fiscal Policy and House Price Shocks.
Notes: This figure plots the IRF of aggregate consumption to a $1,000 fiscal transfer that immediately follows a house price shock of -25% (left) or +25% (right). The transparent lines plot to the baseline case in Figure 4, and are included for reference.

	Exponential	Present Bias
Baseline (No Shocks)		
1 Year MPC	15%	28%
2 Year MPC	26%	41%
3 Year MPC	35%	49%
-25% House Price Shock		
1 Year MPC	18%	28%
2 Year MPC	31%	43%
3 Year MPC	41%	53%
+25% House Price Shock		
1 Year MPC	10%	26%
2 Year MPC	17%	35%
3 Year MPC	24%	40%

Table 7: Fiscal Policy MPCs and House Price Shocks.
Notes: This table presents aggregate MPCs out of a $1,000 fiscal transfer that is given immediately after a ± 25% house price shock.

Income Shocks. Section 6.2 of the main text outlines the effect of aggregate income shocks on monetary and fiscal policy. For this aggregate income shock experiment, the left panel of Figure 18 plots the consumption response to monetary policy and the right panel plots the consumption response to fiscal policy. As described in the main text, the consumption response is almost identical to the baseline case in Section 5.

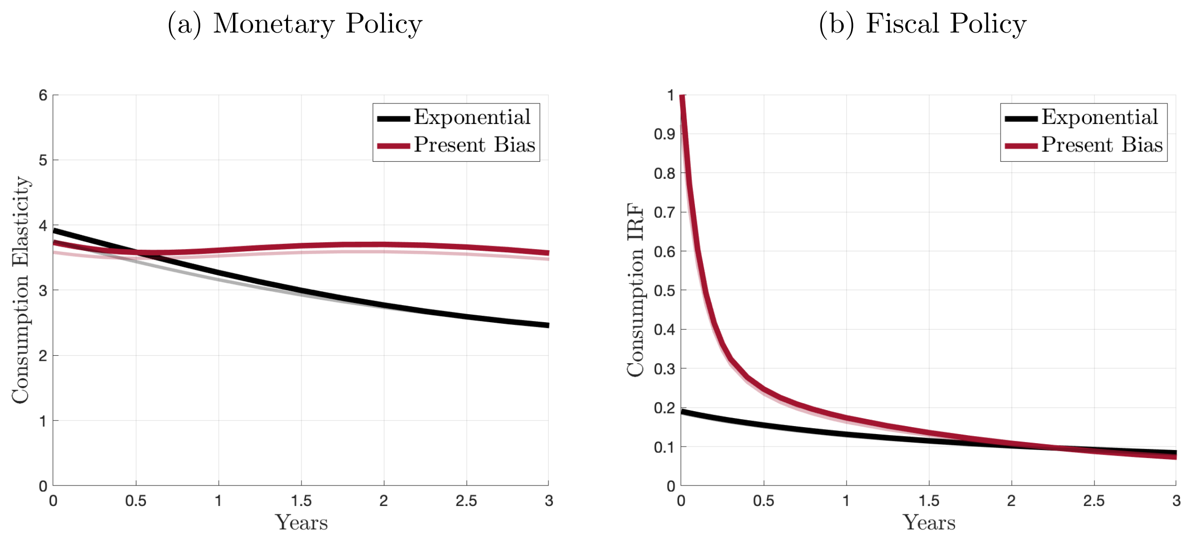


Figure 18: Fiscal and Monetary Policy Following a Negative Income Shock.
Notes: This figure plots the consumption response to monetary (left) and fiscal (right) policy that is implemented immediately following a transitory 5% decline in aggregate income.