

The Impact of Trade: Changes in the Elasticity of Substitution Between Varieties

In the spirit of Krugman ((1979),(1980)), I now examine the impact of trade on two economies of the kind previously described. In this section, I assume as in (Krugman 1979) that the increase in product variety made possible through trade induces an increase in the elasticity of substitution between varieties. I then analyze the effects of this increased elasticity on aggregate productivity and the reallocations of market shares and profits between firms in the industry. I initially assume that there are no additional costs involved with trade. In the next section, I investigate the effects of certain trade costs such as the cost of entry into the export market and some non-tariff barriers. This model shares the same characteristics as those based on Krugman's ((1979),(1980)) seminal work: in the absence of differences between countries in relative factor abundance, taste, and technology, trade nevertheless occurs and increases welfare by providing the benefits of an increase in country size. As with Krugman ((1979),(1980)), the free trade equilibrium attained by two countries of the type previously described (when there are no transport costs) replicates the equilibrium outcome of a single integrated world economy that combines the resources of both countries. This property is driven by the creation of a single world market where all varieties are traded. Consumers in both countries have access to the same bundle of goods at the same aggregate price index P . Regardless of country size or relative wage differences, the cutoff productivity levels in both countries must satisfy the same $h(\varphi^*) = k(\varphi^*)$ condition and must therefore be identical. This, in turn, implies the equality of aggregate productivity levels and wages between the two countries.¹ Each firm then divides its sales between domestic and foreign consumers on the basis of the ratio of its country size to the size of the integrated economy. The balance of payments condition between the two countries is thus trivially satisfied.

If the exposure of a country to trade does not affect the elasticity of substitution between varieties, then this exposure duplicates the effects of an increase in country size. As previously described, none of the firm level equilibrium variables change – only the number of firms and available varieties are affected. Exposure to trade then increases welfare solely through the increased product variety effect. This result, also obtained by (Krugman 1980), is driven by the absence of

¹Although true in this instance, the equality of aggregate productivity between the trading countries is no longer a necessary consequence of technological similarity (as is the case with representative firms). Interestingly, the presence of trade costs in this model will break this equality and confer an aggregate productivity advantage to the larger country (along with a higher relative wage)

any link between an increase in the number of varieties consumed and the elasticity of substitution between the varieties. This is a well known property of Dixit-Stiglitz style models of monopolistic competition and is addressed in the original paper by the authors.² (Dixit & Stiglitz 1977) also show how to change the functional form of utility in order to generate preferences that exhibit the very plausible characteristic of a positive relationship between the number of varieties consumed and the elasticity of substitution. This type of preferences is used in (Krugman 1979) to show how trade can provide the additional welfare benefits of an increase in the elasticity of substitution through lower prices (driven by lower markups). The number of varieties then increases from its autarky level, but not as much as in the constant elasticity case.

It is also possible that a country's exposure to trade generates an increase in the elasticity of substitution through a different mechanism: the characteristics of individual varieties could change in such a way that they become closer substitutes. The monopolistic competition models of trade typically assume that any firm from one country producing the same variety as a firm from the other country will costlessly "switch" to the production of another "unproduced" variety when the two countries open to trade. Similarly, I will assume that exposure to trade induces a costless process of product modifications amongst all firms in both countries. On the other hand, I assume that firms can not just "pick" another variety and are forced to re-design their products. Firms are therefore unable to maintain the same level of product differentiation in a now more crowded product market space. I assume that the symmetry between goods is preserved, but that they are now all closer substitutes in the integrated world market. This is, of course, a quite specific and restrictive assumption. However, its goal is not to realistically model the firm's product selection decision nor to explain *how* trade will generate an increase in the elasticity of substitution. Rather, its goal is to analyze the *effect* of this increase (since this effect is quite plausible – although the symmetry assumption is obviously a massive simplification) on an industry comprised of heterogeneous firms. Hence, this reduced form approach is adopted in order to introduce the link between increases in product variety and substitutability between varieties while preserving the C.E.S. formulation of utility. Preserving the homotheticity of preferences in this model is necessary since firm heterogeneity precludes the use of cost symmetries to obtain equilibrium conditions in the way that was achieved by (Krugman 1979) and (Dixit & Stiglitz 1977). Instead, I rely on the construction

²As implied by its name, the C.E.S. utility function obviously maintains a constant elasticity of substitution between varieties. This property, holding the number of varieties fixed, is different than the one under examination, which refers to the relationship between the elasticity (constant *between* varieties) and the total *number* of varieties consumed.

of an aggregate price index that is only valid when the underlying preferences are homothetic. I henceforth assume that the opening of the countries to trade and the resulting merger of varieties on the world market induces an increase in the elasticity of substitution from its autarky level σ_a to σ . I implicitly assume the existence of an increasing function relating the elasticity change to the change in product variety.

Briefly returning to the benchmark case of representative firms, one can note that an increase in the elasticity of substitution applied to Krugman's ((1980)) model with C.E.S. utility produces the same effects as in his ((1979)) model with non-homothetic preferences: The increase from σ_a to σ causes a rise in the output per firm and an increase in the number of varieties consumed (though not as big an increase as in the constant elasticity case).³ On the other hand, by construction, aggregate productivity must remain fixed at the productivity level common to all firms. I now show how firm heterogeneity introduces an additional benefit from trade through an increase in aggregate productivity. This increase is generated by the shifting of resources towards more productive firms. Although average output per firm increases as in the representative firm case, this increase is not equally spread across firms. In fact, only the more productive firms expand while the less productive ones contract or exit.

Let φ_a^* denote a country's equilibrium productivity cutoff level under autarky. If this country opens to trade with another identical country, it attains a new equilibrium that replicates the closed economy equilibrium of a country with twice the number of workers and a higher elasticity of substitution between varieties of $\sigma > \sigma_a$. This increase in σ shifts up the $k(\varphi)$ curve⁴ and induces an increase in the equilibrium cutoff level to $\varphi^* > \varphi_a^*$. The percentage difference between the market shares of the firms with average and cutoff productivity levels increases for two reasons: given the same ratio of productivity levels between any two firms ($\frac{\varphi_1}{\varphi_2} > 1$), the ratio of these two firms' market shares is now higher due to the increase in the elasticity of substitution: $\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1} > \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma_a-1}$. Furthermore, the increase in σ also induces an increase in the average productivity level relative to any given cutoff level ($\tilde{\varphi}(\varphi^*)$ is higher for any φ^*). There are thus two effects generating an increase in aggregate productivity: the firm selection effect, driven by the increase in the cutoff level φ^* (the least productive firms exit), but also a further reallocation: among the group of firms who remain in the industry after the transition to trade, market shares are further redistributed

³The welfare effect of the increased product variety is ambiguous as I have assumed that the characteristics of the goods have changed.

⁴ $\frac{\partial k(\varphi)}{\partial \sigma} = \frac{1}{1-G(\varphi)} \frac{1}{\varphi^{\sigma-1}} \int_{\varphi}^{\infty} (\ln \xi - \ln \varphi) \xi^{\sigma-1} g(\xi) d\xi > 0$

towards the more productive firms.

This redistribution can be analyzed more precisely by tracking the performance of a particular firm (with productivity level φ) who remains in the industry after the transition to trade ($\varphi \geq \varphi^*$). Let $\Delta s(\varphi) = \ln r(\varphi) - \ln r_a(\varphi)$ denote this firm's percentage market share change where $r_a(\varphi)$ and $r(\varphi)$ denote the firm's revenues before and after the transition to trade (percentage market share changes are equal to percentage revenue changes since aggregate revenue $R = Y = L$ remains constant). This percentage change can then be written:

$$\begin{aligned} \Delta s(\varphi) &= \ln \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} \sigma f \right] - \ln \left[\left(\frac{\varphi}{\varphi_a^*} \right)^{\sigma_a-1} \sigma_a f \right] \\ &= (\ln \sigma - \ln \sigma_a) - (\sigma_a - 1)(\ln \varphi^* - \ln \varphi_a^*) + (\sigma - \sigma_a)(\ln \varphi - \ln \varphi^*) \\ &= \Delta \sigma - (\sigma_a - 1)\Delta \varphi^* - (\sigma - \sigma_a)pct(\varphi), \end{aligned}$$

where $\Delta \sigma > 0$ and $\Delta \varphi^* > 0$ are the percentages increase in the elasticity of substitution and the cutoff productivity level, and $pct(\varphi) \geq 0$ is the percentage difference between φ^* and φ . A firm's percentage market share change is thus an increasing function of its productivity level φ – though this change need not be positive for all firms who remain in the industry. Naturally, this change is negative for any firm who no longer produces after the transition to trade ($\Delta s(\varphi) = -\infty$, if $\varphi_a^* \leq \varphi < \varphi^*$). There will thus exist another cutoff productivity level $\varphi^\dagger \geq \varphi^*$ separating the firms who gain from those who lose market share:⁵

$$\begin{cases} \Delta s(\varphi) \leq 0 & \text{if } \varphi_a^* \leq \varphi \leq \varphi^\dagger, \\ \Delta s(\varphi) > 0 & \text{if } \varphi > \varphi^\dagger. \end{cases}$$

Furthermore, the percentage market share gain of a firm with productivity $\varphi > \varphi^\dagger$ increases with the difference between φ and φ^\dagger : the more productive the firm, the greater the market share gain.

This market share reallocation will obviously also affect the distribution of profits across firms. Since a firm's variable profit is given by $\frac{r(\varphi)}{\sigma}$, the percentage change in this firm's variable profit

⁵Depending on the sign of $\Delta \sigma - (\sigma_a - 1)\Delta \varphi^*$ (which can not be signed without making further assumptions), φ^\dagger will either be equal to or strictly greater than φ^* . If $\varphi^\dagger = \varphi^*$, then firms either remain in the industry and gain market share, or exit.

(denoted $\Delta\pi_v(\varphi)$), provided it still produces, can be written:

$$\begin{aligned}\Delta\pi_v(\varphi) &= \Delta s(\varphi) - \Delta\sigma \\ &= -(\sigma_a - 1)\Delta\varphi^* + (\sigma - \sigma_a)pct(\varphi).\end{aligned}$$

Note that a firm whose market share remains constant ($\Delta s(\varphi) = 0$) will nevertheless incur a profit loss. Of course, even if the new cutoff firm gains market share ($\Delta s(\varphi^*) > 0$), it must still lose profits since these drop to zero. There will thus be yet another cutoff productivity level $\varphi^\ddagger > \varphi^\dagger$ such that $\Delta ms(\varphi^\ddagger) = \Delta\sigma > 0$ and

$$\begin{cases} \Delta\pi_v(\varphi) \leq 0 & \text{if } \varphi_a^* \leq \varphi \leq \varphi^\ddagger, \\ \Delta\pi_v(\varphi) > 0 & \text{if } \varphi > \varphi^\ddagger. \end{cases}$$

Interestingly, all firms with productivity $\varphi > \varphi^\ddagger$ will earn higher profits in the integrated world economy, even though their common markup decreases. As in the case of market shares, the percentage profit gain rises with the firm's productivity level. Finally, note that even though trade increases the product variety available to consumers, the number of firms producing in either country is lower than in autarky since the average firm profit has increased (p_{in} is lower).

This section has highlighted a key difference between this model and its predecessors based on representative firms. If there exists a link between product variety and the elasticity of substitution, then the existence of firm level heterogeneity creates a new transmission channel for the effects of trade. Through this channel, the exposure to trade generates an increase in aggregate productivity without affecting the state of firm level technology (referenced by the distribution $g(\varphi)$). This aggregate productivity increase is driven by a Darwinian evolution within the industry: the more productive firms thrive and expand while the less productive ones contract or exit.

Although this section has focused on the impact of trade, there also exist other phenomena that may affect the elasticity of substitution between varieties in a closed economy. These phenomena could either directly affect this elasticity or do so indirectly through an impact on product variety. In particular, economy wide productivity growth (which can be modeled as an increase in the effective labor force) will increase product variety. Given the assumed link between product variety and the substitutability between varieties, the ensuing increase in the elasticity of substitution will generate the same reallocations that were just described. Technological progress would then

engender reallocative productivity growth. The overall economy would grow, not just by adding firms, but also by shifting its resources towards more efficient firms. Larger economies (vis-a-vis smaller ones) would then exhibit a distribution of firms with a more concentrated range of productivity levels (because of the higher cutoff) but with a greater dispersion of firm sizes over this range: the large economy would have a greater share of production concentrated at a smaller proportion of firms.

References

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