Chapter 23

Leibniz and Optics

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Introduction

Although often overlooked today, optics thrived in the early modern era as a science of first rank engaging many of the best minds of the period and producing some of its most dramatic scientific results.[1] The present essay attempts to shed light on Leibniz's efforts to contribute to the development of early modern optics by focusing on his derivations of the laws of reflection and refraction.[2] The first three sections examine Leibniz's attempts to derive the central laws of geometrical optics in works drawn from his early, middle, and later optical studies.[3] The fourth section briefly considers the broader significance of Leibniz's sophisticated approach to the laws of optics. Connections to more familiar themes from Leibniz's philosophy are drawn along the way.

1. *Leges Reflexionis et Refractionis Demonstratae*, 1671*

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[1] For general studies of seventeenth century optics, see Vasco Ronchi, *The Nature of Light*, trans. V. Barocas (Cambridge, MA: Harvard University Press, 1970); A. I. Sabra, *Theories of Light from Descartes to Newton* (New York: Cambridge University Press, 1981).

[2] This essay overlaps in parts with three longer and more detailed studies by the author: Jeffrey K. McDonough, "Leibniz's Two Realms Revisited," *Noûs* 42, no. 4 (2008); Jeffrey K. McDonough, "Leibniz on Natural Teleology and the Laws of Optics," *Philosophy and Phenomenological Research* 78, no. 3 (2009); Jeffrey K. McDonough, "Leibniz's Optics and Contingency in Nature," *Perspectives on Science* 18, no. 4 (2010).

[3] Many of Leibniz's optical studies are catalogued in LH 37.2. A collection of helpful, if not always reliable, transcriptions of Leibniz's optical writings is available in Ernst Gerland, *Leibnizens Nachgelassene Schriften Physikalischen, Mechanischen Und Technischen Inhalts* (Leipzig: B. G. Teubner, 1906). The definitive edition of Leibniz's optical works will appear (primarily) in Series VIII of the Akademie edition of Leibniz's writings, the first volume of which is now available and contains several intriguing early studies.

Leibniz's earliest studies in optics take place against the background of his first systematic theory of the natural world as presented in his twin studies of 1671, the *Theoria motus abstracti* (TMA) and the *Hypothesis physica nova* (HPN, also known as the *Theoria motus concreti*) (A VI.ii.261-276, 221-257). The first of these works presents a rather surprising account of the fundamental or "private" laws of the natural world, according to which the motions of bodies are determined solely by their conatus with no role assigned to their respective sizes or masses. The second work presents Leibniz's attempt to reconcile these supposed fundamental laws of nature with the dictates of idealized experience, and in particular with the laws of impact then recently made public by Huygens and Wren. Leibniz is explicit both in the HPN as well as in other early writings that the laws of optics are to be counted among the derived or "public" laws of nature (A.VI.ii.228-231, 312).

In the first of a series of three pieces dated by the Akademie editors to the same period as the TMA and HPN, Leibniz offers a succinct derivation of the law of reflection (A.IV.ii.309-10). Referring to Figure 1 below, he proposes to let A be a body traveling along a straight line from point a striking the plane bc at point d.

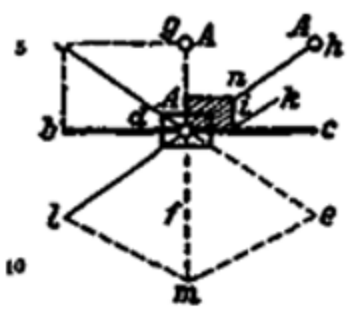

Figure 1

Leibniz argues that at point d, the body A will "try to continue its motion with the same speed in the same direction from d to e" (A.VI.ii.309-310). He maintains, however, that the motion from d to e "can be understood to be composed from two conatus [one] from d to c and [one] from d to f, in such a way that the conatus towards c would be as much stronger than the conatus towards f as the straight line dc is greater than the straight line df" (A.VI.ii.310). From this result, Leibniz is able to demonstrate the core dictate of the law of reflection, namely, that the angle of incidence (adb) must be equal to the angle of reflection (hdc) (A.VI.ii.310).

A similarly elegant derivation of the law of refraction, however, is conspicuously absent from the *Leges Reflexionis et Refractiones*. In the first of the three pieces, Leibniz proposes to prove that "If the incident [ray] penetrates a resisting [medium] from a more resisting [medium] it is refracted from the perpendicular; if from a less resisting [medium], towards the perpendicular" (A.VI.ii.312). The derivation that follows begins as one might expect, with Leibniz imagining, in reference to the Figure 1 above, that body A travels along the straight line hd striking the surface bdc. Having suggested that the impetus of the striking body will be diminished (or increased) in proportion to the resistance of the refracting medium, however, the proof quickly trails off. In the second piece, Leibniz affirms proportionality between, on the one hand, the ratio of the sine of the angle of incidence and the sine of the angle of refraction to, on the other hand, the ratio of the resistances of the relevant mediums (A.VI.ii.313, 318). Having come that far, however, Leibniz's derivation ultimately breaks off abruptly, literally in mid-sentence (A.VI.ii.320-322). The third piece takes up once again the topic of refraction, but makes no attempt to derive the law of refraction itself (A.VI.ii.322-323).

It seems likely that Leibniz's failure to produce an elegant derivation of the law of refraction in the *Leges Reflexionis et Refractiones* is due to his appreciation of the deep difficulties of constructing a proof within the confines of an austere mechanism. It was well known, for example, that a ray of light traveling from one medium to another medium may be refracted either away from a perpendicular drawn at the point of impact (with the ray bending, as it were, in the counterclockwise direction) or towards such a perpendicular (with the ray bending, as it were, in the clockwise direction). Strict mechanists, like Descartes, had a relatively easy time accounting for the case of refraction away from the perpendicular. For in that case, they could suppose that the direction of the refracted ray is determined crucially by the vertical tendency of the ray being reduced by the refracting medium. The case of refraction towards the perpendicular, however, was thought to present a greater difficulty. For in that case, it was harder to imagine a plausible mechanical cause for what, by parallel reasoning, would appear to be an increase in the vertical tendency of the ray as it entered into the refracting medium.

In the *Leges Reflexionis et Refractiones*, Leibniz attempts to address this worry through the postulation of an "elastic force" (*vi Elastica*) (A.VI.ii.314). The rough idea is that the elastic force of a medium may lend an additional downward tendency to a projectile or ray of light moving from one medium to another, and thereby act as the cause of the observed phenomenon of refraction towards the perpendicular. Leibniz maintains that elastic forces are similarly necessary for explaining how the vertical component of the tendency of a ray of light may be reversed in cases of reflection, and he insists that in recognizing the elasticity of reflecting bodies he is able to explain what had been simply taken for granted by his predecessors (A.VI.iv.1404). Because in his early writings Leibniz takes elastic forces themselves to be explained by his distinction between fundamental and derived laws of nature, he could see his earliest system of the world, as sketched in the TMA and HPN, as both lending support to, and in turn being supported by, his earliest accounts of the laws of reflection and refraction (A.VI.ii.228-231).[4]

2. *Unicum Opticae, Catoptricae et Dioptricae Principium*, 1682

Although the *Leges Reflexionis et Refractiones* are dominated by a broadly mechanistic approach to the laws of optics, Leibniz's writings from mid-1670's on also reveal a deep and abiding interest in a radically different approach to deriving the laws of reflection and refraction. That approach is clearly on display in one of Leibniz's most significant scientific writings, the *Unicum Opticae, Catoptricae et Dioptricae Principium* published in the 1682 edition of the *Acta Eruditorum* (Dutens 3.145-150). In it, Leibniz introduces as "the first principle" of optics, catoptrics and dioptrics the rule that "Light radiating from a point reaches an illuminated point by the easiest path," and shows how this "unitary principle" may be used to derive both the laws of reflection and refraction.[5]

In his derivation of the law of reflection, Leibniz argues, in reference to Figure 2 just below, that "in simple optics, the ray directed from the radiating point C to the illuminated point E arrives by the shortest direct path, in the same medium, that is, by the straight line CE":

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[4] For discussion of the place of elasticity in Leibniz's early thought as well as in the problems discussed below, see Herbert Breger, "Elastizität Als Strukturprinzip Der Materie Bei Leibniz," *Studia Leibnitiana* Sonderheft 13 (1982).

[5] Leibniz's 1682 paper is also discussed in: Gerd Buchdahl, *Metaphysics and the Philosophy of Science. The Classical Origins. Descartes to Kant* (Oxford: Blackwell, 1969), 425-34; Hartmut Hecht, "Dynamik Und Optik Bei Leibniz," *NTM International Journal of History and Ethics of Natural Sciences, Technology and Medicine* 4, no. 1 (1996), 83-102.

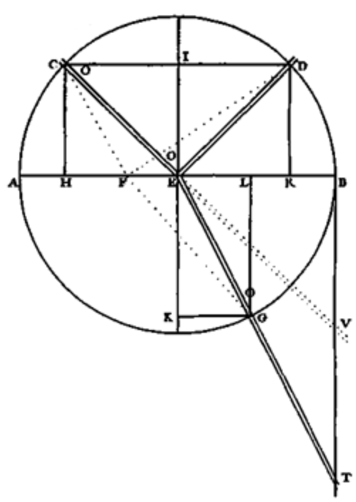

Figure 2

The accompanying proof of the law of reflection is geometric and elegant. Leibniz reasons that, under the given conditions, "the whole path CE + ED becomes the least of all … if E is taken to be such that as a result the angles CEA and DEB are equal, as is evident from geometry" (Dutens 3.145). In a related piece, however, Leibniz hints at a more tantalizing set of metaphysical or "architechtonic" considerations that almost certainly helped to motivate his approach to the laws of optics in the *Unicum Opticae, Catoptricae et Dioptricae Principium*. In his *Metaphysical Definitions and Reflections*, he suggests that since nature always chooses the "optimal means … there ought to be a reason only for a long or short journey," drawing, in effect, the conclusion that a ray of light reflected from C to D must pass through the point E on the grounds that (i) there would have to be a reason for its passing through any point, and (ii) there could be no reason for its passing through some point other than E (A.VI.iv.1405).

Leibniz's derivation of the law of refraction in the *Unicum Opticae, Catoptricae et Dioptricae Principium* is necessarily more involved than his derivation of the law of reflection. Whereas the paths of reflected rays of light typically coincide with shortest reflected paths between two points, this evidently cannot be the case with refracted rays (since refracted rays typically follow bent, rather than straight, paths). Leibniz's solution to this difficulty is to suppose that a refracted ray must always follow the "easiest" path – rather than the shortest path – from one point to another, where the "ease" of a path is a measurable quantity that can be computed by multiplying the distance of the path by its resistance. With this strategy in mind, Leibniz argues, in essence, that the actual refracted path from C to T will be as CET provided that the quantity determined by the length of CE times the resistance of the medium IE plus the length of ET times the resistance of the medium EK is less than the quantity similarly determined from the sum of any other two paths CF and FT.

In casting his proof in terms of "ease," rather than speed, Leibniz hoped to resolve a dispute that had pit Descartes and his defenders against the great mathematician Pierre Fermat. The nub of the dispute concerned the question of whether light travels faster in denser materials, such as water, or faster in rarer materials, such as air. Fermat took the perhaps more intuitive view that light travels faster in rarer materials, and was thus able to argue that a ray of light, such as CET in the diagram above, may follow a quickest path by traveling a greater distance through air (IE) and a shorter distance in water (EK). Cartesians objected. On theoretical grounds, they argued that rays of light must travel faster in denser materials. Descartes, for example, suggests that air acts like a soft body absorbing the motion of a ray of light, while water acts more like a hard body that preserves (even while redirecting) a ray's motion (AT VI 103/CSM 1:163). Cartesians consequently maintained that a path such as CET could not represent the quickest path from C to T and that Fermat's principle must therefore be false. By introducing the notion of "ease," Leibniz hoped to strike a conciliatory middle position between these two opponents, one that would allow him to side with the letter of the Cartesian view that light travels faster in denser materials, while nonetheless preserving the spirit of Fermat's position by insisting that CET is after all a minimal path with respect to ease if not with respect to speed.

The technical innovation represented by Leibniz's introduction of the quantity of "ease," reflects a general conciliatory attitude on his part towards what may be thought of as mechanistic and optimality approaches to deriving the laws of optics. Whereas Descartes, Fermat, and later Cartesians saw two irreconcilable methods for discovering the laws of optics, Leibniz saw two complementary routes to scientific discovery. In keeping with this view, Leibniz affirms throughout his career that it must be possible to derive the laws of optics from broadly mechanistic considerations, and he manifestly believed that such a derivation was readily available in the case of the law of reflection. He also insists, however, from at least the late 1670's on, that the laws of optics may also be derived from considerations of optimality. He thus insists throughout his mature career that "Both methods are good, [and that] both can be useful not only for admiring the skill of the great workman but also for making useful discoveries …" (GP IV 447-448).

3. *Tentamen Anagogicum. Essay Anagogicum dans la recherché des causes*, 1696*

For all the innovation it represents, Leibniz's 1682 paper makes no significant advance on at least one difficulty that had separated proponents of mechanistic and optimality approaches to the laws of optics. That difficulty is perhaps most apparent in cases of reflection involving concave mirrors: in reflecting off of a concave mirror, a ray of light may travel along a path that is in fact longer, slower, and "harder" than other merely possible paths that would involve the ray's reflecting off one of the mirror's "upturned" sides. Fermat and Leibniz had maintained that in such cases a ray of light should be viewed as being optimized with respect to a tangent drawn at the point of reflection, and even in later writings Leibniz continues to insist that "Order demands that curved lines and surfaces be treated as composed of straight lines and planes, [so that] … a ray is determined by the plane on which it falls, which is considered as forming the curved surface at that point" (GP VII 274/L 479). Opponents of the optimization approach to the laws of optics, however, understandably demurred. They saw such appeals to tangent planes as an ad hoc response to a family of clear counterexamples to the proposal that rays of light always follow optimal paths regardless of whether those paths are taken to be shortest, quickest, or easiest paths.

Leibniz's derivations of the laws of optics in the *Tentamen Anagogicum* address this technical problem head on and in the process display the full sophistication of his mature work in optics. The proofs may be thought of as being developed in three steps.[6] In the first step, Leibniz considers, with respect to Figure 3 below, "a curve AB, concave or convex, and an axis ST to which the ordinates of the curve are referred:"

[6] For discussion of Leibniz's proofs in the *Tentamen Anagogicum*, see also: François Duchesneau, *Leibniz et la Méthode de la Science* (Paris: Presses universitaires de France, 1993) 284-310.

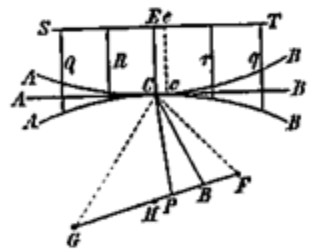

Figure 3

Stating the problem in terms of finding the point C which is unique with respect to its ordinate (i.e. y-axis) value, Leibniz characterizes C as the only point on AB that does not have a corresponding point of the same ordinate value a finite distance away, i.e. as the only point on AB whose "twin" with respect to ST would have to be "infinitely close." As Leibniz shows, it is therefore possible to find C by taking the derivative of an equation describing the line AB and setting it equal to zero. With good justification, Leibniz maintains that this now standard operation for determining local maxima and minima greatly simplifies the calculations employed in his derivations of the laws of reflection and refraction.

In the second step, Leibniz shows how the law of reflection may be derived from the principle that "a ray is directed in the most determined or unique path, even in relation to curves" (GP VII 274/L 479). In reference once again to Figure 3 above, Leibniz considers a ray of light traveling between the fixed points F and G being reflected off a mirror ACB, which might be planar, concave, or convex. Tacitly assuming that the medium through which the light travels is everywhere the same, Leibniz reduces the problem of finding the path unique with respect to "ease" to the problem of finding the point C such that the path FCG is unique with respect to its length. Using the technique set out in the first step, and elementary trigonometry, Leibniz is able to show that for such a path the angle of incidence FCA must be equal to the angle of reflection GCB. Because his derivation is fully applicable to standard cases involving concave and convex mirrors, Leibniz could see his derivation of the law of reflection in the *Tentamen Anagogicum* as a response to the family of counterexamples highlighted just above.

In the third step, Leibniz uses essentially the same strategy in order to derive the law of refraction. In reference to Figure 4 below, Leibniz considers a refracting surface ACB which, again, might be planar, concave, or convex, and lets F and G represent illuminating and absorption points for a ray of light (so that the refracted ray of light is represented by FCG):

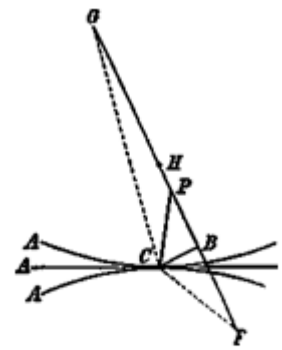

Figure 4

Here once again Leibniz reduces the problem of finding the path unique with respect to ease to the problem of finding the point C such that the path FCG is unique with respect to its length. Employing the same technique as above and using elementary trigonometry, Leibniz is able to show in this case that (a) that the ratio of the sine of incidence to the sine of refraction is inversely proportional to the ratio of incident velocity to the refractive velocity, and (b) that the ratio between the sine of the angle at which a ray of light strikes a refractive surface and the sine of the angle at which the ray is refracted is a constant determined by the mediums involved. Here as well, since Leibniz's proof is fully applicable to standard cases involving concave and convex surfaces, he could see his derivation of the law of refraction as a response to standard counter examples that had been raised against Fermat's approach.

In the *Tentamen Anagogicum*, Leibniz explicitly ties his optimization approach to the laws of optics to his defense of divine teleology. Going in one direction, he maintains that the laws of optics lend support to the belief in a providential creator. The laws of optics, he suggests, cannot plausibly be viewed as being either absolutely necessary or wholly contingent. He concludes that they must therefore "originate in the wisdom of their Author or in the principle of greatest perfection, which has led to their choice" (GP VII.272/L 478). Going in the other direction, he maintains that his optimization approach to the laws of optics shows how reflection on God's ends can yield tangible advancements in the study of nature. Indeed, he goes so far as to suggest the law of refraction must have first been discovered by considering final causes, and that Descartes, the first to publish the law of refraction, must therefore have appropriated his results from the work of Willebrord Snell and his disciples while living in Holland (GP VII.274/L 480). Although the charge of plagiarism may well have been unfair to Descartes, the laws of optics nonetheless arguably provide Leibniz with his best example of how he sees the laws of nature as supporting his commitment to the providential design of the world as well as his best response to Descartes's proposal to "banish from our philosophy the search for final causes" (AT VIIA:15/CSM 1:202).[7]

4. Optics and Optimal Form

Leibniz's sophisticated derivations of the laws of reflection and refraction take on a broader significance when viewed against the backdrop of his general interest in what he terms the "method of optimal forms" (*Methode de Formis Optimis*)" (GP 7:272/L 478). In the simplest of terms, an optimal form is a structure that admits of a locally unique minimum or maximum value, and which may therefore, at least in principle, be treated using the same mathematical techniques that Leibniz helped to pioneer in his sophisticated derivations of the laws of optics.[8] While it had long been recognized that natural phenomena often appear to instantiate optimal forms, the development of the calculus, as well as a growing appreciation of the limitations of strict Cartesian mechanism, led in Leibniz's time to an increased interest in a handful of special problems involving optimal form. A brief survey of three of those problems may help to give a sense of the larger implications of Leibniz's sophisticated approach to the laws of optics.

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[7] For discussion of the connection between Leibniz's optics and teleology see: François Duchesneau, "Hypothèses et Finalité Dans La Science Leibnizienne," *Studia Leibnitiana* 12 (1980); George Gale, "Did Leibniz Have a Practical Philosophy of Science? Or, Does 'Least-Work' Work?," in *Akten Des Ii. Internationalen Leibniz-Kongress, Studia Leibnitiana, Supplementa* 13 (Wiesbaden: F. Steiner Verlag, 1974); George Gale, "Leibniz' Force: Where Physics and Metaphysics Collide," in *Studia Leibnitiana*, *Sonderheft* 13 (Stuttgart: F. Steiner Verlag, 1984); David Hirschmann, "The Kingdom of Wisdom and the Kingdom of Power in Leibniz," *Proceedings of the Aristotelian Society* 88 (1988).

[8] For discussion of the notion of optimal form and its history, see: Don S. Lemons, *Perfect Form: Variational Principles, Methods, and Applications in Elementary Physics* (Princeton, NJ: Princeton University Press, 1997); Robert Woodhouse, *A Treatise on Isoperimetrical Problems, and the Calculus of Variations* (New York: Chelsea Publishing Company, 1810); Wolfgang Yourgrau and Stanley Mandelstam, *Variational Principles in Dynamics and Quantum Theory*, Third ed. (London: Sir Isaac Pitman and Sons, 1968).

A first special problem is treated in Leibniz's "*Demonstrationes Novae de Resistentia Solidorum*" (*New Proofs Concerning the Resistance of Solids*) published in the July 1684 edition of the *Acta eruditorum* (Dutens 3.161-166). It concerns a difficulty introduced by Galileo, namely, the problem of determining the resistance of solid beams to bending under the force of applied weights. Drawing on Hooke's spring law, as well as on Marriotte's assertion that all bodies are flexible to some degree, Leibniz argues that Galileo's proposed formula for the resistance of a beam should be replaced by a new formula that crucially takes into account variations in resistance over the cross-section of a beam as well as over its length.[9] In doing so, he helps to show how weighted beams may be viewed as instances of optimal form. Just as rays of light may be viewed as minimizing speed, distance or ease so bending beams may be viewed as minimizing overall stress energy.[10] With his "*Demonstrationes Novae de Resistentia Solidorum*," Leibniz not only made an important advance with respect to the first of Galileo's "two new sciences," he also showed in convincing fashion how attention to optimal form may bear fruit well outside the domain of optics.

A second special problem was introduced by James Bernoulli in the 1690 May edition of the *Acta eruditorum*, when he challenged his fellow mathematicians and natural philosophers to "find the curve assumed by a loose string hung freely from two fixed points … [assuming] the string is a line which is easily flexible in all parts." At the close of the contest, three correct solutions had been received: one from Johann Bernouli, James's younger brother and perpetual rival, one from Huygens, who coined the term "catenary" to name the resulting curves, and one from Leibniz, who reportedly replied with a solution on the day he received the challenge. The solutions collectively showed that catenaries, like rays of light and bending beams, may also be treated as instances of optimal form. Just as rays of light take optimal paths, and bent beams assume optimal configurations, so hanging chains take on an optimal shape that, in Leibniz's terms, maximizes

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9 More specifically, he argues that, with respect to a cubical beam, the Galilean formula $P_b = 1/2\ P_t$ should be replaced by the formula $P_b = 1/3P_t$, where P_b is the breaking force in bending by terminal load, and P_t is the breaking force in tension. For an historical and technical discussion, see C. Truesdell, *The Rational Mechanics of Flexible or Elastic Bodies, 1638-1788*, vol. 11, pt. 2, *Leonhardi Euleri Opera Omnia* (Turici: Orell Füssli, 1960) 38-42, 59-64.

10 On this point, see especially, Mark Wilson, "From the Bending of Beams to the Problem of Free Will," *A Priori* 4 (2010). See also, Hartmut Hecht, *Mathematik und Naturwissenschaften im Paradigma der Metaphysik* (Stuttgart: B.G. Teubner Verlagsgesellschaft, 1992) 100-04.

descent, or, in modern parlance, minimizes potential energy. As is made especially clear in a letter he wrote to Huygens, dated 14 September 1694, Leibniz's efforts to respond to Bernoulli's challenge served to show once again that the notion of an optimal form, championed in his optical studies, can be utilized to make novel discoveries well outside the domain of optics.[11]

A third special problem was introduced in the1696 June edition of the *Acta eruditorum*, when Johann Bernoulli dared "the most acute mathematicians flourishing in the whole world," to find the path of quickest descent between two points in the vertical plane for a freely falling body. Taking advantage of his calculus, Leibniz was again able to solve the problem of the brachistochrone, as it came to be called, on the day he received it. Of the four other solutions submitted – one from each of the Bernoulli brothers, one from Newton, and one from L'Hopital, two merit special mention. Johann Bernoulli's solution was remarkable for showing how the quickest path of descent could be found by treating a descending body as a ray of light passing through increasingly dense media so that the path of quickest descent could be found by exploiting the already known the laws of refraction. James Bernoulli's solution, although perhaps less imaginative, was equally remarkable. It highlighted the fact that any portion of a path of quickest descent must itself be a path of quickest descent, and that consequently any larger path can be thought of as a path that is such that any of its sub-paths is unique with respect to quickest descent. This important property of optimal forms, noted explicitly by Leibniz in his *Tentamen Anagogicum*, would prove to be crucial to the later development of what has become known as the calculus of variations proper (GP 7: 272/L 478).

Special problems such as those involving bending beams, the catenary and the brachistochrone set the stage for later developments that further extended the spirit of Leibniz's approach to the laws of reflection and refraction. Spurred by Leibniz's treatment of bending beams, James Bernoulli, for example, would take up in greater detail problems of "elastica" and produce the first general equations in the theory of elasticity.[12] Prompted by the debate treated in Leibniz's 1682 paper, Maupertuis would develop his famous, general "principle of least action," according to which "in all the changes that take place in the universe, the sum of the products

[11] Christiaan Huygens, *Oeuvres complètes de Huygens*, vol. 10 (La Haye, M. Niijhoff, 1888), 679. See also GM VII 370-372.
[12] C. Truesdell, *The Rational Mechanics of Flexible or Elastic Bodies*, 63, 88-109.

of each body multiplied by the distance it moves and by the speed with which it moves is the least possible."[13] Such results would later be refined and extended by the next half-generation of natural philosophers resulting in the full flourishing of what would become known as the rational mechanics of the eighteenth century. Thus, Euler, for example, would offer systematic treatments of the special problems discussed just above in the process of drawing out the full implications of Newton's second law and arriving at his definitive statement of the principle of linear motion.[14] Lagrange would pick up the intellectual thread present in the notion of an optimal form and produce the first general variational method for dynamics, publishing his results in his *Mechique Analytic* in 1811.[15] In light of these developments, one might reasonably conclude that, although in surveys of the history and philosophy of science, Leibniz is most often associated with his role in the *vis viva* controversy and his exchange of letters with Samuel Clark, his most enduring scientific legacy might well be his influence on the development of modern rational mechanics, whose founding proponents were, in effect, drawing out and extending many of the implications already present in his relatively accessible optical studies.

Conclusion

Although necessarily incomplete, even the brief discussion offered here of Leibniz's derivations of the laws of optics should be sufficient to suggest

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[13] P. Maupertuis, *Oeuvres De Maupertuis* (Lyons: Jean-Marie Bruyset, 1698-1759), IV 20, II 274. For related discussion, see: François Duchesneau, "L'Épistémologie De Maupertuis Entre Leibniz Et Newton. Physique Et Physiologie," *Revue de synthèse* 113-114 (1984): 7-36; Martial Gueroult, *Leibniz, Dynamique Et Métaphysique, Suivi D'une Note Sur Le Principe De La Moindre Action Chez Maupertuis* (Paris: Aubier-Montaigne, 1967); Hartmut Hecht, "La Quantité De La Force Et Quantité D'action. Dynamique Et Métaphysicque Chez Leibniz Et Maupertuis," in *La Notion De Nature Chez Leibniz. Studia Leibnitiana*, *Sonderheft* 24, ed. Martine de Gaudemar (Stuttgart: Franz Steiner Verlag, 1995); Hartmut Hecht, "Leibniz' Concept of Possible Worlds and the Analysis of Motion in Eighteenth Century Physics," in *Between Leibniz, Newton, and Kant*, ed. W. Lefevre (Dordrecht, the Netherlands: Kluwer Academic Publishers, 2001).

[14] See Leonhardi Euleri, "Décuverte D'un Nouveau Principe De Mécanique," *Mémoires de l'académie des sciences de Berlin* 6 (1752) 185-217. Reprinted in Leonhardi Euleri, *Opera Omnia*, series 2, vol. 5, ed. Joachim Otto Fleckenstein (Turici: Orell Füssli, 1957), 81-108.

[15] C. Truesdell, "A Program toward Rediscovering the Rational Mechanics of the Age of Reason," *Archive for history of exact sciences* 1 (1960): 33.

three general conclusions: First, Leibniz's interest in optics spanned the entire breadth of his career, running from studies concurrent with his earliest systematic treatments of physics, to mature studies as represented most famously by his *Tentamon Anagogicum*. Second, within that long span, Leibniz continued work on and refine his earlier efforts, at first more radically when his views on the physical world shifted dramatically around the time of his years in Paris, and then more cautiously as his physical and metaphysical views continued to ferment. Third, Leibniz's thinking about optics intertwines in interesting and often surprising ways with other threads associated with his thought, from the nature of material bodies and divine choice to the implications of his calculus and related scientific techniques. One may reasonably hope that more of Leibniz's interests in the study of optics and optical phenomena, as well as their broader implications for his wide ranging pursuits, will come to light as the Akademie editors continue their important task of bringing his scientific, medical and technical writings to print.

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